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A Generalized Equilibrium Value-based Approach for Solving Fuzzy Programming Problem

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***Abstract*—Fuzzy programming approach has wide application in many fields such as project management, multi-attribute decision making, and comprehensive evaluation. Its solving methods have attracted many attentions. In this paper we present a new approach, based on the genetic algorithm, for dealing with a programming problem with fuzzy-valued objective function and constraints. Firstly, we propose the concept of generalized equilibrium value of fuzzy number, and analyze the properties, further give the operation rules; secondly, we establish a generalized equilibrium value-based fuzzy programming method combined with genetic algorithm; finally, we analyze the characteristic of the above mentioned method through a non-linear fuzzy programming problems.**

Keywords—fuzzy programming; fuzzy number ranking; triangular fuzzy number; generalized equilibrium value; genetic algorithm

I. INTRODUCTION

A. Review for fuzzy programming

In classical programming problems, all the data are known precisely or given as crisp values. However, with the rapid development of information technology and internet of things, crisp data are inadequate or insufficient to model real-life decision problems. Many decisions in real life cannot be modeled easily in deterministic terms because of imprecision surrounding involved data. In this connection, the Physics Nobel laureate Feynman once wrote: "When dealing with a mathematical model, special attention should be paid to imprecision in data". Zadeh's incompatibility principle [1] stipulating that: "When the complexity of a system increases, our ability to formulate precise and yet meaningful statement on this system decreases up to a threshold beyond which precision and significance become mutually exclusive characteristics", is also instructive in this regards.

This stimulates us to study the programming problems with uncertainty. The uncertain information processing is the bottle neck of solving programming problems. The usual way is to transform uncertain information to a numerical value, based on it, further construct a decision making method. As probability theory is a matured segment we are familiar with, it is not a surprise that early works on mathematical programming under uncertainty was devoted to situations with randomness. In 1955, Dantzing proposed the expected value model of stochastic programming, aimed to get the maximum expected return; in 1959, Charnes and Cooper [2] proposed the chance constraint programming model; in 1970, Liu [3-4] et al. proposed chance-dependent programming model, and further gave the solving method based on stochastic simulation and genetic algorithm; Li and Wang [5] established the generalized expected value model based on the compound quantitative method of the expected value and variance. Nevertheless, uncertainty cannot be equated with randomness. As a matter of fact, there is another different type of uncertainty (fuzziness) which cannot be tackled with probabilistic theories [6]. In this case, the decision makers often apply the fuzziness to improve the chances of success of decision making. Zimmerman [7] first applied fuzzy set theory to conventional linear programming problems. Following this attempt, fuzzy programming has been developed and extended in many applications. Due to the dual nature of set and numerical value, fuzzy number has been widely applied in fuzzy programming problems. Among the different methods for solving fuzzy programming problems, the method based on the ranking and comparison of fuzzy numbers is one of the most convenient [8-10]. In 1970, Bellman and Zadeh [11] first proposed the basic fuzzy decision making model for multi-objective decision making problems. In 1978, Zimmermann [12] established fuzzy linear programming model based on tolerance method. Zadeh [6] discussed the connection between fuzzy set and possibility measure, analyzed the feasibility of constructing the fuzzy decision making methods by possibility theory, and further gave the fuzzy expected value model. In 1999, Liu [13] gave the concept of fuzzy variables and established fuzzy chance-dependent programming model based on possibility measure. In 2010, Li and Jin [14] systematically analyzed the characteristic of fuzzy decision

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making and studied fuzzy programming theories and methods based on synthesis effect.

According to the discussions in-above, fuzzy programming problems focus on the development of the quantitative way for fuzzy number.

B. Ranking for fuzzy number

Fuzzy number, as a tool describing and processing uncertain information, can closely reflect the reality. Consequently, it is very natural and important to introduce fuzzy numbers ranking methods. In 1976, Jain [15] first discussed the comparison of fuzzy numbers. In 1977, Baas and Kwakernaak [16] designed a ranking method based on fuzzy relation. Based on the reference of [15, 16], Baldwin and Guild [17] constructed fuzzy preference method in 1979. Thereafter, many researches gave various ranking methods combined with the structural characteristic of fuzzy set. In 1985, Bortolan and Degani [18] discussed the ranking strategy of fuzzy number. Chen [19] proposed a ranking method based on the maximal and minimal fuzzy set. In 1988, Lee and Li [20] regarded fuzzy number as a possibility variable, and proposed a ranking method based on possibility measure. In 1990, Kim and Park [21] proposed a combination ranking method based on optimization index. In 1992, Liou and Wang [22] established a ranking method based on integral combined with the function representation theory of fuzzy number. In 1998, Cheng [23] established a ranking method based on distance by combining with a metric of fuzzy number space. Based on the interval decomposition theory of fuzzy number, in 2002, Liu et al. [24] took the convex combination of the interval end as the average value, and level importance function as the importance measure of level cuts, and established a concentralized quantitative method of fuzzy number using integral. In 2007, Asady and Zendehnam [25] proposed a ranking method based on distance minimization. In 2009, Abbasbandy and Hajjari [26] proposed a ranking method for triangular fuzzy number, which overcome the disadvantages of that in [25]. In 2012, Li et al. [27] discussed the ranking criteria of fuzzy number based on numerical characteristic, and further gave the corresponding constructing strategy.

As the above literatures mentioned, they often characterized a fuzzy number as a real number, and then realized the comparison by the total ordering of real numbers.

In this paper, we give the concept of generalized equilibrium value for fuzzy number, discuss the properties and operation rules; we then establish a generalized equilibrium value-based fuzzy programming method combined with genetic algorithm; finally, we analyze the characteristic of the above mentioned method through a non-linear fuzzy programming problems.

II. Preliminaries

For the better understanding of this paper, let us briefly review some concepts and results on fuzzy numbers. In this paper, we use R to denote the family of all real numbers, $\mathrm{I}(R)$ the family of all interval numbers, and $\mathrm{F}(R)$ the family of all fuzzy sets. For $A\in \mathrm{F}(R)$, the membership function of A will be denoted by $A(x)$, the λ-cut set by $A_\lambda=\{x\mid A(x)\geq\lambda\}$, the support set by $\operatorname{supp}A=\{x\mid A(x)>0\}$, and the closure of supp$A$ will be denoted by A_0.

Definition 1 [28] Suppose $A\in F(R)$, if it satisfies the following conditions: 1)$A_1\neq\Phi$; 2) for any $\lambda\in(0, 1]$, $A_\lambda\in I(R)$; 3)supp$A=\{x|A(x)>0\}$ is bounded, then we call A a fuzzy number. The family of all fuzzy numbers can be called fuzzy space and denoted by $\tilde{R}$. In particular, ① if there exist $(a,[b_1,b_2],c)\in R$ satisfying: i) $A(x)=0$ for $x<a$ or $x>c$; ii) $A(x)=(x-a)/(b_1-a)$ for $a\leqslant x<b_1$; iii) $A(x)=1$ for $b_1\leqslant x\leqslant b_2$; iv) $A(x)=(x-c)/(b_2-c)$ for $b_2<x\leqslant c$, then we call A trapezoidal fuzzy number, simply denoted as $A=(a,[b_1,b_2],c)$; ② if $b_1=b_2=b$, then we call $A=(a,[b, b],c)$ a triangular fuzzy number, simply denoted as $A=(a, b, c)$.

Obviously, if we take interval number $[a, b]$ as a fuzzy set whose membership function is $[a, b](x)=1$ for any $x\in[a, b]$, and $[a, b](x)=0$ for any $x\notin[a,b]$, and real number a as a fuzzy set whose membership function is $a(x)=1$, and $a(x)=0$ for any $x\neq a$, then the interval numbers and real numbers are special fuzzy numbers, which shows that fuzzy numbers are the extension of the interval numbers and real numbers.

Theorem 1 [29] Let $A,B\in\tilde{R}$, $k\in R$, $f(x, y)$ be a continuous binary function, A_λ, B_λ be the λ-cut sets of A and B. the $f(A,B)\in\tilde{R}$, and $(f(A, B))_\lambda=f(A_\lambda, B_\lambda)=\{f(x, y)|x\in A_\lambda, y\in B_\lambda\}$. In particular, we have:

1) $A+B=B+A$, $A\bullet B=B\bullet A$, $k(A\pm B)=kA\pm kB$;
2) $(A+B)_\lambda=[\underline{a}(\lambda)+\underline{b}(\lambda), \overline{a}(\lambda)+\overline{b}(\lambda)]$,
   $(A-B)_\lambda=[\underline{a}(\lambda)-\overline{b}(\lambda), \overline{a}(\lambda)-\underline{b}(\lambda)]$;
3) $(A\cdot B)_\lambda=[\underline{a}(\lambda)\times\underline{b}(\lambda),\ \overline{a}(\lambda)\times\overline{b}(\lambda)]$, $\underline{a}(\lambda)\geq 0$, $\underline{b}(\lambda)\geq 0$;
4) $(A\div B)_\lambda=[\underline{a}(\lambda)\div\overline{b}(\lambda),\ \overline{a}(\lambda)\div\underline{b}(\lambda)]$, $\underline{a}(\lambda)\geq 0$, $\underline{b}(\lambda)\geq 0$;
5) If $A=(a_1,b_1,c_1)$, $B=(a_2,b_2,c_2)$, then
   $A+B=(a_1+a_2, b_1+b_2, c_1+c_2)$;
   $A-B=(a_1-a_2, b_1-b_2, c_1-c_2)$.
6) For $A=(a_1,b_1,c_1)$, $kA=(ka_1, kb_1, kc_1)$ for any $k\geqslant 0$;
   $kA=(kc_1, kb_1, ka_1)$ for any $k<0$;
7) If $k\neq 0$, then $kA(x)=A(x/k)$, if $k=0$, then $kA=0$.

Fuzzy numbers have many good analytical properties. For details, please see ref. [28].

III. The Generalized Equilibrium Value of Fuzzy Number

Balance state is a quantitative index widely concerned by all kinds of uncertain decision making. In random environment, we can use the frequency of event happening (i.e., the average number of events happening in each test) to describe the probability of the event, and use the mathematical expectation to describe the average values of random variable. In physics, we can use the center of gravity to describe the equilibrium position of a particle system. In game theory, we can make specific countermeasures through the balance of interests. In fuzzy decision making problems, fuzzy numbers were the most commonly used quantitative fuzzy information description method. Although it has many good operation and analytical properties, there does not exist generally accepted ranking model, which cause it was difficult to make the right decision. Therefore, how to establish the fuzzy number ranking method with good interpretability and operability has important theoretical and practical significance. If fuzzy number A is seen as a variable with membership limitation, A(x) is understood as the support degree of element x belonging to A. Similar to the calculation method of center of gravity of a object, we can construct fuzzy number average value [30] measure model as follow (Here, a is a real number),

$$E(A)=\begin{cases}\dfrac{\int_{-\infty}^{+\infty}xA(x)dx}{\int_{-\infty}^{+\infty}A(x)dx}, & A\neq a\\ a, & A=a\end{cases} \tag{1}$$

Because fuzzy number has fuzziness, so in the process of fuzzy decision making, it tends to reflect fuzzy preference of people, therefore, if we merge fuzzy processing preference into decision making, so (1) can be perfect as the following:

$$E_G(A)=\begin{cases}\alpha E_L(A)+(1-\alpha)E_R(A), & A\neq a\\ a, & A=a\end{cases} \tag{2}$$

We call $E_G(A)$ a generalized equilibrium value of A (Here, a is a real number, $E_L(A)$ expresses the left uncertainty, $E_R(A)$ expresses the right uncertainty).

Theorem 2 let $A\in\tilde{R}$, $k\in R$, we have:

1) $E_G(kA)=k\,E_G(A)$; 2) $E_G(k+A)=k+E_G(A)$

Proof: 1) if k=0, then kA=0, we have $E_G(kA(x))=k\,E_G(A(x))=0$. If $A=a$, then $kA=ka$, we have $E_G(kA)=ka=kE_G(A)$.

If $k\neq 0$, $A\neq a$, according to the extension principles, we have:

$$E(kA)=\frac{\int_{-\infty}^{+\infty}x((kA)(x))dx}{\int_{-\infty}^{+\infty}((kA)(x))dx}=\frac{\int_{-\infty}^{+\infty}xA(x/k)dx}{\int_{-\infty}^{+\infty}A(x/k)dx}$$
$$=\frac{\int_{-\infty}^{+\infty}k^2uA(u)du}{\int_{-\infty}^{+\infty}kA(u)du}=k\frac{\int_{-\infty}^{+\infty}uA(u)du}{\int_{-\infty}^{+\infty}A(u)du}=kE(A)$$

So we have $E_L(kA)=kE_L(A)$, and $E_R(kA)=kE_R(A)$,

$$\begin{aligned}E_G(kA)&=\alpha E_L(kA)+(1-\alpha)E_R(kA)\\&=\alpha kE_L(A)+(1-\alpha)kE_R(A)\\&=k[\alpha E_L(A)+(1-\alpha)E_R(A)]=kE_G(A)\end{aligned}$$

2) If $A=a$, then $E(k+A)=k+E(A)$, if $A\neq a$, then

$$E(k+A)=\frac{\int_{-\infty}^{+\infty}x((k+A)(x))dx}{\int_{-\infty}^{+\infty}((k+A)(x))dx}=\frac{\int_{-\infty}^{+\infty}xA(x-k)dx}{\int_{-\infty}^{+\infty}A(x-k)dx}$$
$$=\frac{\int_{-\infty}^{+\infty}(k+u)A(u)du}{\int_{-\infty}^{+\infty}A(u)du}=k+\frac{\int_{-\infty}^{+\infty}uA(u)du}{\int_{-\infty}^{+\infty}A(u)du}=k+E(A)$$

So we have $E_L(k+A)=k+E_L(A)$, and$E_R(k+A)=k+E_R(A)$, therefore,

$$\begin{aligned}E_G(k+A)&=\alpha E_L(k+A)+(1-\alpha)E_R(k+A)\\&=\alpha(k+E_L(A))+(1-\alpha)(k+E_R(A))\\&=[\alpha k+(1-\alpha)k]+[\alpha E_L(A)+(1-\alpha)E_R(A)]\\&=k+E_G(A)\end{aligned}$$

Theorem 3 For a triangular fuzzy number $A=(a,b,c)$, we have

$$E_G(A)=\frac{\alpha(2b+a)}{3}+\frac{(1-\alpha)(2b+c)}{3}=\frac{2b+c+(a-c)\alpha}{3}$$

Proof: 1) when $a=b=c$, the conclusion is obviously true.

2) If $a<c$, then

$$E_L(A(x))=\frac{\int_a^b x(\frac{x-a}{b-a})dx}{\int_a^b\frac{x-a}{b-a}dx}=\frac{\frac{(b-a)^2}{3}+\frac{a(b-a)}{2}}{\frac{b-a}{2}}=\frac{2b+a}{3}$$

$$E_R(A(x))=\frac{\int_b^c x(\frac{x-c}{b-c})dx}{\int_b^c\frac{x-c}{b-c}dx}=\frac{-\frac{(b-c)^2}{3}-\frac{c(b-c)}{2}}{-\frac{b-c}{2}}=\frac{2b+c}{3}$$

So

$$\begin{aligned}E_G(A)&=\alpha E_L(A)+(1-\alpha)E_R(A)\\&=\frac{\alpha(2b+a)}{3}+\frac{(1-\alpha)(2b+c)}{3}=\frac{2b+c+(a-c)\alpha}{3}\end{aligned}$$

Corollary 1 If $A=(a_1,b_1,c_1)$, $B=(a_2,b_2,c_2)$, k, $l\in R$, then we have:

1) $E_G(A+B)=E_G(A+B)$,

2) $E_G(kA+lB)=kE_G(A)+kE_G(A)$.

Proof: 1) From $A+B=(a_1+a_2, b_1+b_2, c_1+c_2)$ and Theorem 2, we can get

$$\begin{aligned}E_G(A+B)&=\frac{\alpha[2(b_1+b_2)+(a_1+a_2)]}{3}+\frac{(1-\alpha)[2(b_1+b_2)+(c_1+c_2)]}{3}\\&=\frac{\alpha(2b_1+a_1)}{3}+\frac{\alpha(2b_2+a_2)}{3}+\frac{(1-\alpha)(2b_1+c_1)}{3}+\frac{(1-\alpha)(2b_2+c_2)}{3}\\&=[\frac{\alpha(2b_1+a_1)}{3}+\frac{(1-\alpha)(2b_1+c_1)}{3}]+[\frac{\alpha(2b_2+a_2)}{3}+\frac{(1-\alpha)(2b_2+c_2)}{3}]\\&=E_G(A)+E_G(B)\end{aligned}$$

2) It can be directly obtained by Theorem 2 and the conclusion 1).

IV. A Generalized Equilibrium Value-based Fuzzy Programming Problem and Its Solving Method

A. Problem Formulation

Formally, the fuzzy program models involve a vector of decision variables, objective functions, and constraints. Generally, the mathematical form can be formulated as follows:

$$\begin{cases}\max f(x), \\ \text{s. t. } g_i(x) \lesssim b_i, i=1,2,\cdots,m.\end{cases} \tag{3}$$

Here, $x=(x_1,x_2,\cdots,x_n)$ is a vector of decision variables, both f and $g_1,g_2,\cdots,g_m$ are n-dimensional fuzzy-valued functions, $\lesssim$ denotes the inequality relationship in the fuzzy sense, $b_i\in E^1$ are the given fuzzy numbers.

This model can be seen in several applications including project management [31, 32], multi-attribute decision making problems [33, 34, 35, 36, 37], supplier selection and order allocation [38]. Without loss of generality, these problems focus on the case of fuzzy objective function and constraints. As a matter of fact, the literatures were addressing the problem of converting fuzzy program into a crisp one.

Therefore the optimal solution does not exist for (3) and we have to seek for some satisfactory solution. Moreover, existing deterministic approaches cannot be applied blindly. Here, we take advantage of the concept of the generalized equilibrium value of a fuzzy number, the original problem is converted into a deterministic one using approximate transformations.

$$\begin{cases}\max E_G(f(x)), \\ \text{s. t. } E_G(g_i(x)) \le E_G(b_i), i=1,2,\cdots,m.\end{cases} \tag{4}$$

Here, $E_G(f(x))$, $E_G(g_i(x))$, $E_G(b_i)$ represents the generalized equilibrium value of $f(x)$, $g_i(x)$, b_i respectively, $x_j\in R,(j=1,2,\cdots,n)$.

Since triangular fuzzy numbers are often used to describe fuzzy information in practical problems, we will assume in this paper that the coefficients are all triangular fuzzy numbers. Due to the intrinsic difference between the operations of triangular fuzzy numbers and those of the real number, optimization problems with triangular fuzzy coefficients cannot be solved by analytical methods. We establish concrete solution methods to our optimization problem by combining with genetic algorithm.

B. Solution algorithm for the problem

The task of global minimization problems is of paramount importance in several areas of applications. In this section, we present a novel evolutionary computation method called Genetic algorithm [39] (GA) to solve the problem proposed above. Here, before introducing the novel algorithm, let us first review the basic GA. GA is among the most popular methods to stochastic global optimization which mimic the natural evolutionary process in order to search the optimum from the feasible region. GA has the benefits of simplicity and superior performance. For many years this technique has been successfully applied to a wide variety of real fields. In GA, each individual in the population is encoded into a chromosome representing a possible solution. It works by generating a random initial population of potential solutions. The fitness of an individual is evaluated with respect to a given objective function. Highly fit individuals in a crossover procedure are given big probability to reproduce new "offspring" solutions. These new "offspring" solutions will share some characteristics with their parents. Mutation is often applied after crossover by altering some genes. This evaluation -selection-crossover-mutation cycle is repeated until a satisfactory solution is found.

In the following, we adopt binary code to represent the real optimized variables. To find the satisfactory solutions to (4), we give the procedures of fuzzy GA.

Step1 Transform (3) into a crisp optimization one, by using the generalized equilibrium value of fuzzy number and search for the global maxima of (2).

Step2 Submit the generalized equilibrium value of fuzzy-valued objective function to the fuzzy GA, in order to search for the satisfactory solutions to (2). Here, it is possible to simply run the algorithm several times. As a stochastic method, fuzzy GA is able to explore different regions during different activations.

- 1) Initialize parameters setting: the population size s, crossover probability p_c, mutation probability p_m and stopping iteration G_{max};
- 2) Set initial evolution iteration G=0, randomly generate s candidate solutions,

  $\bar{X}(G)=(X_1(G),X_2(G),\cdots,X_s(G))$;
- 3) Calculate the fitness value $f(\bar{X}(G))$ of initial candidate solution;
- 4) (Selection Operation) The selection process is performed according to $f(\bar{X}(G))$, update the solution vectors in $\bar{X}(G)$ for the next iteration;
- 5) (Crossover Operation) To increase the diversity of the population, crossover is introduced. In this paper, we adopt arithmetic crossover strategy;
- 6) (Mutation Operation) To avoid trapping into the local premature convergence. In this paper, we adopt uniform mutation strategy.

Step3 Check the stopping criterion. If the stopping criterion (maximum of iterations G_{max}) is satisfied, then quit the iterative process. Otherwise, return to 3) of Step 2.

Remark 1 In Step 2, the fitness value is closely related with the generalized equilibrium value of the fuzzy-valued objective function, so we name our algorithm by FGA-GEV.

Remark 2 In practice, triangle fuzzy number has good operability and interpretability, so the following we will make a example based on the triangle fuzzy numbers.

V. Numerical Example

Example 1 Considering the fuzzy nonlinear programming problem as follows:

$$
\begin{aligned}
\max\ f(x_1,x_2) = &-(0.1, 0.3, 0.8)x_1^2 - (0.2, 0.4, 0.7)x_2^2 + (16.1, 17, 17.3)x_1 \\
&+ (17.7, 18, 18.6)x_2
\end{aligned}
$$

$$
\text{s.t.}\begin{cases}
(1.4, 2, 2.6)x_1 + (2.7, 3, 3.3)x_2 \lesssim (47, 50, 51), \\
(3.8, 4, 4.4)x_1 + (1.6, 2, 2.2)x_2 \lesssim (40, 44, 47), \\
(2.6, 3, 3.2)x_1 + (1.6, 2, 2.2)x_2 \cong (32, 36, 40), \\
x_1, x_2 \gtrsim 0
\end{cases}
$$

For this optimization problem (both coefficients are real numbers), the optimal solutions are x_1=4.8333, x_2=10.75, $max\ f(x_1,x_2)$=222.4329.

The parameters in the FGA-GEV are set as follows: the population size s=80, crossover probability p_c=1, mutation probability p_m=0.0001and stopping iteration G_{max}=100. Then, we implement the FGA-GEV on MATLAB with $\alpha = 0.5$. We can get the iteration curve as Figure 1, and the satisfactory solutions are x_1=4.5503, x_2=11, and the maximal generalized equilibrium value of fuzzy objective function is 217.7872. We make 10 times experiments, the corresponding results are listed in the following TABLE I. By varying the values of decision parameter α, different results can be obtained which are listed in the following TABLE II.

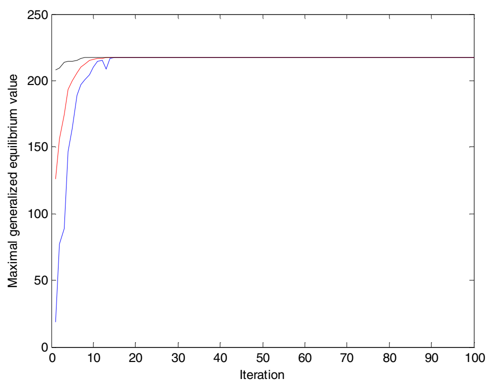

Fig. 1 The 100 iteration curve for Example 1

TABLE I. The 10 Computation Results Obtained by Model (4) when $\alpha = 0.5$

	S. S.	M. G. E. V.	C. T.	C. I.
1	x_1=4.5650 x_2=10.9677	217.7012	0.9683	15
2	x_1=4.5503 x_2=11	217.7872	0.8595	17
3	x_1=4.5503 x_2=11	217.7872	0.8726	13
4	x_1=4.5601 x_2=10.9785	217.7299	0. 9395	13
5	x_1=4.5601 x_2=10.9785	217.7299	0.9906	16
6	x_1=4.5503 x_2=11	217.7872	0.8449	13
7	x_1=4.5552 x_2=10.9892	217.7586	0.8141	16
8	x_1=4.5503 x_2=11	217.7872	0.8307	15
9	x_1=4.5503 x_2=11	217.7872	0.8024	12
10	x_1=4.5503 x_2=11	217.7872	0.8178	13
A.V.	———	217.7643	0.8740	14.3

TABLE II. The Computation Results Obtained by Model (4) Under Different Values of α

α	S. S.	M. G. E. V.	C. T.	C. I.
0	x_1=4.1984 x_2=0.9570	202.9568	0.8452	11
0.1	x_1=4.2522 x_2=10.9892	205.8321	0.8642	11
0.3	x_1=4.3939 x_2=11	211.7108	0.8801	13
0.4	x_1 =4.4819 x_2=10.9892	214.7790	0.8625	13
0.6	x_1 =4.6383 x_2=10.9785	220.7957	0.8049	12
0.8	x_1 =4.7849 x_2=11	227.0739	0.8984	14
1	x_1 =4.9462 x_2=11	233.4603	0.8237	16

In TABLE I-II, S.S. denotes satisfactory solutions, M. G. E. V. denotes Maximal generalized equilibrium value, C. T. denotes Convergence Time, C. I. denotes Convergence iteration, A. V. denotes Average value.

From the results above we see that: 1) The computational results are closely related to the parameter α, which shows FGA-GEV can effectively merge uncertainty decision preferences into the decision process; 2) Despite of the variations of parameters, the convergence time is less than 1 seconds, and the convergence iteration is less than 20, which shows the algorithm have higher computational efficiency and good convergence performance; 3) The computational complexity is equivalent to that of conventional algorithms, so FGA-GEV has good practicability; 4) FGA-GEV has many advantages such as good interpretability and strong operability. Therefore, FGA-GEV is suitable for the optimization problems under uncertain environment.

VI. CONCLUSIONS

In this paper, we present a new method FGA-GEV that can merge uncertainty decision preference into the solving when coping with the fuzzy programming with fuzzy-valued objective function and constraints.

In ranking fuzzy numbers, we propose the concept of generalized equilibrium value, discuss its properties and operation rules. Based on it, the discussed fuzzy programming is transformed into a crisp one. Experiments studies make transition from theory to practice in this setting.

Another line for further development consists of extending our method described here to the programming problems where the optimized variables are fuzzy numbers.

REFERENCES

[1] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications. Academic Press, 1980.

[2] A. Charness and W. W. Cooper, "Chance-constrained programming," Management Science, vol. 6, no. 1, pp. 34–39, Sep 1959.

[3] B. D. Liu, "Dependent-chance programming: A class of stochastic programming," Computers and Mathematics with Applications, vol. 34, no. 12, pp. 89-104, Dec 1997.

[4] B. D. Liu and K. Iwamura, "Modelling stochastic decision systems using dependent-chance programming," European Journal of Operational Research, vol. 101, no. 1, pp. 193-203, Aug 1997.

[5] F. C. Li, L. Wang and Y. Shi, "Generalized expected value model based on compound quantification and its application in transportation problems," International Journal of Innovative Computing, Information and Control, vol. 7, no. 6, pp. 3303-3315, 2011.

[6] L. A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," Fuzzy Sets and Systems, vol. 100, no. supp1, pp. 9-34, 1999.

[7] H. J. Zimmermann, Description and optimization of fuzzy systems 1976, pp. 209-215.

[8] A. Ebrahimnejad, "Sensitivity analysis in fuzzy number linear programming problems," Mathematical and Computer Modelling, vol. 53, no. 9-10, pp. 1878-1888, May 2011.

[9] W. Li, J. J. Luo and C. Y. Deng, "Necessary and sufficient of some strong optimal solutions to the interval linear programming," Linear Algebra and Its Applications, vol. 439, no. 10, pp. 3241-3255, Nov 2013.

[10] B. Farhadinia, "Sensitivity analysis in interval-valued trapezoidal fuzzy number linear programming problems," Applied Mathematical Modelling, vol. 38, no. 1, pp. 50-62, Jan 2014.

[11] R. E. Bellman and L. A. Zadeh, "Decision making in fuzzy environment," Management Science, vol. 17, pp. 141-164, 1970.

[12] H. J. Zimmermann, "Fuzzy programming and linear programming with several functions," Fuzzy Sets and Systems, vol. 1, no. 1, pp. 45-55, Jan 1978.

[13] B. D. Liu, "Dependent chance programming with fuzzy decisions," IEEE Transanctions on Fuzzy Systems, vol. 7, no. 3, pp. 354-360, 1999.

[14] F. C. Li, C. X. Jin and Y. Shi, "Fuzzy programming theory based on synthesizing effect and its application," International Journal of Innovative Computing, Information and Control, vol. 6, no. 8, pp. 3563-3572, Sep 2010.

[15] R. Jain, "Decision making in the presence of fuzzy variables," in Conf. Rec. 1976, IEEE Trans. on Systems, Man and Cybernetics, pp. 698-703.

[16] S. M. Baas and H. Kwakernassk, "Rating and ranking of multiple-aspect alternatives using fuzzy sets," Automatical, vol. 13, no. 1, pp. 47-58, Jan 1977.

[17] J. F. Baldwin and N. C. F. Guild, "Comparison of fuzzy sets on the same decision space," Fuzzy Sets and Systems, vol. 2, no. 3, pp. 213-231, July 1979.

[18] G. Bortolan and R. Degani, "A review of some methods for ranking fuzzy subsets," Fuzzy Sets and Systems, vol. 15, no. 1, pp. 1-19, Feb 1985.

[19] S. H. Chen, "Ranking fuzzy numbers with maximizing set and minimizing set," Fuzzy Sets and Systems, vol. 17, no. 2, pp. 113-129, Nov 1985.

[20] E. S. Lee and R. J. Li, "Comparison of fuzzy number based on the probability measure of fuzzy events," Computers and Mathematics with Application, vol. 15, no. 10, pp. 887-896, 1988.

[21] K. Kim and K. S. Park, "Ranking fuzzy numbers with index of optimism," Fuzzy Sets and Systems, vol. 35, no. 2, pp. 143-150, Apr 1990.

[22] T. S. Liou and M. J. Wang, "Ranking fuzzy numbers with integral value," Computers and Mathematics with Application, vol. 56, no. 9, pp. 2340-2346, Nov 2008.

[23] C. H. Cheng, "A new approach for ranking fuzzy numbers by distance method," Fuzzy Sets and Systems, vol. 95, no. 3, pp. 307-317, May 1998.

[24] M. Liu, F. C. Li and C. Wu, "The order structure of fuzzy numbers based on the level characteristic and its application in optimization problems," Science in China (Series F), vol. 45, pp. 433-441, 2002.

[25] B. Asady and A. Zendehnam, "Ranking fuzzy numbers by distance minimization," Applied Mathematical Modelling, vol. 31, no. 11, pp. 2589-2593, Nov 2007.

[26] S. Abbasbandy and T. Hajjari, "A new approach for ranking of trapezoidal fuzzy numbers," Computers and Mathematics with Applications, vol. 57, no. 3, pp. 413-419, Feb 2009.

[27] F. C. Li, F. Guan and C. X. Jin, "A quantity property-based fuzzy number ranking method for fuzzy decision," International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, vol. 20, no. supp1.1, pp. 133-145, 2012.

[28] P. Diamond and P. Kloeden, Metric space of fuzzy set: theory and applications, 1994.

[29] R. Goetschel and W. Voxman, "Topological properties of fuzzy number," Fuzzy Sets and Systems, vol. 10, no. 1-3, pp. 87-99, 1983.

[30] H. B. Mitchell and P. A Schaefer, "On ordering fuzzy numbers," International Journal of Inteligent Systems, vol. 15, pp. 981-993, 2000.

[31] M. F. Yang and Y. Lin, "Applying fuzzy multi-objective linear programming to project management decisions with the interactive two-phase method," Computers and Industrial Engineering, vol. 66, no. 4, pp. 1061-1069, Dec 2013.

[32] K. K. Damghani, S. S. Nezhad and M. Tavana, "Solving multi-period project selection problems with fuzzy goal programming based on TOPSIS and a fuzzy preference relation," Information Sciences, vol. 252, pp. 42-61, Dec 2013.

[33] D. F. Li and S. P. Wan, "Fuzzy linear programming approach to multiattribute decision making with multiple types of attribute values and incomplete weight information," Applied Soft Computing, vol. 13, no. 11, pp. 4333-4348, Nov 2013.

[34] C. W. Wu and M. Y. Liao, "Fuzzy nonlinear programming approach for evaluating and ranking process yields with imprecise data," Fuzzy Sets and Systems, in press.

[35] Y. R. Fan, G. H. Huang and A. L. Yang, "Generalized fuzzy linear programming for decision making under uncertainty: Feasibility of fuzzy solutions and solving approach," Information Sciences, vol. 241, pp. 12-27, Aug 2013.

[36] Y. Zheng, J. Liu and Z. P. Wan, "Interactive fuzzy decision making method for solving bilevel programming problem," Applied Mathematical Modelling, in press.

[37] M. K. Luhandjula and M. J. Rangoaga, "An approach for solving a fuzzy multiobjective programming problem," European Journal of Operational Research, vol. 232, no. 2, pp. 249-255, Jan 2014.

[38] S. N. Shirkouhi, H. Shakouri, B. Javadi and A. Keramati, "Supplier selection and order allocation problem using a two-phase fuzzy multi-objective linear programming," Applied Mathematical Modelling, vol. 37, pp. 9308-9323, 2013.

[39] J. H. Hollan, "Genetic algorithms and the optimal allocations of trials," SIAMJ of Computing, vol. 2, pp. 88-105, 1973.