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A Consensus and Maximizing Deviation based Approach for Multi-criteria Group Decision Making under Linguistic Setting

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***Abstract*— In practical group decision making (GDM) problems adhere to uncertain and imprecise data, the decision makers may express their preferences using linguistic terms. The aim of this paper is to present a method to assist the consensus process and selection process of multi-criteria GDM (MCGDM) problem under linguistic setting. If the consensus level does not meet predefined requirements, an algorithm is provided to help the decision maker or moderator reach the consensus goal. Once the consensus reaching process is finished, the maximizing deviation method is used to derive the importance weights of the attributes. Then, the linguistic weighted arithmetic averaging (LWAA) operator of 2-tuple linguistic variables is used to obtain the overall assessment value of each alternative and the ranking order of all alternatives can be determined. Finally, one example of personal selection problem is given to show the use of the proposed method.**

I. Introduction

MULTI-CRITERIA decision making (MCDM) problems arise in many practical situations and have drawn much attention in the management and engineering field. Multi-criteria group decision making (MCGDM) problems address decision situations where a group of decision makers express preferences on multiple attributes and attempt to find a common solution and have been widely discussed in recent years [1][2]. There are cases in which the information cannot be expressed precisely in a quantitative form but may be stated only in linguistic terms. For example, when attempting to qualify phenomena related to human perception, we are likely to use words in natural language instead of numerical values. (e.g. when evaluating the "comfort" or "design" of a car, terms like "bad," "poor," "tolerable," "average," or "good" can be used [3]). A more realistic measurement is to use linguistic assessments instead of numerical values [4]–[6]. Linguistic variables are very useful in situations where the decision making problems are too complex or ill-defined to be described properly using conventional quantitative expressions.

A number of studies have emphasized the importance of MCDM with fuzzy or linguistic data [7]. Some methods of MCDM under linguistic environment have been proposed [8]–[11]. Xu [8] presented uncertain linguistic ordered weighted averaging (ULOWA) operator and uncertain linguistic hybrid aggregation (ULHA) operator to solve MCDM with uncertain linguistic information. Wu and Chen [9] developed a method named the maximizing deviation method to determine the optimal relative weights of attributes in multi-attribute GDM with linguistic variables. Xu and Da [10] also proposed an optimization model based on the deviation degree and ideal point of uncertain linguistic variables to derive the attribute weights. Fan [11] provided a method to solve the GDM problem with multi-granularity uncertain linguistic information. In the method, multi-granularity uncertain linguistic information is transformed into trapezoidal fuzzy numbers and an extension of TOPSIS is conducted to rank the alternatives.

To solve GDM problems, two processes are applied before obtaining a final solution: a consensus process and a selection process [12]. From the literature on linguistic decision analysis, there are two general decision models [13]: the first model is based mainly on an aggregation-and-ranking scheme, and the second is based on a consensus reaching oriented solution scheme. Some consensus models with linguistic information have been developed for GDM (see for example, [14]–[16]). In previous researches, few papers have discussed MCGDM consensus reaching processes [18]–[22]. Fu and Yang [17] suggested a MCGDM group consensus model based on an evidential reasoning approach. Parreiras et al. [18] proposed a flexible MCGDM consensus scheme under linguistic assessments. To maximize the soft consensus index, an optimization procedure that searched for the weight of each decision maker's opinion was conducted. Xu [20] investigated the problem of MCGDM consensus in numerical settings, and developed a straightforward algorithm to reach a group consensus. Xu and Wu [21] presented a discrete consensus support model to deal with MCGDM in numerical settings. The above studies have made significant contributions to the MCGDM consensus models. However, some previous methods can only be used in a crisp case. Some methods although considered linguistic setting, they do not fucus on the consensus process. Therefore, group consensus as a basic problem in GDM needs to be considered for the MCGDM under a linguistic setting [22]. As pointed by [24], looking for a simple yet reasonable MCGDM process is still in progress. It is necessary to develop different consensus measures and different consensus models for a specified problem.

The rest of the paper is organized as follows. Section II introduces the concept of linguistic variable and describes the MCGDM problem. In Section III, a method is presented to

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solve the linguistic MCGDM problem. Section IV gives an example to illustrate the proposed method. Finally, Section V concludes the paper.

II. Preliminaries and Problem Description

For the convenience of analysis, this section introduces the concepts of linguistic variables. These basic definitions and notations will be used throughout the paper, unless stated otherwise.

When using linguistic approaches to solve decision problems, techniques are needed to compute with word (CWW). There are main three linguistic computational models: the semantic model, the symbolic model and the 2-tuple linguistic model [23][24]. Suppose that $S=\{s_\alpha|\alpha=0,\cdots,g\}=\{s_0,s_1,s_2,\cdots,s_g\}$ is the linguistic term set accompanied with a pre-ordered structure such that $s_{i_1}<s_{i_2}$ iff $i_1<i_2$. Here, S is a finite and totally ordered discrete term set whose cardinality value is an odd one, such as 7 and 9, where s_α represents a possible value for a linguistic variable. The semantics of linguistic variables is usually represented by fuzzy numbers. For example, the following semantics can be assigned to a set of seven terms via triangular fuzzy numbers [14] (see Fig. 1):

$$\begin{aligned} S=\ & \{N=None\text{=(0,0,0.17)}\\ & VL=Very\ Low\text{=(0,0.17,0.33)}\\ & L=Low\text{=(0.17,0.33,0.5)}\\ & M=Medium\text{=(0.33,0.5,0.67)}\\ & H=High\text{=(0.5,0.67,0.83)}\\ & VH=Very\ High\text{=(0.67,0.83,1)}\\ & P=Perfect=\text{(0.83,1,1)}\}\end{aligned}$$

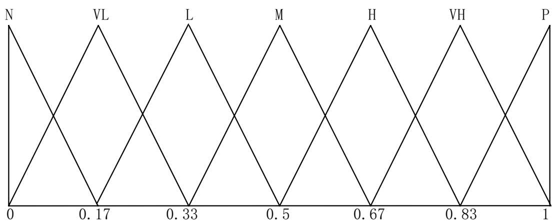

Fig. 1. Set of seven linguistic terms with their semantics

In the symbolic computation process, the discrete linguistic set S is extended to a continuous interval $\overline{S}=\{s_\alpha|\alpha\in[0,g]\}$. $\overline{S}$ is called an extended linguistic term set associated with S [25]. When considering any two linguistic terms $s_\alpha,s_\beta\in\overline{S}$, and $\mu,\mu_1,\mu_2\in[0,1]$, their operational laws are given by

$$s_\alpha\oplus s_\beta=s_{\alpha+\beta},\quad s_\alpha\oplus s_\beta=s_\beta\oplus s_\alpha,\quad \mu s_\alpha=s_{\mu\alpha},$$
$$(\mu_1+\mu_2)s_\beta=\mu_1 s_\beta\oplus\mu_2 s_\beta,\quad \mu(s_\alpha\oplus s_\beta)=\mu s_\alpha\oplus\mu s_\beta.$$

Let $s\in\overline{S}$, and $I(s)$ be denoted as the position index of s and called the gradation of s in $\overline{S}$. For example, $I(s_\alpha)=\alpha$. Let (s_i,x_i) be a 2-tuple linguistic label, where $s_i\in S$, then the corresponding extended linguistic label is $s_{i+x_i}\in\overline{S}$. It has been shown that the continuous label based computation model and the 2-tuple label based computation model are equivalent in their calculated values [15][16]. In the following, for notation simplicity, only the symbolic calculated values in the computation process are used. However, the symbolic aggregation values are explained by the 2-tuple representation model.

The MCGDM problem with linguistic information refers to the problem of the selection or ranking of the alternatives that are associated with incommensurate and conflicting attributes, in which the attribute values given by decision makers are linguistic variables. The following notations are used.

Let $M=\{1,2,\cdots,m\}$, $N=\{1,2,\cdots,n\}$, $P=\{1,2,\cdots,p\}$. Suppose there are $n(n\geq 2)$ potential alternatives denoted by $X=\{X_1,X_2,\cdots,X_n\}$. Each alternative is evaluated with respect to a predefined attribute set $C=\{C_1,C_2,\cdots,C_m\}$. There are a group of decision makers $E=\{E_1,E_2,\cdots,E_p\}(p\geq 2)$. Assume $\lambda=(\lambda_1,\lambda_2,\cdots,\lambda_p)$ is the weight vector of the decision makers, where $\lambda_k\in(0,1)$, $k\in P$, $\sum_{k=1}^p\lambda_k=1$. Suppose that $R_k=(r_{ij}^{(k)})_{n\times m}$ is a linguistic decision matrix given by the decision maker $E_k\in E$, where $r_{ij}^{(k)}$ represents the performance of the alternative X_i over the attribute $C_j\in C$. The problem in this paper is concerned with the ranking of the alternatives or the selection of the most desirable alternative(s) using the linguistic decision matrices R_k, $k=1,2,\cdots,p$.

III. Decision Support Model

In this section, the consensus measure of linguistic variable and linguistic decision matrix are brief introduced. Based on these measures, a consensus reaching process is formally developed. The maximizing deviation method is introduced to obtain the importance weights of the criteria. Then a decision support model based on the consensus process and the maximizing deviation method is presented.

A. Consensus Measure

The distance between s_α and s_β can be defined as follows:

Definition 1: Let s_α and s_β be two linguistic variables. The deviation measure between s_α and s_β is defined as

$$d(s_\alpha,s_\beta)=\frac{|I(s_\alpha)-I(s_\beta)|}{g}=\frac{|\alpha-\beta|}{g},\tag{1}$$

where g is the number of linguistic terms in the set $S/\{s_0\}$.

Similarly, let (r_α,x_α) and (r_β,x_β) be the 2-tuples corresponding to s_α and s_β. Then the distance function of 2-tuples is given by

$$d((r_\alpha,x_\alpha),(r_\beta,x_\beta))=\frac{\left|\triangle^{-1}(r_\alpha,x_\alpha)-\triangle^{-1}(r_\beta,x_\beta)\right|}{g}.\tag{2}$$

Note that for the definitions $\triangle$ and $\triangle^{-1}$, please refer to [23]. It is easy to verify that $0\leq d(s_\alpha,s_\beta)\leq 1$.

Definition 2: Let $\{s_1,s_2,\cdots,s_n\}$ be a set of variables to be aggregated, where $s_i\in\bar{S}$ and $w=\{w_1,w_2,\cdots,w_n\}$ be their associated weights where $w_i\geq 0,\sum_{i=1}^n w_i=1$. The corresponding linguistic 2-tuple of s_i is denoted as

$s_i = (r_i, x_i)$. The linguistic weighted arithmetic averaging (LWAA) operator based on 2-tuples is

$$LWAA_{2-tuple}(s_1, s_2, \cdots, s_n) = \triangle(\sum_{i=1}^{n} \triangle^{-1}(s_i) \cdot w_i). \tag{3}$$

Using the extended linguistic representation model, the definition is

$$LWAA_e(s_1, s_2, \cdots, s_n) = s_\alpha, \text{ where } \alpha = \sum_{i=1}^{n} w_i I(s_i). \tag{4}$$

Using the extended linguistic terms and linguistic 2-tuples operation laws respectively, it follows that

$$\triangle^{-1}(LWAA_{2-tuple}) = I(LWAA_e) = \sum_{i=1}^{n} w_i I(s_i). \tag{5}$$

For notation simplicity, in the sequel, both the $LWAA_{2-tuple}$ operator and the $LWAA_e$ operator are denoted by $LWAA$.

Based on the deviation measure between two linguistic variables, we introduce the similarity measure between two linguistic decision matrices.

Definition 3: Let $A = (a_{ij})_{n\times m}$ and $B = (b_{ij})_{n\times m}$ be two linguistic decision matrices, then the similarity degree between A and B is defined as

$$S(A, B) = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} d(a_{ij}, b_{ij}). \tag{6}$$

Let $R_1, R_2, \cdots, R_p$ be p linguistic decision matrices provided by the p decision makers, where $R_k = (r_{ij}^{(k)})_{n\times n}$, $r_{ij}^{(k)} \in \bar{S}$. Then the weighted combination $R = \lambda_1 R_1 \oplus \lambda_2 R_2 \oplus \cdots \oplus \lambda_p R_p$ is the group linguistic decision matrix $R = (r_{ij})_{n\times n}$, where

$$\begin{aligned} r_{ij} &= LWAA(r_{ij}^{(1)}, r_{ij}^{(2)}, \cdots, r_{ij}^{(p)}) \\ &= \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \cdots \oplus \lambda_p r_{ij}^{(p)}. \end{aligned} \tag{7}$$

Definition 4: Let $R_k = (r_{ij}^{(k)})_{n\times m}$, $k = 1, 2, \cdots, p$ and $R_c = (r_{ij}^{c})_{n\times m}$ be p linguistic decision matrices and the group linguistic decision matrix, respectively. Then, based on the similarity measure between the two linguistic decision matrices, the group consensus index for R_k is defined by

$$GCI(R_k) = 1 - S(R_k, R_c) = 1 - \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} d(r_{ij}^{(k)}, r_{ij}^{c}). \tag{8}$$

If $GCI(R_k) \geq \overline{GCI}$, R_k is called a decision matrix with acceptable consensus, where $\overline{GCI}$ is the consensus level threshold. $\overline{GCI}$ can be determined in advance by the decision makers. If $GCI(R_k) = 1$, then the kth decision maker is in full consensus with the group preference. Otherwise, the larger the value of $GCI(R_k)$, the closer that decision maker is to the group. Depending on the actual situation, the decision makers establish the threshold $\overline{GCI}$ for the deviation degree between the individual linguistic decision matrix and the group linguistic decision matrix. In this paper, when each individual preference is close enough to the group preference at a given level, it is considered a consensus has been achieved.

B. Consensus Reaching Process

Let $R_1, R_2, \cdots, R_p$ and R_c be the p individual linguistic decision matrices and the group linguistic decision matrix, respectively. It seems reasonable to assume that an decision maker who is asked to adjust their preferences is the decision maker who has the minimum consensus index. As with other research [14][18], an implicit hypothesis in the proposed approach is that the decision makers are expected to effectively support the complete decision making process from problem formulation to solution implementation. To reach a predefined consensus level, the following algorithm is designed.

Algorithm 1: Consensus process based on LWAA operator

Input: Individual linguistic decision matrices R_1, R_2, $\cdots$, R_p, the weight vector of the decision makers $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)^T$, the predefined threshold $\overline{GCI}$, the maximum number of iterative times $h_{\max} \geq 1$ and $0 < \gamma < 1$.

Output: Modified linguistic decision matrices $\overline{R}_1$, $\overline{R}_2$, $\cdots$, $\overline{R}_p$, $GCI(\overline{R}_k)$, $k = 1, 2, \cdots, p$, and the number of iterations h.

*Step 1:*Set $h = 0$ and $R_{k,0} = (r_{ij,0}^{(k)})_{n\times m} = (r_{ij}^{(k)})_{n\times m}$.

Step 2: Calculate the group linguistic preference relation $R_{c,h} = (r_{ij,h}^{c})_{n\times m}$ corresponding to $R_{1,h}, R_{2,h}, \cdots, R_{p,h}$, where

$$r_{ij,h}^{c} = LWAA(r_{ij,h}^{(1)}, r_{ij,h}^{(2)}, \cdots, r_{ij,h}^{(p)}).$$

Step 3: Calculate the group consensus index $GCI(R_{k,h})$, $k \in P$ by Definition 4. If $GCI(R_{k,h}) \geq \overline{GCI}$, $\forall k \in P$ or $h \geq h_{\max}$, then go to step 5; otherwise, go on to the next step.

Step 4: Suppose that $GCI(R_{\tau,h}) = \min_k \{GCI(R_{k,h})\}$. Let $R_{k,h+1} = (r_{ij,h+1}^{(k)})_{n\times m}$, where

$$r_{ij,h+1}^{(k)} = \begin{cases} \gamma r_{ij,h}^{(k)} \oplus (1-\gamma) r_{ij,h}^{c} & k = \tau \\ r_{ij,h}^{(k)} & k \neq \tau \end{cases}. \tag{9}$$

Set $h = h + 1$ and go to Step 2.

Step 5: Let $\overline{R}_k = R_{k,h}$, for all $k = 1, 2, \cdots, p$. Output $\overline{R}_1, \overline{R}_2, \cdots, \overline{R}_p$, $GCI(\overline{R}_k)$, for all $k = 1, 2, \cdots, p$, and the number of iterations h.

Step 6: End.

Algorithm 1 is an iterative process. It can improve the consensus level of each individual in the group. When the individual k who has the smallest GCI value implemented the improving strategy, the individual k will have a better GCI value. To demonstrate that Algorithm 1 is convergent, the following theorem is proposed.

Theorem 1: Let $R_1, R_2, \cdots, R_p$ and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)^T$ be the p linguistic decision matrices and the weight vector of the decision makers respectively. Let $R_{l,h}$ be the decision matrix sequences generated by Algorithm 1 for decision maker l. In the hth iteration, suppose that the kth decision maker E_k has the minimum GCI value, then

$$GCI(R_{k,h+1}) > GCI(R_{k,h}). \tag{10}$$

Proof: According to the strategy of Algorithm 1, we have

$$R_{k,h+1} = (r_{ij,h+1}^{(k)})_{n\times m},$$

where

$$r_{ij,h+1}^{(k)} = \gamma r_{ij,h}^{(k)} \oplus (1-\gamma) r_{ij,h}^{c}.$$

Furthermore, we have

$$\begin{aligned} & I(r_{ij,h}^{c}) - I(r_{ij,h+1}^{c}) \\ &= \sum_{l=1}^{p} \lambda_l I(r_{ij,h}^{(l)}) - \sum_{l=1}^{p} \lambda_l I(r_{ij,h+1}^{(l)}) \\ &= \lambda_k (I(r_{ij,h}^{(k)}) - I(r_{ij,h+1}^{(k)})) \\ &= \lambda_k (I(r_{ij,h}^{(k)}) - (\gamma I(r_{ij,h}^{(k)}) + (1-\gamma) I(r_{ij,h}^{c}))) \\ &= \lambda_k (1-\gamma)(I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c})). \end{aligned} \tag{11}$$

Therefore

$$\begin{aligned} & \left| I(r_{ij,h+1}^{(k)}) - I(r_{ij,h+1}^{c}) \right| \\ &= \left| \gamma I(r_{ij,h}^{(k)}) + (1-\gamma) I(r_{ij,h}^{c}) - I(r_{ij,h+1}^{c}) \right| \\ &= \left| \gamma (I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c})) + (I(r_{ij,h}^{c}) - I(r_{ij,h+1}^{c})) \right| \\ &= \left| \gamma (I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c})) + \lambda_k (1-\gamma)(I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c})) \right| \\ &= \left| (\gamma + \lambda_k(1-\gamma))(I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c})) \right| \\ &< \left| I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c}) \right|. \end{aligned}$$

Since

$$d(r_{ij,h}^{(k)}, r_{ij,h}^{c}) = \frac{\left| I(r_{ij,h}^{(k)}) - I(r_{ij,h}^{c}) \right|}{g},$$

we have

$$d(r_{ij,h+1}^{(k)}, r_{ij,h+1}^{c}) < d(r_{ij,h}^{(k)}, r_{ij,h}^{c}). \tag{12}$$

Consequently,

$$S(R_{k,h+1}, R_{c,h+1}) < S(R_{k,h}, R_{c,h}).$$

That is,

$$GCI(R_{k,h+1}) > GCI(R_{k,h}).$$

This completes the proof of Theorem 1.

Theorem 1 guarantees that for the decision maker E_k, the consensus level of this round is better than that of the last round. Generally, after implementing the process finite times, the group can achieve a predefined consensus level.

Note 1: The parameter γ controls the extent of the modification in every round. In practice, γ is determined through simulation experiment. At the same time, the convergence rate of the process depends on both the size of the group of decision makers and the set of alternatives, and mostly on the decision makers' willingness to compromise.

C. Maximizing Deviation Method

Once the consensus process is terminated, we obtain the final group linguistic matrix which is also denoted as $R_c = (r_{ij}^{c})_{n\times m}$. Assume $w = (w_1, w_2, \cdots, w_m)^T$ is the attribute weight vector to be determined. From R_c, the overall assessment value of alternative X_i can be written as

$$\begin{aligned} Z_i(w) &= LWAA(r_{i1}^{c}, r_{i2}^{c}, \cdots, r_{im}^{c}) \\ &= w_1 r_{i1}^{c} \oplus w_2 r_{ij}^{c} \oplus \cdots \oplus w_m r_{im}^{c}. \end{aligned} \tag{13}$$

The basic idea of the maximizing deviation method is that the criterion or attribute with a larger deviation value among alternatives should be considered as a more important criterion or attribute [9]. For the case where each alternative takes similar value on an attribute, such an attribute does not add meaningful value to the overall assessment value of each alternative since each alternative adds a similar value. Especially, if all available alternatives score about equally with respect to a given attribute, then such an attribute will be judged unimportant. In other word, such an attribute should be assigned a very small weight. Based on this idea, an optimization method could be developed to determine the attribute weights under the assumption that attribute weights are completely unknown.

For the attribute C_j, the deviation of alternative X_i to all the other alternatives can be defined as

$$F_{ij} = \sum_{l=1}^{n} d(r_{ij}^{c}, r_{lj}^{c}) w_j = \frac{1}{g} \sum_{l=1}^{n} |I(r_{ij}^{c}) - I(r_{lj}^{c})| w_j. \tag{14}$$

Further, the deviation value of all alternatives to other alternatives over the attribute C_j is denoted by

$$F_j = \sum_{i=1}^{n} F_{ij} = \frac{1}{g} \sum_{i=1}^{n} \sum_{l=1}^{n} |I(r_{ij}^{c}) - I(r_{lj}^{c})| w_j. \tag{15}$$

Based on the idea described above, the weight vector is obtained by solving the following non-linear programming model:

$$\begin{aligned} \min \quad & F(w) = \sum_{j=1}^{m} F_j = \frac{1}{g} \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{l=1}^{n} |I(r_{ij}^{c}) - I(r_{lj}^{c})| w_j \\ s.t. \quad & w_j \geq 0,\ j \in M \ \sum_{j=1}^{m} w_j^2 = 1. \end{aligned} \tag{16}$$

The attribute weights w_j, $j \in M$ can be derived by solving model (16) using Lagrangian multiplier method. After further normalization of the obtained weights, $w_j, j \in M$ is rewritten as

$$w_j = \frac{\sum_{i=1}^{n} \sum_{l=1}^{n} d(r_{ij}^{c}, r_{lj}^{c})}{\sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{l=1}^{n} d(r_{ij}^{c}, r_{lj}^{c})} \tag{17}$$

Therefore, once the group linguistic decision matrix is obtained, Eq. (17) is used to get the attribute weights.

Note 2: In the proposed maximizing deviation method, the LWAA operators is utilized to get the overall assessment value. However, there are some other aggregation operators that can be used such as the linguistic weighted geometric averaging (LWGA) operator. Different to the ordered weighted averaging (OWA) like operator where a weight w_i is associated with a particular ordered position i of the arguments, LWAA and LWGA operators associate each argument with a particular weight. Thus, when we have to consider the attribute weight in the aggregation, the operators like LWAA or LWGA should be used, otherwise the OWA-like operators are useful. It depends on the predetermined mechanism used in the group decision-making process.

D. The Decision Framework for Linguistic MCGDM

The decision framework for linguistic MCGDM problems is shown in Fig. 2. As mentioned earlier, there are in general two processes to conduct before obtaining a final solution: the consensus process and the selection process. Before the selection process, a consensus process is conducted to make a maximum degree of agreement solution between the group members. The selection process involves the aggregation of individual decision matrices and the exploitation of the group decision matrix. Note that the proposed consensus reaching process can automatically adjust each linguistic decision matrix to meet the predefined consensus level. Therefore, in some situations, the role of the moderator can be replaced by a designed system. The maximizing deviation method is used to determine the objective weights of the attributes and thus it alleviates the influence of the decision makers in the selection process. This model can be used to facilitate the decision makers and the moderator in the decision process when they expect to achieve a consensus solution.

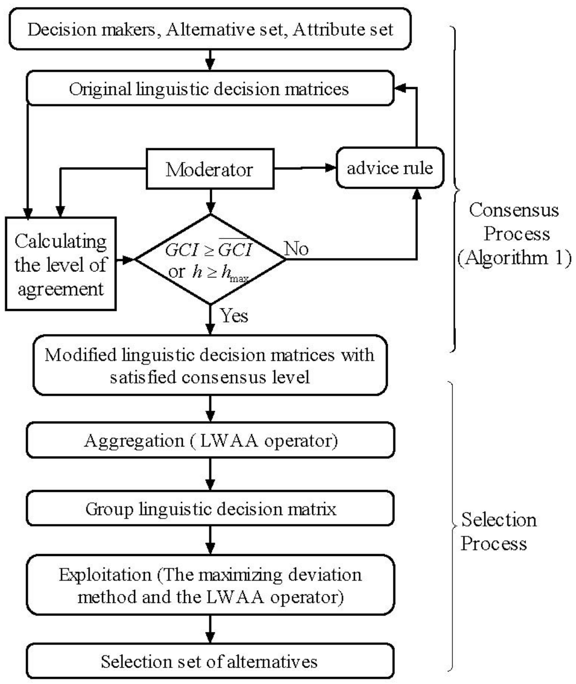

Fig. 2. The decision framework for linguistic MCGDM

IV. Numerical examples

In this section, a problem of personal selection is used to illustrate the developed approach. This example is adapted from Chen [26].

Suppose that a software company desires to hire a system analysis engineer. After preliminary screening, three candidates X_1, X_2 and X_3 remain for further evaluation. A committee of three decision-makers, E_1, E_2 and E_3 has been formed to conduct the interview and to select the most suitable candidate. Five benefit criteria are considered:

(1) C_1: emotional steadiness;
(2) C_2: oral communication skill;
(3) C_3: personality;
(4) C_4: past experience;
(5) C_5: self-confidence.

The three possible candidates are to be assessed using the following linguistic term set

$$S = \{s_0 = extremely\ poor,\ s_1 = very\ poor,\ s_2 = poor,\ s_3 = slightly\ poor,\ s_4 = fair,\ s_5 = slightly\ good,\ s_6 = good,\ s_7 = very\ good,\ s_8 = extremely\ good\}.$$

Based on the 2-tuple linguistic representation model, the three decision makers give the linguistic decision matrices as shown in Table I-III.

TABLE I
LINGUISTIC DECISION MATRIX R_1

	C_1	C_2	C_3	C_4	C_5
X_1	$(s_3,0)$	$(s_4,0)$	$(s_6,0)$	$(s_4,0)$	$(s_8,0)$
X_2	$(s_6,0)$	$(s_5,0)$	$(s_7,0)$	$(s_5,0)$	$(s_7,0)$
X_3	$(s_5,0)$	$(s_8,0)$	$(s_7,0)$	$(s_7,0)$	$(s_6,0)$

TABLE II
LINGUISTIC DECISION MATRIX R_2

	C_1	C_2	C_3	C_4	C_5
X_1	$(s_5,0)$	$(s_6,0)$	$(s_7,0)$	$(s_7,0)$	$(s_6,0)$
X_2	$(s_3,0)$	$(s_4,0)$	$(s_5,0)$	$(s_6,0)$	$(s_7,0)$
X_3	$(s_4,0)$	$(s_6,0)$	$(s_6,0)$	$(s_5,0)$	$(s_6,0)$

TABLE III
LINGUISTIC DECISION MATRIX R_3

	C_1	C_2	C_3	C_4	C_5
X_1	$(s_4,0)$	$(s_7,0)$	$(s_6,0)$	$(s_5,0)$	$(s_8,0)$
X_2	$(s_6,0)$	$(s_5,0)$	$(s_7,0)$	$(s_6,0)$	$(s_4,0)$
X_3	$(s_4,0)$	$(s_7,0)$	$(s_8,0)$	$(s_6,0)$	$(s_7,0)$

To select the most suitable alternative(s), the proposed approach is applied and the computational procedure is given as follows:

Stage 1: The consensus reaching process.

Without loss of generality, assume $\lambda = (1/3, 1/3, 1/3)^T$ is the weight vector of the decision makers. The current group linguistic decision matrix is shown in Table IV. The current consensus indices for each decision maker are as follows

$$GCI(R_1) = 0.9250,\ GCI(R_2) = 0.9111,$$
$$GCI(R_3) = 0.9361.$$

TABLE IV

GROUP LINGUISTIC DECISION MATRIX R

	C_1	C_2	C_3	C_4	C_5
X_1	(s_4,0)	(s_6,-1/3)	(s_6,1/3)	(s_5,1/3)	(s_7,1/3)
X_2	(s_5,0)	(s_5,-1/3)	(s_6,1/3)	(s_6,-1/3)	(s_6,0)
X_3	(s_4,1/3)	(s_7,0)	(s_7,0)	(s_6,0)	(s_6,1/3)

If $\overline{GCI} = 0.95$, it is known that all the linguistic decision matrices do not meet the predefined consensus level. So, Algorithm 1 is used to modify the original linguistic decision matrices. Setting $\gamma = 0.9$, the simulation results are shown in Fig. 3. The algorithm is terminated after 12 iterations. Overall, E_1, E_2, E_3 modified their preferences 4, 7 and 1 times, respectively. The final group consensus indices are

$$GCI(R_1) = 0.9512,\ GCI(R_2) = 0.9501,$$
$$GCI(R_3) = 0.9515.$$

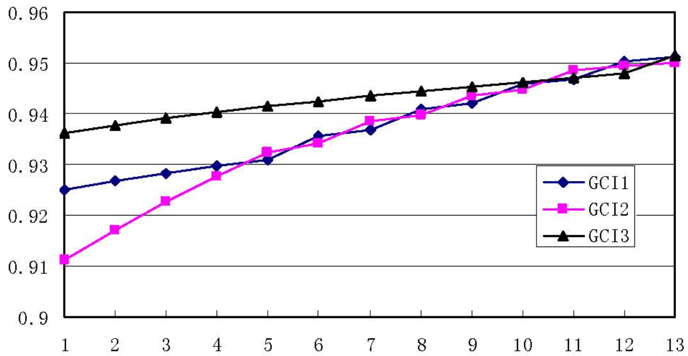

Fig. 3. The group consensus indices of Algorithm 1

All the decision makers have higher consensus indices which are larger than the predefined consensus level. The modified linguistic decision matrices are omitted here and the final group linguistic decision matrix, R_{new}, is shown in Table V.

TABLE V

GROUP LINGUISTIC DECISION MATRIX R_{new}

	C_1	C_2	C_3	C_4	C_5
X_1	(s_4,-0.08)	(s_6,-0.23)	(s_6,0.24)	(s_5,0.16)	(s_8,-0.48)
X_2	(s_5,0.27)	(s_5,-0.24)	(s_7,-0.48)	(s_6,-0.23)	(s_6,-0.28)
X_3	(s_4,0.33)	(s_7,0.09)	(s_7,0.19)	(s_6,0.09)	(s_6,0.43)

Stage 2: The selection process.

Based on R_{new}, the maximizing deviation method is used to derive the importance weights of attributes. From Eq. (17), we have

$$w = (0.1847, 0.3164, 0.1281, 0.1266, 0.2442)^T.$$

Further, by utilizing the LWAA operator, the overall assessment value for each alternative is

$$Z_1 = (s_6, -0.16), Z_2 = (s_5, 0.43),$$
$$Z_3 = (s_6, 0.30).$$

Rank all the alternatives $X_i(i = 1, 2, 3)$ in accordance with $Z_i(i = 1, 2, 3)$. The ranking of the alternatives is $X_3 \succ X_1 \succ X_2$. Thus the best candidate for this job is X_3.

V. CONCLUDING REMARKS

Many practical decision-making problems involves the multiplicity of criteria for judging the alternatives and a group of decision makers who provide the preferences over the alternatives. In this paper, a consensus and maximizing deviation based approach has been proposed to solve such problems under linguists setting. The computation process of the proposed approach is illustrated by an example. The use of the proposed method could be extend to support situations in which the preference information is in other forms, e.g., interval numerical number, triangular fuzzy number, intuitionistic fuzzy number or hybrid certain and uncertain information. How to deal with GDM problem with linguistic information using the interval type-2 fuzzy sets is the future work.

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