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Interval type-2 fuzzy modeling and chaotic synchronization of two different memristor-based Lorenz circuits

Tsung-Chih Lin[1*] Fu-Yu Huang[1]
[1]Department of Electronic Engineering, Feng-Chia University, Taichung, Taiwan

***Abstract*—**This paper considers the interval type-2 T-S fuzzy modeling and chaotic synchronization of two different memristor-based Lorenz circuits. In this paper, memristor-based Lorenz circuit is constructed by adding a flux-controlled memristor and phase portraits of the state variables are presented. In the meantime, an interval type-2 Takagi-Sugeno (T-S) fuzzy modeling of memristor-based Lorenz circuit proposed and the numerical simulations of the system's solution and the phase portraits are presented. Finally, the synchronization between two different memristor-based Lorenz circuits is achieved by using the proposed fuzzy controller.

***Keywords*—**Interval type-2 fuzzy system, memristor, Lorenz circuit, chaotic synchronization.

I. INTRODUCTION

In 1971, Leon O. Chua [1] predicted the forth fundamental passive circuit element named memristor which means "memory resistor". There are six combinations of four basic circuit variables, i, v, q and φ, of which five relationships have been found. Three of them are given by the passive circuit elements, resistor $R = dv/di$, capacitor $C = dq/dv$ and inductor $L = d\varphi/di$. The other two are the definition of current $i = dq/dt$ and Faraday's law $v = d\varphi/dt$. Chua believes memristor shows the relation between q and φ, which can be described as charge-controlled memristor $M(q) = d\varphi/dq$ or flux-controlled memristor $W(\varphi) = dq/d\varphi$.

For the flux-controlled memristor, a monotonically increasing and piecewise linear characteristic is assumed [2], [3]. The memristor constitutive relation as shown in Fig. 1 can be expressed as

$$q(\varphi) = b\varphi + 0.5(a-b)(|\varphi+1| - |\varphi-1|) \tag{1}$$

where $a,\ b > 0$. Therefore, the memristance can be obtained as

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi} = \begin{cases} a, & |\varphi| < 1 \\ b, & |\varphi| > 1 \end{cases} \tag{2}$$

Chaos synchronization in master-slave systems which is first proposed by Pecora and Carroll [4] has attracted much attention because of its potential applications image encryption, biological systems, information processing and secure communication [5]-[8]. There are many different schemes which have been developed for chaos synchronization such as backstepping method [9], adaptive control [10], [11], fuzzy control [12], [13], impulsive control method [14] and linear and nonlinear feedback control [15], [16]. Recently, active researches have been extensively carried out in the T-S fuzzy logic systems as a valuable tool for analysis and design of fuzzy control system [17]. In order to handle high levels of uncertainty and linguistic uncertainty, interval type-2 fuzzy logic controller is proposed [18]-[21]. In this paper, the interval type-2 T-S fuzzy modeling and chaotic synchronization of two different memristor-based Lorenz circuits are developed.

The remaining part of this paper is organized as follows. The interval type-2 fuzzy modeling of the memristor-based Lorenz circuit is performed in Section II. The control input vector for Synchronization of two different memristor-based Lorenz circuits is derived in Section III. Section IV gives the simulation example to explain the usefulness of the proffered design scheme. The convictive conclusions of the advocated design method are provided in Section V.

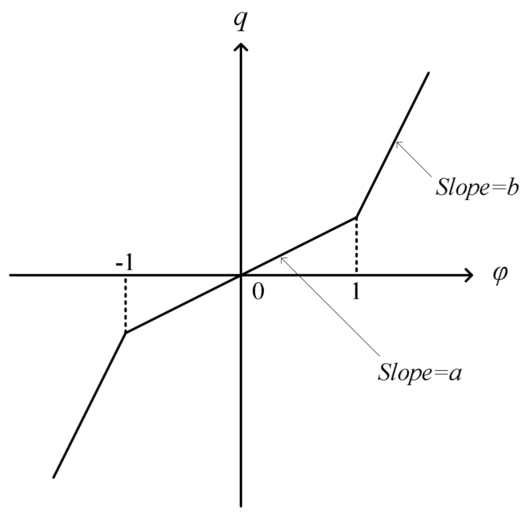

Fig. 1. The constitutive relation of the flux-controlled memristor.

II. INTERVAL TYPE-2 FUZZY MODELING OF THE MEMRISTOR-BASED LORENZ CIRCUIT

Recently, several nonlinear oscillators based on Chua's circuit are proposed by Itoh and Chua, in which the Chua's diode was replaced by monotone increasing piecewise linear memristors [3] and a new kind of memristor-based Lorenz circuit is presented [17] as shown in Fig. 2.

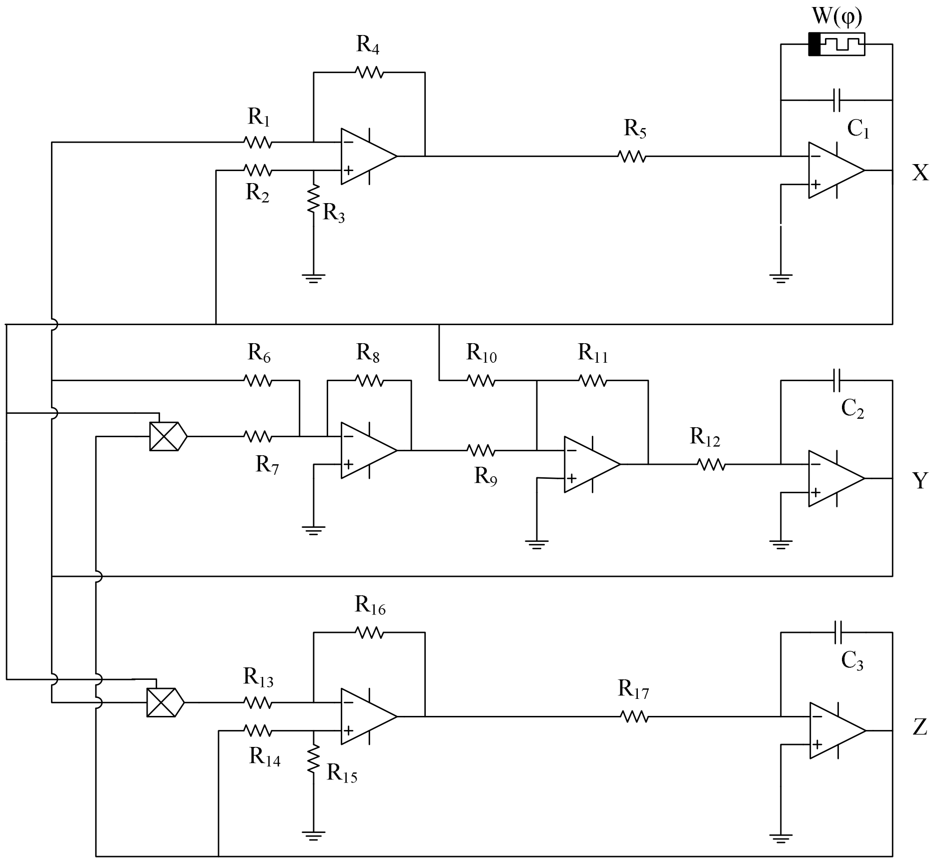

Fig. 2. Memristor-based Lorenz circuit.

Based on basic circuit theories, the circuit equations can be derived as follows.

$$\begin{cases} \dot{X}(t) = -\dfrac{(R_1+R_4)R_3}{R_1(R_2+R_3)R_5C_1}X(t) + \dfrac{R_4}{R_1R_5C_1}Y(t) \\ \qquad -\dfrac{1}{C_1}W(\varphi(t))X(t) \\ \dot{Y}(t) = \dfrac{R_{11}}{R_{10}R_{12}C_2}X(t) - \dfrac{R_8R_{11}}{R_6R_9R_{12}C_2}Y(t) \\ \qquad -\dfrac{R_8R_{11}}{R_7R_9R_{12}C_2}X(t)Z(t) \\ \dot{Z}(t) = \dfrac{R_{16}}{R_{13}R_{17}C_3}X(t)Y(t) \\ \qquad -\dfrac{(R_{13}+R_{16})R_{15}}{R_{13}(R_{14}+R_{15})R_{17}C_3}Z(t) \\ \dot{\varphi}(t) = -X(t) \end{cases} \tag{3}$$

Let $\alpha_1 = \dfrac{(R_1+R_4)R_3}{R_1(R_2+R_3)R_5C_1}$, $\alpha_2 = \dfrac{R_4}{R_1R_5C_1}$, $\alpha_3 = \dfrac{R_{11}}{R_{10}R_{12}C_2}$, $\alpha_4 = \dfrac{(R_{13}+R_{16})R_{15}}{R_{13}(R_{14}+R_{15})R_{17}C_3}$ and $\dfrac{R_{16}}{R_{13}R_{17}C_3} = \dfrac{R_8R_{11}}{R_6R_9R_{12}C_2} = \dfrac{R_8R_{11}}{R_7R_9R_{12}C_2} = \dfrac{1}{C_1} = 1$, for simplicity, the system state vector is denoted as $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)] = [X(t), Y(t), Z(t), \varphi(t)]$. Therefore, the state representation of (3) can be obtained as

$$\begin{cases} \dot{x}_1(t) = -\alpha_1 x_1(t) + \alpha_2 x_2(t) - W(x_4(t))x_1(t) \\ \dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - \alpha_4 x_3(t) \\ \dot{x}_4(t) = -x_1(t) \end{cases} \tag{4}$$

where $W(x_4(t)) = \begin{cases} a, & |x_4(t)| \le 1 \\ b, & |x_4(t)| > 1 \end{cases}$ is the piecewise linear function of $W(x_4(t))$. If all variable values are chosen as $\alpha_1 = 8$, $\alpha_2 = 15$, $\alpha_3 = 28$, $\alpha_4 = 8/3$, $a = 5$, $b = 8$ and initial value vector is $x(0) = [10^{-4}, 0, 0, 0]$, the system (3) generates chaotic behaviors and 2-scroll attractor for $x_2(t)$ and $x_3(t)$ is given in Fig. 3.

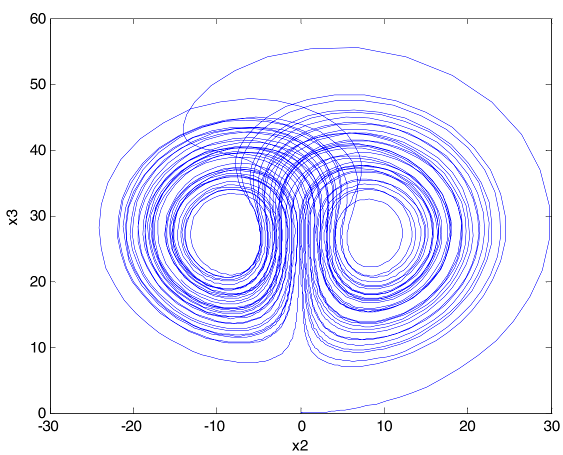

Fig. 3. The chaotic attractor of the system (2).

Moreover, the phase portrait of the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ is described in Fig. 4.

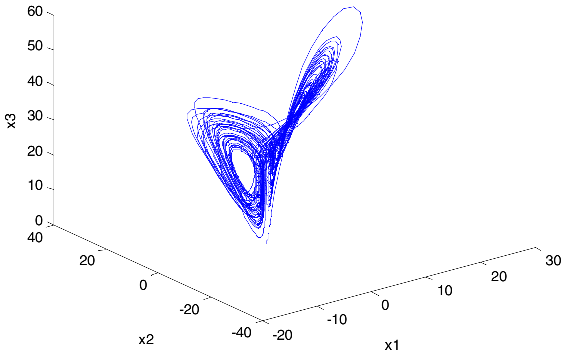

Fig. 4. The phase portrait of the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$.

Suppose that state of $x_1(t)$ is described as $x_1(t) \in [-\xi,\ \xi]$, $\xi > 0$, and the interval type-2 fuzzy membership function of $x_1(t)$ is selected as shown in Fig. 5.

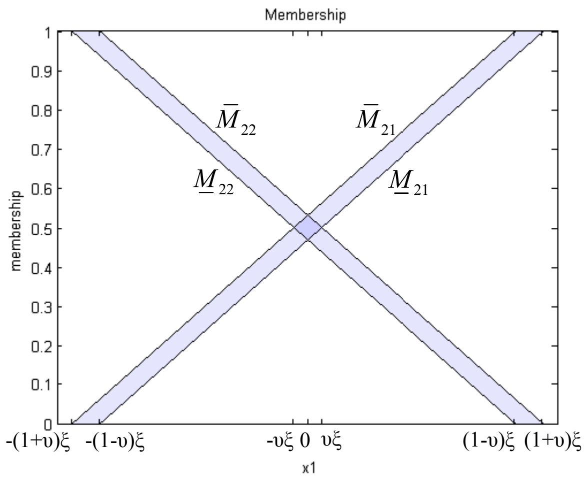

Fig. 5. The membership function of $x_1(t)$.

where

$$\bar{M}_{21} = \frac{1}{2}\left(1 + \frac{x_1(t) + \xi\upsilon}{\xi}\right), \quad \underline{M}_{21} = \frac{1}{2}\left(1 + \frac{x_1(t) - \xi\upsilon}{\xi}\right),$$

$$\bar{M}_{22} = \frac{1}{2}\left(1 - \frac{x_1(t) - \xi\upsilon}{\xi}\right) \text{ and } \underline{M}_{22} = \frac{1}{2}\left(1 - \frac{x_1(t) + \xi\upsilon}{\xi}\right).$$

The interval type-2 T-S fuzzy model of the system (4) is expressed in the following IF-THEN form:

For $\dot{x}_1(t) = -\alpha_1 x_1(t) - W(x_4(t))x_1(t) + \alpha_2 x_2(t)$:

Rule 1: If $|x_4(t)| \le 1$, then

$$\dot{x}_1(t) = -\alpha_1 x_1(t) - a x_1(t) + \alpha_2 x_2(t) \tag{5}$$

Rule 2: If $|x_4(t)| > 1$, then

$$\dot{x}_1(t) = -\alpha_1 x_1(t) - b x_1(t) + \alpha_2 x_2(t) \tag{6}$$

For $\dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) - x_1(t)x_3(t)$, and $\dot{x}_3(t) = x_1(t)x_2(t) - \alpha_4 x_3(t)$:

Rule 1: If $x_1(t)$ is $[\underline{M}_{21}, \bar{M}_{21}]$, then

$$\dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) - \xi x_3(t) \tag{7}$$

$$\dot{x}_3(t) = \xi x_2(t) - \alpha_4 x_3(t) \tag{8}$$

Rule 2: If $x_1(t)$ is $[\underline{M}_{22}, \bar{M}_{22}]$, then

$$\dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) + \xi x_3(t) \tag{9}$$

$$\dot{x}_3(t) = -\xi x_2(t) - \alpha_4 x_3(t) \tag{10}$$

Therefore, the first linear system construed by fuzzy rules (5), (7) and (8) can be obtained as follows:

$$\begin{cases} \dot{x}_1(t) = (-\alpha_1 - a)x_1(t) + \alpha_2 x_2(t) \\ \dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) - \xi x_3(t) \\ \dot{x}_3(t) = \xi x_2(t) - \alpha_4 x_3(t) \\ \dot{x}_4(t) = -x_1(t) \end{cases} \tag{11}$$

Similarly, the second linear system construed by fuzzy rules (6), (9) and (10) can be obtained as follows:

$$\begin{cases} \dot{x}_1(t) = (-\alpha_1 - b)x_1(t) + \alpha_2 x_2(t) \\ \dot{x}_2(t) = \alpha_3 x_1(t) - x_2(t) + \xi x_3(t) \\ \dot{x}_3(t) = -\xi x_2(t) - \alpha_4 x_3(t) \\ \dot{x}_4(t) = -x_1(t) \end{cases} \tag{12}$$

Consequently, the final left most and right most outputs of the interval type-2 T-S fuzzy memristor-based Lorenz system can be characterized as follows:

$$\begin{aligned} \dot{x}_L(t) &= \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & \bar{M}_{21} & 0 & 0 \\ 0 & 0 & \bar{M}_{21} & 0 \\ 0 & 0 & 0 & M_{11} \end{bmatrix} \begin{bmatrix} -\alpha_1 x_1(t) - a x_1(t) + \alpha_2 x_2(t) \\ \alpha_3 x_1(t) - x_2(t) - \xi x_3(t) \\ \xi x_2(t) - \alpha_4 x_3(t) \\ -x_1(t) \end{bmatrix} \\ &+ \begin{bmatrix} M_{12} & 0 & 0 & 0 \\ 0 & \underline{M}_{22} & 0 & 0 \\ 0 & 0 & \underline{M}_{22} & 0 \\ 0 & 0 & 0 & M_{12} \end{bmatrix} \begin{bmatrix} -\alpha_1 x_1(t) - b x_1(t) + \alpha_2 x_2(t) \\ \alpha_3 x_1(t) - x_2(t) + \xi x_3(t) \\ -\xi x_2(t) - \alpha_4 x_3(t) \\ -x_1(t) \end{bmatrix} \\ &= \Theta_{L1} A_1 x(t) + \Theta_{L2} A_2 x(t) \end{aligned} \tag{13}$$

and

$$\begin{aligned} \dot{x}_R(t) &= \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & \underline{M}_{21} & 0 & 0 \\ 0 & 0 & \underline{M}_{21} & 0 \\ 0 & 0 & 0 & M_{11} \end{bmatrix} \begin{bmatrix} -\alpha_1 x_1(t) - a x_1(t) + \alpha_2 x_2(t) \\ \alpha_3 x_1(t) - x_2(t) - \xi x_3(t) \\ \xi x_2(t) - \alpha_4 x_3(t) \\ -x_1(t) \end{bmatrix} \\ &+ \begin{bmatrix} M_{12} & 0 & 0 & 0 \\ 0 & \bar{M}_{22} & 0 & 0 \\ 0 & 0 & \bar{M}_{22} & 0 \\ 0 & 0 & 0 & M_{12} \end{bmatrix} \begin{bmatrix} -\alpha_1 x_1(t) - b x_1(t) + \alpha_2 x_2(t) \\ \alpha_3 x_1(t) - x_2(t) + \xi x_3(t) \\ -\xi x_2(t) - \alpha_4 x_3(t) \\ -x_1(t) \end{bmatrix} \\ &= \Theta_{R1} A_1 x(t) + \Theta_{R2} A_2 x(t) \end{aligned} \tag{14}$$

where $A_1 = \begin{bmatrix} -\alpha_1 - a & \alpha_2 & 0 & 0 \\ \alpha_3 & -1 & -\xi & 0 \\ 0 & \xi & -\alpha_4 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$, and

$$A_2 = \begin{bmatrix} -\alpha_1 - b & \alpha_2 & 0 & 0 \\ \alpha_3 & -1 & \xi & 0 \\ 0 & -\xi & -\alpha_4 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

The final output of the interval type-2 T-S fuzzy memristor-based Lorenz system can be obtained as follows:

$$\begin{aligned} \dot{x}(t) &= \frac{1}{2}\left(\dot{x}_L(t) + \dot{x}_R(t)\right) \\ &= \frac{1}{2}\left(\Theta_{L1} A_1 x(t) + \Theta_{L2} A_2 x(t) + \Theta_{R1} A_1 x(t) + \Theta_{R2} A_2 x(t)\right) \\ &= \frac{1}{2}\sum_{i=1}^{2}\left(\Theta_{Li} + \Theta_{Ri}\right) A_i x(t) \end{aligned} \tag{15}$$

where

$$\Theta_{L1} = diag\left[M_{11} \quad \bar{M}_{21} \quad \bar{M}_{21} \quad M_{11}\right]$$
$$\Theta_{L2} = diag\left[M_{12} \quad \underline{M}_{22} \quad \underline{M}_{22} \quad M_{12}\right]$$
$$\Theta_{R1} = diag\left[M_{11} \quad \underline{M}_{21} \quad \underline{M}_{21} \quad M_{11}\right]$$
$$\Theta_{R2} = diag\left[M_{12} \quad \bar{M}_{22} \quad \bar{M}_{22} \quad M_{12}\right]$$

We can see that based on this interval type-2 T-S fuzzy model, the memristor-based Lorenz system can be represented by only four linear subsystems to exhibit complex chaotic behaviors. Hence, the interval type-2 T-S fuzzy model of the memristor-based Lorenz system with center-average defuzzier is expressed as

$$\dot{x}(t) = \sum_{i=1}^{2} \beta_i A_i x(t) \tag{16}$$

where $\beta_i = \left(\Theta_{Li} + \Theta_{Ri}\right)\left(\Theta_{L1} + \Theta_{L2} + \Theta_{R1} + \Theta_{R2}\right)^{-1}$.

Let's reconsider the memristor-based Lorenz circuit (4) and all system parameters are selected as $\alpha_1 = 15$, $\alpha_2 = 16$, $\alpha_3 = 46$, $\alpha_4 = 4$, $a = 3$, $b = 8$ and initial conditions are $x_1(0) = 0.01$, $x_2(0) = 0.01$, $x_3(0) = 0.01$, $x_4(0) = 0.01$. By using Matlab calculation, we have $-27.2267 \le x_1(t) \le 30.5893$ and we assign $\xi = 32$, $\upsilon = 1/16$. The comparisons of the state outputs between interval type-2 fuzzy model (16) and circuit (4) are shown in Fig. 6. The chaotic attractor x_2 vs. x_3 and the phase portrait of the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$ are given in Fig. 7 and Fig. 8, respectively.

From the dynamic behaviors, we can see that the memristor-based Loren circuit can be exactly approximated by the interval type-2 fuzzy model.

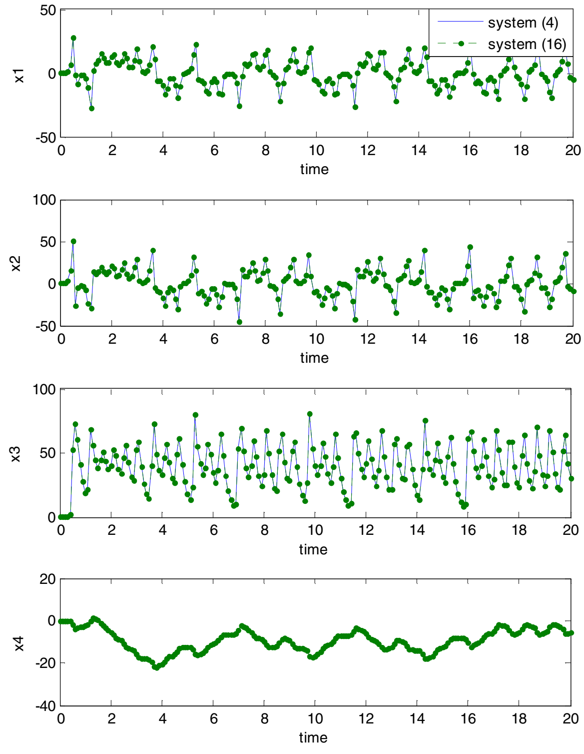

Fig.6. State trajectories $x_1(t) \sim x_4(t)$ of interval type-2 fuzzy model (16) and circuit (4).

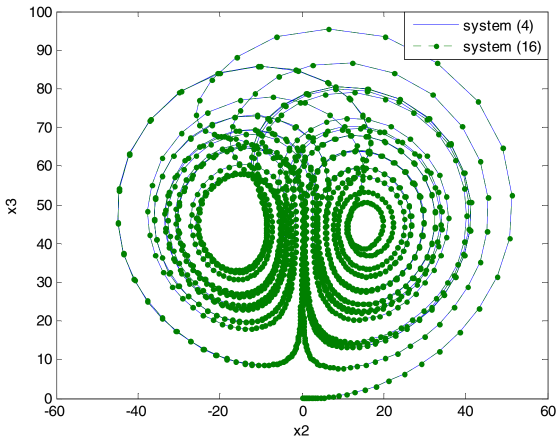

Fig. 7. The chaotic attractor x_2 vs. x_3.

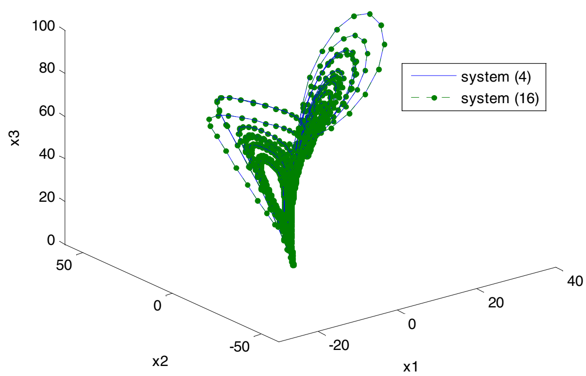

Fig. 8. the phase portrait of the state variables $x_1(t)$, $x_2(t)$ and $x_3(t)$.

III. SYNCHRONIZATION OF TWO DIFFERENT MEMRISTOR-BASED LORENZ CIRCUITS

Consider the following two memristor-based Lorenz circuits, the interval type-2 T-S fuzzy model of which are described by (17) and (18).

The master system in (14) is rewritten as:

$$\dot{x}(t)=\sum_{i=1}^{2}\beta_{xi}A_{xi}x(t) \tag{17}$$

The slave system with state feedback control vector is stated as:

$$\dot{y}(t)=\sum_{i=1}^{2}\beta_{yi}A_{yi}y(t)+Bu(t) \tag{18}$$

where B is a diagonal matrix and $u(t)=[u_1(t),u_2(t),u_3(t),u_4(t)]^T$ is the control input vector. The main objective is to design the fuzzy controllers $u_i(t),\ i=1\sim 4$ such that the synchronization refer to that the state of the master system asymptotically synchronizes with the slave system at time t, namely,

$$\lim_{t\to\infty}\|x(t)-y(t)\|=\lim_{t\to\infty}\|e(t)\|=0 \tag{19}$$

where $x(t)$ and $y(t)$ are the state vectors of the master system and slave system, respectively, and $e(t)=x(t)-y(t)$ is the synchronization error vector.

According to the systems (17) and (18), we have the synchronization error dynamical system

$$\begin{aligned}\dot{e}(t)&=\dot{x}(t)-\dot{y}(t)\\&=\sum_{i=1}^{2}\beta_{xi}A_{xi}x(t)-\sum_{i=1}^{2}\beta_{yi}A_{yi}y(t)-Bu(t)\end{aligned} \tag{20}$$

Obviously, chaotic synchronization of systems (17) and (18) will be achieved if the error dynamically system (20) has an asymptotically stable equilibrium point $e(t)=0$. Following the preceding consideration, the sufficient condition for the asymptotic stability of the synchronization error system (20) can be acquired by the following theorem.

Theorem: If the fuzzy controller is given in (21) and the following conditions (22) can be satisfied, the synchronization error dynamical system (20) is asymptotically stable, i.e., the chaotic synchronization between slave system (18) and master system (17) can be realized.

$$u(t)=\sum_{i=1}^{2}\beta_{xi}\Psi_{xi}x(t)-\sum_{i=1}^{2}\beta_{yi}\Psi_{yi}y(t) \tag{21}$$

$$A_{xi}-B\Psi_{xi}=A_{yi}-B\Psi_{yi}=F<0 \tag{22}$$

where Ψ_{xi} and Ψ_{yi} are feedback gain.

Proof:

Substituting the fuzzy controller (21) into (20), we can get

$$\begin{aligned}\dot{e}(t)=&\sum_{i=1}^{2}\beta_{xi}\left(A_{xi}-B\Psi_{xi}\right)x(t)\\&-\sum_{i=1}^{2}\beta_{yi}\left(A_{yi}-B\Psi_{yi}\right)y(t)\end{aligned} \tag{23}$$

If there exist feedback gains Ψ_{xi} and Ψ_{yi} such that (22) is satisfied, the overall system can be linearized by the fuzzy controller (21) as

$$\dot{e}(t)=Fe(t) \tag{24}$$

As a result, the synchronization error dynamical system (24) is asymptotically stable and the feedback gains Ψ_{xi} and Ψ_{yi} derived from (22) as

$$\begin{cases}\Psi_{xi}=B^{-1}\left(A_{xi}-F\right)\\ \Psi_{yi}=B^{-1}\left(A_{yi}-F\right)\end{cases} \tag{25}$$

This completes the proof.

IV. SIMULATION EXAMPLE

In this section, we will apply the proposed control scheme to synchronize two different memristor-based Lorenz circuits.

Example: The parameters and initial states of master and slave systems are selected as:

$$\sigma_{x1}=15,\sigma_{x2}=16,\sigma_{x3}=46,\sigma_{x4}=4,a_x=3,b_x=8,$$
$$x_1(0)=0.01,x_2(0)=0.01,x_3(0)=0.01,x_4(0)=0.01$$
$$-27.2267\le x_1(t)\le 30.5893 \quad \therefore \xi_x=32,\upsilon_x=1/16$$

and

$$\sigma_{y1}=10,\sigma_{y2}=15,\sigma_{y3}=32,\sigma_{y4}=4,a_y=0.3,b_y=0.8,$$
$$y_1(0)=0.1,y_2(0)=0.1,y_3(0)=0.1,y_4(0)=0.1 \quad ,$$
$$-30.4166\le y_1(t)\le 32.5842 \quad \therefore \xi_y=34,\upsilon_y=1/17$$

respectively.

The phase portraits and chaotic behaviors of the state variables before fuzzy controller applied are given in Fig. 9 and the dynamical behaviors of $x_1(t)\sim x_4(t)$ and $y_1(t)\sim y_4(t)$ are shown in Fig. 10.

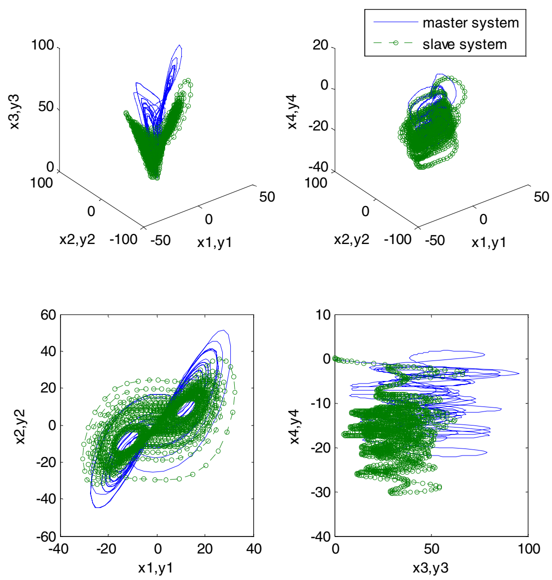

Fig. 9. The phase portraits and chaotic behaviors of the state variables before fuzzy controller applied.

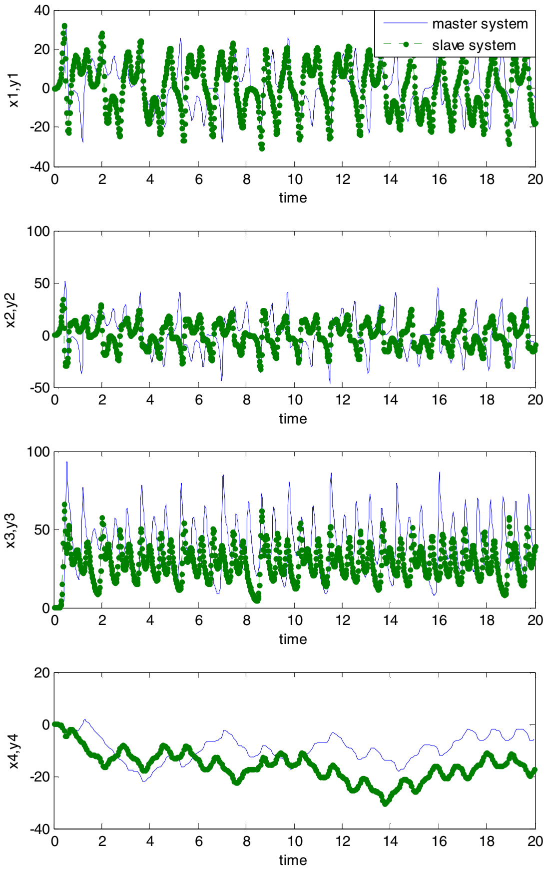

Fig. 10. The dynamical behaviors of $x_1(t) \sim x_4(t)$ and $y_1(t) \sim y_4(t)$ before fuzzy controller applied.

From Fig. 9 and Fig. 10, it is obvious that master and slave systems are not synchronized at all before fuzzy controller applied. If $F = -I$ and $B = I$ are chosen, the feedback gains can be determined as

$$\Psi_{x1} = \begin{bmatrix} -17 & 16 & 0 & 0 \\ 46 & 0 & -32 & 0 \\ 0 & 32 & -3 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix},$$

$$\Psi_{x2} = \begin{bmatrix} -22 & 16 & 0 & 0 \\ 46 & 0 & 32 & 0 \\ 0 & -32 & -3 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix},$$

$$\Psi_{y1} = \begin{bmatrix} -9.3 & 15 & 0 & 0 \\ 32 & 0 & -34 & 0 \\ 0 & 34 & -3 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix},$$

$$\Psi_{y2} = \begin{bmatrix} -9.8 & 15 & 0 & 0 \\ 32 & 0 & 34 & 0 \\ 0 & -34 & -3 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

The fuzzy controller (21) is applied to slave system as described in (18). The phase portraits and chaotic behaviors of the state variables after fuzzy controller applied are given in Fig. 11 and the dynamical behaviors of $x_1(t) \sim x_4(t)$ and $y_1(t) \sim y_4(t)$ are shown in Fig. 12. The synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ which fully demonstrate the effectiveness of the advocated method are given in Fig. 13. The control inputs $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ are shown in Fig. 14. Fig.11~Fig. 13 show that master and slave systems can be synchronized very fast when fuzzy controller is applied to slave system.

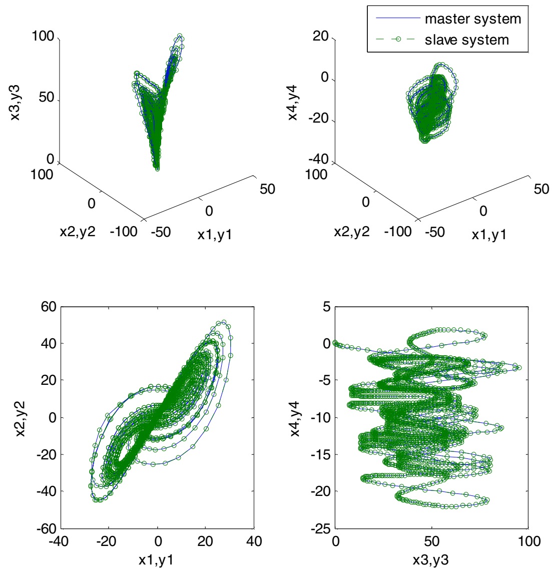

Fig. 11. The phase portraits and chaotic behaviors of the state variables after fuzzy controller applied.

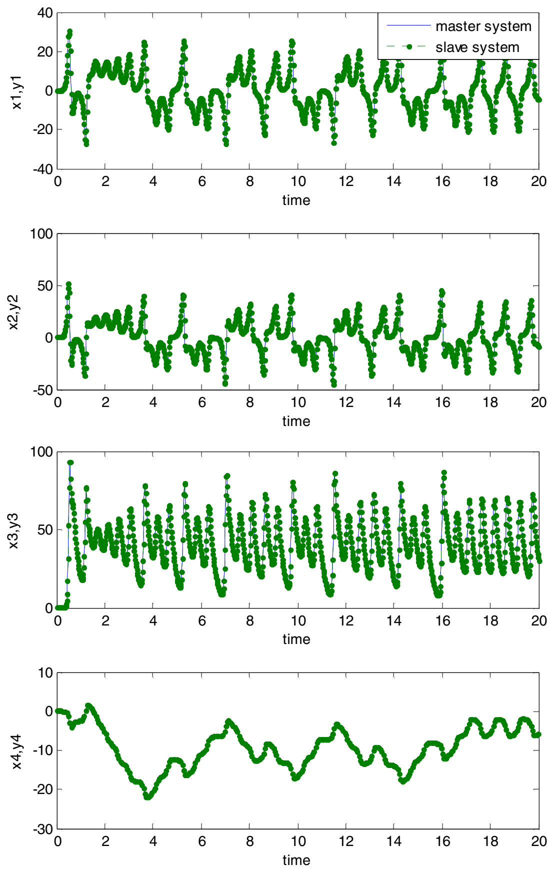

Fig. 12. The dynamical behaviors of $x_1(t) \sim x_4(t)$ and $y_1(t) \sim y_4(t)$ after fuzzy controller applied.

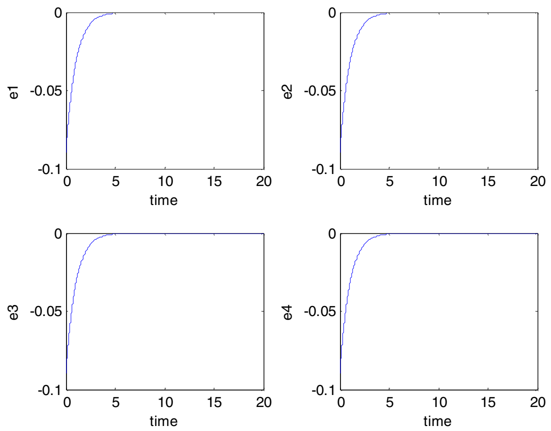

Fig. 13. Synchronization errors $e_1(t) \sim e_4(t)$.

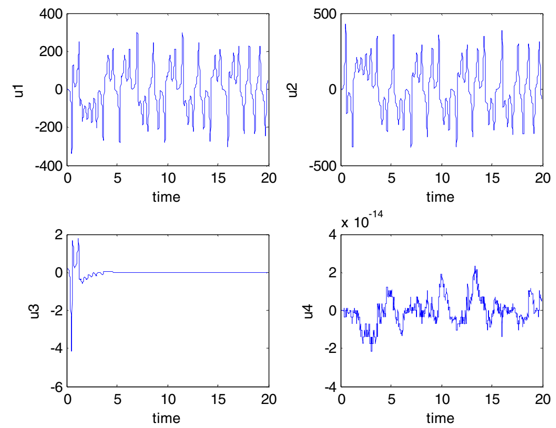

Fig. 14. The control inputs $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$.

V. CONCLUSIONS

In this paper, the interval type-2 T-S fuzzy modeling of memristor-based Lorenz circuit is proposed and the synchronization issue between two different memristor-based Lorenz circuits is developed. Numerical simulations are provided to show that the asymptotical stability of the zero equilibrium point of the synchronization error can be guaranteed and to illustrate the effectiveness of the scheme proposed in this work.

VI. REFERENCES

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