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Granular Cognitive Maps Reconstruction

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***Abstract*—Cognitive Maps are abstract knowledge representation framework, suitable to model complex systems. Cognitive Maps are visualized with directed graphs, where nodes represent phenomena and edges represent relationships. Granular Cognitive Maps are augmented Cognitive Maps, which use knowledge granules as information representation model. Conceptually, GCMs originated as an extension of Fuzzy Cognitive Maps. The contribution presented in this paper is a methodology for Granular Cognitive Map reconstruction. The goal of the procedure is to construct a weights matrix - and thereby the GCM, which outputs best describe the phenomena of interest. The article addresses the conflict between generality and specificity of various Granular Cognitive Maps. Balance between generality and specificity is the most important architectural aspect of a model built with knowledge granules. A series of experiments illustrates, how various optimization techniques allow improvement in map's quality without a loss in map's precision.**

I. Introduction

Granular Computing is a general approach to knowledge representation and exploration, which is using granules such as sets, classes, clusters, groups, and intervals to build a flexible computational model for complex applications. The core ideas behind Granular Computing have been functioning in sciences for long time, usually under different notions. The name - Granular Computing does not refer to one particular algorithm or method. It is a widely understood approach, some may say paradigm, to information understanding.

Knowledge granules describe units and clusters of information with a given precision. The precision of modeled phenomena relies on system's architecture. Depending on application, we aim at less or more precise phenomena description. The main benefit from granular knowledge representation scheme is enhanced modeling ability. Knowledge granules describe information very flexibly, [16], [17].

In this paper we discuss Granular Cognitive Maps (GCM) - kind of Cognitive Maps, which uses knowledge granules as information representation model. Granular Cognitive Maps model phenomena and relation within phenomena. The contribution described in this paper is a methodology for Granular Cognitive Map reconstruction procedure. The proposed procedure constructs a weights matrix with input training data, without prior knowledge about the shape of this map. So far, the research on maps reconstruction has not been transfered to Granular Cognitive Maps. The original aspect discussed in this paper is a methodology for Granular Cognitive Map reconstruction.

The paper is structured as follows. Section II is a brief literature overview on Cognitive Maps. Section III covers the methodology of our approach. Section IV illustrates the developed methodology with a series of experiments. Finally, Section V covers conclusions and future research plans.

II. Literature Overview

In this section we present a brief literature review on the topic of Cognitive Maps and Granular Cognitive Maps.

Cognitive Maps are graph-alike knowledge modeling tool, comprising of nodes and edges between the nodes. Nodes represent phenomena, edges represent relations within phenomena. Cognitive Maps have been applied to modeling for the first time in 1976 by a political scientist, R. Axelrod, [1]. Originally, Cognitive Maps have been used to describe very simplistic systems. Primarily, relations between map's nodes were of only three different kinds: positive (+1), negative (-1) and none (0). Positive evaluation of a connection from node A to B means that an increase of the value in node A causes the increase in the value in node B. Such generic framework has been functioning in the sciences for over 10 years.

Major shift in the research on Cognitive Maps has occurred in 1986, when B. Kosko has published his paper on Fuzzy Cognitive Maps, [4]. FCMs are an extension of generic Cognitive Maps. The new model provided more complex framework for relationships modeling. In FCMs relations between map's nodes are expressed with real numbers from the $[-1, 1]$ interval. The value of a given connection corresponds to the degree of positive or negative relation between two nodes. The higher the absolute value of given connection, the stronger the relationship. 0 informs about lack of relation.

Exploration of FCM is performed with a set of activations and targets, expressed with real numbers from the $[0, 1]$ interval. The core element of each Cognitive Map, not only Fuzzy one, is the$n \times n$ weights matrix, which gathers connections between n map's nodes.

The model of FCMs, due to its attractive modeling capabilities, has been applied in several areas. Most important are time series prediction ([3], [13]) and classification ([7], [8]).

Subsequently Cognitive Maps have been adapted to model knowledge with the use of other information representation schemes, including Intuitionistic Cognitive Maps: [6] and Granular Cognitive Maps: [11], [12]. The latter model of Granular Cognitive Maps is present in sciences for relatively short

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time. GCMs join the benefits of Granular Computing with Cognitive Maps. Granular Cognitive Maps augment generic map connections to granular connections.

One of the most important topics in the research on Cognitive Maps is Cognitive Map construction. The goal of such procedure is to generate weights matrix suitable to model phenomena of interest. This topic has originated as an important aspect of applied modeling with FCMs. Before the discussed issue has appeared, Cognitive Maps' shapes were typically given by an expert or by a group of experts. Manual construction offers limited possibilities, as the quality of such map depends on the human factor.

Research in the area of FCM reconstruction resulted in several noteworthy approaches, including:

- Differential Hebbian Learning, [2],
- Nonlinear Hebbian Learning, [9],
- genetic strategies for FCM learning, [5], [14], [15],
- swarm intelligence-based FCM learning, [10].

Named methods aimed at FCMs weights matrix learning with the use of different optimization procedures.

The research on Granular Cognitive Maps, especially in the context of GCM reconstruction, is at relatively early stage, comparing to the research on FCMs. The paper [12] introduces the key concepts on knowledge mining with GCMs, but the authors focus on the switch between FCMs to GCMs and on adaptation of FCM-related methodology of map reconstruction to GCMs. In this paper we discuss an approach, in which the GCM is reconstructed without the knowledge of prior FCM describing training data.

III. METHODOLOGY

A. Granular Cognitive Maps

Granular Cognitive Maps use knowledge granules for information representation model. GCMs may be perceived as an augmentation of Fuzzy Cognitive Maps, where we use more abstract, flexible, and general knowledge granules.

GCM models phenomena of interest and relations between them, which are manifested by observed *causes* and *results*. Observed causes are presented to GCM, which should respond with granular results. Causes presented to a GCM are named *activations* and results are named *targets* or *goals*. Since perfect modeling is rather impossible, GCM *responses* are expected to cover as many results as possible.

GCM is characterized by its weights matrix $\mathbf{W}$. It is the crux of the GCM - weights describe connections between nodes in the map. Connections in GCM are represented with knowledge granules. Input information (activations) is scaled to the $[0, 1]$ interval, alike in FCMs. GCM responses are represented with knowledge granules.

Let us determine notation for Granular Cognitive Maps based on up-to-date research:

- n, N - numbers of phenomena and observations,
- $\mathbf{W}$ - square matrix of granular weights with n rows and n columns,
- $\mathbf{W}_{i\cdot}$, $\mathbf{W}_{\cdot j}$ - i-th row and j-th columns of $\mathbf{W}$,
- $\mathbf{w}_{ij}$ - item of $\mathbf{W}$ in i-th row and j-th column (single knowledge granule),
- X and G - matrices of (N) observations with n rows and N columns, called activations (X) and targets or goals (G), respectively,
- $\mathbf{Y}$ - matrix of granular GCM responses, n rows and N columns,
- $X_{i\cdot}$, $X_{\cdot j}$, $G_{i\cdot}$, $G_{\cdot j}$, $\mathbf{Y}_{i\cdot}$, $\mathbf{Y}_{\cdot j}$, x_{ij}, g_{ij}, $\mathbf{y}_{ij}$ - rows, columns and items in corresponding matrices X, G and $\mathbf{Y}$,
- the pair $X_{\cdot j}$ and $G_{\cdot j}$ is called (j-th) observation,
- $X_{\cdot j}$, $G_{\cdot j}$ $\mathbf{Y}_{\cdot j}$ are named (j-th) activation, target and response, respectively.

B. Modeling with Granular Cognitive Maps

Conceptually, GCMs are an augmentation of Fuzzy Cognitive Maps. In GCMs weights matrix contains granular connections. GCM is explored according to the formula:

$$\mathbf{Y} = f(\mathbf{W} \star X) \tag{1}$$

where boldface denotes just matrices of granular items. Observations $\mathbf{X}$, $\mathbf{G}$, weights $\mathbf{W}$ and responses $\mathbf{Y}$ are granules, e.g. intervals, triangular or parabolic sets based on fuzzy sets or their extensions (intuitionistic, balanced, second type etc.) and an operator $\star$ is a specific operator, applicable to chosen representation model of knowledge granules. f is a sigmoid mapping applied individually to elements of $\mathbf{W} \star X$:

$$f(z) = \frac{1}{1 + e^{-\tau z}} \tag{2}$$

with positive value of the parameter τ. In this study the value of τ is arbitrarily set to 2.5, based on experiments.

The switch from FCM to GCM is on the level of weights and with granules assumed to be intervals. In order to transform FCM to GCM we augment numeric weights matrix W to granular weights matrix $\mathbf{W}$. Since observations (activations and targets) are assumed to be still numerical, we keep normal font of their symbols: X and G. However, in consequence of weights granularity, GCM responses become granular as well and are denoted in boldface $\mathbf{Y}$. For single activation $X_{\cdot j}$, GCM's map response $\mathbf{Y}_{\cdot j}$ is:

$$\mathbf{Y}_{\cdot j} = f(\mathbf{W} \star X_{\cdot j}) \tag{3}$$

In the discussed model weights are assumed to be intervals included in the bipolar unit interval $[-1, 1]$. For given i-th row and j-th column of weight's matrix $\mathbf{W}$, weight $\mathbf{w}_{ij}$ is the interval denoted as $[w_{ij}^-, w_{ij}^+]$ and satisfies inequalities $-1 \leqslant w_{ij}^- \leqslant w_{ij}^+ \leqslant 1$. Therefore, we can decompose the granular weights matrix $\mathbf{W}$ into two numeric matrices W^- and W^+ of left and right ends of intervals.

Responses $\mathbf{Y}$ of a GCM modeled by Formula (1) are defined as intervals. Alike weight, GCM's response $\mathbf{y}_{ij}$ is the interval denoted as $[y_{ij}^-, y_{ij}^+]$ that satisfies inequalities $0 \leqslant y_{ij}^- \leqslant y_{ij}^+ \leqslant 1$. We can decompose the granular responses

$\mathbf{Y}$ into two numeric matrices Y^- and Y^+ of left and right limits of intervals. Namely, the i-th item $\mathbf{y}_{ij}$ of the response to j-th activation $X_{\cdot j}$ is the interval $\mathbf{y}_{ij} = [y_{ij}^-, y_{ij}^+]$ with ends defined as follows:

$$y_{ij}^- = f(W_{i\cdot}^- * X_{\cdot j}) = f\Big(\sum_{k=1}^{n} w_{ik}^- \cdot x_{kj}\Big)$$
$$y_{ij}^+ = f(W_{i\cdot}^+ * X_{\cdot j}) = f\Big(\sum_{k=1}^{n} w_{ik}^+ \cdot x_{kj}\Big) \qquad (4)$$

It is worth to notice that inequalities $0 \leqslant y_{ij}^- \leqslant y_{ij}^+ \leqslant 1$ are assured by Formula (4), nonnegative values of activations and source inequalities of weights $w_{kl}^- \leqslant w_{kl}^+$ for all $k, l \in [1, n]$.

C. Generality vs. specificity

For a Cognitive Map built on knowledge granules it is necessary to consider several important architectural issues. Knowledge granules have the ability to represent information in a very general fashion. This feature is on one hand very attractive - modeling capabilities of the system are widened. On the other hand, too general knowledge representation scheme is in conflict with precision of the model. The more general data representation, the less accurate model outputs and predictions.

Knowledge granules generalize information units. While constructing a GCM we have to resolve the conflict between generality and specificity. Map shall describe a system of phenomena as precisely as possible. At the same time, we want the model to represent and include certain variability - or in other words generality. The need for generality is not only, but also caused by imperfectness of information.

If we use interval representation for knowledge granules, balance between precision and generality is manipulated with lengths of intervals. The longer the interval, the more general given knowledge granule and in consequence the wider spectrum of phenomena it reflects. The shorter the knowledge granule interval, the higher precision.

In this paper we discuss weights granulation, which then propagates to responses as a secondary effect of their granulation. It is worth to underline that in discussed model, granules of map's responses are fully determined by weights granules and activations, c.f. Formulas (1), (3) and (4).

The simplest approach to interval-based granulation relies on setting the same length κ of all weights' granules, i.e. $\kappa = w_{ij}^+ - w_{ij}^-$, where $\mathbf{w}_{ij} = [w_{ij}^-, w_{ij}^+]$ for all $i, j \in [1, n]$. In such the case, generality/specificity is controlled only by the single parameter κ. Obviously, flexibility of such granularity architecture is low.

The approach applied in this paper allows constraint optimization of granules sizes. Granules lengths are flexible - to certain extent. Sum of all weights' granules cannot exceed fixed threshold. As a consequence, if we increase sizes of selected granules, lengths of other granules have to be simultaneously decreased. Moreover, we may add an upper limit on the length of a single granule. This outcome directly concerns granules of weights and indirectly (by propagation) granules of map's responses. Of course, our interest is in managing granularity of responses more than granularity of weights. However, in our model, granularity of responses is a secondary effect of granularity of weights. Therefore, weights granularity is directly controlled with analysis of aimed granularity of responses. In other words, in our approach we distinguish:

- local granularity and
- global granularity.

Local granularity admits variability of granules' sizes while global granularity keeps the whole system inside assumed average level of generality/specificity. Specifically, we assume that average length of weights' interval does not exceed a given value κ and we allow length of some intervals to be increased, but not exceed certain size, i.e. κ multiplied by some constant ϱ. This is described by the following conditions for given matrix of weights' granules $\mathbf{W}$:

$$\max\left\{ w_{ij}^+ - w_{ij}^- \,:\, i, j \in [1, n] \right\} \;\leqslant\; \varrho \cdot \kappa$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n}(w_{ij}^+ - w_{ij}^-) \;\leqslant\; n^2 \cdot \kappa \qquad (5)$$

where $\mathbf{w}_{ij} = [w_{ij}^-, w_{ij}^+]$ and κ and ϱ are constants as above.

D. Quality of modeling

In order to assess the quality of a GCM we calculate how well granular map responses cover targets.

In case of FCMs quality of modeling is defined by distance (error) between targets and responses. In contrast, *coverage* of targets by map responses is a basic criterion to evaluate the quality of the GCM. Meaning of covering a single target by a corresponding response depends on granules definition. There are two basic approaches to calculate coverage of the whole targets:

- step-alike coverage,
- gradual coverage.

In step-alike coverage approach, the final coverage factor is increased by a constant value each time a target value *falls* into granule of corresponding response. Otherwise, the final coverage factor is not increased. Therefore, step-alike coverage has locally binary character. In contrast, gradual coverage approach assumes locally *smooth* coverage change.

In this study we assume that, for interval-based granules, a target value is covered by a corresponding map's response, if and only if the target belongs to the interval. Namely, the target g_{ij} is covered by the response Y_{ij} if and only if the following inequalities hold $y_{ij}^- \leqslant g_{ij} \leqslant y_{ij}^+$.

We can distinguish two most important kinds of coverages:

- weak coverage,
- strict coverage.

For weak coverage, the final coverage factor is increased by a constant value, say 1, each time a target g_{ij} falls into the corresponding granular map response $\mathbf{y}_{ij} = [y_{ij}^-, y_{ij}^+]$. Let us define local coverage factor cvg_{ij} as follows:

$$cvg_{ij} = \begin{cases} 1 & y_{ij}^- \leqslant g_{ij} \leqslant y_{ij}^+ \\ 0 & otherwise \end{cases} \qquad (6)$$

then, the coverage factor CVG_j for j-th observation is defined by the formula:

$$CVG_j = \sum_{i=1}^{n} cvg_{ij} \tag{7}$$

and finally, the next formula defines the unscaled weak coverage factor:

$$CVG_{weak} = \sum_{j=1}^{N} CVG_j = \sum_{j=1}^{N}\sum_{i=1}^{n} cvg_{ij} \tag{8}$$

Strict coverage factor CVG_{strict} counts observations, for which all target values are covered by granules of corresponding map's response. This factor is increased if and only if for given j-th observation the coverage factor CVG_j equals to n. The formula for unscaled strict coverage is as follows:

$$CVG_{strict} = \sum_{j=1}^{N} \left\lfloor \frac{CVG_j}{n} \right\rfloor \tag{9}$$

Unscaled weak coverage can assume values from $[0, n \cdot N]$ while unscaled strict coverage assumes values from $[0, N]$. In order to represent coverages on a comparative and scale $[0, 1]$ we have to compute *mean* values of these factors dividing them by $n * N$ and by N, respectively:

$$MWC = \frac{CVG_{weak}}{N \cdot n} \qquad MSC = \frac{CVG_{strict}}{N} \tag{10}$$

E. Reconstructing Granular Cognitive Maps

The objective of the article is to present developed approach to GCM reconstruction. The goal of this procedure is to produce granular weights' matrix that describes phenomena of interest most faithfully. We use the term "reconstruction" on purpose. We treat the real system (phenomena and relations) as the ideal model and the goal is to reconstruct it with GCM.

GCM reconstruction procedure restores granular weights matrix $\mathbf{W}$ and chosen granularity parameters through maximization of coverage. Optimization procedure aims at fitting weights and optionally granularity parameters so that the map output covers the greatest number of targets. Figure 1 illustrates the our methodology for GCM reconstruction and validation. Three synthetic datasets are used:

- train not distorted,
- train distorted,
- test.

Train not distorted is the ideal dataset, which we never have when we deal with real data. Perfect information in nature gets distorted. Usually we distinguish two types of distortions: random and systematical. Systematical distortions are generated for example by malfunctioning measuring devices. Random distortions happen, e.g. by human mistake.

Therefore, in the experiments on synthetic data we disturb perfect data and we use distorted train dataset for model training. Distorted train dataset contains (on purpose) a lot of 0's and 1's. These values cannot be covered by map responses, because of asymptotic properties of the sigmoid function. It will be shown, that even if coverage on distorted data is not excellent, the model describes very well the perfect data.

On the input we have:

- W - initial FCM weights matrix.
- X - activations matrix with input data for training.
- TGT_D - distorted targets.

Weights matrix for the FCM gets augmented to granular weights matrix $\mathbf{W_{fin}}$. Such augmentation allows to estimate base values of coverages prior to the map reconstruction. Coverage before GCM reconstruction is the baseline coverage that we want to improve.

We adjust granular weights alone or with granularity parameters to obtain the highest coverage. Optimization procedure is based on Particle Swarm Optimization. As a result we receive new set of granular weights and optionally new values of granularity parameters ϵ - length of knowledge granules and γ - symmetry parameter. Both ϵ andγ are $n \times n$ matrices with separate values of parameters fro each granular weight $\mathbf{w_{ik}}$.

Quality of the map is assessed on both training and testing datasets. We evaluate how reconstructed GCM (comprising of weights and granularity parameters) covers data. In Section IV we present a comparative overview how optimization of different elements of a GCM improves coverage.

IV. Reconstructing Granular Cognitive Maps

In this section we discuss a series of experiments conducted according to the methodology illustrated in Figure 1. We are interested in improvement of coverage. Therefore, we compare baseline coverage (prior to optimization) with coverage after the GCM reconstruction. Note that the baseline coverage is not just random, it is a coverage provided by an augmented FCM. The research presented in paper [12] proposes an augmentation from FCM to GCM. The baseline coverage, which we try to improve with our procedure is coverage discussed in the aforementioned paper. Procedure proposed in this article allows to improve the coverage without any loss in map's specificity.

The optimization concerns intervals representing knowledge granules of weights. Maximal possible length of the interval is 2 (weights can assume values from the $[-1, 1]$).

We discuss GCMs built on several different levels of specificity. Figures 3, 2 and 4 present coverage versus length of intervals and symmetry of knowledge granules. Values of ϵ and γ for GCMs prior to optimization are directly corresponding to OX and OY axes. For GCMs after reconstruction, values of the two parameters are a subject of constraints.

When we optimize granularity parameter ϵ we allow knowledge granules intervals to vary. Moreover, at all time sum of all knowledge granules for weights has to be smaller or equal to $n^2 * \epsilon$. As a result of the optimization procedure we increase coverage and maintain the same balance between generality and specificity. In other words we increase the quality of the coverage provided by the map without a loss in map's precision. The aforementioned restrictions on interval lengths were discussed in greater detail in Section III, Formula (5).

For each optimization strategy we have conducted 100 experiments for different values of γ and different restrictions

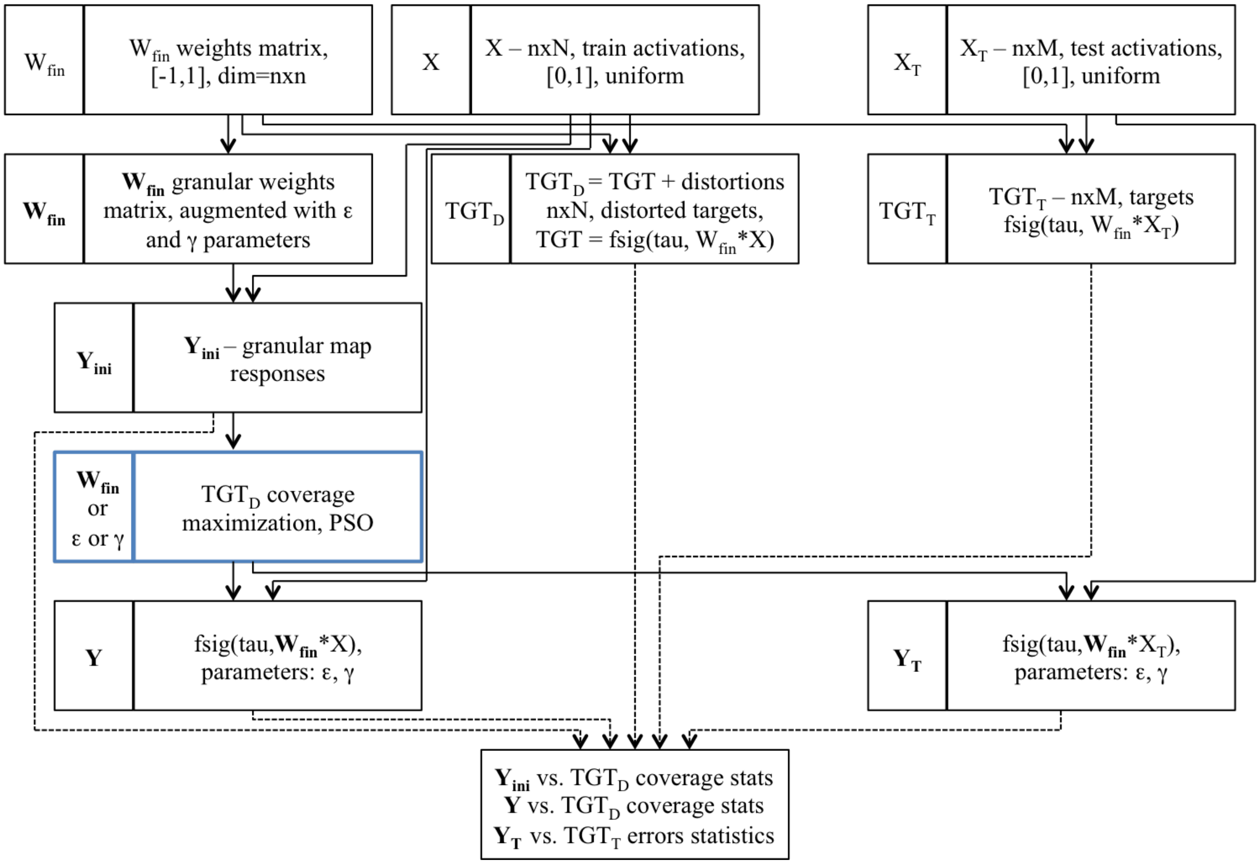

Fig. 1. Granular Cognitive Map reconstruction and validation procedure.

on ϵ. In each case we reconstructed GCMs with γ from 0.1 to 1 by 0.1 and restrictions on the ϵ also from 0.1 to 1 by 0.1.

GCMs have been trained on distorted train datasets. Quality of the procedure is assessed with mean weak coverage (MWC) and mean strict coverage (MSC) - see Formulas (10). Results are presented numerically in tables and visually in figures. We do not expect to achieve good results on the distorted training datasets. It is because of 0s and 1s, which cannot be covered by map responses. MWC and MSC on not distorted dataset, which is the "perfect" data, shall be satisfying. High coverages for not distorted train data means that the model describes well the real system. We expect that for not distorted train datasets, with the growth of the generality factor the coverage will increase. Finally, we assess quality of the procedure on test datasets, which are not related to train datasets in any way. Test datasets are synthetic random values, so we expect that the coverage will not be smooth.

Table I contains mean weak and mean strict coverages for 100 GCMs before their reconstruction procedure.

TABLE I. BASELINE MEAN COVERAGES (BEFORE RECONSTRUCTION).

	train not distorted	train distorted	test
MWC	0.700	0.415	0.644
MSC	0.238	0.003	0.085

Discussed GCM reconstruction schemes are based on:

- adjustment of weights (Subsection IV-A),
- adjustment weights and ϵ (Subsection IV-B),
- adjustment of weights, ϵ and γ (Subsection IV-C).

In Figures 2, 3, and 4 we illustrate how named optimization strategies improve coverage with maintaining given maps' balance between specificity and generality. Each Figure shows baseline coverage (in paler color) against improved coverage. In addition, Tables II, III, and IV inform about numerical differences in MWC and MSC for the whole series of 100 experiments with the respective three strategies of optimization. With each listed procedure we were able to improve the coverage. There are no qualitative differences between obtained results. Differences are quantitative.

A. *Optimization of granular weights*

In this subsection we focus on GCM reconstruction strategy that readjusts weights matrix $\mathbf{W_{fin}}$. We optimize (move) centers of weights' granules. In this approach to GCM reconstruction the two granularity parameters remain at a fixed level. There is the same ϵ and the same γ for all weights. Figure 2 illustrates weak and strict coverage before and after optimization of weights matrix.

Such strategy does not take full advantage of the assumed granular information representation model, but it is successful. In each case coverage has improved. For the ideal train dataset coverage reaches 1. It is most difficult to achieve high coverage on distorted train dataset. Distorted train set contains 0s and 1. Coverage characteristics on test dataset, as we have expected, is less smooth. Nevertheless, in each case there is improvement in coverage after weights optimization.

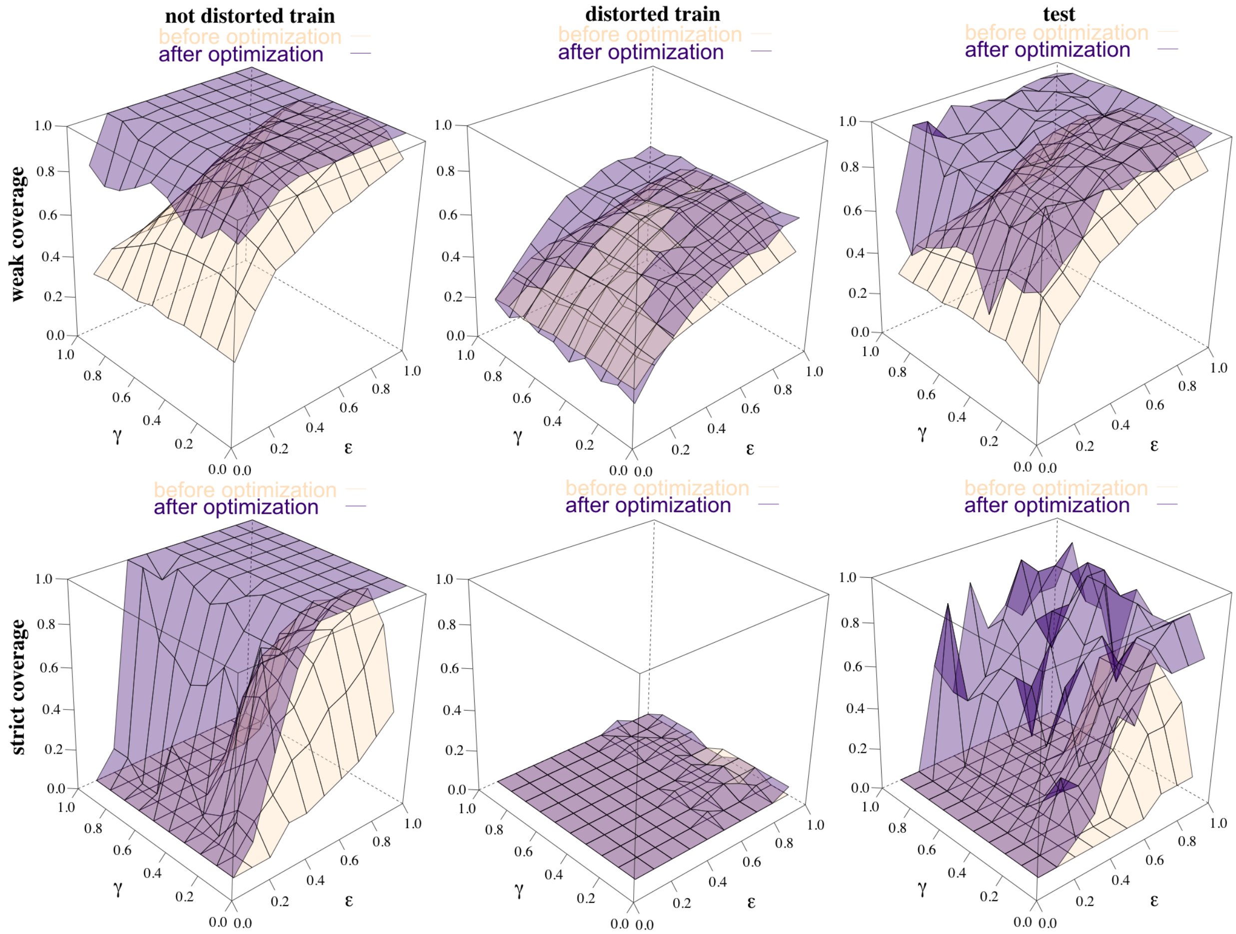

Fig. 2. Weak (top row) and strict (bottom row) mean coverage on not distorted train dataset (first column), distorted train dataset (second column) and test dataset (third column) before and after optimization of granular weights.

Table II gathers means of strict and weak coverages obtained for 100 GCMs trained during the course of this experiment.

TABLE II. MWC AND MSC AFTER ADJUSTMENT OF WEIGHTS.

	train not distorted	train distorted	test
MWC	0.966	0.443	0.866
MSC	0.764	0.008	0.467

Coverage heavily depends on the choice of ϵ. The larger the ϵ, the greater the coverage. Without optimization we are unable to achieve very high coverage. Optimization procedure allows to produce granular responses, which without a total decrease in model specificity cover targets very well. In conclusion, the proposed methodology of GCM reconstruction performs well.

B. Simultaneous optimization of granular weights and ϵ

In this subsection we combine two optimization subjects. Granular Cognitive Map is reconstructed through a simultaneous optimization of granular weights and ϵ matrix. Quality of reconstructed maps is assessed by weak and strict coverage statistics plotted in Figure 3.

Table III presents mean weak and strict coverages for 100 GCMs reconstructed by optimization of $\mathbf{W}$ and ϵ.

TABLE III. MEAN COVERAGES AFTER WEIGHTS AND ϵ ADJUSTMENT.

	train ND	train D	test
MWC	0.975	0.4532	0.879
MSC	0.778	0.010	0.449

The reconstructed GCM is of better quality, than before. Coverage on the ideal dataset for all values of γ reaches 1 even for relatively small sizes of allowed knowledge granules (when $\sum_{i=1}^{n}\sum_{j=1}^{n}\epsilon_{ij} \leqslant (n^2 * 0.4)$). This is quite rigorous restriction, because the maximal length of the ϵ_{ij} is 2. We are able to build a map that maintains an upstanding balance between specificity and generality. As a result of optimization we significantly increased coverage and maintained the same generality of the whole map.

C. Simultaneous optimization of granular weights, ϵ and γ

Finally, we present improvement in coverage after we simultaneously optimize weights and both granularity parameters: ϵ and γ.

Figure 4 shows that coverage after optimization of all relevant GCM elements has significantly increased. The improvement in coverage is the highest. Nevertheless, results are only slightly better, than for the previous optimization scheme (weights and ϵ). At the same time, computational cost has

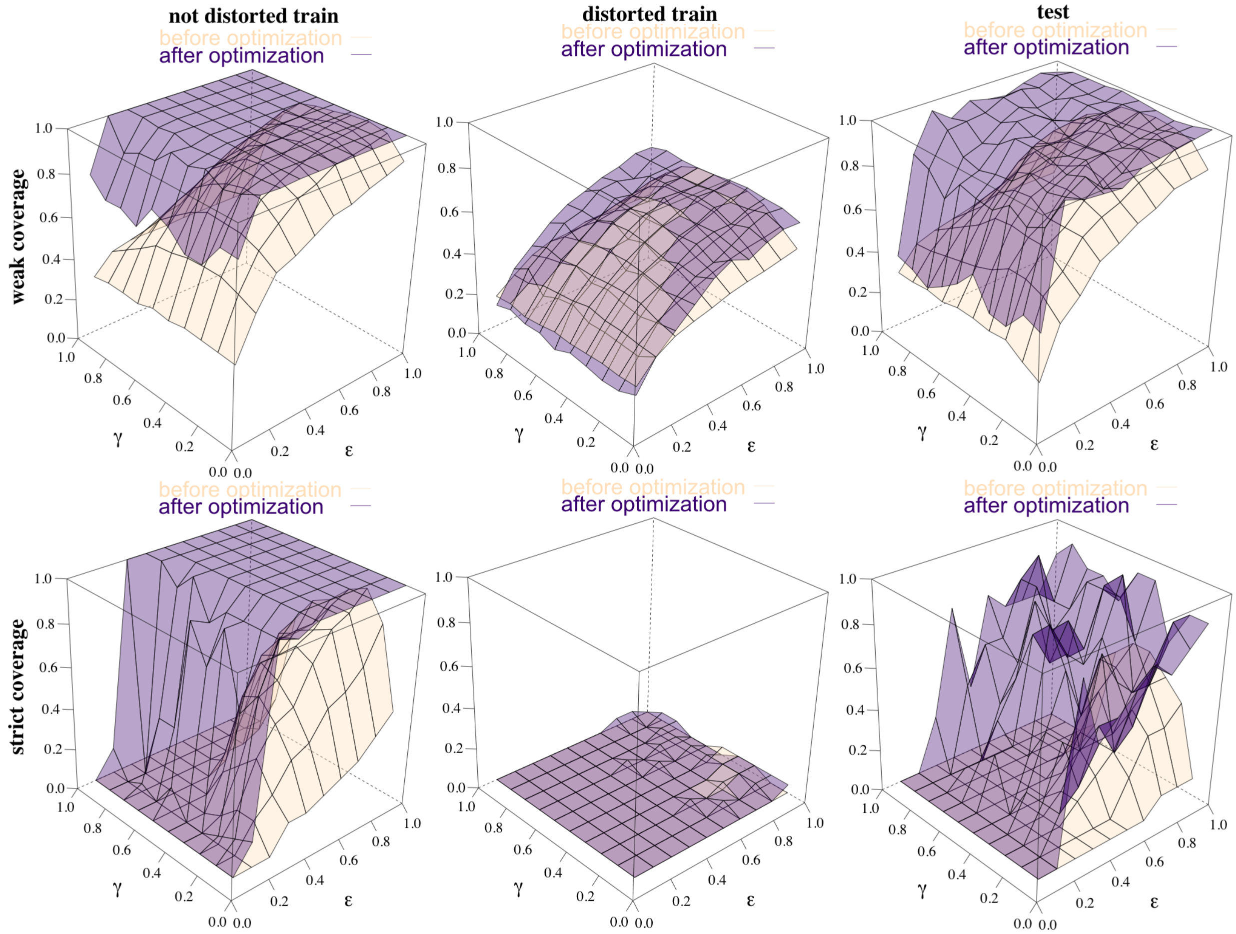

Fig. 3. Improvement in coverage after adjustment of weights and ϵ.

increased. We parallelized computations, but still total time required to calculate coverage for Figure 4 was over 24 hours.

Moreover, after analysis of these results we came to a conclusion that for real life data and interval-based granules representation model we will keep a fixed level of γ close to 0.5, the exact value will be selected with crossvalidation. Optimization of γ is more justifiable for other granules representation models and for different strategies of optimization.

Table IV presents averaged weak and averaged strict coverages for 100 GCMs after reconstruction procedure.

TABLE IV. MEAN COVERAGES AFTER RECONSTRUCTION (W, ϵ, γ).

	train not distorted	train distorted	test
MWC	0.972	0.445	0.874
MSC	0.783	0.011	0.487

In conclusion, the proposed procedure of Granular Cognitive Map reconstruction performs well. We have shown that, produced models faithfully describes the data.

V. CONCLUSION

In the article a methodology for Granular Cognitive Map reconstruction has been introduced. The goal of the procedure is to produce granular weights matrix and relevant granularity parameters that describe phenomena of interest most flexibly.

We have shown that assumed granular knowledge representation model allows to represent aggregates of knowledge with a desired balance between precision and generality. We have introduced restrictions on two levels of granularity: global and local. Restrictions, considered as local concern single knowledge granules. Restrictions at the global level concern the whole model - the whole GCM.

In this paper intervals represent knowledge granules. Granularity is introduced on the level of weights and is propagated to map's responses. Granular map responses are intervals. We investigate how map responses cover targets. As a result of GCM reconstruction we expect map responses to cover targets to greater extent without a loss in the precision. We have proposed three strategies for GCM reconstruction, which allow to achieve this goal. Theoretical analysis of the discussed issues was supported with a series of experiments.

Proposed procedures perform very well. In each case baseline coverage has improved. The more elements of the GCM we readjust, the better quality of the map we get.

The main contribution discussed in this paper is Granular Cognitive Map reconstruction procedure that allows to build from scratch a good GCM, that maintains fixed balance between specificity and generality and covers data very well. In future research authors plan to apply developed methodology to describe real-life datasets, for example time series.

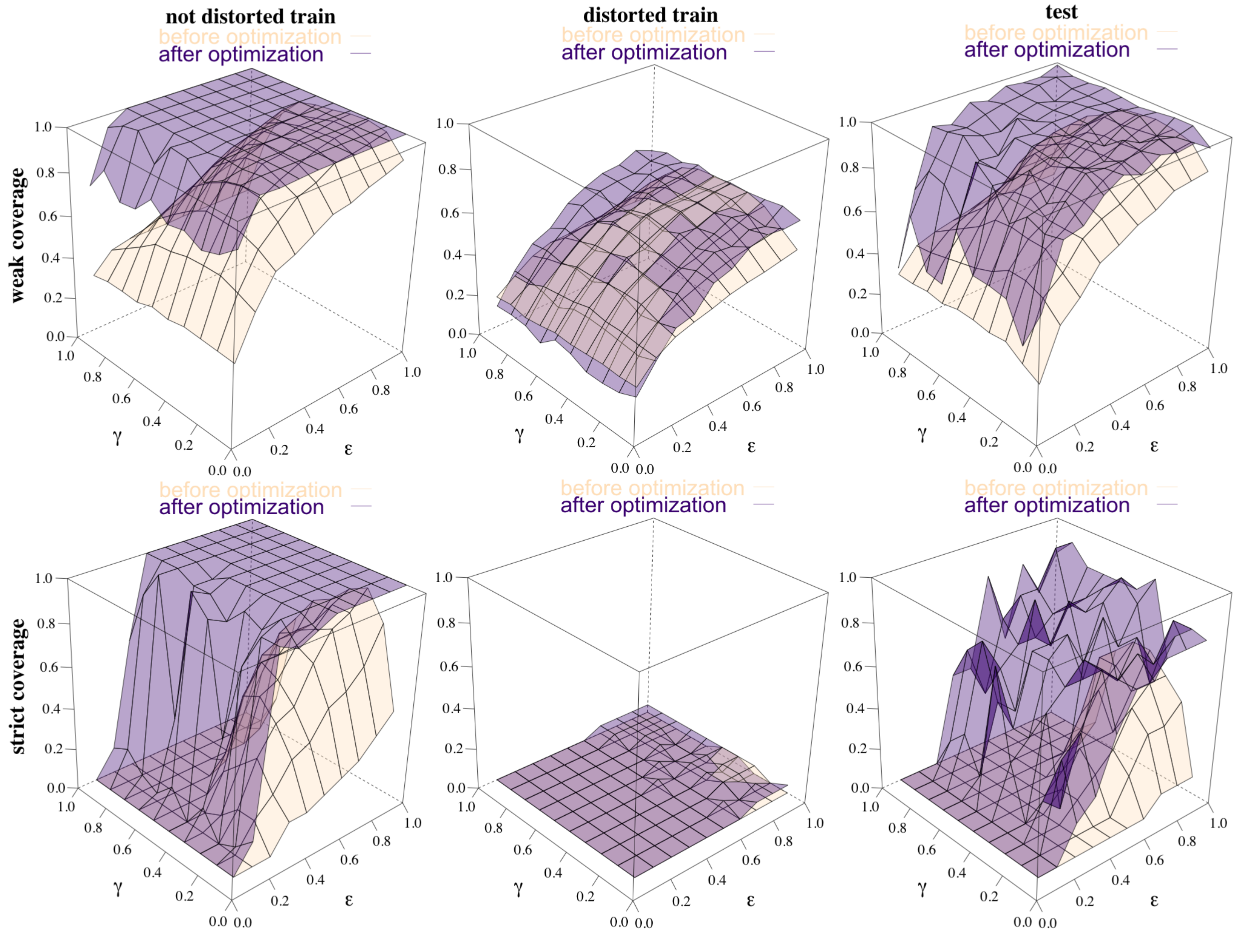

Fig. 4. Baseline MWC and MSC coverages (light color) against improved MWC and MSC coverages (darker color) on not distorted, distorted and test datasets. Optimization concerned weights, ϵ, and γ.

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