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Building Linguistic Random Regression Model from the Perspective of Type-2 Fuzzy Set

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***Abstract*—Information given in linguistic terms around real life sometimes is vague in meaning, as type-1 fuzzy set was introduced to modulate this uncertainty. Meanwhile, same word may result in various meaning to people, indicating the uncertainty also exist when associated with the membership function of a type-1 fuzzy set. Type-2 fuzzy set attempt to express the hybrid uncertainty of both primary and secondary fuzziness, in order to address regression problems, we built a type-2 Linguistic Random Regression Model based on credibility theory. Confidence intervals are constructed for fuzzy input and output, and the proposed regression model give a rise to a nonlinear programming problem focus on a well-trained model, which would be helpful and useful in linguistic assessment cases. Finally, a numerical example is provided.**

Keywords: Type-2 fuzzy set, Linguistic rules, Regression model, Creditability theory, Confidence interval

I. Introduction

Fuzzy sets play a pivotal role in computing with words being casted in the setting of granular computing [1]. The essence of granular computing is to carry out computing that exploits information granules [2]. Information granules are regarded as collections of elements that can be perceived and treated together because of their similarity, functional properties, or spatial or temporal adjacency [3], [4], [5], [6]. In this sense, fuzzy logic becomes instrumental as an effective vehicle to manipulate information granules.

It becomes apparent that experts with much professional experiences are capable of making assessment using their intuition and experiences. In such cases,experts may express judgements with linguistic terms. The difficulty in the direct measurement of certain characteristics makes their estimation highly imprecise and this situation implies the use of fuzzy sets [7], [8]. There have been a number of well-documented cases in some of which fuzzy regression analysis has been effectively used.

Nevertheless, most of the existing studies on modeling fuzzy regression analysis have focused on data consisting of numeric values, interval-like numbers, or fuzzy numbers without taking randomness into consideration. In practical situations, there exists a genuine need to cope with data that involve the factors of fuzziness and probability. For example, let us discuss experts' evaluation of products. Assume that we have 100 samples of the same agricultural product. Suppose five inspectors (experts) evaluate the products on the basis of ten attributes. Each expert grades each piece according to his experiences and expertise. These gradings are given linguistically, e.g., good, very good, bad, and very bad, or about 5, about 6, etc. When different inspectors give different grades, the grading can be understood stochastic in its nature, i.e., the differences among the five inspectors can be treated statistically, but each grade itself should be treated by considering the formalism of fuzzy sets. When we intend to build a multi-attribute model of the experts' evaluation, we have to consider this twofold uncertainty, i.e., uncertainty due to both fuzziness and randomness. Therefore, in the example considered here, fuzzy random data should be employed to evaluate the products. Moreover, if we measure the change of the fuzzy random values using their confidence intervals, we can handle the multi-attribute problem by taking advantage of statistical analysis.

Information in real life may contain linguistic vagueness. Traditional set theory uses characteristic function to define whether an element belongs to a certain set (event) and does not trying to dealing with such uncertainty. Fuzzy set (type-1 fuzzy set) was first introduced in 1965 by Lofti A Zadeh [8]. After that, Watada and Tanaka expanded a fuzzy quantification method in 1987 [9]. From then on, it is able to describe an artificial membership function with its output called primary membership grades, to which extend one element belongs to a certain set (event).

On the background that the membership function of a type-1 fuzzy set may also have uncertainty associated with it, Lofti A. Zadeh invented type-2 fuzzy sets(type-2 fuzzy set) in 1975 [10]. A type-2 fuzzy set enables us to implement fuzziness about the membership function into fuzzy set theory and is a way to address the above concern of type-1 fuzzy sets head-on. However, type-2 fuzzy set did not become popular immediately because of its complexity of calculation. type-2 fuzzy sets are difficult to understand and use because: (1) the three-dimensional nature of type-2 fuzzy sets makes them difficult to handel. (2) using type-2 fuzzy sets is computationally more complicated than using type-1 fuzzy sets. Thus, the conception was only investigated by a few researchers; for example, Mizumoto and Tanaka [11] discussed what kinds of algebraic structures the grades of type-2 fuzzy sets form under join, meet and negation; Dubois and Prade [12] investigated the operations in a fuzzy-valued logic. It is not until recent days that type-2 fuzzy sets have been applied successfully to type-2 fuzzy logic systems to handle linguistic and numerical uncertainties [13], [14], [15], [16], [17].

On the other hand, various fuzzy regression models were introduced to cope with qualitative data coming from fuzzy environments where human (expert) subjective estimates are used. The first fuzzy linear regression model was proposed by Tanaka [7]. Tanaka [18], Tanaka and Watada [19], Watada and Tanaka [20] presented possibilistic regression based on the concept of possibility measure. Chang [21] discussed a fuzzy least-squares regression, by using weighted fuzzy-arithmetic and the least-squares fitting criterion. Watada [22] developed models of fuzzy time-series by exploiting the concept of intersection of fuzzy numbers.

Most of the existing studies on modeling fuzzy regression analysis have focused on data consisting of numeric values or type-1 fuzzy variables without type-2 hybrid uncertainty into consideration. In practical situations, there exists a growing need to cope with data in presence of more complicated uncertainty. For example, in capital markets, one candlestick used to model financial series data can be viewed as a fuzzy set because it is a combination of a line-chart and a bar-chart, in that each bar represents the range of price movement over a given time interval. While the membership function of the prices set of financial instruments is hard to define given that prices' structure in periods varies a lot with much uncertainty. In this case, type-2 fuzzy set will help a lot with no doubt.

Motivated by the above reasoning, the objective of this paper is to introduce a class of linguistic type-2 fuzzy regression model based on creditability theory to deal with type-2 fuzzy inputs and outputs. We use creditability theory introduced by Liu [23] to define the expected value of a type-2 fuzzy variable. After that, we transfer the type-2 fuzzy variable into type-2 fuzzy expected value and build an type-2 fuzzy expected value regression model. A well-trained model could get the evaluation itself for the data already entered and output the linguistic data after the intermediate steps defuzzification and refuzzification.

The remainder of this paper is organized as follows. In Section 2, we cover some preliminaries of creditability theory, linguistic model and type-2 fuzzy sets. Then we define the expected value of type-2 fuzzy set and type-2 fuzzy variable. Notice that these two conceptions of expected values are different. Section 3 a linguistic type-2 fuzzy regression has been built and formulates a type-2 fuzzy expected value regression model. In section 4 we introduce a numerical example for this model and extend the applications of the model. Finally, concluding remarks are presented in Section 5.

II. Historical Background

A. Type-2 Fuzzy Set

Type-2 fuzzy sets were first described by Zadeh as a development for his fuzzy set theory [24]. According to [25] type-2 fuzzy sets are "sets whose membership grades are themselves type-1 fuzzy sets". A type-2 fuzzy set, denoted by $\tilde{A}$, can be defined at the universe X as

$$\tilde{A} = \int_{x \in X} \int_{x \in J_x} \mu_{\tilde{A}}/(x,u) \tag{1}$$

where $J_x \subseteq [0,1]$ is the set of primary membership grades of $x \in X$, with $u \in J_x, \forall x \in X$, and $\mu_{\tilde{A}}(x,u)$ is the type-2 membership function [26] [27]. Since type-2 membership functions are defined in R^3 [28] [29], obviously there is a lot of obstacles for drawing, handling and understanding them [26].

B. Creditability Theory

Credibility measure [30] is an average of the possibility and the necessity measure, i.e., $Cr\{\cdot\} = (Pos\{\cdot\} + Nec\{\cdot\})/2$, and it is a self-dual set function, i.e., $Cr\{A\} = 1 - Cr\{A^c\}$for any A in $P(\Gamma)$. The motivation behind the introduction of the credibility measure is to develop a certain measure, which is a sound aggregate of the two extreme cases, such as the possibility (which expresses a level of overlap and is highly optimistic in this sense) and necessity (which articulates a degree of inclusion and is pessimistic in its nature). Based on credibility measure, the expected value of a fuzzy variable is presented as follows:

$$E[Y] = \int_0^{\infty} Cr\{Y \geq r\}dr - \int_{-\infty}^{0} Cr\{Y < r\}dr \tag{2}$$

provided that at least one integral is finite.

Let ε be a fuzzy random variable with expected value e. Then, the variance of ε is defined by $V[\varepsilon] = E[(\varepsilon - e)^2]$.

C. Linguistic Fuzzy Random Regression Model

In making assessments regarding some objects, we use multi-attribute evaluation. The difficulty in the direct measurement of certain characteristics makes their estimation highly imprecise and this situation results in the use of fuzzy values and linguistic values. Often, experts use a linguistic word to judge an object from various features and characteristics. And the whole process is pursued in linguistic way. For instance, although it is possible to measure numerical value, it is difficult to analytically interpret the obtained numerical value in terms of possible influence. This result might have impacted on further decision making.

To cope with linguistic variables, we define processes of vocabulary translation and vocabulary matching which convert linguistic expressions into membership functions defined in the unit interval. That is, human words can be translated (formalized) into fuzzy sets (fuzzy numbers, to be more specific) which are afterward employed in a fuzzy reasoning scheme. Fuzzy regression analysis [7], [6], [31] is employed to deal with the mapping and assessment process [32], [33] of experts which are realized from linguistic variables of features and characteristics of an objective into the linguistic expression articulating the total assessment.

III. Linguitic Fuzzy Regression Model wih Confidence Intervals

A. De-Linguitic

We built a model based on the relationship between the assessments given for different attributes and the overall assessment of the object totally. Watada et al. [34] propose fuzzy random regression model with confidence interval to deal with situations under hybrid uncertainty. The data given by experts are shown in Table I such as good, bad, extremely bad, as fuzzy random numbers.

An event has its population including the finite or infinite number of samples with probability. Generally such probability is not known explicitly. We employ it as the linguistic assessment result percentage. Such as, 50 experts evaluate the object good, and 50 percentage evaluate the object very good, then the probability is 0.5, 0.5 respectively.

Then, we translate attributes from linguistic values L_i into fuzzy grades X_L making use of triangular membership functions:

$$X_L \equiv (a, b, c) \tag{3}$$

where X_L denotes the representative value of the fuzzy event, a is the central value and b, c are the left-side bound and right-side bound, respectively. The estimation of the total assessment is written by the following fuzzy assessment function:

$$Y_i = f(X_{L_{i1}}, X_{L_{i2}}, \cdots, X_{L_{iK}}) \tag{4}$$

where $i = 1, 2...N$, the number of experts, K is the number of the attributes of the object. Then X_L is obtained from the vocabulary dictionary of experts. From this dictionary we can convert the linguistic words to fuzzy variable random numbers.

B. Vertical Slice Centroid Type-Reduction

Vertical Slice Centroid Type-Reduction (VSCTR) is a highly intuitive method employed by John [35]; the paper of Lucas et al. [36] renewed interest in this strategy. In this approach the type-2 fuzzy set is cut into vertical slices, each of which is defuzzified as a type-1 fuzzy set. By pairing the domain value with the defuzzified value of the vertical slice, a type-1 fuzzy set is formed, which is easily defuzzified to give the defuzzified value of the type-2 fuzzy set. Though chronologically preceding it, this method is a generalisation of the Nie-Tan method for interval type-2 fuzzy sets [37].

In VSCTR we calculate only the centroids C_j of the j^{th} vertical slice of $\tilde{B}$. These calculated centroids become the new memberships of elements $y \in Y$. Thus the type-reduced set is obtained by

$$C_{\tilde{B}} = \int_{y \in Y} C_j / y = \int_{y \in Y} \frac{\int_{u \in J_y} u \times f_y(u)}{\int_{u \in J_y} f_y(u)} / y \tag{5}$$

C. Type-2 Credibility Based-Interval Regression

All the linguistic data have been converted to fuzzy random variable data. We need to build a fuzzy regression model for fuzzy random data, which is based on the possibilities linear model.

Fuzzy Random Regression Model with Confidence Interval:Table I is the format of data that come from linguistic words, where input data X_{iK} and output data Y_i, for all $i = 1, 2, ..., n$ and $k = 1, 2, ..., K$. They are all fuzzy random variables, which defined as:

TABLE I
LINGUISTIC VALUES OF EACH SAMPLE ω GIVEN BY EXPERTS

sample	Input Attribute k-th Value			Output Value
	1	$\cdots$	K	
1	(L_{11}, p_{11})	$\cdots$	(L_{1K}, p_{1K})	$(\ _{1}, p_1)$
2	(L_{21}, p_{21})	$\cdots$	(L_{2K}, p_{2K})	$(\ _{2}, p_2)$
3	(L_{31}, p_{31})	$\cdots$	(L_{3K}, p_{3K})	$(\ _{3}, p_3)$
4	$(good, 0.2)$	$\cdots$	$(very\ good, 0.1)$	$(good, 0.1)$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
ω	$(L_{\omega 1}, p_{\omega 1})$	$\cdots$	$(L_{\omega K}, p_{\omega K})$	$(\ _{\omega}, p_\omega)$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
N	(L_{N1}, p_{N1})	$\cdots$	(L_{NK}, p_{NK})	$(\ _{N}, p_N)$

where $L_{\omega K}$ and $\ _{\omega}$ denote linguistic values of input k-th attribute and output value of ω-th sample, respectively.

$$\begin{aligned} Y_i &= \bigcup_{t=1}^{M_{Yi}} \{(Y_i^t, Y_i^{t,l}, Y_i^{t,r}), p_i^t\}, \\ X_{iK} &= \bigcup_{t=1}^{M_{XiK}} \{(X_{iK}^t, X_{iK}^{t,l}, X_{iK}^{t,r}), q_{iK}^t\} \end{aligned} \tag{6}$$

respectively. This means that all values are given as fuzzy numbers with probabilities, where fuzzy variables $(Y_i^t, Y_i^{t,l}, Y_i^{t,r})$ and $(X_{iK}^t, X_{iK}^{t,l}, X_{iK}^{t,r})$ are associated with probability p_i^t and q_{iK}^t, for $i = 1, 2, , N$, $k = 1, 2, , K$ and $t = 1, 2, , M_{Yi}$ and M_{XiK} respectively.

Let us denote fuzzy linear regression model with fuzzy coefficients $\bar{A}_1, \cdots, \bar{A}_K$ as follows:

$$\bar{Y}_i = \bar{A}_1 X_{i1} + \cdots + \bar{A}_K X_{iK} \tag{7}$$

And then we need to determine the optimal fuzzy parameters $\bar{A}_i$. Two optimization criteria are considered. One concerns the fitness of the fuzzy regression model, h. The other one deals with fuzziness captured by the fuzzy regression model 7. Let us elaborate on the detailed formulation of these criteria.

In this study, we employ the confidence-interval based inclusion, which combines the expectation and variance of fuzzy random variables and the fuzzy inclusion relation satisfied at level h, to deal with the model (7) as discussed in [3], [31]. There are also some other ways to define the fuzzy random inclusion relation $\subset_h$, which will yield more complicated fuzzy random regression models. For instance, in order to retain more complete information of the fuzzy random data, we can use the fuzzy inclusion relation directly for the product between a fuzzy parameter and a fuzzy value at some probability level.

Before building the fuzzy random regression model with confidence interval, we define the confidence interval that is induced by the expectation and variance of a fuzzy random variable based on the credibility theory. When we consider the one-sigma confidence interval of each fuzzy random variable, we can express it as the following interval:

$$\begin{aligned} I(e_{X_{iK}}, \sigma_{X_{iK}}) = \Big[& E(X_{iK}) - \sqrt{Var(X_{iK})}, \\ & E(X_{iK}) + \sqrt{Var(X_{iK})} \Big] \end{aligned} \tag{8}$$

After then, in order to obtain linguistic expression, we need to match the obtained fuzzy numbers to the most appropriate linguistic words (Vocabulary Matching). First we consider the one-sigma confidence interval of each fuzzy random variable, and it is expressed as follows:

$$\begin{aligned} I(e_{X_{iK}}, \sigma_{X_{iK}}) &= [e_{X_{iK}} - \sigma_{X_{iK}}, e_{X_{iK}} + \sigma_{X_{iK}}] \\ I(e_{Y_i}, \sigma_{Y_i}) &= [e_{Y_i} - \sigma_{Y_i}, e_{Y_i} + \sigma_{Y_i}] \end{aligned} \tag{9}$$

Then, the new confidence-interval-based fuzzy random regression mode is built as follows:

$$\left.\begin{array}{ll} min_{\bar{A}} & J(\bar{A}) = \sum_{k=1}^{K} (\bar{A}_k^r - \bar{A}_k^l) \\ subject\ to & \bar{A}_k^r \geq \bar{A}_k^l \\ & \bar{A}_i = \sum_{k=1}^{K} \bar{A}_k I(e_{X_{iK}}, \sigma_{X_{iK}}) \supset_h I(e_{Y_i}, \sigma_{Y_i}) \end{array}\right\}. \tag{10}$$

where $i = 1, 2, ..., N$, $k = 1, 2, ..., K$, and the $\subset_h$ denotes the fuzzy inclusion relation realized at level h.

Since the product of a fuzzy number (fuzzy coefficient) and an interval (confidence interval) is influenced by the signs of each component, in order to solve the model (10), we need to take into account all the cases corresponding to different combinations to the signs of the fuzzy coefficients, as well as the σ-confidence intervals of the fuzzy random data.

D. Solution of the Regression Model

The solution of model (10) can be rewritten as a problem of N samples with one-output and K-input interval values. This problem is difficult to solve, since it consists of NK products between the fuzzy coefficients and confidence intervals. In order to solve the proposed model (10), we can employ a vertices method, as given shortly, i.e., these multidimensional vertices are taken as new sample points with fuzzy output numbers. In the sequel, we can solve this problem using the conventional method. Nevertheless, this problem suffers from combinatorial explosion that becomes very much visible when the number of variables increases.

Type-2 fuzzy regression model can be developed to include the mean interval values of all samples in the model. Therefore, it is sufficient and necessary to consider only both two vertices of the end points on the interval of each dimension of a sample. For example, one sample with one input interval feature can be expressed with two vertices of the end points on the interval with a fuzzy output value. As a consequence, in this Model, if we denote I_{ik}^L and I_{ik}^U as the left and right end points of the expected primary grades intervals of the input X_{ik}, respectively, that is

$$\begin{aligned} I_{iK}^L &= E(X_{iK}) - \sqrt{Var(X_{iK})}, \\ I_{iK}^U &= E(X_{iK}) + \sqrt{Var(X_{iK})} \end{aligned} \tag{11}$$

for $i = 1, 2, \cdots, N; k = 1, 2, \cdots, K$; the original Model can be converted into the following conventional fuzzy regression model by making use of the vertices method:

$$\left.\begin{array}{lll} min_{\bar{A}} & J(\bar{A}) = \sum_{k=1}^{K} (\bar{A}_k^r - \bar{A}_k^l) & \\ subject\ to & \bar{A}_k^r \geq \bar{A}_k^l & \\ (1) \rightarrow & \bar{Y}_i = \sum_{k=1}^{K} \bar{A}_1 \cdot I_{i1}^L + \bar{A}_2 \cdot I_{i2}^L + \cdots & \\ & +\bar{A}_K \cdot I_{iK}^L \supset_h I(e_{Y_i}, \sigma_{Y_i}) & \\ (2) \rightarrow & \bar{Y}_i = \sum_{k=1}^{K} \bar{A}_1 \cdot I_{i1}^U + \bar{A}_2 \cdot I_{i2}^L + \cdots & \\ & +\bar{A}_K \cdot I_{iK}^L \supset_h I(e_{Y_i}, \sigma_{Y_i}) & \\ (3) \rightarrow & \bar{Y}_i = \sum_{k=1}^{K} \bar{A}_1 \cdot I_{i1}^L + \bar{A}_2 \cdot I_{i2}^U + \cdots & \\ & +\bar{A}_K \cdot I_{iK}^L \supset_h I(e_{Y_i}, \sigma_{Y_i}) & \\ \vdots & \vdots & \vdots \\ (2^K) \rightarrow & \bar{Y}_i = \sum_{k=1}^{K} \bar{A}_1 \cdot I_{i1}^U + \bar{A}_2 \cdot I_{i2}^U + \cdots & \\ & +\bar{A}_K \cdot I_{iK}^U \supset_h I(e_{Y_i}, \sigma_{Y_i}) & \end{array}\right\}. \tag{12}$$

The regression model (12) can be easily solved by exhaustive way. Unfortunately, this problem cannot be solved within a reasonable computing time when K becomes even moderately large. For example, when we have 1000 features and 10000 samples, the linear programming problem will come with $2 \times 10000 \times 2^{1000}$ constraints and 1000 nonnegative constraints. Given this, we have to resort to some heuristic strategies.

E. Heuristic Method

We use the new notations for $\bar{A}_k = [\bar{a}_k, \underline{a}_k]$, for $k = 1, 2, \cdots, K$,in (10) and indicate step(n) of the algorithm(see [31]) by a suffix, say $\bar{A}_k^{(n)} = [\bar{a}_k^{(n)}, \underline{a}_k^{(n)}]$. Depending on different sign of A_k, the product of fuzzy number $\bar{A}_k$ and $I(e_{X_{ik}}, \sigma_{X_{ik}})$ involves three cases, for $i = 1, 2, \cdots, N$, an α-level set of fuzzy degree of a structural attribute at the level h^0 is denoted as follows:

$$(\bar{A}_k)_{h^0} = [\underline{a}_k, \bar{a}_k] \tag{13}$$

for each i and k, due to the signs of confidence interval and (13) the interval representing the product $(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0}$ requires several cases to be considered separately, as shown in Table II, where $\bar{e}_{ik} = E[X_{ik}] + Var[X_{ik}]$ and $\underline{e}_{ik} = E[X_{ik}] - Var[X_{ik}]$.

IV. A Numerical Example

Here is a simple example from linguistic evaluation about present market value and future value for 4 different enterprises and they have some inner relations, i.e. supply chain, then we could build a regression model. The given numbers are from the authoritative rating agencies, for instance, the revenue by company A was 5 million in objective way and 6 million in optimal way, in the future the company would have a objective value about 7 million and if the situation continues,

TABLE II
DIFFERENT CASES OF THE PRODUCT

Case	Condition	Result
Case I	$\bar{e}_{ik} \geq \underline{e}_{ik} \geq 0$	
I-a	$\bar{a}_k \geq \underline{a}_k \geq 0$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\underline{a}_k \cdot \underline{e}_{ik}, \bar{a}_k \cdot \bar{e}_{ik}]$
I-b	$\bar{a}_k \geq 0 \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\underline{a}_k \cdot \bar{e}_{ik}, \bar{a}_k \cdot \bar{e}_{ik}]$
I-c	$0 \geq \bar{a}_k \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\underline{a}_k \cdot \bar{e}_{ik}, \bar{a}_k \cdot \underline{e}_{ik}]$
Case II	$0 \geq \bar{e}_{ik} \geq \underline{e}_{ik}$	
II-a	$\bar{a}_k \geq \underline{a}_k \geq 0$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\bar{a}_k \cdot \underline{e}_{ik}, \underline{a}_k \cdot \bar{e}_{ik}]$
II-b	$\bar{a}_k \geq 0 \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\bar{a}_k \cdot \underline{e}_{ik}, \underline{a}_k \cdot \underline{e}_{ik}]$
II-c	$0 \geq \bar{a}_k \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\bar{a}_k \cdot \bar{e}_{ik}, \underline{a}_k \cdot \underline{e}_{ik}]$
Case III	$\bar{e}_{ik} \geq 0 \geq \underline{e}_{ik}$	
III-a	$\bar{a}_k \geq \underline{a}_k \geq 0$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\bar{a}_k \cdot \underline{e}_{ik}, \bar{a}_k \cdot \bar{e}_{ik}]$
III-b	$\bar{a}_k \geq 0 \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [\underline{a}_k \cdot \bar{e}_{ik}, \underline{a}_k \cdot \underline{e}_{ik}]$
III-c	$0 \geq \bar{a}_k \geq \underline{a}_k$	$(\bar{A}_k \cdot I(e_{X_{ik}}, \sigma_{X_{ik}}))_{h^0} = [a_k^\star \cdot e_{ik}^\star, a_k^{\star\star} \cdot e_{ik}^{\star\star}]$

$note\ that\ a_k^\star \cdot e_{ik}^\star = min\{\underline{a}_k \cdot \bar{e}_{ik}, \bar{a}_k \cdot \underline{e}_{ik}\},\ a_k^{\star\star} \cdot e_{ik}^{\star\star} = max\{\underline{a}_k \cdot \bar{e}_{ik}, \bar{a}_k \cdot \underline{e}_{ik}\}$

TABLE VI
AFTER TRANSFER THE LINGUISTIC TERMS TO TYPE-2 FUZZY SET

X_{11}	(0.4/0.6+0.6/0.7)/5	+	(0.7/0.7+0.3/0.1)/6
X_{21}	(0.7/0.4+0.3/0.6)/4	+	(0.6/0.8+0.4/0.5)/5
X_{31}	(0.3/0.7+0.7/0.5)/15	+	(0.4/0.9+0.6/0.4)/18
X_{41}	(0.3/0.3+0.7/0.4)/20	+	(0.5/0.3+0.5/0.4)/24
X_{12}	(0.4/0.7+0.6/0.9)/7	+	(0.7/0.3+0.3/0.5)/9
X_{22}	(0.9/0.3+0.1/0.1)/3	+	(0.7/0.5+0.3/0.2)/14
X_{32}	(0.8/0.5+0.2/0.6)/18	+	(0.6/0.7+0.4/0.3)/22
X_{42}	(0.9/0.6+0.1/0.7)/25	+	(0.6/0.9+0.4/0.4)/28
$_1$	(0.8/0.7+0.2/0.6)/15	+	(0.6/0.7+0.4/0.4)/17
$_2$	(0.7/0.4+0.3/0.3)/9	+	(0.4/0.2+0.6/0.3)/7
$_3$	(0.5/0.5+0.5/0.4)/28	+	(0.6/0.4+0.4/0.7)/30
$_4$	(0.7/0.7+0.3/0.9)/35	+	(0.7/0.5+0.3/0.8)/40

TABLE VII
INPUT DATA AND OUTPUT DATA

1	$X_{11} = (5,4,6)_T, 0.66; (6,5,7)_T, 0.52$
2	$X_{21} = (4,3,5)_T, 0.46; (5,4,6)_T, 0.68$
3	$X_{31} = (15,13,17)_T, 0.56; (18,16,20)_T, 0.60$
4	$X_{41} = (20,18,22)_T, 0.37; (24,22,26)_T, 0.35$
1	$X_{12} = (7,6,8)_T, 0.82; (9,8,10)_T, 0.36$
2	$X_{22} = (3,2,4)_T, 0.28; (4,3,5)_T, 0.41$
3	$X_{32} = (18,16,20)_T, 0.52; (22,20,24)_T, 0.54$
4	$X_{42} = (25,22,28)_T, 0.61; (28,25,31)_T, 0.70$
1	$_1 = (15,13,17)_T, 0.68; (17,15,19)_T, 0.58$
2	$_2 = (9,7,11)_T, 0.37; (7,5,9)_T, 0.26$
3	$_3 = (28,25,31)_T, 0.35; (30,27,33)_T, 0.52$
4	$_4 = (35,32,38)_T, 0.76; (40,37,43)_T, 0.59$

optimistically with the excellent campaign 9 million would be reachable. And the Table V focuses on the comprehensive market value from now to the future, it has a affinity bond with the market performance. Suppose that we have ten excellent analysts and they hold the different evaluations about the data, which were given by the rating agencies, evaluate both present and future concluded in optimistic condition and objective condition, the details shown in Table III, IV and V.

Count the frequencies of different evaluation and apply them the numerical weighs, in extend that is (extreme good,0.9), (very good,0.7), (good,0.6), (normal,0.5), (bad,0.4), (very bad,0.3), (extreme bad,0.1), and in possibility, (huge,0.9), (very large,0.8), (large,0.7), (considerable,0.6), (fair,0.5), (modest,0.4), (small,0.3), (very small,0.2) and (tiny,0.1) in the charge, say $rule$ 1. Eventually, we get the type-2 fuzzy set for this case, see Table VI. And we need to short the uncertainty on the secondary grade at the same time adding the confidence interval for the values instead. For more desirable calculating and more precise model we introduced. Using the VSCTR for the data and we get the type-1 prior considering them into confidence intervals.

The fuzzy regression model with confidence interval for the given data reads as follows:

$$\bar{Y}_i = \bar{A}_1 I[e_{X_{i1}}, \sigma_{X_{i1}}] + \bar{A}_2 I[e_{X_{i2}}, \sigma_{X_{i2}}]$$

where $I[e_{X_{ik}}, \sigma_{X_{ik}}]$, for $k = 1, 2$, and the one-sigma confidence intervals shown in (8). Since $N = 4, K = 2$, from the model (10), and assuming $(\bar{A}_k)_{h^0} = [\bar{A}_k^l, \bar{A}_k^r], k = 1\ and\ 2$, the model can be built. First of all, we need to calculate all the $I[e_{X_{ik}}, \sigma_{X_{ik}}]$, and $I[e_{Y_k}, \sigma_{Y_k}]$, for $i = 1, 2, 3, 4, k = 1\ and\ 2$. We need to calculate the pairs $(e_{X_{ik}}, \sigma_{X_{ik}})$ and e_{Y_k}, σ_{Y_k}.

Hence, the confidence intervals for the input data and output data are obtained in the follow equations:

$$\begin{aligned} I[e_{X_{ik}}, \sigma_{X_{ik}}] &= I[e_{X_{ik}} - \sigma_{X_{ik}}, e_{X_{ik}} + \sigma_{X_{ik}}] \\ I[e_{Y_i}, \sigma_{Y_i}] &= I[e_{Y_i} - \sigma_{Y_i}, e_{Y_i} + \sigma_{Y_i}] \end{aligned} \tag{14}$$

for i = 1,2,3,4 and k = 1,2. They are listed in Tables VII and VIII, respectively.

We make use of Algorithm 1 to construct a regression model. Noting that $K = 2$, all the confidence intervals are positive, and we need to set

$$(\bar{A}_k^{(1)} \cdot I[e_{X_{ik}}, \sigma_{X_{ik}}])_{h^0} = [\underline{a}_k^{(1)} \cdot e_{ik}, \bar{a}_k^{(1)} \cdot e_{ik}]$$

and from the Algorithm, we get the following linear programming:

TABLE III
OBJECTIVE VALUE AND OPTIMAL VALUE ASSESSMENT BY 10 ANALYSTS FOR PRESENT

Present		View about the Objective Value and Opportunity for the Ideal Value Held by Different Analysts									
Value of company		1	2	3	4	5	6	7	8	9	10
A	5	good	very good	good	good	very good	very good	very good	good	very good	very good
	6	large	tiny	large	tiny	large	large	large	large	large	large
B	4	bad	bad	bad	good	bad	good	bad	bad	bad	good
	5	fair	very large	very large	fair	fair	very large	very large	fair	very large	very large
C	15	very good	normal	normal	normal	very good	normal	normal	normal	very good	normal
	18	huge	huge	huge	modest	modest	modest	modest	huge	modest	modest
D	20	very bad	very bad	very bad	bad	bad	bad	bad	bad	bad	bad
	24	small	modest	modest	small	modest	small	small	modest	small	modest

TABLE IV
OBJECTIVE VALUE AND OPTIMAL VALUE ASSESSMENT BY 10 ANALYSTS FOR FUTURE

Future		View about the Objective Value and Opportunity for the Ideal Value by Different Analysts									
Company		1	2	3	4	5	6	7	8	9	10
A	7	very good	very good	extreme good	extreme good	extreme good	extreme good	very good	very good	extreme good	extreme good
	9	small	small	fair	fair	fair	small	small	small	small	small
B	3	bad	bad	bad	bad	bad	bad	bad	bad	bad	very bad
	4	fair	fair	very small	fair	fair	very small	fair	very small	fair	fair
C	18	normal	normal	good	normal	good	normal	normal	normal	normal	normal
	22	large	small	small	large	small	large	small	large	large	large
D	25	good	good	good	good	good	very good	good	good	good	good
	28	modest	huge	huge	huge	modest	modest	huge	huge	modest	huge

TABLE V
COMPREHENSIVE MARKET PERFORMANCE ABOUT THE FOUR COMPANIES FOR PRESENT AND FUTURE

Comprehensive		Comprehensive Market Performance about the Four Companies for Present and Future									
Evaluation		1	2	3	4	5	6	7	8	9	10
A	15	very good	very good	good	very good	good	very good	very good	very good	very good	very good
	17	large	large	large	modest	modest	large	modest	large	modest	large
B	9	bad	bad	very bad	bad	very bad	bad	bad	very bad	bad	bad
	7	small	very small	very small	small	small	very small	very small	small	small	small
C	28	normal	28,bad	bad	bad	normal	normal	bad	bad	normal	normal
	30	modest	modest	modest	large	large	modest	modest	large	modest	large
D	35	good	good	good	extreme good	good	extreme good	extreme good	good	good	good
	40	fair	fair	fair	very large	fair	very large	fair	very large	fair	fair

$$
\left.\begin{array}{ll}
min & J(\bar{A}) = \bar{a}_1^{(1)} - \underline{a}_1^{(1)} + \bar{a}_2^{(1)} - \underline{a}_2^{(1)} \\
subject\ to & \bar{a}_1^{(1)} \geq \underline{a}_1^{(1)}, \bar{a}_2^{(1)} \geq \bar{a}_2^{(1)} \\
& [\underline{a}_1^{(1)}, \bar{a}_1^{(1)}] \cdot 5.44 + [\underline{a}_2^{(1)}, \bar{a}_2^{(1)}] \cdot 7.61 \\
& \qquad \supseteq [13.10, 18.74] \\
& [\underline{a}_1^{(1)}, \bar{a}_1^{(1)}] \cdot 4.60 + [\underline{a}_2^{(1)}, \bar{a}_2^{(1)}] \cdot 8.17 \\
& \qquad \supseteq [6.85, 9.49] \\
& [\underline{a}_1^{(1)}, \bar{a}_1^{(1)}] \cdot 16.55 + [\underline{a}_2^{(1)}, \bar{a}_2^{(1)}] \cdot 20.04 \\
& \qquad \supseteq [26.17, 32.43] \\
& [\underline{a}_1^{(1)}, \bar{a}_1^{(1)}] \cdot 21.94 + [\underline{a}_2^{(1)}, \bar{a}_2^{(1)}] \cdot 26.60 \\
& \qquad \supseteq [24.82, 49.52]
\end{array}\right\} . \quad (15)
$$

Solving the linear programming, we obtain:

$$
\begin{aligned}
min\ J(\bar{A}) &= \bar{a}_1^{(1)} - \underline{a}_1^{(1)} + \bar{a}_2^{(1)} - \underline{a}_2^{(1)} \\
&= 0.21 - 0.21 + 2.32 - 0.77 = 1.55.
\end{aligned}
$$

Next, we move to Step 2. Since the $\underline{a}_k^{(1)}$ and $\bar{a}_k^{(1)}$ are nonnegative, then according to case I-a in Table ii, we can determine

$$
(\bar{A}_k^{(2)} \cdot I[e_{X_{ik}}, \sigma_{X_{ik}}])_{h^0} = [\underline{a}_k^{(2)} \cdot e_{ik}, \bar{a}_k^{(2)} \cdot e_{ik}]
.
$$

Then we determine $\underline{a}_k^{(2)}$ and $\bar{a}_k^{(2)}$, by solving the following

TABLE VIII
CONFIDENCE INTERVALS OF THE INPUT DATA AND OUTPUT DATA

i	$(e_{X_{i1}},\sigma_{X_{i1}})$	$I[e_{X_{i1}},\sigma_{X_{i1}}]$
1	(5.44,0.66)	[4.78,6.10]
2	(4.60,0.63)	[3.97,5.23]
3	(16.55,4.41)	[12.14,20.96]
4	(21.94,3.84)	[18.10,25.78]
i	$(e_{X_{i2}},\sigma_{X_{i2}})$	$I[e_{X_{i2}},\sigma_{X_{i2}}]$
1	(7.61,1.33)	[6.28,8.94]
2	(3.69,0.38)	[3.31,4.07]
3	(20.04,5.65)	[14.39,25.69]
4	(26.60,6.61)	[19.99,33.21]
i	(e_{Y_i},σ_{Y_i})	$I[e_{Y_i},\sigma_{Y_i}]$
1	(15.92,2.82)	[13.10,18.74]
2	(8.17,1.32)	[6.85,9.49]
3	(29.20,3.03)	[26.17,32.43]
4	(37.19,12.33)	[24.82,49.52]

linear programming:

$$\left.\begin{array}{ll} min & J(\bar{A})=\bar{a}_1^{(1)}-\underline{a}_1^{(1)}+\bar{a}_2^{(1)}-\underline{a}_2^{(1)} \\ subject\ to & \bar{a}_1^{(1)}\geq\underline{a}_1^{(1)}\geq 0,\bar{a}_2^{(1)}\geq\bar{a}_2^{(1)}\geq 0 \\ & \\ & [\underline{a}_1^{(2)}\cdot 4.84,\bar{a}_1^{(2)}\cdot 6.04]+[\underline{a}_2^{(2)}\cdot 6.48, \\ & \qquad \bar{a}_2^{(2)}\cdot 8.74]\supseteq[13.10,18.74] \\ & [\underline{a}_1^{(2)}\cdot 4.05,\bar{a}_1^{(2)}\cdot 5.15]+[\underline{a}_2^{(2)}\cdot 3.14, \\ & \qquad \bar{a}_2^{(2)}\cdot 4.24]\supseteq[6.85,9.49] \\ & [\underline{a}_1^{(2)}\cdot 12.75,\bar{a}_1^{(2)}\cdot 20.35]+[\underline{a}_2^{(2)}\cdot 14.71, \\ & \qquad \bar{a}_2^{(2)}\cdot 25.37]\supseteq[26.17,32.43] \\ & [\underline{a}_1^{(2)}\cdot 16.61,\bar{a}_1^{(2)}\cdot 27.27]+[\underline{a}_2^{(2)}\cdot 21.55, \\ & \qquad \bar{a}_2^{(2)}\cdot 31.65]\supseteq[24.86,49.52] \end{array}\right\}. \quad (16)$$

Solving this linear programming, we obtain:

$$\begin{aligned} min\ J(\bar{A}) &= \bar{a}_1^{(2)}-\underline{a}_1^{(2)}+\bar{a}_2^{(2)}-\underline{a}_2^{(2)} \\ &= 0.39-0.39+1.83-0.89=0.94. \end{aligned}$$

Since $\underline{a}_k^{(2)}\cdot\underline{a}_k^{(1)}\geq 0,\bar{a}_k^{(2)}\cdot\bar{a}_k^{(1)}\geq 0$, we move directly to Step 5 in heuristic method. We check all the vertices, in this case, 16 vertices in the regression that are obtained by (16) and we could found some vertices do not satisfy the range $I[e_{Y_i},\sigma_{Y_i}]$, given the limited pages, we would not listed them all, and after the check, we need to add at most 32 qualifications if the lower range or the upper range do not met. In our example we add 13 qualifications in (17). And from (17), the optimal value of $J(\bar{A})$ is obtained:

$$\begin{aligned} min\ J(\bar{A}) &= \bar{A}_1^r-\bar{A}_1^l+\bar{A}_2^r-\bar{A}_2^l \\ &= 0.66-0.66+2.48-0.39=2.09. \end{aligned}$$

Then, we terminate the Algorithm and return the optimal solution:

$$\bar{A}_1^l=\bar{A}_1^r=0.66,\bar{A}_2^l=0.39,\bar{A}_2^r=2.48.$$

$$\left.\begin{array}{ll} min & J(\bar{A})=\bar{a}_1^{(1)}-\underline{a}_1^{(1)}+\bar{a}_2^{(1)}-\underline{a}_2^{(1)} \\ subject\ to & \bar{a}_1^{(1)}\geq\underline{a}_1^{1}\geq 0,\bar{A}_2^{(1)}\geq\bar{A}_2^{(1)}\geq 0 \\ & \\ & \underline{a}_1^{(2)}\cdot 4.78+\underline{a}_2^{(2)}\cdot 6.28\leq 13.10 \\ & \underline{a}_1^{(2)}\cdot 3.97+\underline{a}_2^{(2)}\cdot 3.31\leq 6.85 \\ & \underline{a}_1^{(2)}\cdot 12.14+\underline{a}_2^{(2)}\cdot 14.39\leq 26.17 \\ & \underline{a}_1^{(2)}\cdot 18.10+\underline{a}_2^{(2)}\cdot 19.99\leq 24.86 \\ & 18.74\leq\bar{a}_1^{(2)}\cdot 6.10+\bar{a}_2^{(2)}\cdot 8.94 \\ & 9.49\leq\bar{a}_1^{(2)}\cdot 5.23+\bar{a}_2^{(2)}\cdot 4.07 \\ & 32.23\leq\bar{a}_1^{(2)}\cdot 20.96+\bar{a}_2^{(2)}\cdot 25.69 \\ & 49.52\leq\bar{a}_1^{(2)}\cdot 25.78+\bar{a}_2^{(2)}\cdot 33.21 \\ & \star The\ added\ portion\ is\ followed. \\ & \underline{a}_1^{(2)}\cdot 20.96+\underline{a}_2^{(2)}\cdot 14.39\leq 26.17 \\ & \underline{a}_1^{(2)}\cdot 25.78+\underline{a}_2^{(2)}\cdot 19.99\leq 24.86 \\ & \underline{a}_1^{(2)}\cdot 18.10+\underline{a}_2^{(2)}\cdot 33.21\leq 24.86 \\ & 18.74\leq\bar{a}_1^{(2)}\cdot 4.78+\bar{a}_2^{(2)}\cdot 6.28 \\ & 18.74\leq\bar{a}_1^{(2)}\cdot 4.78+\bar{a}_2^{(2)}\cdot 8.94 \\ & 18.74\leq\bar{a}_1^{(2)}\cdot 6.10+\bar{a}_2^{(2)}\cdot 6.28 \\ & 9.49\leq\bar{a}_1^{(2)}\cdot 3.97+\bar{a}_2^{(2)}\cdot 3.31 \\ & 9.49\leq\bar{a}_1^{(2)}\cdot 3.97+\bar{a}_2^{(2)}\cdot 4.07 \\ & 9.49\leq\bar{a}_1^{(2)}\cdot 5.23+\bar{a}_2^{(2)}\cdot 3.31 \\ & 32.23\leq\bar{a}_1^{(2)}\cdot 12.14+\bar{a}_2^{(2)}\cdot 14.39 \\ & 49.52\leq\bar{a}_1^{(2)}\cdot 25.78+\bar{a}_2^{(2)}\cdot 19.99 \\ & 49.52\leq\bar{a}_1^{(2)}\cdot 18.10+\bar{a}_2^{(2)}\cdot 19.99 \end{array}\right\}. \quad (17)$$

Thus, the fuzzy random regression model with confidence interval is given in the form:

$$\begin{aligned} \bar{Y}_i &= \bar{A}_1 I[e_{X_{i1}},\sigma_{X_{i1}}]+\bar{A}_2 I[e_{X_{i2}},\sigma_{X_{i2}}] \\ &= \bar{A}_1 I[e_{X_{i1}},\sigma_{X_{i1}}]+((\bar{A}_2^l+\bar{A}_2^r)/2,\bar{A}_2^l,\bar{A}_2^r)I[e_{X_{i2}},\sigma_{X_{i2}}] \\ &= 0.66 I[e_{X_{i1}},\sigma_{X_{i1}}]+[1.44,0.39,2.48]I[e_{X_{i2}},\sigma_{X_{i2}}]. \end{aligned}$$

We could recover the regression model in linguistic rule, we use E for evaluation, and R for range that $R=0.66\cdot(e_{X_{i1}}+\sigma_{X_{i1}}-(e_{X_{i1}}-\sigma_{X_{i1}})+1.44\cdot(e_{X_{i2}}+\sigma_{X_{i2}}-(e_{X_{i2}}-\sigma_{X_{i2}}))$, which means the standard level of the company, and calculate the difference between $\bar{Y}_i$ and e_{Y_i}, the ratio of the difference and R, the numerical evaluation obtained.

$$E=\frac{1.44\cdot e_{X_{i2}}+0.66\cdot e_{X_{i1}}-e_{Y_i}}{R}$$

transfer E to word guiding by $rule$ 2, that is when E greater than 1, it was splendid good, if below 0, junk level perhaps, to the rest (0,0.33], overvalued, (0.33,0.67), deserved, [0.67,1), undervalued. then we got our result. that the four companies evaluation in sequence of A, B , C, and D should be: overvalued, deserved, undervalued, superb.

V. Conclusion

In this paper we built a model for transferring linguistic data to type-2 fuzzy data and importing random regression model, in the end recover the result to linguistic data. After talking about type-2 fuzzy set and reduce function as well as linguistic transform, we use expectations and variances of fuzzy random variables to construct the confidence interval based fuzzy random data. The proposed vertices method can convert the original fuzzy random regression to a conventional fuzzy regression, with the heuristic algorithm, integrates linear programming and vertices checking, which enables us to handle the proposed regression by solving a series of linear programming problems. An illustrative example was provided to demonstrate the solution process.

The method can be implemented to several applications, it works on the non-meta linguistic data handling, lots of evaluations could be qualifiable, and helps the decision maker do the more appropriate choice. And also the fuzzy random multi-attribute evaluation for production, further applications will be discussed in our forthcoming studies.

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References

[1] L. A. Zadeh, "What is Computing with Words," *Computing with Words in Information/Intelligent Systems*, pp. VIII-IX, 2006.

[2] L. A. Zadeh, "Fuzzy Sets and Information Granularity," *Advances in Fuzzy Set Theory and Applications*, pp. 3-18, 1979.

[3] S. Li, S. Imai, J. Watada, "Building Linguistic Random Regression Model and its Application," *Proceedings of the International Conference on Knowledge-Based Intelligent Information and Engineering Systems KES-IDT*, 2012.

[4] W. Pedrycz, *Granular Computing: An Emerging Paradigm*, 2001.

[5] W. Pedrycz, "Computational Intelligence as an Emerging Paradigm of Software Engineering," *Proceedings of SEKE 2002, Ischia, Italy, July 15-19*, pp. 7-14, 2002.

[6] J. Watada, W. Pedrycz, "A Fuzzy Regression Approach to Acquisition of Linguistic Rules," *Handbook of Granular Computing*, pp. 719-732, 2008.

[7] H. Tanaka, S. Uejima, K. Asai, "Linear regression analysis with fuzzy model," *IEEE Transactions on Systems, Man and Cybernetics SMC-12*, pp. 903-907, 1982.

[8] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338-353, 1965

[9] J. Watada, H.Tanaka, "Fuzzy quantification methods," *Proceedings,the 2nd IFSA Congress*, pp. 66-69, 1987.

[10] L. A. Zadeh, "the Concept of a Linguistic Variable and Its Application to Approximate Rea-soning-1," *Information Sciences*, vol. 8, no. 3, pp. 199-249, 1975.

[11] M. Mizumoto and K. Tanaka, "Some properties of fuzzy sets of type-2," *Information and Control*, vol. 31, no. 4, pp. 312-340, 1976.

[12] D. Dubois and H. Prade, "Operations in a fuzzy-valued logic," *Information and Control* vol. 43 no. 2, pp. 224-240, 1979.

[13] R. I. John, P. R. Innocent, and M. R. Barnes, "Type-2 fuzzy sets and neuro-fuzzy clustering or radiographic tibia images," *Proc.6th Int. Conf. on Fuzzy Systems*, pp. 1375-1380, 1997.

[14] R. I. John and C. Czarnecki, "A Type 2 adaptive fuzzy inference system," *Proc. IEEE Conf. Systems, Man, Cybernetics*, vol. 2, pp. 2068-2073, 1998.

[15] Q. Liang and J. M. Mendel, "Interval type-2 fuzzy logic systems: theory and design," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 535-549, 2000.

[16] Q. Liang and Jerry M. Mendel, "MPEG VBR video traffic modeling and classification using fuzzy techniques," *IEEE Trans. Fuzzy Syst.*, vol. 9, pp. 183-193, 2001.

[17] K. C. Wu, "Fuzzy interval control of mobile robots," *Comput. Elect. Eng.*, vol. 22, pp. 211-229, 1996.

[18] H. Tanaka, I. Hayashi, and J. Watada, "Possibilistic linear regression for fuzzy data," *European Journal of Operational Research*, vol. 40, no. 3, pp. 389-396, 1989.

[19] H. Tanaka and J. Watada, "Possibilistic linear systems and their application to the linear regression model," *Fuzzy Sets and Systems*, vol. 27, no. 3, pp. 275-289, 1988.

[20] J. Watada and H. Tanaka, "The perspective of possibility theory in decision making," *Multiple Criteria Decision Making Toward Interactive Intelligent Decision Support Systems, VIIth Int. Conf.*, pp. 328-337, 1986.

[21] Y. H. O. Chang, "Hybrid fuzzy least-squares regression analysis and its reliability measures," *Fuzzy Sets and Systems*, vol. 119, no. 2, pp. 225-246, 2001.

[22] J. Watada, "Possibilistic time-series analysis and its analysis of consumption," *Fuzzy Information Engineering*, pp. 187-200, 1996.

[23] B. Liu, *Theory and Practice of Uncertain Programming*, 2002.

[24] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-I," *Information Sciences*, vol. 8, pp. 199-249, 1975.

[25] N. K. Karnik and J. M. Mendel, "Operations on type-2 fuzzy sets," *Fuzzy Sets and Systems*, vol. 122, pp. 327-348, 2001.

[26] J. M. Mendel and R. I. B. John, "Type-2 fuzzy sets made simple," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 2, pp. 117-127, 2002.

[27] X. Chen, R. Harrison, Y. Q. Zhang, and Y. Qiu, "A multi-SVM fusion model using type-2 FLS," *IEEE International Conference on Fuzzy Systems*, pp. 6611-6615, 2006.

[28] U. Pareek and I. N. Kar, "Estimating compressor discharge pressure of gas turbine power plant using type-2 fuzzy logic systems," *IEEE International Conference on Fuzzy Systems*, pp. 3113-3118, 2006.

[29] R. John, J. M. Mendel, and J. Carter, "The extended sup-star composition for type-2 fuzzy sets made simple," *IEEE International Conference on Fuzzy Systems*, pp. 7212-7216, 2006.

[30] B. Liu, Y. K. Liu, "Expected Value of Fuzzy Variable and Fuzzy Expected Value Models," *IEEE Transactions on Fuzzy Systems* vol. 10 no. 4, pp. 445-450, 2002.

[31] J. Watada, S. Wang, W. Pedrycz, "Building Confidence-interval-based Fuzzy Random Regression Model," *IEEE Transactions on Fuzzy Systems* vol. 11, no. 6, pp. 1273-1283, 2009.

[32] J. Watada, "Multiattribute Decision-making," *Applied Fuzzy System*, pp. 244-252, 1994.

[33] J. Watada, "The Thought and Model of Linguistic Regression," *Proceedings of the 9th World Congress of International Fuzzy Systems*, pp. 340-346, 2001.

[34] S. Imai, S. Wang, J. Watada, "Regression Model Based on Fuzzy Random Variables," *KES 2008, Part III. LNCS (LNAI)*, vol. 5179, pp. 127-135, 2008.

[35] R. I. B. John, "Perception Modelling Using Type-2 Fuzzy Sets," *PhD Thesis, De Montfort University*, 2000.

[36] L. A. Lucas, T. M. Centeno and M. R. Delgado, "General type-2 fuzzy inference systems: analysis, design and computational aspects," *Proceedings of FUZZ-IEEE 2007*, pp. 1743-1747, 2007.

[37] M. Nie, W. W. Tan, "Towards an efficient type-reduction method for interval type-2 fuzzy logic systems," *Proceedings of FUZZ-IEEE 2008*, pp. 1425-1432, 2008.