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A Hybrid Weighted Aggregation Method Based on Consistency and Consensus in Group Decision Making

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***Abstract*—A recent study in Science indicated that the confidence of a decision maker played an essential role in group decision making problems. In order to make use of the information of each individual's confidence of the current decision problem, a new hybrid weighted aggregation method to solve a group decision making peoblem is proposed in this paper. Specifically, the hybrid weight of each expert is generated by a convex combination of his/her subjective experience-based weight and objective problem-domain-based weight. The experience-based weight is derived from the expert's historical experiences and the problem-domain-based weight is characterized by the confidence degree and consensus degree of each expert's opinions in the current decision making process. Based on the hybrid weighted aggregation method, all the experts' opinions which are expressed in the form of fuzzy preference relations are consequently aggregated to obtain a collective group opinion. Some valuable properities of the proposed method are discussed. A nurse manager hiring problem in a hospital is employed to illustrate that the proposed method provides a rational and valid solution for the group decision making problem when the experts are not willing to change their initial preferences, or the cost of change is high due to time limitation.**

***Index Terms*— Group decision making, consistency, consensus, aggregation**

I. INTRODUCTION

MAKING a decision by a group is a widespread process in daily life. Various group decision making (GDM) models have been proposed in the literatures. The traditional way of reaching a solution for GDM problems consists of two steps ([1]-[2]): (i) an aggregation phase that combines individual preferences by appropriate aggregation operators; and (ii) an exploitation phase that transforms the collective information into a ranking sequence to obtain a solution set of alternatives for the decision problem. In the aggregation phase, the weighted average method is the most widely used operator due to its simplicity and ease of interpretation [3, 4]. Given that the weight represents the importance of each decision maker (DM), such weights are usually taken as prior knowledge and determined by the DMs or moderators subjectively. For example, an expert who consistently makes more accurate decisions relative to other DMs in the past would be assigned a higher weight. Nonetheless, consistency in the past does not always reflect correct judgment for the present problems. The correctness of judgement from an expert depends on the problem domain as well. Therefore, the importance of the DMs are also depended on his/her current judgements on the specific problems.

A recent study published in Science indicated that the confidence of individuals in a group can be a valid predictor of accuracy in decision-making problems ([5], [6]). Based on this idea, we seek to integrate the information of each individual's confidence degree of the problem at hand with his/her experience, thus leading to a proposed hybrid weight of each DM by considering his/her current confidence level and historical experience.

In the process of group decision analysis, the pairwise comparison method of alternatives or criteria is widely used [1]-[2]. DMs state their preferences in a sequence by comparing two alternatives or criteria at a time, and then a comparison matrix is formed. Consistency of the comparison matrix plays an important role since humans usually are not able to have the intrinsic logical ability or confidence to be always consistent in making paired comparisons [7]. When the pairwise comparison matrix is highly inconsistent, the derived priority weights may not reflect the DMs' preferences correctly, and consequently an incorrect decision outcome may occur ([8], [9]). From the viewpoint of information processing, a higher degree of inconsistency implies a higher degree of uncertainty and a higher chance to yield an incorrect answer. Therefore, an indicator for obtaining the right answer can be measured by the level of certainty or confidence with which the decision was made. The degree of inconsistency pertaining to the pairwise comparison matrix

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can be regarded as a degree of confidence. When the degree of inconsistency is high, an expert can be considered as less certain on his/her assessment; therefore less confident.

Another important issue in GDM problems is the consensus degree that is being used to characterize the mutual agreement among the experts in a group. A high degree of consensus on the solution set of alternatives is important to guarantee the acceptance of the group decision by all DMs. The consensus measure has a dual purpose: (i) it evaluates the agreement of all DMs, which is often used to guide the consensus process until the final collective solution is achieved, and ii) it can be regarded as an indicator of the degree of importance of DM in a decision problem. Many theoretical consensus models have been proposed in the literatures([1]-[2],[11]-[15]), but they neglected the measure of consistency. To overcome this problem, some researches ([16]-[19]) have combined both consistency and consensus measures into the group decision process. Both measures are used to design a feedback mechanism to generate advices for the experts on how they should change and complete their fuzzy preference relations. However, in some cases, the experts are not willing to change their initial preferences, or the cost of change is high due to time limitation. Therefore, in order to make a more rational choice under such circumstances, we attempt to combine both consensus and consistency measures to generate a hybrid weight for each expert, and then the collective decision can be obtained based on the hybrid weighted aggregation method in the aggregation process. In the GDM models which included the feedback process, the collective decision based on the hybrid weighted aggregation method can be regarded as the initial collective group decision, based on which the iteration of the revision proceeds. While the feedback is impractical, the temporal collective decision can be regarded as the final decision, which is more precise and more valid than the collective result obtained by using the simple weighted average operator in the existing group decision models.

The paper is organized as follows. A hybrid weight of each expert in a group is generated by means of a convex combination of his/her subjective and objective weights, as explained in Section II. In Section III, a hybrid weighted aggregation method to reach a collective decision is proposed. Some important properties of the method are discussed. In Section IV, the proposed method and its corresponding properties are illustrated with an example. A summary that contains concluding remarks is presented in Section V.

II. The Hybrid Weight of Each Individual

The hybrid weight of an expert consists of three parts: (i) the experience-based weight, which is derived from historical data; (ii) the confidence-based weight, which is used to measure the confidence of the expert pertaining to his/her current decision; and (iii) the consensus-based weight, which indicates the support he/she obtained in the current decision making problem. The procedure for obtaining the hybrid weight of each expert is studied, as follows.

Given a set of alternatives $X = \{x_1, x_2, ..., x_n\}$, m experts, $E = \{E_1, E_2, ..., E_m\}$, are invited to provide their opinions by using the fuzzy preference relation (additive-based pairwise comparison matrix). Let P^k be the preference on the alternative set X given by E_k. Then, P^k is denoted by $P^k = (p_{ij}^k)_{n\times n}$, where $p_{ij}^k \in [0,1]$ indicates x_i is p_{ij}^k times more important than x_j, and $p_{ij}^k + p_{ji}^k = 1$, $\forall i, j = 1,2,...,n$.

A. The Experience-based Weight

The experience-based weight measures the relative importance of the expert in a group. Such weight can be derived from historical data. For example, if an expert made accurate decisions in previous decision making problems, his/her experience-based weight is usually assigned with higher value. Therefore, it can be measured by the ratio of the correct decisions one expert made in previous related decision making problems. In some other cases, the experience-based weight of an expert is related to his/her social position or privilege. As an instance, the president of a company is usually more important than its common staff. Therefore, we utilize the concept of the relative importance weight defined in [10]. The relative importance weight consists of three steps: (i) select the most experienced (with the highest influence) expert from a group of DMs comprising m experts, and assign him/her a weight of one. In other words, suppose *i-th* expert is the most experienced one, then a highest weight is assigned as r_i=1; (ii) compare the *j-th* expert with *i-th* expert, and obtain a relative weight for the *j-th* expert, r_j, $j=1,2,...,m$. Therefore, we have max$\{r_1,r_2,...,r_m\}$=1 and min$\{r_1,r_2,...,r_m\}$>0; (iii) define the relative importance weight of the *i-th* expert, as the experience-based weight, $w_i^{\exp}$:

$$w_i^{\exp} = r_i / \sum_{i=1}^{m} r_i \text{ , } i=1,2,...,m \tag{1}$$

Clearly, $0 \le w_i^{\exp} \le 1$ and $\sum_{i=1}^{m} w_i^{\exp} = 1$. If the influence and experience of each expert is equal, i.e., $r_i=r_j$, $i, j=1,2,...,m$ then $w_1^{\exp} = w_2^{\exp} = ... = w_m^{\exp} = 1/m$.

B. The Confidence-based Weight

The confidence-based weight of an expert is also called a consistence-based weight under the pairwise comparative mode. The consistency of a pairwise comparison matrix assesses the ability of a DM to provide a logical judgements on making comparison between alternatives/criteria. Different measures have been proposed to evaluate the consistency of a pairwise comparison matrix. As an example, transivity is one of the most measures concerning the consistency. Herrera-Viedma et. al have made comparisons among eight transitivity properties and pointed out that the additive transitivity was acceptle to characterize the consistency of a pairwise comparison matrix [22]. For the sake of simplicity, we employ the additive transitivity to measure consistency of a completed pairwise comparison matrix in this paper since all judgements have been confined to the interval [0,1] in a complete pariewise matrix, although it seems not proper for deriving missing values [23].

For any given additive-based pairwise comparison matrix P=$(p_{ij})_{n\times n}$ provided by an expert, P is said to be consistent if the following additive transitivity is satisfied [20]:

$$(p_{ij}-0.5)+(p_{jk}-0.5)=p_{ik}-0.5 \quad \forall i,j,k=1,2,...,n \tag{2}$$

Due to additive reciprocity ($(p_{ij}+p_{ji}=1,\forall i,j)$ of matrix P, Eq. (2) can be written as:

$$p_{ij}=p_{ik}+p_{kj}-0.5 \quad \forall i,j,k=1,2,...,n \tag{3}$$

If P is consistent, then $p_{ij}-(p_{ik}+p_{kj}-0.5)=0$, $\forall i,j,$ $k=1,2,...,n$. Therefore, for any $p_{ij},\forall i,j=1,2,...,n$, we have $\sum_{k=1,k\neq i,j}^{n}|p_{ij}-(p_{ik}+p_{kj}-0.5)|=0$. In other words, each p_{ij} can be estimated by the other *n-2* intermediate alternative x_k, $k\in\{1,2..,n\}/\{i,j\}$. The overall estimated value of p_{ij} is obtained by averaging all possible estimated values. If P is inconsistent, the average deviation between p_{ij} and its corresponding estimated values can be defined as the inconsistency degree of P:

$$\rho=\frac{2}{n(n-1)(n-2)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1,k\neq i,j}^{n}|p_{ij}-(p_{ik}+p_{kj}-0.5)| \tag{4}$$

where ρ is the inconsistency degree of P, which implies the uncertainty of P, i.e., the confidence of the expert on decision matrix P. The larger the value of ρ , the more uncertain of P and the less confidence of the expert. Therefore, the relative confidence-based weight of the *i-th* expert is defined by

$$w_i^{conf}=\frac{1-\rho_i}{\sum_{i=1}^{m}(1-\rho_i)} \tag{5}$$

Clearly, w_i^{conf} satisfies the following two conditions: (i) $0\leq w_i^{conf}\leq 1$ and (ii) $\sum_{i=1}^{m}w_i^{conf}=1$. When all pairwise comparison matrices provided by the experts are consistent, then $w_1^{conf}=w_2^{conf}=...=w_m^{conf}=1/m$.

C. The Consensus-based Weight

The consensus-based weight is used to indicate the relative agreement degree of an expert acquired in the current decision making problem. The expert(s) with more supporters tends to be more important in the consensus-based GDM problems. Consequently, the corresponding opinion is of much importance. In this paper, the agreement degree between two experts is defined by the similarity between their opinions.

Let $P^{(l)}=(p_{ij}^{l})_{n\times n}$ and $P^{(k)}=(p_{ij}^{k})_{n\times n}$ be two additive reciprocal preference matrices in the pairwise comparison mode, which are given by the *l-th* and *k-th* experts. The similarity degree between P^k and P^l is defined as

$$S_{kl}\ (P^k,P^l)=1-\frac{1}{n(n-1)/2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|p_{ij}^{k}-p_{ij}^{l}| \tag{6}$$

where $0\leq S_{kl}(P^k,P^l)\leq 1$.

If the *l-th* and *k-th* experts have the same opinion, then $S_{kl}(P^k,P^l)=1$.

After all similarity degrees between two experts are measured by Eq. (6), an agreement matrix (AM) is constructed. It provides an insight into the agreement degree between the experts, i.e,.

$$AM=\begin{pmatrix} 1 & S_{12}\ ...\ S_{1l}\ ...\ S_{1m} \\ ... & ... \\ S_{k1} & S_{k2}\ ...\ S_{kl}\ ...\ S_{km} \\ ... & ... \\ S_{m1} & S_{m2}\ ...\ S_{ml}\ ...\ 1 \end{pmatrix},$$

where $S_{kl}=\begin{cases} S_{kl}(P^k,P^l), & k\neq l \\ 1, & k=l \end{cases}$, and $S_{kl}=S_{lk}$ for any *l ,k=1,2,..,m*. Therefore, the consensus degree of the *k-th* expert is defined by $\mu_k=\frac{1}{m-1}\sum_{l=1,l\neq k}^{m}S_{lk}$. If all experts have the same opinion on the alternatives, then $\mu_i=\mu_j$, for any *i, j=1,2,..,m.* The consensus-based weight of the *i-th* expert can be defined by the corresponding relative agreement degree

$$w_i^{cons}=\frac{\mu_i}{\sum_{i=1}^{m}\mu_i}=\frac{\sum_{j=1,j\neq i}^{m}S_{ij}}{\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}S_{ij}} \tag{7}$$

Obviously, $0\leq w_i^{cons}\leq 1,\forall i=1,2,..m$ and $\sum_{i=1}^{m}w_i^{cons}=1$.

D. The hybrid weight of an expert

A hybrid weight for the *i-th* expert is generated by a convex combination ofthe subjective experience-based weight and the objective confidence-based and consensus-based weights, denoted by

$$\omega_i=(1-\alpha)w_i^{\exp}+\alpha[\beta w_i^{conf}+(1-\beta)w_i^{cons}] \tag{8}$$

where $0\leq\alpha,\beta\leq 1$, *i=1,2,...,m*. Parameter α is used to represent the tradeoff between the subjective weight that reflects the importance of the expert based on historical data and the objective weights that reflect the importance of the expert based on the current decision making problem. Parameter β is used to indicate the balance between the importance of the consistence and consensus degrees of the opinion provided by each expert.

Theorem 1. The hybrid weight obtained by Eq. (8) satisfies the following two properties: 1) $0\leq\omega_i\leq 1,\forall i$ and 2) $\sum_{i=1}^{m}\omega_i=1$

Proof. 1) Based on Eqs. (5) and (7), *for any i=1,2,..,m*, we have $0\leq w_i^{conf}\leq 1$, and $0\leq w_i^{cons}\leq 1$.

Since $0\leq\beta\leq 1$, $\beta w_i^{conf}+(1-\beta)w_i^{cons}$ is a convex combination of w_i^{conf} and w_i^{cons} . As a result, we have

$\beta w_i^{conf}+(1-\beta)w_i^{cons}\geq\min\{w_i^{conf},w_i^{cons}\}\geq 0$

and $\beta w_i^{conf}+(1-\beta)w_i^{cons}\leq\max\{w_i^{conf},w_i^{cons}\}\leq 1$.

Therefore, $0\leq\beta w_i^{conf}+(1-\beta)w_i^{cons}\leq 1$.

Similarly, since $0 \le \alpha \le 1$,

$\omega_i = (1-\alpha) w_i^{\exp} + \alpha[\beta w_i^{conf} + (1-\beta) w_i^{cons}]$ is a convex combination of $w_i^{\exp}$ and $\beta w_i^{conf} + (1-\beta) w_i^{cons}$. Given that $0 \le w_i^{\exp} \le 1$, $0 \le \beta w_i^{conf} + (1-\beta) w_i^{cons} \le 1$.

As a result, we prove that $0 \le \omega_i \le 1, \forall i$.

2) Based on Eq. (8), we have

$$\sum_{i=1}^{m} \omega_i = \sum_{i=1}^{m} ((1-\alpha) w_i^{\exp} + \alpha[\beta w_i^{conf} + (1-\beta) w_i^{cons}])$$
$$= (1-\alpha) \sum_{i=1}^{m} w_i^{\exp} + \alpha\beta \sum_{i=1}^{m} w_i^{conf} + \alpha(1-\beta) \sum_{i=1}^{m} w_i^{cons}$$
$$= 1 - \alpha + \alpha\beta + \alpha(1-\beta) = 1. \ \square$$

III. Properties of the Hybrid Weighted Aggregation Method

Based on the hybrid weight of each expert, the collective decision can be obtained by the hybrid weighted aggregation method as follows:

$$P^c = f(P^1, P^2, ..., P^m) = \sum_{k=1}^{m} \omega_k P^k \tag{9}$$

where ω_k is determined by Eq. (8), denoting the importance of the *k-th* expert E_k, and P^k is the opinion provided by E_k, $k=1,2,..,m$.

It should be noted that the hybrid weight aggregation method preserves some important properties which makes it valuable in the practical GDM problems, as follows.

Property1. Agreement preservation [21]:
If $P^k = P^l \ \ \forall k, l = 1, 2, ..., m$, then $P^c = P^l$.

Proof. Based on Eq. (9), the collective decision, P^c, can be calculated by

$P^c = \sum_{k=1}^{m} \omega_k P^k$. If $P^k = P^l \ \ \forall k, l = 1, 2, ..., m$, then

$P^c = \sum_{k=1}^{m} \omega_k P^l = P^l \sum_{k=1}^{m} \omega_k = P^l$. $\square$

Remark: Property 1 indicates that if the assessments from all experts are identical, then the collective result should be the common assessment.

Property 2. Order independency [21]:

If $\{i_1, i_2, ..., i_m\}$ is a permutation of {1, 2,…, m}, then $P^c = f(P^1, P^2, ..., P^m) = f(P^{i_1}, P^{i_2}, ..., P^{i_m})$.

Remark: Property 2 implies that the result of the hybrid weighted aggregation method is independent on the order of the aggregated opinions.

Property 3. Let the inconsistency degree of the opinion from the *k-th* expert be ρ_k (*k=1, 2,…, m*), and the inconsistency degree of the collective opinion be ρ_c, then $\rho_c \le \max_k \{\rho_k\}$.

Proof. Based on Eq. (9), we have

$$P^c = \sum_{k=1}^{m} \omega_k P^k$$
$$= \sum_{k=1}^{m} [(1-\alpha) w_k^{\exp} + \alpha(\beta w_k^{conf} + (1-\beta) w_k^{cons})] P^k.$$

Without loss of generality, let $\alpha = 1$, i.e., considering only the current experts' opinions, then

$\omega_k = \beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}$ and $P^c = \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] P^k$.

Suppose that the opinion of the k_0*-th* expert has the maximum degree of inconsistency, i.e., $\rho_{k_0} = \max_k \{\rho_k\}$. Based on Eq. (4), for any *k=1,2…,m*, we have

$$\rho_{k_0} = \frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{h=1, h\ne i,j}^{n} | p_{ij}^{k_0} - (p_{ih}^{k_0} + p_{hj}^{k_0} - 0.5) |$$
$$\ge \frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{h=1, h\ne i,j}^{n} | p_{ij}^{k} - (p_{ih}^{k} + p_{hj}^{k} - 0.5) |.$$

and

$$\rho^c = \frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{h=1, h\ne i,j}^{n} | p^c{}_{ij} - (p^c{}_{ih} + p^c{}_{hj} - 0.5) |$$

where $p_{ij}^c = \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{ij}^k$,

$p_{ih}^c = \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{ih}^k$,

and $p_{hj}^c = \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{hj}^k$.

Therefore, $\forall i, j = 1, 2, ..., n$, we have|

$$p^c{}_{ij} - (p^c{}_{ih} + p^c{}_{hj} - 0.5) |$$
$$= | \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{ij}^k$$
$$-(\sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{ih}^k$$
$$+ \sum_{k=1}^{m} [\beta \omega_k^{conf} + (1-\beta) \omega_k^{cons}] p_{hj}^k \text{-0.5})|$$
$$= | \beta \sum_{k=1}^{m} w_k^{conf} [p_{ij}^k - (p_{ih}^k + p_{hj}^k - 0.5)]$$
$$+(1\text{-} \beta) \sum_{k=1}^{m} w_k^{cons} [p_{ij}^k - (p_{ih}^k + p_{hj}^k - 0.5)] |$$
$$\le \beta \sum_{k=1}^{m} w_k^{conf} | p_{ij}^k - (p_{ih}^k + p_{hj}^k - 0.5) |$$
$$+ (1-\beta) \sum_{k=1}^{m} w_k^{cons} | p_{ij}^k - (p_{ih}^k + p_{hj}^k - 0.5) |$$
$$\le \beta \sum_{k=1}^{m} w_k^{conf} | p_{ij}^{k_0} - (p_{ih}^{k_0} + p_{hj}^{k_0} - 0.5) |$$
$$+ (1-\beta) \sum_{k=1}^{m} w_k^{cons} | p_{ij}^{k_0} - (p_{ih}^{k_0} + p_{hj}^{k_0} - 0.5) |$$
$$= | p_{ij}^{k_0} - (p_{ih}^{k_0} + p_{hj}^{k_0} - 0.5) | (\beta \sum_{k=1}^{m} w_k^{conf} + (1-\beta) \sum_{k=1}^{m} w_k^{cons})$$
$$= | [p_{ij}^{k_0} - (p_{ih}^{k_0} + p_{hj}^{k_0} - 0.5)] |.$$

Therefore,

$$\frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{h=1, h\ne i,j}^{n} | p^c{}_{ij} - (p^c{}_{ih} + p^c{}_{hj} - 0.5) |$$
$$\le \frac{2}{n(n-1)(n-2)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sum_{h=1, h\ne i,j}^{n} | p^{k_0}{}_{ij} - (p^{k_0}{}_{ih} + p^{k_0}{}_{hj} - 0.5) |,$$

i.e., $\rho_c \le \rho_{k_0} = \max_k \{\rho_k\}$. $\square$

Property 4. If an expert's assessment is less consistent, then his/her assessment is less important.

Remark: This property indicates that the importance of an expert's assessment is related to his/her relative confidence-based weight. When the assessment is less consistent, its degree of consistence is lower, i.e., the expert *provides the estimation with less confidence. Therefore, his/her relative confidence-based weight is less. As a result, the collective decision is less influenced by less consistent opinions.*

Property 5. If an expert's assessment is far from the others, then his/her corresponding estimation is less important.

Remark: This property indicates that the importance of an expert's assessment is related to his/her relative consensus-based weight. When the assessment is less supported, its degree of consensus is lower. Therefore, the relative consensus-based weight of the expert is less. As a result, the collective decision is less influenced by less consensus opinions.

Property 6. Let the agreement degree of the *k-th* expert be μ_k (*k=1, 2,…, m*), and the agreement degree of the collective opinion be μ_c, then $\mu_c \geq \min_k\{\mu_k\}$. In other words, the agreement degree of the collective opinion based on the hybrid weighted aggregation method is greater than the one from the expert with the minimum agreement degree.

Proof. Denote that $\mu_{k_0} = \min_k\{\mu_k\}$, based on the Eq. (6) and $\mu_k = \frac{1}{m-1}\sum_{l=1,l\neq k}^{m} S_{lk}$, we have

$$\mu_{k_0} = \frac{1}{m-1}\sum_{l=1,l\neq k_0}^{m}\left(1-\frac{1}{n(n-1)/2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|p_{ij}^{k_0}-p_{ij}^{l}|\right)$$

$$=1-\frac{1}{m-1}\sum_{l=1,l\neq k_0}^{m}\left(\frac{1}{n(n-1)/2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|p_{ij}^{k_0}-p_{ij}^{l}|\right)$$

and

$$\mu_c = \frac{1}{m}\sum_{l=1}^{m} S(P^c, P^l)$$

$$=1-\frac{1}{m}\sum_{l=1}^{m}\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\sum_{k=1}^{m}\omega_k p_{ij}^{k}-p_{ij}^{l}\right|$$

$$=1-\frac{1}{m}\sum_{l=1}^{m}\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\sum_{k=1,k\neq l}^{m}\omega_k p_{ij}^{k}-\sum_{k=1,k\neq l}^{m}\omega_k p_{ij}^{l}\right|$$

$$=1-\frac{1}{m}\sum_{l=1}^{m}\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|\sum_{k=1,k\neq l}^{m}\omega_k (p_{ij}^{k}-p_{ij}^{l})\right|$$

$$\geq 1-\frac{m-1}{m}\sum_{l=1}^{m}\frac{1}{m-1}\sum_{k=1,k\neq l}^{m}\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}|p_{ij}^{k}-p_{ij}^{l})|$$

$$=1-\frac{m-1}{m}\sum_{l=1}^{m}(1-\mu_l) \geq 1-\frac{m-1}{m}\sum_{l=1}^{m}(1-\mu_{k_0})$$

$$=(2-m)+(m-1)\mu_{k_0} \geq (2-m)\,\mu_{k_0}+(m-1)\mu_{k_0}$$

$$=\mu_{k_0} = \min_k\{\mu_k\}.\square$$

IV. An Illustrative Example

To illustrate the hybrid weighted aggregation method proposed in Section III, a nurse manager hiring problem in a hospital is investigated in this section, as follows.

Good nurse managers should be competent in their specialties, and be capable of fulfilling management responsibilities, such as planning and budgeting. In a hospital, the success of a nurse manager depends on the acceptance of his/her competence and personality by subordinates, peers, and superiors. Therefore, four candidates (possible nurse managers) are being evaluated by a group of three experts consisits of their potential peer (P1), superior (P2) and subordinate (P3). The accessments are provided by using fuzzy preference relations in paired comparisons. Their assessments are listed as follows:

$$P^1 = \begin{pmatrix} 0.5 & 0.8 & 0.7 & 0.9 \\ 0.2 & 0.5 & 0.4 & 0.6 \\ 0.3 & 0.6 & 0.5 & 0.7 \\ 0.1 & 0.4 & 0.3 & 0.5 \end{pmatrix},\ P^2 = \begin{pmatrix} 0.5 & 0.4 & 0.3 & 0.1 \\ 0.6 & 0.5 & 0.6 & 0.2 \\ 0.7 & 0.4 & 0.5 & 0.1 \\ 0.9 & 0.8 & 0.9 & 0.5 \end{pmatrix}$$

$$\text{and } P^3 = \begin{pmatrix} 0.5 & 0.2 & 0.7 & 0.9 \\ 0.8 & 0.5 & 0.3 & 0.3 \\ 0.3 & 0.7 & 0.5 & 0.1 \\ 0.1 & 0.7 & 0.9 & 0.5 \end{pmatrix}$$

All three experts stated that they had carefully examined their opinions before they provided their assessments, and they are not willing to change them anymore. Based on the preferences provided by the groups, we need to determine the overall evaluation of the four alternatives. Since no feedback is involved during the decision making process, the hybrid weighted aggregation method is employed.

The experience-based weight of each expert should be determined in order to generate the hybrid weight of each expert. Due to the great influence of the superior, the *2nd* expert is assigned the highest weight value, i.e., r_2=1. The *1st* and the *3rd* experts had the weights as r_1=0.9 and r_3=0.8. Therefore, the experience-based weight of each expert is calculated based on Eq. (1) respectively, i.e., $w_1^{\exp}$ =0.33, $w_2^{\exp}$ =0.37, and $w_3^{\exp}$ =0.30.

Considering the degree of consistency of each expert based on their assessments, it is obvious that each column in P^1 can be obtained from any other columns by adding a fixed constant. Therefore, the *1st* expert provided a perfectly consistent judgment matrix. The *3rd* expert provided a very inconsistent assessment. The *2nd* expert provided opinions between these two extremes. Based on Eqs. (4) and (5), the relative confidence-based weight of each expert is calculated, i.e, w_1^{conf} =0.45, w_2^{conf} =0.40, and w_3^{conf} =0.15 respectively. Clearly, the *3rd* expert had the lowest degree of confidence pertaining to his opinion since he/she provided the most inconsistent assessments.

Based on the agreement matrix constructed by Eq. (6), the average agreement degree of each expert is calculated, i.e., μ_1 =0.63, μ_2 =0.62, and μ_3 =0.72. Therefore, the consensus-based weight of each expert is obtained, i.e, w_1^{cons} =0.33, w_2^{cons} =0.31, and w_3^{cons} =0.36.

After obtaining all the required weights of the experts, the hybrid weight is generated by Eq. (8). Consequently, the collective decision can be obtained by the hybrid weighted aggregation method. The collective decision is denoted by P^c, which is obviously a pairwise comparison matrix, where $p_{ij}^c = f(p_{ij}^1, p_{ij}^2, p_{ij}^3) = \sum_{k=1}^{3} \omega_k p_{ij}^k$ and $\omega_k = (1-\alpha) w_k^{\exp} + \alpha[\beta w_k^{conf} + (1-\beta) w_k^{cons}]$, $\alpha \in [0,1]$ is a parameter to adjust the importance of the experience-based weight; $\beta \in [0,1]$ is a parameter to balance the importance of the consistency and the consensus of the expert's opinions. The final solution is expressed by the priority weight of each alternatives.

To further investigate the influence of the degrees of consistency and consensus of experts' opinions, let α=1, i.e, we only focus on the information provided by the current decision making problem. The priority weight of each alternative derived from the collective decision is calculated by using the Dominance Degree method [2]. The relationship between the consensus degree and the consistence degree of the collective decisions based on different values of β is shown in Figure 1. The solid line represents the consistence degree of the collective decision, while the dash line represents the consensus degree of the collective decision generated by the hybrid weighted aggregation method.

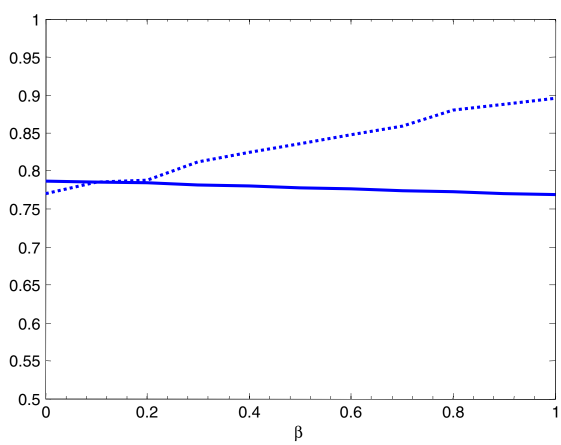

Fig. 1. The consensus and the consistence degrees of the collective decisions

Clearly, Fig. 1. shows that the consistence degree of the generated collective decision increases with the value of β, while its consensus degree slightly decreases with the value of β. The minimum consensus degree of the collective decision is greater than any experts' initial assessments. In other words, the results show that if we increase the importance of consistency of each expert's opinion, the generated collective decision would be more consistent with the cost of slightly decreasing the consensus degree.

In order to verify the properties in Section III, the degrees of inconsistence and agreement of P^i (i=1, 2, 3) and P^c is calculated. The results are shown in Table 1, where the degrees of consistency and consensus are obtained by averaging the values based on the different β settings.

Table 1 shows that the inconsistence degree of the collective decision obtained by the hybrid weight aggregation method is lower than that from the most inconsistent expert, i.e., property 3 is verified. The agreement degree of the collective decision is higher than the opinion of the *2nd* expert in the group, i.e., property 6 is verified.

TABLE I
THE CONSISTENCY AND CONSENSUS DEGREES OF OPINIONS

	Inconsistency Degree ρ_i	Consensus Degree μ_i
P^1	0	0.633
P^2	0.083	**0.617**
P^3	**0.65**	0.717
P^c	0.1649*	0.7782*

V. Conclusions

Owing to the great influence of an DM's confidence on the group decision and the significant role of consensus between DMs, a hybrid weighted aggregation method has been proposed in this paper. It aims to generate a more effective and practical group decision for tackling GDM problems. The proposed method provides a rational choice when DMs are reluctant to change their original opinions, or the feedback mechanism is impractical due to the lack of the further information in decision making proceeds. Alternatively, when the feedback mechanism is in place, the collective decision based on the hybrid weighted aggregation method can also be regarded as the initial decision, based on which the iterations of the revision proceed. Our future work will focus on the changes of the confidence degree of each DM by taking the information exchange between the DMs into account.

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