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Uniformly Strongly Prime Fuzzy Ideals

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***Abstract*—In this paper we define the concept of uniformly strongly prime fuzzy ideal for associative rings with unity. This concept is proposed without dependence of level cuts. We show a pure fuzzy demonstration that all uniformly strongly prime fuzzy ideals are a prime fuzzy ideal according to the newest definition given by Navarro, Cortadellas and Lobillo [1] in 2012. Also, some properties about fuzzy strongly prime radical and their relations with Zadeh's extension are shown.**

I. Introduction

In 1971, Rosenfeld [2] introduced fuzzy sets in the realm of group theory and formulated the concept of *fuzzy subgroups* of a group. Since then, many researchers have been engaged in extending the concepts/results of abstract algebra to a broader framework of the fuzzy setting. Thus, in 1982, Liu [3] defined and studied fuzzy subrings as well as fuzzy ideals. Subsequently, Liu himself (see [4]), Mukherjee and Sen [5], Swamy and Swamy [6], and Zhang Yue [7], among others, fuzzified certain standard concepts/results on rings and ideals. Those works were further carried out by Kumar [8] and [9].

In 1973, Formanek [10] proved that: "If D is a domain and G is a suitable free product of a group, then the group ring DG is primitive". In the same year, Lawrence in his master's thesis showed that a generalization of Formanek's result was possible, in which *the domain could be replaced by a prime ring with a finiteness condition* called *strongly prime*. Although the condition of strongly primeness was used for primitive group rings, it became more interesting. As a consequence, in 1975, Lawrence and Handelman [11] began to study the properties of strongly prime rings and then many results were discovered, for example: "every prime ring may be embedded in a strongly prime ring; all strongly prime rings are nonsingular; and only the Artinian strongly prime rings have a minimal right ideal". The class of **uniformly strongly prime rings** was proposed by Lawrence in [11] and studied by Olson [12].

As it is known, ideals are the main object in the investigation of rings. The same occurs to the fuzzy ring theory. Prime ideals can provides important information about fuzzy rings. In order to investigate this kind of ideal, the notion of *prime fuzzy ideals* was introduced, but this definition is not suitable for noncommutative rings. In 2012, Navarro, Cortadellas and Lobillo [1] exposed this problem. They proposed a definition for prime fuzzy ideal which is compatible with noncommutative rings and is α-cut (or level cut) independent, but compatible with α-cuts (see definition 12 and proposition 3).

If we choose to introduce a new fuzzy concept based on α-cuts it may impose some restrictions to the fuzzy setting or fuzzy results may be likely to depend on crisp results. Also, it reveals that any advantage of extension from classical to fuzzy setting and pure fuzzy results may be not detected. In 2013, Bergamaschi and Santiago [13] proposed **Strongly Prime Fuzzy(SP)** ideals for commutative and noncommutative rings with unity, and investigated their properties, but this approach is still based on α-cuts.

This paper goes one step further. Here we investigate some properties of a fuzzy version of *uniformly strongly prime ideals (USPI)*. Unlike the method used to define strongly prime fuzzy ideals (SPFI) in [13], here we provide a pure fuzzy definition for (USPI). In other words, this definition **is not based on α-cuts**. This definition was possible because the crisp concept of USPI is more suitable to translate it for the fuzzy environment. Thus, it was possible to discover some pure fuzzy results for such structure. For example, the demonstration of corollary 9 shows that all uniformly strongly prime fuzzy ideals are a prime fuzzy ideal in accordance with the newest definition of prime fuzzy ideal given in [1].

This paper has the following structure: section 2, which not only provides an overview about the ring and fuzzy ring theory, but it also contains the definition and results of (uniformly) strongly prime rings and (uniformly) strongly prime ideals; section 3, which has the main results introduced in Bergamaschi and Santiago [13]; section 4, which provides a definition of uniformly strongly prime fuzzy ideal. Also, it is proved that this definition is compatible with α-cuts; section 5, which introduces the notion of radical strongly prime and shows its relation with Zadeh's [14] extension; section 6, which provides the final remarks.

II. Preliminaries

This section explains some definitions and results that will be required in the next section. All rings are associative with identity and usually denoted by R.

Definition 1: [15] A prime ideal in an arbitrary ring R is any proper ideal P such that, whenever I, J are ideals of R with[1] $IJ \subseteq P$, either $I \subseteq P$ or $J \subseteq P$. Equivalently P is prime whenever[2] $xRy \subseteq P$ for some $x, y \in R$, then $x \in P$

[1] $IJ = \{x : \ x = ij, i \in I, j \in J\}$.
[2] $xRy = \{xry : \ r \in R\}$.

or $y \in P$. A *prime ring* is a ring in which the ideal (0) is a prime ideal.

Note that a prime ring must be nonzero. Also, in commutative rings, the definition above is equivalent to: if $ab \in P$, then $a \in P$ or $b \in P$. Thus, if P has the latter property, then it is called completely prime. In arbitrary rings, every *completely prime* is prime, but the converse is not true(see [13])

In some papers, *completely prime* means *strongly prime*, but *those concepts are different* in the context of non-commutative rings. So, every completely prime ideal is strongly prime, but the converse is not true.

The next definitions are required to build up the concept of strongly prime ideal. We recall that one of the basic concepts in algebraic geometry are *algebraic varieties via rings of functions*. The important part in this theory is a correspondence between certain ideals and subvarieties that arise from annihilation. For example, if $g(x) = 0$ or $ax = 0$, we say that x has been annihilated by g or a.

Definition 2: [11] Let A be a subset of a ring R. The right annihilator of A is written as follows $An_r(A) = \{r \in R : \ Ar = (0)\}$.

If A is a right ideal of a ring R, then $An_r(A)$ is an ideal of R.

Definition 3: [11] A ring R is called *Right Strongly Prime* if for each nonzero $x \in R$ there exists a finite subset F_x of R such that the right annihilator of xF_x is zero.

The set F_x is called the right insulator of x. Handelman and Lawrence worked on rings with identity. However, Parmenter, Stewart and Wiegandt [16] have shown that the definition 3 for associative rings is equivalent to:

Definition 4: A ring R is said to be *right strongly prime* if each nonzero ideal I of R contains a finite subset F which has right annihilator zero.

In other words, a right insulator in a ring R is a finite subset $F \subseteq R$ such that $Fr = 0 \quad r \in R$, implies $r = 0$. The ring R is said to be right strongly prime if every nonzero ideal (two-sided) contains an insulator. The left strongly prime is defined analogously. Handelman and Lawrence [11] provided an example to show that these two concepts are distinct.

From this point forward, we will call **right strongly prime** ideals shortly by **strongly prime** (SP).

Example 1: If I is an ideal in a Field F, then $I = (0)$ or $I = F$. Thus, if $I \neq (0)$, then $1 \in I$ and $An_I(1) = (0)$. Therefore F is strongly prime.

It is not hard to prove that Z_n the integers mod n (n not prime) is not strongly prime.

Definition 5: An ideal I of a ring R is strongly prime if R/I is strongly prime ring.

It is not hard to prove that an ideal I of a ring R is strongly prime iff for every $x \in R - I$ there exists a finite subset F of R such that $r \in I$ whenever $r \in R$ and $xFr \subseteq I$.

Example 2: If p is a prime number, then pZ is a strongly prime ideal in the ring of integers Z. In fact, Z/pZ is a field (see the last example).

Proposition 1: [11] Let R be a ring. If P is a strongly prime ideal, then P is a prime ideal.

Definition 6: A ring is a *bounded right strongly prime of bound n*, denoted $SP_r(n)$, if each nonzero element has an insulator containing no more than n elements and at least one element has no insulator with fewer than n elements.

Definition 7: A ring is called **Uniformly Right Strongly Prime** if the same insulator may be chosen for each nonzero element.

Note that every uniformly right SP ring is $SP_r(n)$, since each insulator must be a finite set. Olson [12] showed that the concept of uniformly SP is two-sided and a ring R is uniformly strongly prime iff there exists a finite subset $F \subseteq R$ such that for any two nonzero elements x and y of R, there exists $f \in F$ such that $sfy \neq 0$. Equivalently R is uniformly SP iff there exists a finite subset $F \subseteq R$ such that $xFy = 0$ implies $x = 0$ or $y = 0$. Moreover, in the next Lemma, Olson [12] gives us some equivalent conditions for uniformly SP:

Lemma 1: [12] The following are equivalent:

i) R is uniformly SP;

ii) There exists a finite subset $F \subseteq R$ such that $xFy = 0$ implies $x = 0$ or $y = 0$, where $x, y \in R$;

iii) For every $a \neq 0, a \in R$, there exists a finite set $F \subset (a)$ such that $xFy = 0$ implies $x = 0$ or $y = 0$, where $x, y \in R$;

vi) For every $a \neq 0, a \in R$ there exists a finite set $F \subset (a)$ such that $xFx = 0$ implies $x = 0$, where $x \in R$;

v) For every ideal $I \neq 0$, there exists a finite set $F \subset I$ such that $xFy = 0$ implies $x = 0$ or $y = 0$, where $x, y \in R$;

vi) For every ideal $I \neq 0$, there exists a finite set $F \subset I$ such that $xFx = 0$ implies $x = 0$, where $x \in R$;

vii) For every $a \neq 0, a \in R$ there exists a finite set $F \subset R$ such that $xFaFx = 0$ implies $x = 0$, where $x \in R$;

viii) For every $a \neq 0, a \in R$ there exists a finite set $F \subset R$ such that $xFaFy = 0$ implies $x = 0$ or $y = 0$, where $x, y \in R$.

Clearly, if R is a field, then R is uniformly strongly prime ring, where $F = \{1\}$.

Definition 8: An ideal I of a ring R is uniformly strongly prime if R/I is uniformly strongly prime ring.

As Z/p (integers mod p) is a field (p prime), then we have $Z/\langle p\rangle$ unif. strongly prime ring. Thus, $\langle p\rangle = \{x \in Z : \ x = pq\}$ is unif. strongly prime ideal in Z.

The next result provide another characterization of uniformly strongly prime ideal.

Proposition 2: **An ideal I of a ring R is uniformly strongly prime iff there exists a finite set $F \subseteq R$ such that $xFy \subseteq I$ implies $x \in I$ or $y \in I$, where $x, y \in R$.**

Proof: If I is uniformly stronlgy prime, then there exists $F_* \subseteq R/I$ a finite set according to last definition. Let $F \subseteq R$ a finite set such that $\bar{F} = F_*$. Suppose $xFy \in I$, then $\bar{x}\bar{F}\bar{y} = 0$. As $\bar{F} = F_*$ and R/I is uniformly strongly prime, we have $\bar{x} = 0$ or $\bar{y} = 0$. Therefore, $x \in I$ or $y \in I$. On the other side, let $F_* = \bar{F}$. If $\bar{x}F_*\bar{y} = 0$, then $xFy \subseteq I$ and $x \in I$ or $y \in I$. Thus, $\bar{x} = 0$ or $\bar{y} = 0$. ■

Definition 9 (Zadeh's Extension): Let f be a function from set X into Y, and let μ be a fuzzy subset of X. Define the fuzzy subset $f(\mu)$ of Y in the following way: For all $y \in Y$,

$$f(\mu)(y) = \begin{cases} \vee\{\mu(x) : \ x \in X, f(x) = y\}, \\ \quad \text{if } f^{-1}(y) \neq \emptyset \\ \\ 0, \quad \text{otherwise.} \end{cases}$$

If λ is a fuzzy subset of Y, we define the fuzzy subset of X by $f^{-1}(\lambda)$ where $f^{-1}(\lambda)(x) = \lambda(f(x))$.

Definition 10 (Level subset): Let μ be any fuzzy subset of a set S and let $\alpha \in [0, 1]$. The set $\{x \in X : \ \mu(x) \geq \alpha\}$ is called a *level subset* of μ which is symbolized by μ_α.

Clearly, if $t > s$, then $\mu_t \subseteq \mu_s$.

Definition 11: A fuzzy subset I of a ring R is called a fuzzy ideal of R if for all $x, y \in R$ the following requirements are met:

1) $I(x - y) \geq I(x) \wedge I(y)$;

2) $I(xy) \geq I(x) \vee I(y)$.

Theorem 2: [17] A fuzzy subset I of a ring R is a fuzzy ideal of R iff the level subsets I_α, ($\alpha \in [0, 1]$), are ideals of R.

Definition 12: [1] Let R be a ring with unity. A non-constant fuzzy ideal $P : R \longrightarrow [0, 1]$ is said to be prime if for any $x, y \in R$, $\bigwedge P(xRy) = P(x) \vee P(y)$.

Proposition 3: [1] Let R be an arbitrary ring with unity and $P : R \longrightarrow [0, 1]$ be a non-constant fuzzy ideal of R. The following conditions are equivalent:

(i) P is prime;

(ii) P_α is prime for all $P(1) < \alpha \leq P(0)$;

(iii) R/P_α is a prime ring for all $P(1) < \alpha \leq P(0)$;

(iv) For any fuzzy ideal J, if $J(xry) \leq P(xry)$ for all $r \in R$, then $J(x) \leq P(x)$ or $J(y) \leq P(y)$.

Here we stop to present some of the previous results from literature in Algebra and Fuzzy Algebra. In what follows we show some of the advances proposed by the authors in the field of Fuzzy Algebra.

III. Strongly Prime fuzzy ideals

The next results were proved by Bergamaschi and Santiago in [13].

Definition 13: (Strongly prime fuzzy ideal) Let R be an arbitrary ring with unity. A non-constant fuzzy ideal $P : R \longrightarrow [0, 1]$ is said to be strongly prime iff P_α is strongly Prime for any $P(1) < \alpha \leq P(0)$.

Theorem 3: Every strongly prime fuzzy is prime fuzzy.

The converse of this theorem is not true. See the next example:

Example 3: Let R be the ring of 2×2 over real numbers. Consider the fuzzy ideal:

$$P(x) = \begin{cases} 1 & if\ x\ is\ zero\ matrix, \\ 0 & otherwise \end{cases}$$

P is prime fuzzy, since $P_\alpha = \{0\}$ for $P(1) < \alpha \leq P(0)$. Nevertheless, $P_\alpha = \{0\}$ is not strongly prime. Indeed, let $X = Y = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Observe that $XP_\alpha Y \subseteq P_\alpha$ and $XY \in P_\alpha$, but $X \notin P_\alpha$.

Theorem 4: Let R be a finite ring with unity. P is a strongly prime fuzzy iff P is prime fuzzy.

The next two results show that Zadeh's extension preserves prime fuzzy and strongly prime fuzzy whenever f is an isomorphism.

Proposition 4: Let $f : R \longrightarrow S$ be an isomorphism of rings. If P is a prime fuzzy ideal of R, then $f(P)$ is a prime fuzzy ideal of S.

Theorem 5: Let $f : R \longrightarrow S$ be an isomorphism of rings. If P is a Strongly Prime fuzzy ideal of R, then $f(P)$ is a strongly prime fuzzy ideal of S.

Proposition 5: Any strongly prime fuzzy ideal contains a minimal strongly prime fuzzy ideal.

Proposition 6: Any strongly prime fuzzy ideal contains properly another strongly prime fuzzy ideal.

Theorem 6: Let R be a ring with unity. Any fuzzy maximal ideal is strongly prime fuzzy ideal.

IV. Uniformly Strongly Prime Fuzzy Ideals

In this section we fuzzify (to give the definition of) the uniformly strongly prime ideal and show that the resulting definition, although α-cut independent is compatible with α-cuts.

Definition 14: Let R be an associative ring with unity. A non-constant fuzzy ideal $I : R \longrightarrow [0, 1]$ is said to be **uniformly strongly prime fuzzy ideal** if there exists a finite subset F such that $\bigwedge I(xFy) = I(x) \vee I(y)$, for any $x, y \in R$. If F is not a finite set, then we say that I is **almost uniformly strongly prime fuzzy ideal**.

Example 4: Let $R = Z$ be the ring of integers and $I(x) = \begin{cases} 1 & if\ x = 0 \\ 0 & otherwise \end{cases}$.

Let $F = \{1\}$. Thus, we have $\bigwedge I(xFy) = I(x) \vee I(y)$ for any $x, y \in R$.

Note that in the last example we have $I_\alpha = (0)$ for all $I(1) < \alpha \leq I(0)$. By the Lemma 1.ii) I_α is a unif. strongly prime ideal.

Proposition 7: **If I is a uniformly strongly prime fuzzy ideal of R, then I_α is uniformly strongly ideal of R for all $I(1) < \alpha \leq I(0)$.**

Proof: Suppose I is a uniformly strongly fuzzy ideal. For any I_α, let F be a finite set in I such that $xFy \subseteq I_\alpha$. Hence, $I(x) \vee I(y) = I(xFy) \geq \alpha$ implies $x \in I_\alpha$ or $y \in I_\alpha$. Therefore, I_α is unif. strongly ideal. ■

Corollary 7: If I is uniformly strongly prime fuzzy ideal of R, then I is strongly prime fuzzy ideal of R.

Proof: Suppose I uniformly, then by proposition 7 we have the α-cuts uniformly, hence strongly. ■

Observe that, according with the last proposition, we have all results from the last section for uniformly strongly.

Corollary 8: If I is a uniformly strongly prime fuzzy ideal, then I is stronly prime fuzzy ideal.

Proof: Straightforward. ■

Corollary 9: **If I is a uniformly strongly prime fuzzy ideal, then I is a prime fuzzy ideal.**

Proof: First proof: since I is unif. strongly prime fuzzy ideal, then I_α is unif. strongly prime ideal for all α-cut.

Second proof: since I is unif. strongly prime fuzzy ideal, there exists a finite set F, where $\bigwedge I(xFy) = I(x) \vee I(y)$, for any $x, y \in R$. Note that $xFy \subseteq xRy$. Hence, $\bigwedge I(xFy) \geq \bigwedge I(xRy)$. Therefore, $\bigwedge I(xFy) = I(x) \vee I(y) \geq \bigwedge I(xRy)$. ■

Proposition 8: If I is a fuzzy ideal of R such that I_α is unif. strongly prime ideal for all $I(1) < \alpha \leq I(0)$, then I is **almost** unif. stronly prime fuzzy ideal.

Proof: Let F_α be a finite set for each I_α and $F = \bigcup F_\alpha$. Suppose that $x_0, y_0 \in R$ such that $\bigwedge I(x_0Fy_0) > I(x_0) \vee I(y_0)$. Consider $t = \bigwedge I(x_0Fy_0) > I(x_0) \vee I(y_0)$. As I_t is unif. strongly prime[3] if $t > I(x_0) \vee I(y_0)$ implies $\bigwedge I(x_0F_ty_0) < t$. Therefore, $I(xFy) > I(x) \vee I(y)$ for any $x, y \in R$. ■

Observe that, if I is a unif. strongly prime fuzzy ideal of R, then according to proposition 7, $I_* = \{x \in R : I(x) = I(0)\}$ is a unif. strongly prime ideal. Therefore, $R/I \cong R/I_*$ (see [18]) is unif. strongly prime ring. We can conclude that in the fuzzy environment we have the following result: I is unif. strongly prime fuzzy ideal, then R/I is unif. strongly prime ring.

[3] If $xF_ty \subseteq I_t$, then $x \in I_t$ or $y \in I_t$, i.e. $\bigwedge I(xF_ty) \geq t$ implies $I(x) \vee I(y) \geq t$

V. The Uniformly Strongly Radical of a fuzzy ideal

In this section we define the unif. strongly radical of a fuzzy ideal and prove some canonical properties. Also, we prove that the unif. strongly prime radical is a unif. strongly prime ideal and some facts associated with Zadeh's extension.

Definition 15: Let I be a fuzzy ideal of R. **The uniformly strongly radical** of I is $\sqrt[u]{I} = \bigcap_{P \in \mathcal{S}_I} P$, where $\mathcal{S}_I$ is the family of all uniformly strongly prime fuzzy ideals P of R such that $I \subseteq P$.

Proposition 9: **If I is a fuzzy ideal of a ring R, then $\sqrt[u]{I}$ is a uniformly strongly prime fuzzy ideal of R.**

Proof: Consider $F = \bigcap_{P \in \mathcal{S}_I} F_P$. Clearly F is a finite set. Given $x, y \in R$, hence,

$$\bigwedge \sqrt[u]{I}(xFy) = \bigwedge \left(\bigcap_{P \in \mathcal{S}_I} P(xFy) \right) = \bigwedge \left(\bigwedge_{P \in \mathcal{S}_I} P(xFy) \right) = \bigwedge_{P \in \mathcal{S}_I} \left(\bigwedge P(xFy) \right) = \bigwedge_{P \in \mathcal{S}_I} (P(x) \vee P(y)) = \bigwedge_{P \in \mathcal{S}_I} P(x) \vee \bigwedge_{P \in \mathcal{S}_I} P(y) = \sqrt[u]{I}(x) \vee \sqrt[u]{I}(y).$$

Clearly, $\sqrt[u]{I}$ is an ideal, and if I is a unif. strongly prime fuzzy ideal, then $\sqrt[u]{I} = I$. ∎

Proposition 10: If I, J are a fuzzy ideal of a ring R, then:

(i) if $I \subseteq J$, then $\sqrt[u]{I} \subseteq \sqrt[u]{J}$;

(ii) $\sqrt[u]{\sqrt[u]{I}} = \sqrt[u]{I}$;

(iii) $I_\alpha \subseteq (\sqrt[u]{I})_\alpha$;

(iv) If I is unif. strongly prime fuzzy ideal, then $\sqrt[u]{I_\alpha} = (\sqrt[u]{I})_\alpha$;

(v) $\sqrt[u]{I \cap J} \subseteq \sqrt[u]{I} \cap \sqrt[u]{J}$.

Proof: (i) $\sqrt[u]{J} = \bigcap_{P \in \mathcal{S}_J} P \supseteq \bigcap_{P \in \mathcal{S}_I} P = \sqrt[u]{I}$. (ii) It is easy to see that $\sqrt[u]{I} \subseteq \sqrt[u]{\sqrt[u]{I}}$. On the other side, let's show $\mathcal{S}_I \subseteq \mathcal{S}_{\sqrt[u]{I}}$. In fact, let $P \in \mathcal{S}_I$, then $P \supseteq I$ using (i) $P = \sqrt[u]{P} \supseteq \sqrt[u]{I}$. (iii),(iv) and (v) is straightforward. ∎

Proposition 11: Let $f : R \longrightarrow S$ be a homomorphism of rings and I a fuzzy ideal of R. Then:

1) $f(I) \subseteq f(\sqrt[u]{I}) \subseteq \sqrt[u]{f(\sqrt[u]{I})}$;

2) $I \subseteq f^{-1}(\sqrt[u]{f(I)})$.

Proof: 1) Straightforward.

2) As $f(I) \subseteq \sqrt[u]{f(I)}$, then $f^{-1}(f(I)) \subseteq f^{-1}(\sqrt[u]{f(I)})$. Thus, $I \subseteq f^{-1}(f(I)) \subseteq f^{-1}(\sqrt[u]{f(I)})$.

∎

Proposition 12: Let $f : R \longrightarrow S$ be a homomorphism of rings and I a SP fuzzy ideal of R. Then, $f(\sqrt[u]{I}) \subseteq \sqrt[u]{f(I)}$.

Proof: As I is SP fuzzy $\sqrt[u]{I} = I$, then $\sqrt[u]{f(I)} = \sqrt[u]{f(\sqrt[u]{I})}$. Thus, $f(\sqrt[u]{I}) \subseteq \sqrt[u]{f(\sqrt[u]{I})} = \sqrt[u]{f(I)}$. ∎

VI. Final Remarks

This paper provides a fuzzy version of uniformly strongly prime fuzzy ideal, based on ideas developed by the authors in [13]. But here it was possible to investigate the strongly structure without dependence of level. This was possible because the definition of uniformly strongly prime ring is a little simpler than strongly prime ring. The investigation of uniformly strongly prime fuzzy properties may help us in the future to discover a definition for strongly prime fuzzy ideals without dependence of α-cuts

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