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Adaptive Robust Tracking Control of Surface Vessels Using Dynamic Constructive Fuzzy Neural Networks

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***Abstract*— In this paper, an adaptive robust dynamic constructive fuzzy neural control (AR-DCFNC) scheme for trajectory tracking of a surface vehicle with uncertainties and unknown time-varying disturbances is proposed. System uncertainties and unknown dynamics are identified online by a dynamic constructive fuzzy neural network (DCFNN) which is implemented by employing dynamically constructive fuzzy rules according to the structure learning criteria. The entire AR-DCFNC system is globally asymptotical stable.**

I. Introduction

DUE to model uncertainties and unknown disturbances imposed on a surface vehicle, the approximation-based control methods are highly desired to realize online adaptation and robustness to unknown dynamics. Considering unmodelled dynamics, the NN-based model reference adaptive control (MRAC) was proposed in [1] for trajectory tracking of surface vehicles. Yang *et al.* [2] developed an adaptive fuzzy robust tracking control algorithm for a ship autopilot system to maintain the ship on a predetermined heading, whereby stability is guaranteed by using the input-to-state stability (ISS) approach and small gain theorem. Tee and Ge [3] addressed the problem of tracking a desired trajectory for fully actuated ocean vessels by the combination of feedforward NN and domination design techniques which allows time-varying disturbances to be handled. Recently, Dai *et al.* [4] presented a stable adaptive NN tracking controller for the ocean surface ship in uncertain dynamical environments in the framework of backstepping and Lyapunov synthesis.

The FNN can enhance the learning capability of FIS by incorporating the NN topology, which allows all free parameters to be adaptively updated according to performance criteria [5], [6]. Note that adaptive laws only consider parameter learning without structure update, i.e., the number of fuzzy rules or hidden nodes must be determined *a priori*, although the resulting performance is acceptable due to the truth that convergence of the tracking error does not necessarily imply convergence (or even robustness) of the estimated parameters [7]. It implies that the approximation accuracy would be much pooror if inadequate fuzzy rules, i.e. too many or too few, are predefined.

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To circumvent the foregoing problem, the self-organizing FNN (SOFNN) with structure and parameter updated simultaneously have been proposed in [8]–[10] and references therein, which can automatically generate fuzzy rules in addition to parameter update. It has also attracted researchers to incorporate the SOFNN into adaptive approximation-based control schemes [11]–[14]. To the best of our knowledge, there does not exist any SOFNN-based control scheme for tracking a surface vehicle.

In this paper, an adaptive robust dynamic constructive fuzzy neural control (AR-DCFNC) scheme for tracking surface vehicles in the presence of uncertainties and unknown disturbances is proposed. In the AR-DCFNC, a dynamic constructive fuzzy neural network (DCFNN) is developed by dynamically generating and pruning fuzzy rules according to structure learning criteria. The DCFNN is then used to approximate uncertain dynamics together with unknown disturbances, and thereby contributing to a SCFNN-based adaptive controller by employing a sliding mode and projection-based adaptive laws for parameters. In addition, a robust supervisory controller including uncertainty estimator is designed to suppress the DCFNN-based approximation error.

II. Problem Formulation

As shown in Fig. 1, two coordinate frames, i.e., earth-fixed OX_oY_o and body-fixed frames AXY, of surface vehicles are commonly used to clearly formulate the problem to be resolved in this paper. The axes OX_o and OY_o directs to the North and East respectively, while axes AX and AY direct to fore and starboard respectively. Assuming that the vessel is port-starboard symmetric, the distance x_g between geometric center A and gravity center G is allocated along the axis AX. Let $\boldsymbol{\eta} = [x, y, \psi]^T$ be the 3-DOF position (x, y) and heading angle (ψ) of the vessel in an earth-fixed inertial frame, and let $\boldsymbol{\nu} = [u, v, r]^T$ be the corresponding linear velocities (u, v), i.e. surge and sway velocities, and angular rate (r), i.e. yaw, in the body-fixed frame. The dynamic model of the surface vehicle can be described as follows:

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu} \tag{1a}$$

$$\overline{\mathbf{M}}\dot{\boldsymbol{\nu}} + \overline{\mathbf{C}}(\boldsymbol{\nu})\boldsymbol{\nu} + \overline{\mathbf{D}}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} + \mathbf{R}^T(\psi)\mathbf{b} \tag{1b}$$

where, $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ and $\mathbf{b}(t) = [b_1(t), b_2(t), b_3(t)]^T$ are control input and unknown time-varying external environ-

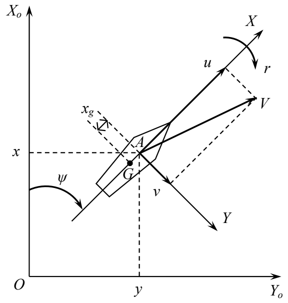

Fig. 1. The earth-fixed OX_oY_o and body-fixed AXY coordinate frames.

mental disturbances due to wind, waves and ocean currents in the body-fixed frame, and the matrix $\mathbf{R}(\psi)$ is the 3-DOF rotation rotation matrix defined as follows:

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

with the following properties:

$$\mathbf{R}^T(\psi)\mathbf{R}(\psi) = \mathbf{I}, \text{ and } \|\mathbf{R}(\psi)\| = 1, \ \forall\ \psi \in [0, 2\pi] \tag{3}$$

Here, the inertia matrix $\overline{\mathbf{M}} = \overline{\mathbf{M}}^T > 0$, the skew-symmetric matrix $\overline{\mathbf{C}}(\boldsymbol{\nu}) = -\overline{\mathbf{C}}(\boldsymbol{\nu})^T$ of Coriolis and centripetal terms and the damping matrix $\overline{\mathbf{D}}(\boldsymbol{\nu})$ are given by,

$$\overline{\mathbf{M}} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} \tag{4a}$$

$$\overline{\mathbf{C}}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & c_{13}(\boldsymbol{\nu}) \\ 0 & 0 & c_{23}(\boldsymbol{\nu}) \\ -c_{13}(\boldsymbol{\nu}) & -c_{23}(\boldsymbol{\nu}) & 0 \end{bmatrix} \tag{4b}$$

$$\overline{\mathbf{D}}(\boldsymbol{\nu}) = \begin{bmatrix} d_{11}(\boldsymbol{\nu}) & 0 & 0 \\ 0 & d_{22}(\boldsymbol{\nu}) & d_{23}(\boldsymbol{\nu}) \\ 0 & d_{32}(\boldsymbol{\nu}) & d_{33}(\boldsymbol{\nu}) \end{bmatrix} \tag{4c}$$

where, $m_{11} = m - X_{\dot{u}}$, $m_{22} = m - Y_{\dot{v}}, m_{23} = mx_g - Y_{\dot{r}}$, $m_{32} = mx_g - N_{\dot{v}}, m_{33} = I_z - N_{\dot{r}}; c_{13}(\boldsymbol{\nu}) = -m_{11}v - m_{23}r, c_{23}(\boldsymbol{\nu}) = m_{11}u; d_{11}(\boldsymbol{\nu}) = -X_u - X_{|u|u}|u| - X_{uuu}u^2, d_{22}(\boldsymbol{\nu}) = -Y_v - Y_{|v|v}|v| - Y_{|r|v}|r|, d_{23}(\boldsymbol{\nu}) = -Y_r - Y_{|v|r}|v| - Y_{|r|r}|r|, d_{32}(\boldsymbol{\nu}) = -N_v - N_{|v|v}|v| - N_{|r|v}|r|, d_{33}(\boldsymbol{\nu}) = -N_r - N_{|v|r}|v| - N_{|r|r}|r|$. Here, m is the mass of the vessel, I_z is the moment of inertia about the yaw rotation, $Y_{\dot{r}} = N_{\dot{v}}$, and symbols X_*, Y_*, N_* denote corresponding hydrodynamic derivatives.

By substituting (1a) into (1b), the dynamic model (1a)-(1b) can be rewritten in the following Lagrange form:

$$\mathbf{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} + \mathbf{D}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} - \mathbf{b} = \mathbf{R}(\boldsymbol{\eta})\boldsymbol{\tau} \tag{5}$$

where,

$$\mathbf{M}(\boldsymbol{\eta}) = \mathbf{R}(\boldsymbol{\eta})\overline{\mathbf{M}}\mathbf{R}^T(\boldsymbol{\eta}) \tag{6a}$$

$$\mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \mathbf{R}(\boldsymbol{\eta})\left(\overline{\mathbf{C}} - \overline{\mathbf{M}}\mathbf{S}\right)\mathbf{R}^T(\boldsymbol{\eta}) \tag{6b}$$

$$\mathbf{D}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \mathbf{R}(\boldsymbol{\eta})\overline{\mathbf{D}}\mathbf{R}^T(\boldsymbol{\eta}) \tag{6c}$$

$$\mathbf{S}(\dot{\psi}) = \begin{bmatrix} 0 & -\dot{\psi} & 0 \\ \dot{\psi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6d}$$

with the following properties:

$$\mathbf{M}(\boldsymbol{\eta}) = \mathbf{M}^T(\boldsymbol{\eta}) > \mathbf{0} \tag{7a}$$

$$\dot{\mathbf{M}}(\boldsymbol{\eta}) - 2\mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = -\left[\dot{\mathbf{M}}(\boldsymbol{\eta}) - 2\mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\right]^T \tag{7b}$$

Given desired smooth trajectories $\boldsymbol{\eta}_d(t)$, $\dot{\boldsymbol{\eta}}_d(t)$ and $\ddot{\boldsymbol{\eta}}_d(t)$, our objective is to design a control law for (5) with uncertainties and unknown disturbances such that all signals in the resulting control system are uniformly bounded and the tracking errors $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t)$ are asymptotically stable, where $\mathbf{e}(t) = \boldsymbol{\eta}(t) - \boldsymbol{\eta}_d(t)$.

In this context, the model (5) can be rewritten as follows:

$$\mathbf{M}(\boldsymbol{\eta})\ddot{\mathbf{e}} = -\mathbf{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}}_d - \mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} - \mathbf{D}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} + \mathbf{b} + \mathbf{R}(\boldsymbol{\eta})\boldsymbol{\tau} \tag{8}$$

Define a sliding surface as

$$\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}\mathbf{e} \tag{9}$$

where $\boldsymbol{\Lambda} = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$ is diagonal positive definite matrix. Substituting (9) into (8), we have

$$\mathbf{M}\dot{\mathbf{s}} + (\mathbf{K} + \mathbf{C})\mathbf{s} = \mathbf{K}\mathbf{s} - \mathbf{f}(\mathbf{z}) + \mathbf{R}\boldsymbol{\tau} \tag{10}$$

where $\mathbf{K} = diag(k_1, k_2, \cdots, k_n)$ is diagonal positive definite matrix and

$$\mathbf{f}(\mathbf{z}) = \mathbf{M}\dot{\boldsymbol{\eta}}_r + \mathbf{C}\boldsymbol{\eta}_r + \mathbf{D}\dot{\boldsymbol{\eta}} - \mathbf{b} \tag{11}$$

with $\mathbf{z} = [\dot{\boldsymbol{\eta}}_r^T, \dot{\boldsymbol{\eta}}^T, \boldsymbol{\eta}_r^T, \boldsymbol{\eta}^T]^T \triangleq [z_1, z_2, \cdots, z_m]^T \in U_z \subset R^m$ and $\boldsymbol{\eta}_r = \dot{\boldsymbol{\eta}}_d - \boldsymbol{\Lambda}\mathbf{e}$.

Accordingly, if the nonlinear dynamics $\mathbf{f}(\mathbf{z})$ is known, an ideal controller can be designed as

$$\boldsymbol{\tau}^* = \mathbf{R}^T\big(-\mathbf{K}\mathbf{s} + \mathbf{f}(\mathbf{z})\big) \tag{12}$$

However, the smooth vector field $\mathbf{f}(\mathbf{z})$ is actually uncertain and perturbed by time-varying external disturbances $\mathbf{b}(t)$. In this context, our objective of this paper is to design an adaptive robust controller for (5) with the ability to not only identify online the unknown dynamics $\mathbf{f}(.)$ but also attenuate approximation errors, such that $\boldsymbol{\eta}$ and $\dot{\boldsymbol{\eta}}$ of surface vehicles can track arbitrary smooth reference trajectory $\boldsymbol{\eta}_d$ and its first derivative $\dot{\boldsymbol{\eta}}_d$, respectively.

III. Dynamic Constructive Fuzzy Neural Network

In this section, the dynamic constructive fuzzy neural network (DCFNN) is proposed to approximate the unknown dynamics $\mathbf{f}(\mathbf{z})$ in (11).

A. Architecture of DCFNN

As shown in Fig. 2, the architecture of the DCFNN is comprised of four layers, i.e., input, membership, rule and output, which contribute to the fuzzy rule base as follows:

$$\begin{aligned} &\text{IF } z_1 \text{ is } A_1^l \text{ and } z_2 \text{ is } A_2^l \ \cdots\ z_m \text{ is } A_m^l, \\ &\text{THEN } f_1(\mathbf{z}) = w_1^l, f_2(\mathbf{z}) = w_2^l, \cdots, f_n(\mathbf{z}) = w_n^l \end{aligned} \tag{13}$$

where $A_i^l, i = 1, 2, \cdots, m$ and $w_j^l, j = 1, 2, \cdots, n$ are input fuzzy sets and output fuzzy singletons, respectively. Given N

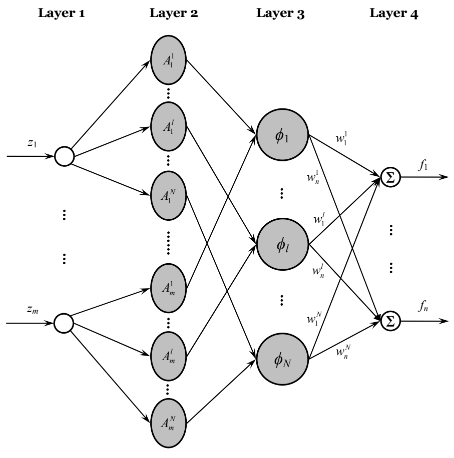

Fig. 2. Architecture of the DCFNN.

fuzzy rules, the overall output of the DCFNN can be obtained as follows:

$$\mathbf{f}_{\mathrm{F}}(\mathbf{z}) = [f_1, f_2, \cdots, f_n]^T = \mathbf{W}^T \mathbf{\Phi}\left(\mathbf{z}; \mathbf{c}, \boldsymbol{\sigma}\right) \tag{14}$$

where $\mathbf{f}_{\mathrm{F}} : U_z \subset R^m \to R^n$, and the output weight matrix $\mathbf{W}$ and regressor $\mathbf{\Phi}\left(\mathbf{z}; \mathbf{c}, \boldsymbol{\sigma}\right)$ are respectively defined as

$$\mathbf{W} = [\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \cdots, \boldsymbol{\omega}_n] \in R^{N \times n},\ \boldsymbol{\omega}_j = \left[w_j^1, w_j^2, \cdots, w_j^N\right]^T,$$
$$\mathbf{\Phi} = [\phi_1, \phi_2, \cdots, \phi_N]^T \in R^N.$$

Here, the fuzzy basis function (FBF) is defined by

$$\begin{aligned} \phi_l &= \exp\left(-\sum_{i=1}^{m} \frac{(z_i - c_i^l)^2}{(\sigma_i^l)^2}\right) \\ &= \exp\Big(-(\mathbf{z} - \mathbf{c}_l)^T \mathbf{\Sigma}_l^{-2} (\mathbf{z} - \mathbf{c}_l)\Big) \end{aligned} \tag{15}$$

where, $\mathbf{\Sigma}_l = diag\left(\sigma_1^l, \sigma_2^l, \cdots, \sigma_m^l\right) \in R^{m \times m}$; $\mathbf{c} = [\mathbf{c}_1^T, \mathbf{c}_2^T, \cdots, \mathbf{c}_N^T]^T \in R^{mN}$, $\mathbf{c}_l = [c_1^l, c_2^l, \cdots, c_m^l]^T$ are FBF center vectors; and $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1^T, \boldsymbol{\sigma}_2^T, \cdots, \boldsymbol{\sigma}_N^T]^T \in R^{mN}$, $\boldsymbol{\sigma}_l = [\sigma_1^l, \sigma_2^l, \cdots, \sigma_m^l]^T$ are FBF width vectors.

Based on the universal approximation ability of the FNN, there exists an optimal FNN using N^* fuzzy rules which can identify the nonlinear function $\mathbf{f}(\mathbf{z})$ with the minimal functional approximation errors (MFAEs), i.e.,

$$\begin{aligned} \mathbf{f}(\mathbf{z}) &= \mathbf{f}_{\mathrm{F}}^*(\mathbf{z}; \mathbf{W}, \mathbf{c}, \boldsymbol{\sigma}) + \boldsymbol{\varepsilon}_f(\mathbf{z}, N^*) \\ &= \mathbf{W}^T \mathbf{\Phi}\left(\mathbf{z}; \mathbf{c}, \boldsymbol{\sigma}\right) + \boldsymbol{\varepsilon}_f(\mathbf{z}, N^*) \end{aligned} \tag{16}$$

where, $\boldsymbol{\varepsilon}_f = [\varepsilon_{f1}, \varepsilon_{f2}, \cdots, \varepsilon_{fn}]^T \in R^n$ is the MFAE and satisfies $|\varepsilon_{fj}| \le \overline{\varepsilon}_{fj}$, and the optimal parameters $\mathbf{W}^*, \mathbf{c}^*, \boldsymbol{\sigma}^*$ are derived from

$$(\mathbf{W}^*, \mathbf{c}^*, \boldsymbol{\sigma}^*) = \arg\min_{\mathbf{W}, \mathbf{c}, \boldsymbol{\sigma}} \left(\max_{\mathbf{z} \in U_z} \|\mathbf{f}_{\mathrm{F}}(\mathbf{z}; \mathbf{W}, \mathbf{c}, \boldsymbol{\sigma}) - \mathbf{f}(\mathbf{z})\|\right)$$

with the parameters being bounded as $\|\boldsymbol{\omega}_j\| \le \overline{\omega}_j$, $\|\boldsymbol{\omega}_j\|_1 \le \overline{\omega}_j'$, $\|\mathbf{c}\| \le \overline{c}$ and $\|\boldsymbol{\sigma}\| \le \overline{\sigma}$.

The structure of traditional fuzzy or neural approximator especially used in control paradigm is usually predefined by trial and error in advance while only parameters are updated by adaptive laws, and thereby leading to fixed-structure linear-in-parameter (LIP) or nonlinear-in-parameter (NLIP) identifier [7]. In this context, unsuitable number of fuzzy rules or hidden nodes would deteriorate the approximation ability. Hence, it is highly desired to realize dynamic constructive fuzzy rules or hidden nodes according to the nonlinear dynamics to be modeled.

B. Dynamic Constructive Scheme

The DCFNN begins with no any fuzzy rules, i.e., $\mathbf{c}(0) = \emptyset$, $\boldsymbol{\sigma}(0) = \emptyset$, $\mathbf{W}(0) = \emptyset$, $N(0) = 0$. According to the novelty of current observation $\mathbf{z}(t)$ to the existing FBF-s together with the desired tracking errors, the dynamic constructive scheme decides to generate new fuzzy rules or to prune redundant ones in the whole structure learning process. Without loss of generality, consider the input $\mathbf{z}(t)$ at time instant t, i.e., $\mathbf{c}(t-1) = [\mathbf{c}_1^T, \mathbf{c}_2^T, \cdots, \mathbf{c}_{N(t-1)}^T]^T$, $\boldsymbol{\sigma}(t-1) = [\boldsymbol{\sigma}_1^T, \boldsymbol{\sigma}_2^T, \cdots, \boldsymbol{\sigma}_{N(t-1)}^T]^T$ and $\mathbf{W}(t-1) = [\mathbf{w}_1^T, \mathbf{w}_2^T, \cdots, \mathbf{w}_{N(t-1)}^T]^T$ where $\mathbf{w}_l = [w_1^l, w_2^l, \cdots, w_n^l]^T$.

Calculate the distance between the current input $\mathbf{z}(t)$ and the existing FBFs as follows:

$$d_l = \|\mathbf{z}(t) - \mathbf{c}_l\| \tag{17}$$

and find the nearest center

$$l^\dagger = \arg\min_{l=1,2,\cdots,N(t-1)} d_l \tag{18}$$

together with the FBFs having high firing strengths as

$$l^\ddagger \in L_r \triangleq \left\{l \middle| \phi_l\big(\mathbf{z}(t); \mathbf{c}_l, \boldsymbol{\sigma}_l\big) \ge \Delta_r\right\} \tag{19}$$

where $\Delta_r \in [0, 1]$ is a threshold of firing strength.

Define a hyperelliposoid set with respect to the sliding surface $\mathbf{s}$ as follows:

$$\Omega_s\left(d\right) = \left\{\mathbf{s} \middle| \sum_{j=1}^{n} k_j \left(|s_j| - \frac{d}{2k_j}\right)^2 \le \sum_{j=1}^{n} \frac{d^2}{4k_j}\right\} \tag{20}$$

where $k_j > 0, j = 1, 2, \cdots, n$ are diagonal elements of matrix $\mathbf{K}$ and $d \ge 0$ is the threshold of approximation error.

1) Generating of Rules: Consider the criteria for rule generating as follows:

$$d_{l^\dagger} \ge d_{\mathrm{th}} \text{ and } \mathbf{s}(t) \notin \Omega_s\left(\overline{d}(t)\right) \tag{21}$$

where $d_{\mathrm{th}} > 0$ is the predefined distance and $\overline{d}(t)$ is the decreasing upper bound for approximation error given by

$$\overline{d}(t) = \max_{j=1,2,\cdots,n} \lambda_j \left(\frac{1}{2k_j} + \sqrt{\sum_{i=1}^{n} \frac{1}{4k_i k_j}}\right)^{-1} \overline{\delta}_j(t) \tag{22}$$

here, $\lambda_j, j = 1, 2, \cdots, n$ are diagonal elements of matrix $\mathbf{\Lambda}$ and $\overline{\delta}_j(t)$ is the upper bound for the tracking error given by

$$\overline{\delta}_j(t) = \overline{\delta}_j(0) \exp(-\overline{\gamma}_j t) + \varsigma_j \tag{23}$$

with $\overline{\delta}_j(0), \overline{\gamma}_j, \varsigma_j > 0$.

If (21) holds, a new rule node needs to be generated as follows:

$$\begin{cases} \mathbf{c}_{N(t)} &= \mathbf{z}(t) \\ \boldsymbol{\sigma}_{N(t)} &= \gamma \cdot \mathrm{abs}\big(\mathbf{z}(t) - \mathbf{c}_{l^\dagger}\big) \\ \mathbf{w}_{N(t)} &= \mathbf{0} \end{cases} \tag{24}$$

where $\gamma > 0$ and $N(t) = N(t-1) + 1$.

2) Pruning of Rules: Consider the criteria for rule pruning as follows:

$$|L_r| > N_a \text{ and } \mathbf{s}(t) \in \Omega_s\left(\underline{d}(t)\right) \tag{25}$$

where $N_a \in Z^+$ is the predefined number of fuzzy rules with high firing strength simultaneously, and $\underline{d}(t)$ is the decreasing lower bound for approximation error given by

$$\underline{d}(t) = \max_{j=1,2,\cdots,n} \lambda_j \left(\frac{1}{2k_j} + \sqrt{\sum_{i=1}^{n} \frac{1}{4k_i k_j}}\right)^{-1} \underline{\delta}_j(t) \tag{26}$$

here $\underline{\delta}_j(t)$ is the lower bound for the tracking error given by

$$\underline{\delta}_j(t) = \underline{\delta}_j(0)\exp(-\underline{\gamma}_j t) \tag{27}$$

with $\underline{\delta}_j(0), \underline{\gamma}_j > 0$.

If (25) holds, redundant fuzzy rules need to be pruned as follows:

$$\mathbf{c}_{l^\circ} = \emptyset,\ \boldsymbol{\sigma}_{l^\circ} = \emptyset,\ \mathbf{w}_{l^\circ} = \emptyset,\ l^\circ \in L_r^\circ \subset L_r \tag{28}$$

where $\phi_{l^\circ} \geq \phi_{l^\ddagger}, l^\circ \in L_r^\circ, l^\ddagger \in L_r \backslash L_r^\circ$, $|L_r^\circ| = |L_r| - N_a$ and $N(t) = N(t) - |L_r| + N_a$.

IV. Adaptive Robust Dynamic Constructive Fuzzy Neural Control

By employing the proposed DCFNN approximation in (16), an adaptive robust dynamic constructive fuzzy neural control (AR-DCFNC) scheme is designed for tracking the MIMO nonlinear system (5) as follows:

$$\boldsymbol{\tau}_{\mathrm{F}} = \mathbf{R}^T\left[-\mathbf{K}\mathbf{s} + \widehat{\mathbf{f}}(\mathbf{z}) + \boldsymbol{\tau}_r\right] \tag{29}$$

where $\widehat{\mathbf{f}}_{\mathrm{F}}(\mathbf{z}) \triangleq \widehat{\mathbf{W}}^T\widehat{\boldsymbol{\Phi}} = \widehat{\mathbf{W}}^T\boldsymbol{\Phi}(\mathbf{z}(t);\widehat{\mathbf{c}},\widehat{\boldsymbol{\sigma}})$ is the DCFNN-based approximation which is parameterized by parameter estimates $\widehat{\mathbf{W}}, \widehat{\mathbf{c}}, \widehat{\boldsymbol{\sigma}}$ and derives from the adaptive laws in the following subsections. The robustness term $\boldsymbol{\tau}_r$ is designed as

$$\boldsymbol{\tau}_r = (1-\alpha(t))\boldsymbol{\tau}_s + \alpha(t)\widehat{\boldsymbol{\tau}}_s \tag{30}$$

here, the weighting parameter $\alpha(t)$ is derived from

$$\alpha(t) = \begin{cases} 1, & \text{if } \mathbf{s} \in \Omega_s\left(\underline{d}(t)\right) \\ \frac{\overline{\mu}(t)}{\underline{\mu}(t)+\overline{\mu}(t)}, & \text{if } \mathbf{s} \notin \Omega_s\left(\underline{d}(t)\right) \text{ and } \mathbf{s} \in \Omega_s\left(\overline{d}(t)\right) \\ 0, & \text{if } \mathbf{s} \notin \Omega_s\left(\overline{d}(t)\right) \end{cases} \tag{31}$$

with

$$\underline{\mu}(t) = \left|\sqrt{\sum_{j=1}^{n} k_j\left(|s_j| - \frac{\underline{d}(t)}{2k_j}\right)^2} - \sqrt{\sum_{j=1}^{n}\frac{\underline{d}^2(t)}{4k_j}}\right| \tag{32}$$

$$\overline{\mu}(t) = \left|\sqrt{\sum_{j=1}^{n} k_j\left(|s_j| - \frac{\overline{d}(t)}{2k_j}\right)^2} - \sqrt{\sum_{j=1}^{n}\frac{\overline{d}^2(t)}{4k_j}}\right| \tag{33}$$

and the sliding control term $\boldsymbol{\tau}_s$ and its estimate $\widehat{\boldsymbol{\tau}}_s$ are designed as follows:

$$\tau_{s,j} = \boldsymbol{\varpi}_j^T\boldsymbol{\psi}_j \mathrm{sgn}(s_j),\ j = 1,2,\cdots,n \tag{34}$$

$$\widehat{\tau}_{s,j} = \widehat{\boldsymbol{\varpi}}_j^T\boldsymbol{\psi}_j \mathrm{sgn}(s_j),\ j = 1,2,\cdots,n \tag{35}$$

where $\boldsymbol{\varpi}_j = [\overline{\omega}_j' + \overline{\varepsilon}_{fj}, \overline{\omega}_j, \overline{c}, \overline{\sigma}]^T$ and $\widehat{\boldsymbol{\varpi}}_j$ is the corresponding estimate which is updated as follows:

$$\dot{\widehat{\boldsymbol{\varpi}}}_j = -\alpha(t)\eta_\tau\boldsymbol{\psi}_j|s_j| \tag{36}$$

with $\eta_\tau > 0$ and

$$\boldsymbol{\psi}_j = \left[1, \left\|\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}_{\mathbf{c}}'\widehat{\mathbf{c}} - \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\widehat{\boldsymbol{\sigma}}\right\|, \left\|\widehat{\boldsymbol{\omega}}_j^T\boldsymbol{\Phi}_{\mathbf{c}}'\right\|, \left\|\widehat{\boldsymbol{\omega}}_j^T\boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\right\|\right]^T \tag{37}$$

where $\boldsymbol{\Phi}_{\mathbf{c}}'$ and $\boldsymbol{\Phi}_{\boldsymbol{\sigma}}'$ are Jacobian matrices derived from

$$\boldsymbol{\Phi}_{\mathbf{c}}' = \frac{\partial\boldsymbol{\Phi}}{\partial\mathbf{c}}\bigg|_{\substack{\mathbf{c}=\widehat{\mathbf{c}} \\ \boldsymbol{\sigma}=\widehat{\boldsymbol{\sigma}}}} = diag\left(\boldsymbol{\phi}_{\mathbf{c}_1}^T, \cdots, \boldsymbol{\phi}_{\mathbf{c}_N}^T\right) \in R^{N\times mN} \tag{38}$$

$$\boldsymbol{\Phi}_{\boldsymbol{\sigma}}' = \frac{\partial\boldsymbol{\Phi}}{\partial\boldsymbol{\sigma}}\bigg|_{\substack{\mathbf{c}=\widehat{\mathbf{c}} \\ \boldsymbol{\sigma}=\widehat{\boldsymbol{\sigma}}}} = diag\left(\boldsymbol{\phi}_{\boldsymbol{\sigma}_1}^T, \cdots, \boldsymbol{\phi}_{\boldsymbol{\sigma}_N}^T\right) \in R^{N\times mN} \tag{39}$$

here, $\boldsymbol{\phi}_{\mathbf{c}_l}^T = \left[\frac{\partial\phi_l}{\partial c_1^l}, \cdots, \frac{\partial\phi_l}{\partial c_m^l}\right]$ and $\boldsymbol{\phi}_{\boldsymbol{\sigma}_l}^T = \left[\frac{\partial\phi_l}{\partial\sigma_1^l}, \cdots, \frac{\partial\phi_l}{\partial\sigma_m^l}\right]$.

A. DCFNN Approximation

In the AR-DCFNC (29), the unknown dynamics $\mathbf{f}(\mathbf{z})$ is identified online by the proposed DCFNN with adaptive parameters in addition to dynamic structure according to (16) with $N(t)$ fuzzy rules as follows:

$$\mathbf{f}(\mathbf{z}) = \widehat{\mathbf{W}}_f^T\boldsymbol{\Phi}\left(\mathbf{z};\widehat{\mathbf{c}},\widehat{\boldsymbol{\sigma}}\right) + \widehat{\boldsymbol{\varepsilon}}_f(\mathbf{z}) \tag{40}$$

where $\widehat{\boldsymbol{\varepsilon}}_f$ is actual approximation error determined by

$$\begin{aligned}\widehat{\boldsymbol{\varepsilon}}_f &= \mathbf{f}(\mathbf{z}) - \widehat{\mathbf{f}}_{\mathrm{F}}(\mathbf{z}) = \mathbf{f}_{\mathrm{F}}^*(\mathbf{z}) - \widehat{\mathbf{f}}_{\mathrm{F}}(\mathbf{z}) + \boldsymbol{\varepsilon}_f(\mathbf{z}) \\ &= \mathbf{W}^{*T}\boldsymbol{\Phi}^* - \widehat{\mathbf{W}}^T\widehat{\boldsymbol{\Phi}} + \boldsymbol{\varepsilon}_f \end{aligned} \tag{41}$$

where $\boldsymbol{\Phi}^* = \boldsymbol{\Phi}\left(\mathbf{z};\mathbf{c}^*,\boldsymbol{\sigma}^*\right) \in R^{N^*}$ and $\widehat{\boldsymbol{\Phi}} = \boldsymbol{\Phi}\left(\mathbf{z};\widehat{\mathbf{c}},\widehat{\boldsymbol{\sigma}}\right) \in R^N$. Without loss of generality, assume $N^* \geq N$ and $\boldsymbol{\Phi}^* = [\boldsymbol{\Phi}_1^*, \mathbf{0}], \boldsymbol{\Phi}_1^* \in R^N$, i.e. $\mathbf{W}^{*T}\boldsymbol{\Phi}^* = (\mathbf{W}_1^*)^T\boldsymbol{\Phi}_1^*$, where $\mathbf{W}_1^* \in R^{N\times m}, \boldsymbol{\Phi}_1^* = \boldsymbol{\Phi}\left(\mathbf{z};\mathbf{c}_1^*,\boldsymbol{\sigma}_1^*\right) \in R^N$. From (41),

$$\widehat{\boldsymbol{\varepsilon}}_f = \left(\widetilde{\mathbf{W}} + \widehat{\mathbf{W}}\right)^T\boldsymbol{\Phi}_1^* - \widehat{\mathbf{W}}^T\widehat{\boldsymbol{\Phi}} + \boldsymbol{\varepsilon}_f \tag{42}$$

where $\widetilde{\mathbf{W}} = \mathbf{W}_1^* - \widehat{\mathbf{W}}$ and $\widetilde{\boldsymbol{\Phi}} = \boldsymbol{\Phi}_1^* - \widehat{\boldsymbol{\Phi}}$ are the output weight errors and regressor error, respectively. By applying the Taylor series expansion of $\boldsymbol{\Phi}(.)$ to $(\widehat{\mathbf{c}},\widehat{\boldsymbol{\sigma}})$ in (42), we have

$$\begin{aligned}\widehat{\boldsymbol{\varepsilon}}_f =& \left(\widetilde{\mathbf{W}} + \widehat{\mathbf{W}}\right)^T\left(\widehat{\boldsymbol{\Phi}} + \boldsymbol{\Phi}_{\mathbf{c}}'\widetilde{\mathbf{c}} + \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\widetilde{\boldsymbol{\sigma}} + \mathbf{h}(\mathbf{z};\widetilde{\mathbf{c}},\widetilde{\boldsymbol{\sigma}})\right) \\ & - \widehat{\mathbf{W}}^T\widehat{\boldsymbol{\Phi}} + \boldsymbol{\varepsilon}_f \\ =& \left(\widetilde{\mathbf{W}} + \widehat{\mathbf{W}}\right)^T\left(\widehat{\boldsymbol{\Phi}} + \boldsymbol{\Phi}_{\mathbf{c}}'\widetilde{\mathbf{c}} + \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\widetilde{\boldsymbol{\sigma}}\right) - \widehat{\mathbf{W}}^T\widehat{\boldsymbol{\Phi}} \\ & + \mathbf{W}_1^*\mathbf{h}(\mathbf{z};\widetilde{\mathbf{c}},\widetilde{\boldsymbol{\sigma}}) + \boldsymbol{\varepsilon}_f \\ =& \widetilde{\mathbf{W}}^T\left(\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}_{\mathbf{c}}'\widehat{\mathbf{c}} - \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\widehat{\boldsymbol{\sigma}}\right) + \widehat{\mathbf{W}}^T\left(\boldsymbol{\Phi}_{\mathbf{c}}'\widetilde{\mathbf{c}} + \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\widetilde{\boldsymbol{\sigma}}\right) + \mathbf{d}_s \end{aligned} \tag{43}$$

where,

$$\mathbf{d}_s = \widetilde{\mathbf{W}}^T\left(\boldsymbol{\Phi}_{\mathbf{c}}'\mathbf{c}_1^* + \boldsymbol{\Phi}_{\boldsymbol{\sigma}}'\boldsymbol{\sigma}_1^*\right) + \mathbf{W}_1^*\mathbf{h}(\mathbf{z};\widetilde{\mathbf{c}},\widetilde{\boldsymbol{\sigma}}) + \boldsymbol{\varepsilon}_f \tag{44}$$

here, $\widetilde{\mathbf{c}} = \mathbf{c}_1^* - \widehat{\mathbf{c}}, \widetilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}_1^* - \widehat{\boldsymbol{\sigma}}$, $\mathbf{h}(\mathbf{z};\widetilde{\mathbf{c}},\widetilde{\boldsymbol{\sigma}})$ is the high order term of $\widetilde{\mathbf{c}}$ and $\widetilde{\boldsymbol{\sigma}}$.

In this context, the residual approximation error $\mathbf{d}_s = [d_{s1}, d_{s2}, \cdots, d_{sn}]^T$ is bounded as

$$|d_{sj}| \triangleq \boldsymbol{\varpi}_j^{*T}\boldsymbol{\psi}_j \leq \boldsymbol{\varpi}_j^T\boldsymbol{\psi}_j,\ j = 1,2,\cdots,n \tag{45}$$

where $\boldsymbol{\varpi}_j^* = [\|\boldsymbol{\omega}_j^*\|_1 + |\varepsilon_{fj}|, \|\boldsymbol{\omega}_j^*\|, \|\mathbf{c}_1^*\|, \|\boldsymbol{\sigma}_1^*\|]^T$.

B. Adaptive Laws

Choose the adaptive laws as follows:

$$\dot{\widehat{\boldsymbol{\omega}}}_j = \begin{cases} -\eta_w s_j \left(\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\widetilde{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widetilde{\boldsymbol{\sigma}}\right), & \text{if } \widehat{\boldsymbol{\omega}}_j \notin \Omega_w^j \\ -\eta_w s_j \left(\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\widetilde{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widetilde{\boldsymbol{\sigma}}\right) \\ +\eta_w s_j \widehat{\boldsymbol{\omega}}_j^T \left(\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\widetilde{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widetilde{\boldsymbol{\sigma}}\right)\widehat{\boldsymbol{\omega}}_j / M_w^j, & \text{if } \widehat{\boldsymbol{\omega}}_j \in \Omega_w^j \end{cases} \tag{46}$$

$$\dot{\widehat{\mathbf{c}}} = \begin{cases} -\eta_c (\boldsymbol{\Phi}'_{\mathbf{c}})^T \widehat{\mathbf{W}}\mathbf{s}, & \text{if } \widehat{\mathbf{c}} \notin \Omega_c \\ -\eta_c (\boldsymbol{\Phi}'_{\mathbf{c}})^T \widehat{\mathbf{W}}\mathbf{s} + \eta_c \widehat{\mathbf{c}}^T (\boldsymbol{\Phi}'_{\mathbf{c}})^T \widehat{\mathbf{W}}\mathbf{s}\widehat{\mathbf{c}}/M_c, & \text{if } \widehat{\mathbf{c}} \in \Omega_c \end{cases} \tag{47}$$

$$\dot{\widehat{\boldsymbol{\sigma}}} = \begin{cases} -\eta_\sigma (\boldsymbol{\Phi}'_{\boldsymbol{\sigma}})^T \widehat{\mathbf{W}}\mathbf{s}, & \text{if } \widehat{\boldsymbol{\sigma}} \notin \Omega_\sigma \\ -\eta_\sigma (\boldsymbol{\Phi}'_{\boldsymbol{\sigma}})^T \widehat{\mathbf{W}}\mathbf{s} + \eta_\sigma \widehat{\boldsymbol{\sigma}}^T (\boldsymbol{\Phi}'_{\boldsymbol{\sigma}})^T \widehat{\mathbf{W}}\mathbf{s}\widehat{\boldsymbol{\sigma}}/M_\sigma, & \text{if } \widehat{\boldsymbol{\sigma}} \in \Omega_\sigma \end{cases} \tag{48}$$

where $\eta_w, \eta_c, \eta_\sigma > 0$ and the sets $\Omega_w^j, \Omega_c, \Omega_\sigma$ are defined as,

$$\Omega_w^j = \left\{\widehat{\boldsymbol{\omega}}_j \middle| \, \|\widehat{\boldsymbol{\omega}}_j\| \geq M_w^j \text{ and } s_j \widehat{\boldsymbol{\omega}}_j (\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\widetilde{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widetilde{\boldsymbol{\sigma}}) < 0\right\} \tag{49}$$

$$\Omega_c = \left\{\widehat{\mathbf{c}} \middle| \, \|\widehat{\mathbf{c}}\| \geq M_c \text{ and } \widehat{\mathbf{c}}^T (\boldsymbol{\Phi}'_{\mathbf{c}})^T \widehat{\mathbf{W}}\mathbf{s} < 0\right\} \tag{50}$$

$$\Omega_\sigma = \left\{\widehat{\boldsymbol{\sigma}} \middle| \, \|\widehat{\boldsymbol{\sigma}}\| \geq M_\sigma \text{ and } \widehat{\boldsymbol{\sigma}}^T (\boldsymbol{\Phi}'_{\boldsymbol{\sigma}})^T \widehat{\mathbf{W}}\mathbf{s} < 0\right\} \tag{51}$$

where $M_w^j, j = 1, 2, \cdots, n$, M_c and M_σ are corresponding upper bounds.

C. Stability Analysis

Theorem 1: Consider the surface vehicle (5) with the proposed AR-DCFNC scheme using (29), and the adaptive laws for parameter updates using (46)-(48), where the online approximation $\widehat{\mathbf{f}}_{\mathrm{F}}$ is realized by the DCFNN (40). Then, the tracking errors $\mathbf{e}(t)$ are globally asymptotical stable.

Proof: Consider the Lyapunov function as follows:

$$V(t) = \frac{1}{2}\Bigg[\mathbf{s}^T\mathbf{M}\mathbf{s} + \eta_w^{-1}\sum_{j=1}^{n}\widetilde{\boldsymbol{\omega}}_j^T\widetilde{\boldsymbol{\omega}}_j + \eta_c^{-1}\widetilde{\mathbf{c}}^T\widetilde{\mathbf{c}} + \eta_\sigma^{-1}\widetilde{\boldsymbol{\sigma}}^T\widetilde{\boldsymbol{\sigma}} + \eta_\tau^{-1}\sum_{j=1}^{n}\widetilde{\boldsymbol{\varpi}}_j^T\widetilde{\boldsymbol{\varpi}}_j\Bigg] \tag{52}$$

where $\widetilde{\boldsymbol{\varpi}}_j = \boldsymbol{\varpi}_j^* - \widehat{\boldsymbol{\varpi}}_j$.

Applying the control law (29) to (5) yields

$$\mathbf{M}\dot{\mathbf{s}} = -(\mathbf{K} + \mathbf{C})\mathbf{s} - \Big[\widetilde{\mathbf{W}}^T\left(\widehat{\boldsymbol{\Phi}} - \boldsymbol{\Phi}'_{\mathbf{c}}\widehat{\mathbf{c}} - \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widehat{\boldsymbol{\sigma}}\right) + \widehat{\mathbf{W}}^T\left(\boldsymbol{\Phi}'_{\mathbf{c}}\widetilde{\mathbf{c}} + \boldsymbol{\Phi}'_{\boldsymbol{\sigma}}\widetilde{\boldsymbol{\sigma}}\right) + \mathbf{d}_s\Big] + (1 - \alpha(t))\boldsymbol{\tau}_s + \alpha(t)\widehat{\boldsymbol{\tau}}_s \tag{53}$$

Differentiating V with respect to time t and using (53) and adaptive laws (46)-(48), we have

$$\begin{aligned} \dot{V}(t) =& \mathbf{s}^T\mathbf{M}\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} - \eta_w^{-1}\sum_{j=1}^{n}\widetilde{\boldsymbol{\omega}}_j^T\dot{\widehat{\boldsymbol{\omega}}}_j - \eta_c^{-1}\widetilde{\mathbf{c}}^T\dot{\widehat{\mathbf{c}}} \\ & - \eta_\sigma^{-1}\widetilde{\boldsymbol{\sigma}}^T\dot{\widehat{\boldsymbol{\sigma}}} - \eta_\tau^{-1}\sum_{j=1}^{n}\widetilde{\boldsymbol{\varpi}}_j^T\dot{\widehat{\boldsymbol{\varpi}}}_j \\ \leq & -\mathbf{s}^T\mathbf{K}\mathbf{s} - \eta_\tau^{-1}\sum_{j=1}^{n}\widetilde{\boldsymbol{\varpi}}_j^T\dot{\widehat{\boldsymbol{\varpi}}}_j \\ & + \mathbf{s}^T\left[(1 - \alpha(t))\boldsymbol{\tau}_s + \alpha(t)\widehat{\boldsymbol{\tau}}_s - \mathbf{d}_s\right] \end{aligned} \tag{54}$$

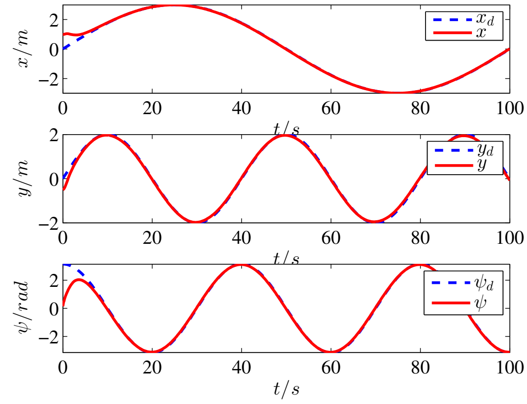

Fig. 3. Desired and actual states x, y and ψ.

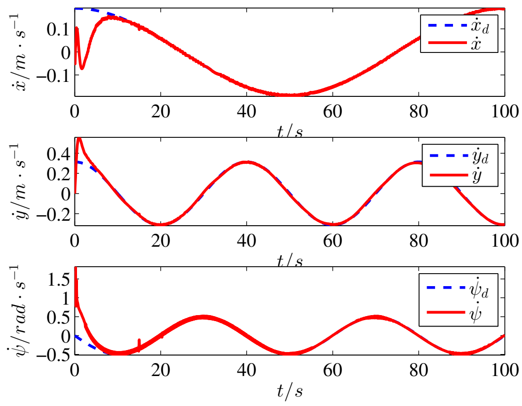

Fig. 4. Desired and actual states $\dot{x}$, $\dot{y}$ and $\dot{\psi}$.

Using (34)-(36), we have

$$\dot{V}(t) \leq -\mathbf{s}^T\mathbf{K}\mathbf{s} \tag{55}$$

If initial conditions $\mathbf{s}(0)$, $\widehat{\mathbf{W}}(0)$, $\widehat{\mathbf{c}}(0)$, $\widehat{\boldsymbol{\sigma}}(0)$ and $\widehat{\boldsymbol{\varpi}}(0)$ are bounded, $0 \leq V(0) < \infty$, and thereby all signals in the system are bounded for all $t > 0$. Moreover,

$$\int_0^\infty \mathbf{s}^T(t)\mathbf{K}\mathbf{s}(t)dt \leq V(0) - V(\infty) \leq V(0) < \infty \tag{56}$$

By the Barbalat's lemma, $\lim_{t\to\infty}\mathbf{s}(t) = \mathbf{0}$, which implies that $\mathbf{e}(t) \to \mathbf{0}$ while $t \to \infty$. This concludes the proof. ■

V. Simulation Studies

In order to demonstrate the effectiveness of the proposed AR-DCFNC scheme, we conduct simulation studies on a surface vehicle called CyberShip II [15]. our objective is to track exactly the smooth trajectory $\boldsymbol{\eta}_d(t)$ given by

$$\boldsymbol{\eta}_d(t) = \begin{bmatrix} 3\sin(0.02\pi t) \\ 2\sin(0.05\pi t) \\ \pi\cos(0.05\pi t) \end{bmatrix} \tag{57}$$

The unknown external environmental disturbances $\mathbf{b}(t)$ are assumed to be governed by $\mathbf{b}(t) = \sin(0.1\pi t) \times [1, 1, 1]^T$

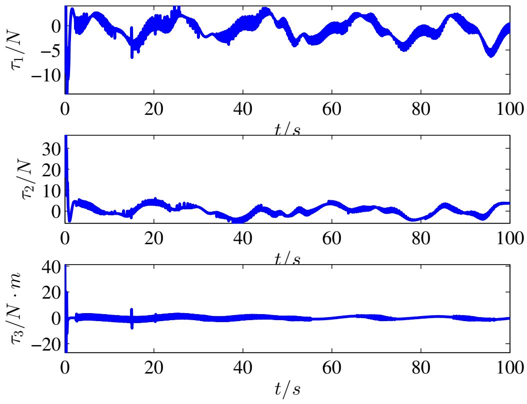

Fig. 5. Control forces τ_1, τ_2 and torque τ_3.

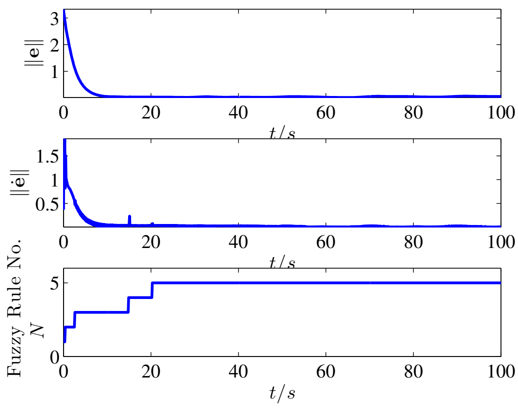

Fig. 6. Online tracking errors and corresponding fuzzy rule numbers.

and the initial conditions of the vessel are set as $\mathbf{z}(0) = [1(m), -0.5(m), 0(rad), 0(m/s), 0(m/s), 0(rad/s)]^T$.

Accordingly, the design parameters of the proposed SARFNC are chosen as follows: $\boldsymbol{\Lambda} = diag(0., 0.5, 0.5)$, $\mathbf{K} = diag(25, 25, 25)$, $\underline{\delta}_j(0) = 0.05$, $\overline{\delta}(0) = 0.1$, $\varsigma_j = 0.02$, $\underline{\lambda}_j = \overline{\lambda}_j = 0.5$, $\gamma = 1$, $d_{\text{th}} = 2$, $\Delta_r = 0.6$, $N_a = 4$, $M_w^j = 10$, $M_c = 15$, $M_\sigma = 15$, $\eta_w = 100$, $\eta_c = 5$, $\eta_\sigma = 5$, $\eta_r = 1$, $\boldsymbol{\varpi} = [5, 5, 5, 5]^T$.

The states $\boldsymbol{\eta}$ and $\dot{\boldsymbol{\eta}}$ together with their desired targets are shown in Fig. 3 and Fig. 4, respectively, from which we can see that the actual states are able to track the desired ones with rapid transient responses and high steady-state accuracy. The corresponding control forces and torque $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ from the AR-DCFNC are shown in Fig. 5. The remarkable control performance of the AR-DCFNC actually results from the online approximation ability of the DCFNN which is shown in Fig. 6, which shows that the DCFNN with 5 fuzzy rules guarantees convergent tracking errors.

VI. Conclusions

In this paper, we have proposed an adaptive robust dynamic constructive fuzzy neural control (AR-DCFNC) scheme for trajectory tracking of surface vehicles in the presence of system uncertainties and unknown time-varying disturbances. In the AR-DCFNC, system uncertainties and unknown dynamics can be identified online by a dynamic constructive fuzzy neural network (DCFNN) which is implemented by dynamically generating and pruning fuzzy rules according to the structure learning criteria. It has been further proven that the tracking errors of the AR-DCFNC control system are globally asymptotical stable. Finally, the simulation results demonstrate that the AR-DCFNC achieves remarkable performance of trajectory tracking with compact fuzzy rules.

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