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New Linguistic Aggregation Operators for Decision Making

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Abstract— **This paper presents new operators named as linguistic reweighted arithmetic averaging (LRAA) and linguistic reweighted geometric averaging (LRGA) to aggregate information in group multi criteria decision making problems under linguistic settings. These operators are equipped with a capacity to deduce weight of a criterion in commensuration with its ability to discriminate among the alternatives. The properties of the operators are given. The proposed concepts are illustrated through a case study in group multi criteria decision making.**

I. Introduction

Several studies have been made in the recent times in the area of group multi criteria decision making (MCDM) under uncertainty [1-6]. In these works, decision makers (DM) give an evaluation of suppliers against various criteria in terms of fuzzy membership values [1-4] or intuitionistic membership values [5-6], followed by the aggregation of information and the ranking of the alternatives on the basis of the aggregated information. The difficulty of assigning membership values lies in the inherent nature of the problem of judging the suppliers with only limited information and a limited knowledge. Moreover, each DM may have his own personal experience, priorities and state of mind, and so forth. Hence the DMs' evaluations may differ substantially, especially when they have to express their opinions through numbers.

However, in many real life situations, such as negotiation processes and supply chain management etc., it is easy for an expert to arrive at a vague judgment in linguistic terms, while evaluating an alternative against a criterion. In the decision making processes under linguistic settings, the experts express their opinions in linguistic terms rather than in numerical form. For example, with limited knowledge at hand, one can opine that weather is "pleasant" or "cold" or "chilly" but may not be able to judge the temperature quantitatively. A few studies in the area of group decision making have the evaluations by the DMs presented by means of different linguistic preference representation structures [7-10]. A consensus model based on a linguistic framework in group decision making has been proposed in [7-8]. Xu [9] has developed a few aggregation operators, such as the linguistic weighted geometric averaging (LWGA) operator and linguistic hybrid geometric averaging (LHGA) operator and developed a method based on these operators for group decision making based on linguistic preference relations. A new operator, linguistic weighted arithmetic averaging (LWAA) has been introduced in [10].

In most of the MCDM problems in the linguistic environment, the aggregation operators are deployed to aggregate the linguistic information. In all such applications, the choice of prior weights has a major impact on the final order of the alternatives. However, crucial information in the pattern of evaluations of alternatives is often ignored. For example, a supplier may fare well in comparison to its other counterparts against a particular criterion; or in a particular year when most suppliers have posted a loss in their profits, one of them has weathered the situation and delivered a significant jump in its profits. In our view, such criteria must be identified against which there are spikes in the performance of few alternatives, or are having high amount of variation in the evaluation of alternatives. These criteria are most informative in the decision making at the final ranking of the alternatives. Also, a criterion in which all the suppliers have been evaluated almost equally, is bound to have a feeble impact on the final order of the alternatives, regardless of the importance or the high a priori weight value of the criterion.

In this paper, we have deduced the weight information from the pattern of evaluations of suppliers against the criteria. Taking account of this deduced weight information and the original a priori weights, we propose two new aggregation operators, named as linguistic re-weighted arithmetic averaging (LRAA) and linguistic re-weighted geometric averaging (LRGA) operators. These operators give equal importance to both the a priori weights and the deduced weights while computing the weighted average.

The rest of this paper is organized as follows. Section II discusses the basic notations, operational laws and the existing aggregation operators with regard to the decision making under linguistic settings. Section III presents the two new operators, LRAA and LRGA. The basic properties of the new operators are also given. In Section IV, an algorithm is developed using the proposed operators for the multi criteria decision making. Section V presents a case study of group multi criteria decision making by using this algorithm. Section VI gives the conclusions of the paper.

II. Decision Making under Linguistic Settings

The process of multiple criteria decision making with linguistic information is an approximate method in which the evaluation of the alternatives against criteria takes the form of linguistic variables. Let us consider a finite and an ordered discrete linguistic label set $S = \{s_{-t}, \dots, s_{-1}, s_0, s_1, s_2, \dots, s_t\}$, where t is a positive integer and s_i $(i = -t, \dots, -1, 0, 1, \dots, t)$ represents a possible value for the linguistic variable such that $s_i > s_j$ if $i > j$.

Example 2.1: The possible linguistic terms in the set S to compare two suppliers against the domain-knowledge based criterion are taken as

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$S = \{s_{-4} = ext.poor, s_{-3} = v.poor, s_{-2} = poor, s_{-1} = slightly\ poor, s_0 = medium, s_1 = slightly\ good, s_2 = good, s_3 = very\ good, s_4 = ext\ good\}$

The terms in S are called the linguistic terms. The cardinality of S is deliberately chosen to be small [11] so as to preserve the vagueness in the evaluations of experts and to rank the alternatives according to their respective evaluations. The fuzzy decision making model imposes a precision index in the evaluation of the alternatives that are often not possible to attain, given the constraints like lack of time, knowledge to judge quantitatively and information about the alternatives. S must satisfy the properties [12]:

the set is ordered: $s_i \geq s_j$ if $i \geq j$;

there is a negation operator: $neg\ (s_i) = s_j\ s.t.\ j = t + 1 - i$;

Max operator: $\max(s_i, s_j) = s_i\ if\ s_i \geq s_j$;

Min operator: $\min(s_i, s_j) = s_i\ if\ s_i \leq s_j$.

To preserve the given information, the discrete term set S is extended to a continuous term set, $\overline{S} = \{S_\gamma | \gamma \in [-q, q]\}$, where $q\ (q > t)$ is a large positive integer [13]. If $S_\gamma \in S$ then S_γ is termed as an original linguistic label, otherwise it is termed as a virtual linguistic label. In Example 2.1, the values such as $s_{-4}\ and\ \ s_2$ are original linguistic labels. The other linguistic labels like $s_{-4.5}$ or $s_{2.5}$ which do not belong to S are referred to as virtual linguistic labels. While working with original linguistic values, we need to apply various operations like aggregation etc. on these values, as a result of which these virtual linguistic values are generated. A few definitions regarding linguistic structures are given as follows.

Definition 2.1: Consider any two linguistic terms $S_\alpha, S_\beta \in \overline{S}$ and $\mu, \mu_1, \mu_2 \in [0, 1]$, a few operational laws are as follows [9, 13]:

i. $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$ (1)

ii. $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$ (2)

iii. $(s_\alpha)^\mu = s_{\alpha^\mu}$ (3)

iv. $(s_\alpha)^{\mu_1} \otimes (s_\alpha)^{\mu_2} = (s_\alpha)^{\mu_1+\mu_2}$ (4)

v. $(s_\alpha \otimes s_\beta)^\mu = (s_\alpha)^\mu \otimes (s_\beta)^\mu$ (5)

vi. $\lambda s_\alpha = s_{\lambda\alpha},\ \lambda \in [0,1]$ (6)

Definition 2.2 : Let $\{s_{\alpha_1}, s_{\alpha_2}, \ldots\ldots, s_{\alpha_n}\}$ be a collection of linguistic arguments, a linguistic weighted geometric averaging (LWGA) operator [9] is a mapping, i.e., LWGA: $\overline{S}^n \to \overline{S}$, and is defined as

$$LWGA_w\ (s_{\alpha_1}, \ldots, s_{\alpha_n}) = (s_{\alpha_1})^{w_1} \otimes \ldots \otimes (s_{\alpha_n})^{w_n} = \left(s_{\alpha_1^{w_1}}\right) \otimes \ldots \otimes \left(s_{\alpha_n^{w_n}}\right) = s_\alpha \quad (7)$$

where $\alpha = \prod_{j=1}^{n} \alpha_j^{w_j}$; $w = (w_1, \ldots, w_n)^T$ is the weighting vector for s_{α_j} such that $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, and $s_{\alpha_j} \in \overline{S}$

Example 2.2: Compute $LWGA_w\ (s_3, s_2, s_3, s_4)$, given the weight vector $w = (0.2, 0.4, 0.1, 0.3)^T$

$$LWGA_w\ (s_3, s_2, s_3, s_4) = (s_3)^{0.2} \otimes (s_2)^{0.4} \otimes (s_3)^{0.1} \otimes (s_4)^{0.3} = (s_{3^{0.2}}) \otimes (s_{2^{0.4}}) \otimes (s_{3^{0.1}}) \otimes (s_{4^{0.3}}) = s_{5.19}$$

Definition 2.3: A linguistic weighted arithmetic averaging (LWAA) operator [10] is a mapping, i.e., LWAA: $\overline{S}^n \to \overline{S}$, given by $LWAA_w\ (s_{\alpha_1}, s_{\alpha_2}, \ldots\ldots, s_{\alpha_n}) = w_1 s_{\alpha_1} \oplus w_2 s_{\alpha_2} \ldots \oplus\ w_n s_{\alpha_n} = s_\alpha$ (8)

Example 2.3: Compute $LWAA_w\ (s_3, s_2, s_3, s_4)$, given the weight vector $w = (0.2, 0.4, 0.1, 0.3)^T$.

It is computed from:

$LWAA_w\ (s_3, s_2, s_3, s_4) = 0.2(s_3) \oplus 0.4(s_2) \oplus 0.1(s_3) \oplus 0.3(s_4) = (s_{3\times 0.2}) \oplus (s_{2\times 0.4}) \oplus (s_{3\times 0.1}) \oplus (s_{4\times 0.3}) = s_{2.9}$.

III. The Proposed New Operators

We present here the linguistic reweighted arithmetic averaging (LRAA) and linguistic reweighted geometric averaging (LRGA) operators which deduce the weights from the observational data. These weights are essential for the solution of MCDM problems and the aggregation makes use of the weights to produce the aggregated argument vector. The existing linguistic aggregation operators [7], [9] consider only a priori weight vector, and the crucial information that can be deduced from the pattern of the observational data is ignored. These operators deduce the weight of a criterion depending upon the extent of variation in the performance of the suppliers in the criterion.

This section is divided into two parts. The first part gives the definitions of the proposed operators. In the second part presents the theorems and properties of the proposed operators.

A. *Linguistic Reweighted Arithmetic Averaging Operators*

Let us contemplate on a group MCDM problem in the linguistic settings; where $X = \{x_1, x_2, \ldots, x_n\}$ is a set of alternatives and, $u = \{u_1, u_2, \ldots, u_p\}$ is a set of criteria against which the alternatives are evaluated, and $E = \{e_1, e_2, \ldots, e_m\}$ represents a set of decision makers. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ be the weight vector of decision makers, where $\lambda_k \geq 0$; $k = 1, 2, \ldots, m$; $\sum_{k=1}^{m} \lambda_k = 1$. Let $w = (w_1, w_2, \ldots, w_p)^T \in W$ be the weight vector for the criteria, where $w_j \geq 0$; $j = 1, 2, \ldots, p$; $\sum_{j=1}^{p} w_j = 1$; W is a set of weights. The decision maker $e_k \in E$, evaluates an alternative x_j against a criterion u_i and assigns a linguistic term from the set $S = \{s_\alpha | \alpha = -t, \ldots, -1, 0, 1, \ldots, t\}$ as performance of x_j against the criterion u_i, and constructs the linguistic evaluation matrix, $A_k = \left(a_{ij}^{(k)}\right)_{p\times n}$. The element $a_{ij}^{(k)} \in S$ of A_k gives the evaluation of x_j against the criterion u_i such that $s_{-t} \leq a_{ij}^{(k)} \leq s_t$, $a_{ij}^{(k)} \oplus a_{ji}^{(k)} = s_0$, $a_{ii}^{(k)} = s_0$.

We elucidate the application of LRAA and LRGA operators to the linguistic group MCDM problem by the following definitions:

Definition 3.1: Let the linguistic terms be denoted by the vector $(s_{\alpha_{i1}}, s_{\alpha_{i2}}, \ldots\ldots, s_{\alpha_{in}})$ for the evaluation of alternatives from the set X against the criterion u_i. Then the linguistic mean for u_1 is a mapping, LM: $\overline{S}^n \to \overline{S}$, given by:

$$LM(s_{\alpha_{i1}}, \ldots, s_{\alpha_{in}}) = s_{\alpha_{i1}} \oplus \ldots \oplus s_{\alpha_{in}} = s_{\alpha_{mean}} \quad (9)$$

Where $\alpha_{mean} = \frac{1}{n}\sum_{j=1}^{n} \alpha_{ij}$. This definition can be applied to perform both criteria-wise as well as expert wise aggregations.

Definition 3.2: The deviation of a linguistic variable s_α is defined as $\delta(s_\alpha) = s_\alpha - s_{mean} = |\alpha - mean|$ (10)

Definition 3.3: The linguistic reweighted geometric averaging (LRGA) operator of dimension n is a mapping $\overline{S}^n \to \overline{S}$, given by

$$LRGA\left(s_{\alpha_{i1}}, \dots, s_{\alpha_{in}}\right) = \varpi_1\left(s_{\alpha_{i1}}\right)^{w_1} \otimes \dots \otimes \varpi_n\left(s_{\alpha_{in}}\right)^{w_n} = s_\alpha \quad (11)$$

where, $s_{\alpha_{ij}} \in \overline{S}$; $\alpha = \prod_{j=1}^{n} \varpi_j\left(\alpha_{ij}\right)^{w_j}$

$$\varpi_i = \xi_i \Big/ \sum_{j=1}^{p} \xi_j, \quad \varpi_i \in [0,1], \quad j \in p, \quad \sum_{j=1}^{p} \varpi_j = 1$$

$$\xi_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(s_{\alpha_{ij}} - s_{mean}\right)^2}$$

Definition 3.4: The linguistic reweighted arithmetic averaging (LRAA) operator of dimension n is a mapping $\overline{S}^n \to \overline{S}$, given by $LRAA\left(s_{\alpha_{i1}}, \dots, s_{\alpha_{in}}\right) = w_1\varpi_1\left(s_{\alpha_{i1}}\right) \oplus \dots \oplus w_n\varpi_n\left(s_{\alpha_{in}}\right) = s_\alpha$ (12)

Where, $s_{\alpha_{ij}} \in \overline{S}$; $\alpha = \sum_{j=1}^{n} w_j \varpi_j \alpha_{ij}$

$$\varpi_i = \xi_i \Big/ \sum_{j=1}^{p} \xi_j, \quad \varpi_i \in [0,1], \quad j \in p, \quad \sum_{j=1}^{p} \varpi_j = 1$$

$$\xi_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(s_{\alpha_{ij}} - s_{mean}\right)^2}$$

Note:- In MCDM problems, often the a priori information is missing or is not accurate. In such a scenario, the a priori weight vector may be replaced by $\left(\frac{1}{p}, \frac{1}{p}, \dots, \frac{1}{p}\right)$.

B. Properties of Proposed Operators

A few properties of the proposed operators are now discussed. These properties qualify the new operators as aggregation operators.

Theorem 3.5: Assume that F is an LRGA operator. Let $S_1 = \left(s_{\alpha_{11}}, \dots, s_{\alpha_{p1}}\right)$ and $S_2 = \left(s'_{\alpha_{12}}, \dots, s'_{\alpha_{p2}}\right)$ be the two linguistic decision vectors for the alternatives x_1 and x_2 respectively such that for each pair of ij, $s_{\alpha_{ij}} \geq s'_{\alpha_{ij}}$. Next a priori weight vector w and deduced weight vector ϖ are applied on S_1 and S_2.

Then, $LRAA(S_1) \geq LRAA(S_2)$.

Proof: From (11), we have $LRGA(S_1) = \varpi(S_1)^w$; $LRGA(S_2) = \varpi(S_2)^w$. The proof follows from the property $s_{\alpha_{ij}} \geq s'_{\alpha_{ij}}$.

Theorem 3.6: Let $S_1 = \left(s_{\alpha_{11}}, s_{\alpha_{21}}, \dots, s_{\alpha_{p1}}\right)$ and $S_2 = \left(s'_{\alpha_{12}}, s'_{\alpha_{22}}, \dots, s'_{\alpha_{p2}}\right)$ be the two linguistic decision vectors for the alternatives x_1 and x_2 respectively such that for each pair of ij, $s_{\alpha_{ij}} \geq s'_{\alpha_{ij}}$. The a priori weight vector w and deduced weight vector ϖ would remain same for both S_1 and S_2. Then, $LRAA(S_1) \geq LRAA(S_2)$.

Proof: From (12), we have $LRAA(S_1) = w\varpi S_1$; $LRAA(S_2) = w\varpi S_2$. Then the proof follows from the property, $s_{\alpha_{ij}} \geq s'_{\alpha_{ij}}$.

Corollary: $LRAA(s_1, \dots, s_n) \geq LRAA(s'_1, \dots, s'_n)\ if\ s_i \geq s'_i$
$LRAA(s_1, .., s_n) \geq LRGA(s'_1, \dots, s'_n)\ if\ s_i \geq s'_i$

Proof: Since $s_i \geq s'_i$, the following condition holds: $(s_1, s_2, \dots, s_n) \geq (s'_1, s'_2, \dots, s'_n)$. Also, the vectors w and ϖ are equal for the two alternatives, therefore an operator exhibits the above kind of symmetry."

Theorem 3.7: The operators LRGA and LRAA are idempotent in the sense that if $S = b, \forall i$, then LRGA$(s_1, \dots, s_n) = \varpi b^w$;LRAA$(s_1, \dots, s_n) = w\varpi b$

Proof: The proof directly follows from (11) and (12).

Theorem 3.8: The operators LRGA and LRAA are commutative in the sense that the order of s_i in S does not affect the final aggregated value.

Proof: From (11),

$$LRGA\left(s_{\alpha_{i1}}, \dots, s_{\alpha_{in}}\right) = \varpi_1\left(s_{\alpha_{i1}}\right)^{w_1} \otimes \dots \otimes \varpi_n\left(s_{\alpha_{in}}\right)^{w_n} = s_\alpha$$

where, $\alpha = \prod_{j=1}^{n} \varpi_j\left(\alpha_{ij}\right)^{w_j}$. Also,

$LRGA\left(s_{\alpha_{in}}, \dots, s_{\alpha_{i1}}\right) = \varpi_n\left(s_{\alpha_{in}}\right)^{w_n} \otimes \dots \otimes \varpi_1\left(s_{\alpha_{i1}}\right)^{w_1} = s_\alpha$.

Similarly, the property of commutativity holds good also for LRAA operator.

IV. Algorithm for Group Decision Making

Decision making in the current socio-economic environment is characterized by the uncertainty due to lack of information, complete knowledge and time. In such a scenario group decision making aids the decision making process by reducing the possibility of errors in the judgment of a single DM. As a result, in real life situations, many decisions are made only in groups. As in [3-6], quantitative evaluation is performed by a group of DMs. However, often it is quite difficult to judge an alternative quantitatively in terms of numbers whereas it is a simpler task to evaluate an alternative in linguistic terms. Moreover in the case of linguistic domain, there is a limited scope of variations creeping into the evaluations by DMs on the account of personal psychological aspects such as experience, learning, situation, state of mind, so on and so forth. Note that the group decision making is done under the linguistic settings in [9,10].

Our contention is that there is often useful information that can be derived from the observational data. This information should also be accounted for in the final decision. For example, in a MCDM problem, when most alternatives are ranked in the same range in their performance against a criterion, there should be a special consideration for the alternative that has performed better than its counterparts. In other words, such criteria, where a few alternatives outsmart the majority of the counterparts should have a higher weight. These criteria could play a major role in discriminating the alternatives. The criteria are bound to have lesser effect on the outcome if a priori weights play only a little role. However, in our approach we also take into account the a priori weights in addition to the weights deduced from the observational data.

In this section, we present an algorithm for group decision making under the linguistic settings in which the proposed operators, self-learning linguistic weighted geometric averaging (LRAA) and self-learning linguistic weighted geometric averaging (LRGA), are deployed to deduce the weights of the criteria and then to aggregate the information so as to arrive at the final ranking of the alternatives.

Consider a group MCDM problem in linguistic settings; where the set of alternatives is $X = \{x_1, x_2, \dots, x_n\}$, $u = \{u_1, u_2, \dots, u_p\}$ is the set of criteria against which the alternatives are evaluated, and $E = \{e_1, e_2, \dots, e_m\}$ represents a set of decision makers. Let $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$ be the weight vector of decision makers, where $\lambda_k \geq 0;\ k = 1, 2, \dots, m;\ \sum_{k=1}^{m} \lambda_k = 1$. Let $w = (w_1, w_2, \dots, w_p)^T \in W$ be the weight vector for the criteria, where $w_j \geq 0;\ j = 1, 2, \dots, p;\ \sum_{j=1}^{p} w_j = 1$; W is the set of weight information.

Algorithm:

Step 1: The decision maker $e_k \in E$, evaluates an alternative x_i from the set X in its performance against a criterion and assigns a linguistic term from the set $S = \{s_\alpha | \alpha = -t, \dots, -1, 0, 1, \dots, t\}$ as her evaluation of x_i against the criterion, and constructs the linguistic evaluation matrix, $A_k = (a_{ij}^{(k)})_{p \times n}$. The element $a_{ij}^{(k)} \in S$ of A_k gives the evaluation of x_j against the criterion u_i such that $s_{-t} \leq a_{ij}^{(k)} \leq s_t$, $a_{ij}^{(k)} \oplus a_{ji}^{(k)} = s_0$, $a_{ii}^{(k)} = s_0$. A sample matrix $[A^{(k)}]_{p \times n}$ is shown in Table 1.

Table 1. Evaluation matrix $[A^{(k)}]_{p \times n}$ for k[th] decision maker

Alternatives	x_1	---	x_n
Criteria			
u_1	$a_{11}^{(k)}$	---	$a_{1n}^{(k)}$
---	----	---	---
u_p	$a_{p1}^{(k)}$	---	$a_{pn}^{(k)}$

Step 2: Construct the linguistic aggregated evaluation matrix D by fusing all the opinions of the DMs, taking into account the weight vector for the DMs, $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m)^T$. The i-jth entry of $D = (d_{ij})_{p \times n}$ is obtained by aggregating each of the $(a_{ij}^{(k)})$ using either LRGA or LRAA operators, as per (7) or (8).

$d_{ij} = LRGA_\lambda (a_{ij}^{(1)}, \dots, a_{ij}^{(m)})$ or $LRAA_\lambda (a_{ij}^{(1)}, \dots, a_{ij}^{(m)})$

The matrix D is shown in Table 2. Each row in matrix D indicates the linguistic evaluation of various suppliers against each of the criteria. Similarly, each column of D gives the evaluation of each of the suppliers against various criteria.

Table 2. Linguistic aggregated evaluation matrix D

Alternatives	x_1	---	x_n
Criteria			
u_1	d_{11}	---	d_{1n}
---	----	---	---
u_p	d_{p1}	---	d_{pn}

Step 3: Apply (9) to compute the linguistic mean, μ for each row of matrix D. The linguistic mean for i[th] row is given by μ_i.

Step 4: Construct linguistic deviation matrix H by taking the difference of the linguistic mean, $\mu_i\ (i = 1, \dots, p)$ and $D = (d_{ij})_{p \times n}, \forall i, j$. The i-j[th] entry of $H = (h_{ij})_{p \times n}$ is obtained by taking the difference of $(d_{ij})_{p \times n}$ and μ_i in accordance with (10). The linguistic deviation matrix H is shown in Table 3.

Table 3. Linguistic deviation matrix H

Alt.	x_1	---	x_n
Cri.			
u_1	h_{11}	---	h_{1n}
---	----	---	---
u_p	h_{p1}	---	h_{pn}

where, $h_{ij} = |d_{ij} - \mu_i|$

Step 5: Compute the mean deviation in the evaluation of alternatives against a criterion, applying (12) on each row of matrix H (in Table 3) that corresponds to a criterion each. The value of ξ_i would indicate the mean deviation value for i[th] row. The value of $\xi_i = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (h_{ij})^2}$, directly determines the weight of the i[th] criterion.

Step 6: Deduce the weight vector $\varpi = (\varpi_1, \varpi_2, \dots, \varpi i, \dots, \varpi p$ for the criteria replacing the value of ξi in (12).

$$\varpi_i = \xi_i \Big/ \sum_{j=1}^{p} \xi_j, \quad \varpi_i \in [0, 1], \quad j \in p, \quad \sum_{j=1}^{p} \varpi_j = 1$$

Step 7: Utilize LRGA or LRAA operator to perform the weighted aggregation on each column of the matrix D given in Table 2. Replace the values of the a priori weight vector $w = (w_1, w_2, \dots, w_p)^T$, the deduced weight vector $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_p)$ and the linguistic evaluations for supplier x_i, given in i[th] column of Table 2, in (11) or (12) in order to compute the final value of linguistic evaluation for the supplier x_i. The final linguistic evaluation value for supplier x_i is given by Sc_i.

$$LRGA\ (d_{1j}, d_{2j}, \dots, d_{pj}) = \varpi_1 (d_{1j})^{w_1} \otimes \dots \otimes \varpi_p (d_{pj})^{w_p} = Sc_j$$

$$LRAA\ (d_{1j}, d_{2j}, \dots, d_{pj}) = w_1 \varpi_1 (d_{1j}) \oplus \dots \oplus w_p \varpi_p (d_{pj}) = Sc_j$$

Step 8: Determine the final ranking of the alternatives in terms of their suitability as per decreasing order of the values of Sc_i (for supplier x_i) computed in Step 7.

V. CASE STUDY

In this section, a group multi criteria decision making problem is contemplated for the supplier selection to illustrate the proposed approach and the concepts of LRAA and LRGA operators. The problem involves the prioritization of a set of suppliers.

Example:

The steering committee of ABC Oil Corporation needs to select the most suitable supplier from a set of set bidders, x_i (i = 1, 2, 3, 4, 5, 6) in the area of ERP implementation for development and implementation of an inclusive ERP solution across the organization. The selection committee from the ABC Oil Corporation consists of three decision makers (DMs): e_1 – the vice president, outsourcing, e_2 - the vice president, information technology and e_3 – the vice-president, operations whose weight vector is λ = (0.3, 0.4, 0.3). The committee focuses on assessing the potential of the bidders in the following areas: u_1 – Financial Strength and Stability, u_2 – ERP Implementation Experience in Mining Domain, u_3 – People and Quality, and u_4 – Infrastructure Services Experience. The weightage of the criteria is: w = (0.21, 0.26, 0.28, 0.25). The DMs evaluate the prospective suppliers with respect to the criteria u_i (i = 1, 2, 3, 4) and assign the linguistic terms in the set S = {s_{-4} = extremely poor, s_{-3} = very poor, s_{-2} = poor, s_{-1} = slightly poor, s_0 = medium, s_1 = slightly good, s_2 = good, s_3 = very good, s_4 = extremely good}. In order to evaluate the prospective suppliers, the selection committee computes the score for each of the suppliers.

We apply the algorithm in Section 4 through the following steps:

Step 1: The decision maker $e_k \in E$ evaluates alternative x_i from the set X against a criterion and assigns a linguistic term from the set S as her evaluation of x_i against the criterion, and construct the linguistic evaluation matrix, $A_k = \left(a_{ij}^{(k)}\right)_{p\times n}$. The linguistic evaluation matrices $[A^{(1)}]_{4\times 6}$, $[A^{(2)}]_{4\times 6}$ and $[A^{(3)}]_{4\times 6}$ are shown in Tables 4 to 6 respectively.

Step 2: We utilize the LRAA operator to construct the linguistic aggregated evaluation matrix $D = (d_{ij})_{4\times 6}$ by combining the opinions of the DMs in Tables 4 – 6. The weight vector for the DMs, λ = (0.3, 0.4, 0.3) is taken into account. We apply (8) to $A^{(1)}$, $A^{(1)}$ and $A^{(3)}$ to obtain the aggregated matrix $[D]_{4\times 6}$.

$$a_{ij} = \text{LRAA}_\lambda\left(a_{ij}^{(1)},\quad a_{ij}^{(2)},\quad a_{ij}^{(3)}\right),\qquad i = 1,2,3,4,\quad j = 1,2,3,4,5,6$$

The entry a_{ij} indicates the overall evaluation of j^th alternative against i^th criterion.

$$a_{11} = \text{LRAA}_\lambda\left(a_{11}^{(1)}, a_{11}^{(2)}, a_{11}^{(3)}\right) = (0.3 * s_0) \oplus (0.4 * s_3) \oplus (0.3 * s_{-1}) = s_{(0.3*0)+(0.4*3)+(0.3*-1)} = s_{0.9}$$

Similarly, the remaining values are given in matrix $[D]_{4\times 6}$ in Table 7.

Step 3: We apply (9) to compute the linguistic mean, μ for each row of matrix D. The linguistic mean for i^th row is given by μ_i.

$$\mu_1 = \frac{1}{6}\sum_{j=1}^{6} d_j = \frac{1}{6}(0.9 + 0.4 + (-0.3) + 0.8 + (-4) + 4) = 0.3$$

Similarly, $\mu_2 = 1.03$ $\quad \mu_3 = -0.26$ $\quad \mu_4 = 0.31$.

Step 4: We construct the linguistic deviation matrix H by taking the difference of the linguistic mean, μ_i $(i = 1, \dots, p)$ and $D = (d_{ij})_{4\times 6}, \forall i, j$, given in Table 7. The i-j^th entry of $H = (h_{ij})_{4\times 6}$ is obtained by taking the difference of $(d_{ij})_{4\times 6}$ and μ_i in accordance with (12). The linguistic deviation matrix H is given in Table 8.

$$h_{11} = |0.9 - 0.3| = 0.6$$

Similarly, the remaining values to generate matrix $[H]_{4\times 6}$ are given in Table 8.

Step 5: The mean deviation ξ_i in the evaluation of alternatives against a criterion is computed by applying (12) on i^th row of matrix H (in Table 8) that corresponds to criterion u_i. The value of ξ_i directly determines the weight of the i^th criterion.

$$\xi_1 = \sqrt{\frac{1}{6}\sum_{j=1}^{6}(0.6)^2 + (0.1)^2 + (0.6)^2 + (0.5)^2 + (4.3)^2 + (3.7)^2} = 2.03$$

Similarly, the other values computed are as follows:

$\xi_2 = 3.13$ $\qquad \xi_3 = 2.69$ $\qquad \xi_4 = 2.71$

Step 6: The weight vector $\varpi = (\varpi_1,\ \varpi_2,\ \varpi_3,\ \varpi_4)$ is deduced for the criteria replacing the value of ξ_i in (12).

$$\varpi_1 = \xi_1 \Big/ \textstyle\sum_{j=1}^{4} \xi_1 = 0.19$$

Similarly, the other values computed are as follows:

$\varpi_2 = 0.29$ $\qquad \varpi_3 = 0.25$ $\qquad \varpi_4 = 0.25$

Step 7: The weighted aggregation is performed on each column of the matrix D (Table 7) using the LRAA operator and vectors w, ϖ and values in Table 7. The final linguistic evaluation value for supplier x_i is given by Sc_j, $j = 1, 2, 3, 4, 5, 6$.

$$\text{LRAA}\,(d_{11}, \dots, d_{41}) = w_1\varpi_1(d_{11}) \oplus \dots \oplus w_4\varpi_4(d_{41}) = Sc_1$$

$$Sc_1 = LRAA\,(0.9, 4, 1.2, 2.9) = 0.60$$

Similarly, the other values computed are as follows:

$Sc_2 = 0.30$, $\quad Sc_3 = -0.23$, $\quad Sc_4 = -0.12$, $\quad Sc_5 = -0.99$, $\quad Sc_6 = 0.99$

Step 8: The alternatives are ranked as per decreasing order of Sc_i. The ranking is

$$x_6 > x_1 > x_2 > x_4 > x_3 > x_5$$

Discussion of Results

Alternatives x_6 and x_5 with extreme values are found to be the first and the last choice for the alternatives. High linguistic terms for alternative x_1 in linguistic aggregated evaluation matrix in Table 7 translates into a high score bringing x_1 at the top of the final order of suppliers, leaving the two extreme cases of x_6 and x_5. Taking into account the linguistic terms in Table 7, a priori and deduced weight vectors, w and ϖ, the result found matches with the intuitive one.

VI. CONCLUSIONS

Two new operators meant for the weighted aggregation of information under the linguistic settings are introduced and their properties are investigated in considerable detail. The potential of operators for dealing with the MCDM problems is highlighted through a case-study of MCDM application. The operators take account of both the a priori weight information and the weights deduced from the observational data on the basis of the variations in the data. We see that the proposed operators are quite useful in discriminating among alternatives in decision making problems. The linguistic settings add to their usefulness.

Table 4. Linguistic evaluation matrix$[A^{(1)}]_{4\times 6}$ for e_1

Alternatives / *Criteria*	x_1	x_2	x_3	x_4	x_5	x_6
c_1	s_0	s_{-2}	s_1	s_{-1}	s_{-4}	s_4
c_2	s_4	s_4	s_{-2}	s_3	s_{-4}	s_4
c_3	s_{-1}	s_{-3}	s_0	s_{-4}	s_{-4}	s_4
c_4	s_4	s_3	s_{-4}	s_{-2}	s_{-4}	s_4

Table 5. Linguistic evaluation matrix$[A^{(2)}]_{4\times 6}$ for e_2

Alternatives / *Criteria*	x_1	x_2	x_3	x_4	x_5	x_6
c_1	s_3	s_{-4}	s_{-3}	s_{-2}	s_{-4}	s_4
c_2	s_4	s_4	s_{-1}	s_4	s_{-4}	s_4
c_3	s_3	s_0	s_3	s_{-1}	s_{-4}	s_4
c_4	s_2	s_0	s_{-1}	s_0	s_{-4}	s_4

Table 6. Linguistic evaluation matrix$[A^{(3)}]_{4\times 6}$ for e_3

Alternatives / *Criteria*	x_1	x_2	x_3	x_4	x_5	x_6
c_1	s_{-1}	s_{-2}	s_2	s_1	s_{-4}	s_4
c_2	s_4	s_2	s_{-4}	s_1	s_{-4}	s_4
c_3	s_1	s_{-1}	s_0	s_{-4}	s_{-4}	s_4
c_4	s_3	s_3	s_{-2}	s_0	s_{-4}	s_4

Table 7. Linguistic aggregated evaluation matrix $[D]_{4\times 6}$

Alternatives	x_1	x_2	x_3	x_4	x_5	x_6
c_1	$s_{0.9}$	$s_{0.4}$	$s_{-0.3}$	$s_{0.8}$	s_{-4}	s_4
c_2	s_4	$s_{3.4}$	$s_{-2.2}$	s_1	s_{-4}	s_4
c_3	$s_{1.2}$	$s_{-1.2}$	$s_{1.2}$	$s_{-2.8}$	s_{-4}	s_4
c_4	$s_{2.9}$	$s_{1.8}$	$s_{-2.2}$	$s_{-0.6}$	s_{-4}	s_4

Table 8. Linguistic Deviation matrix [H]

Alternatives	x_1	x_2	x_3	x_4	x_5	x_6
c_1	$s_{0.6}$	$s_{0.1}$	$s_{0.6}$	$s_{0.5}$	$s_{4.3}$	$s_{3.7}$
c_2	$s_{2.97}$	$s_{2.37}$	$s_{3.23}$	$s_{0.03}$	$s_{5.03}$	$s_{2.97}$
c_3	$s_{1.46}$	$s_{0.94}$	$s_{1.46}$	$s_{2.54}$	$s_{3.74}$	$s_{4.26}$
c_4	$s_{1.59}$	$s_{1.49}$	$s_{2.51}$	$s_{0.91}$	$s_{4.31}$	$s_{3.69}$

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