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Direct Adaptive Fuzzy Tracking Control with Observer and Supervisory Controller for Nonlinear MIMO Time Delay Systems

Chia-Hao Kuo[1] Tsung-Chih Lin[2] Chien-Liang Chen[2]

[1]Ph. D. Program of Electrical and Communications Engineering, Feng-Chia University, Taichung, Taiwan

[2]Department of Electronic Engineering, Feng-Chia University, Taichung, Taiwan

Abstract

This paper proposes a direct adaptive fuzzy H^{∞} control with observer and supervisory controller for uncertain multiple-input multiple-output (MIMO) nonlinear dynamical systems. Since time delays and external disturbances probably are sources of unstable for the system, by combining H^{∞} control theory and fuzzy control to make both fuzzy logic approximation error and external disturbance on the track error are attenuated to a prescribed level. Based on Lyaounov stability criterion, the parameters of the adaptive fuzzy controller can be tuned online by control law and adaptive law. Simulation results show that effectiveness and track performance are acceptable.

Keywords: Observer, supervisory controller, uncertain MIMO nonlinear dynamical system, H^{∞} control theory, Lyaounov stability criterion.

I. Introduction

Time delay often appears in various engineering system, such as chemical engineering process, long-distance transmission of gas, communication system and microwave oscillator etc.. Therefore, time delay systems have broad range of applications. Time delay is projected to become a source of instability, the stabilization problem of time delay systems are true challenge and has received considerable attention [1-7].

Recently years, active research has been carried out in adaptive fuzzy-neural control and adaptive control is a technique of applying systems identification techniques to obtain a model of the process. It has been proven that adaptive fuzzy-neural network (FNN) can approximate any nonlinear function to desired accuracy because of the approximation theorem. For example, [8-15] shows that the adaptive FNN can model unknown functions in dynamic systems.

Many studies are for SISO systems, but reality systems are mostly MIMO system. This paper presents a novel direct adaptive fuzzy neural controller with stats observer and supervisor controller for certain class of unknown MIMO nonlinear dynamical systems. Based on the Lyaounov synthesis approach, the free parameters of the adaptive time delay fuzzy neural controller can be tuned on-line by an observer-based output feedback control law vector and adaptive laws. Also a supervisory controller is designed to be linked with FNN controller [16-20]. If the nonlinear system tends to unstable only by the FNN controller, the supervisory controller will be activated to work with the FNN controller to stabilize the overall system. Hence it is very thrifty design methodology in respect of control capability.

This paper is organized as follows. The problem formulation is first made in section II. The description of adaptive time delay FNN system is in section III. Direct adaptive control law design based on time delay FNN controller with observer and supervisory controller of unknown nonlinear MIMO dynamical systems is presented in section IV. Section V shows that simulation example and the performance of results. Conclusions are given in section VI.

II. Problem Formulation

Consider the nth-order nonlinear MIMO with time delay systems, expressed by the following set of equations

$$
\begin{aligned}
x_1^{(n_1)} &= f_1(\underline{x}_1,\tau)+\sum_{j=1}^{p} g_{1j}(\underline{x}_1,\tau)u_j+d_1 \\
x_2^{(n_2)} &= f_2(\underline{x}_2,\tau)+\sum_{j=1}^{p} g_{2j}(\underline{x}_2,\tau)u_j+d_2 \\
&\vdots \\
x_p^{(n_p)} &= f_p(\underline{x}_p,\tau)+\sum_{j=1}^{p} g_{pj}(\underline{x}_p,\tau)u_j+d_p \\
y_i &= x_i \\
x_i(t) &= \Xi(t), t\in[-\varsigma,0]
\end{aligned}
\tag{1}
$$

Where $\underline{x}_i=\left[\underline{x}_i,\dot{\underline{x}}_i,\ddot{\underline{x}}_i,\ldots,\underline{x}_i^{(n_i-1)}\right]^T$ is plant state vector, $\underline{u}=\left[u_1,\ldots,u_p\right]^T$ is the control vector; $y=\left[\underline{y}_1,\ldots,\underline{y}_p\right]^T$ is the output vector, $f_i, g_{ij}, (i,j=1,2,\ldots,p)$ are the system functions and $f_i(\underline{x}_i,\tau)=f_i\left[x_i,x_i(t-\tau_1),\ldots,x_i(t-\tau_r)\right]$, $g_{ij}(\underline{x}_i,\tau)=g_{ij}\left[x_i,x_i(t-\tau_1),\ldots,x_i(t-\tau_r)\right]$, d_i is the external disturbances, $\Xi(t)$ is the initial state of the system, τ is time delay, $\varsigma=\max\{\tau|1\le r\}$.

Equation (1) can be rewritten as

$$\begin{bmatrix} x_1^{(n_1)} \\ x_2^{(n_2)} \\ \vdots \\ x_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\underline{x}_1,\tau) \\ f_2(\underline{x}_2,\tau) \\ \vdots \\ f_p(\underline{x}_p,\tau) \end{bmatrix} + G(\underline{x},\tau)\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} \qquad (2)$$

where

$$G(\underline{x},\tau) = \begin{bmatrix} g_{11}(\underline{x},\tau) & \cdots & g_{1p}(\underline{x},\tau) \\ \vdots & \ddots & \vdots \\ g_{p1}(\underline{x},\tau) & \cdots & g_{pp}(\underline{x},\tau) \end{bmatrix}$$

The control objective is to force the system output vector $\underline{y}$ to follow a given bounded reference signal vector $\underline{y}_m$, under the constraint that all signals involved must be bounded. To begin with, the tracking error vector $\underline{e}$ can be defined as

$$\underline{e} = \underline{y}_m - \underline{y} \qquad (3)$$

Let $\underline{k} = [k_1, k_2, \cdots, k_n]^T \in R^n$ to be chosen such that all root of the polynomial $p(s) = s^{nq} + k_n s^{(n-1)q} + \cdots + k_1$ are in the open left half-plane. If the functions $f_i(\underline{x},\tau)$ and $g_{ij}(\underline{x},\tau)$ are known and the systems are free of external disturbances d_i, then equivalent control vector is obtained as

$$\begin{bmatrix} u_1^* \\ u_2^* \\ \vdots \\ u_p^* \end{bmatrix} = G^{-1}(\underline{x},\tau)\left\{ -\begin{bmatrix} f_1(\underline{x},\tau) \\ f_2(\underline{x},\tau) \\ \vdots \\ f_p(\underline{x},\tau) \end{bmatrix} + \begin{bmatrix} y_{m1}^{(n_1)} \\ y_{m2}^{(n_2)} \\ \vdots \\ y_{mp}^{(n_p)} \end{bmatrix} + \begin{bmatrix} k_{1c}^T e_1 \\ k_{2c}^T e_2 \\ \vdots \\ k_{pc}^T e_p \end{bmatrix} \right\} \qquad (4)$$

such that the main objective of control $\lim_{t\to\infty} e(t) = 0$ can be achieved.

But not all system state vectors $\underline{x}$ can be measured, we have to design an observer to estimation the state vectors $\underline{x}$. Replacing the functions $f_i(\underline{x},\tau)$, $g_{ij}(\underline{x},\tau)$ and error vector e_i in equation (4) by estimation functions $f_i(\hat{\underline{x}},\tau)$, $g_{ij}(\hat{\underline{x}},\tau)$ and $\hat{e}_i$, the control vector equation (4) is rewritten as

$$\begin{bmatrix} u_1^* \\ u_2^* \\ \vdots \\ u_p^* \end{bmatrix} = G^{-1}(\hat{\underline{x}},\tau)\left\{ -\begin{bmatrix} f_1(\hat{\underline{x}}_1,\tau) \\ f_2(\hat{\underline{x}}_2,\tau) \\ \vdots \\ f_p(\hat{\underline{x}}_p,\tau) \end{bmatrix} + \begin{bmatrix} y_{m1}^{(n_1)} \\ y_{m2}^{(n_2)} \\ \vdots \\ y_{mp}^{(n_p)} \end{bmatrix} + \begin{bmatrix} k_{1c}^T \hat{e}_1 \\ k_{2c}^T \hat{e}_2 \\ \vdots \\ k_{pc}^T \hat{e}_p \end{bmatrix} \right\} \qquad (5)$$

Furthermore, $f_i(\hat{\underline{x}},\tau)$ and $g_{ij}(\hat{\underline{x}},\tau)$ are unknown and external disturbance $d_i(t) \neq 0$, the ideal control vector (5) can't be executed. We replace control effort vector by the time delay fuzzy logic system. Therefore, the resulting control effort vector can be obtained as

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} = \begin{bmatrix} u_{d1}(\underline{x}_1,\tau|\theta_1, m_1, s_1) \\ u_{d2}(\underline{x}_2,\tau|\theta_2, m_2, s_2) \\ \vdots \\ u_{dp}(\underline{x}_p,\tau|\theta_p, m_p, s_p) \end{bmatrix} + G^{-1}(\underline{x},\tau)\begin{bmatrix} u_{s1} \\ u_{s2} \\ \vdots \\ u_{sp} \end{bmatrix} \qquad (6)$$

where u_{si} is the supervisory controller to be incorporated with FNN controller.

Applying (5) and (6) to (2) and after some simple manipulations, we can obtain the error dynamic equation

$$\begin{bmatrix} e_1^{(n_1)} \\ e_2^{(n_2)} \\ \vdots \\ e_p^{(n_p)} \end{bmatrix} = G(\underline{x},\tau)\left\{ \begin{bmatrix} u_1^* \\ u_2^* \\ \vdots \\ u_p^* \end{bmatrix} - \begin{bmatrix} u_{d1}(\underline{x}_1,\tau|\theta_1, m_1, s_1) \\ u_{d2}(\underline{x}_2,\tau|\theta_2, m_2, s_2) \\ \vdots \\ u_{dp}(\underline{x}_p,\tau|\theta_p, m_p, s_p) \end{bmatrix} \right\}$$
$$- \begin{bmatrix} u_{s1} \\ u_{s2} \\ \vdots \\ u_{sp} \end{bmatrix} - \begin{bmatrix} k_{c1}^T \hat{e}_1 \\ k_{c2}^T \hat{e}_2 \\ \vdots \\ k_{cp}^T \hat{e}_p \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} \qquad (7)$$

The state-space representation of the ith equation in (7) can be presented as

$$\dot{\underline{e}}_i = A_i \underline{e}_i - B_i k_{ci}^T \hat{\underline{e}}_i$$
$$+ B_i\left[\sum_{j=1}^{p} (g_{ij}(\underline{x}_i,\tau)(u_j^* - u_{dj}(\underline{x}_i,\tau|\theta_i, m_i, s_i))) - u_{si} - d_i \right]$$
$$e_i = C_i \underline{e}_i \qquad (8)$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad C_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^T$$

From (8) the error vector $\underline{e}_i$ can be estimated by the following observer

$$\dot{\hat{\underline{e}}}_i = A_i \hat{\underline{e}}_i - B_i k_{ci}^T \hat{\underline{e}} + L_i(\underline{e}_i - \hat{\underline{e}}_i)$$
$$\hat{e}_i = C_i \hat{\underline{e}}_i \qquad (9)$$

where $L_i = [l_{i1}, l_{i2} \ldots l_{in}]^T$ is the observer gain vector to stabilize the observer.

The observation errors are defined as

$$\tilde{\underline{e}}_i = \underline{e}_i - \hat{\underline{e}}_i = \underline{x}_i - \hat{\underline{x}}_i$$
$$\tilde{e}_i = e_i - \hat{e}_i = x_i - \hat{x}_i \qquad (10)$$

Subtracting (9) from (8), we have error dynamics

$$\dot{\tilde{\underline{e}}}_i = (A_i - L_i C_i)\tilde{\underline{e}}_i$$
$$+ B_i\left[\sum_{j=1}^{p} (g_{ij}(\underline{x}_i,\tau)(u_j^* - u_{dj}(\underline{x}_i,\tau|\theta_i, m_i, s_i))) - u_{si} - d_i \right]$$
$$= \Lambda_i \tilde{\underline{e}}_i$$
$$+ B_i\left[\sum_{j=1}^{p} (g_{ij}(\underline{x}_i,\tau)(u_j^* - u_{dj}(\underline{x}_i,\tau|\theta_i, m_i, s_i))) - u_{si} - d_i \right]$$
$$\tilde{e}_i = C_i \tilde{\underline{e}}_i \qquad (11)$$

where Λ_i is strictly Hurwitz (i.e., the roots of the closed-loop system are in the open left half-plane) and we know that there exists a positive definite symmetric $n \times n$ matrix P which satisfies the Lyapunov equation

$$\Lambda_i^T P_i + P\Lambda_i = -Q_i \qquad (12)$$

where Q_i is an arbitrary $n \times n$ positive definite matrix. The observer complete proof can be obtained in [17].

III. Description of adaptive time delay fuzzy neural network

The adaptive time delay FNN system structure is shown in Fig.1.

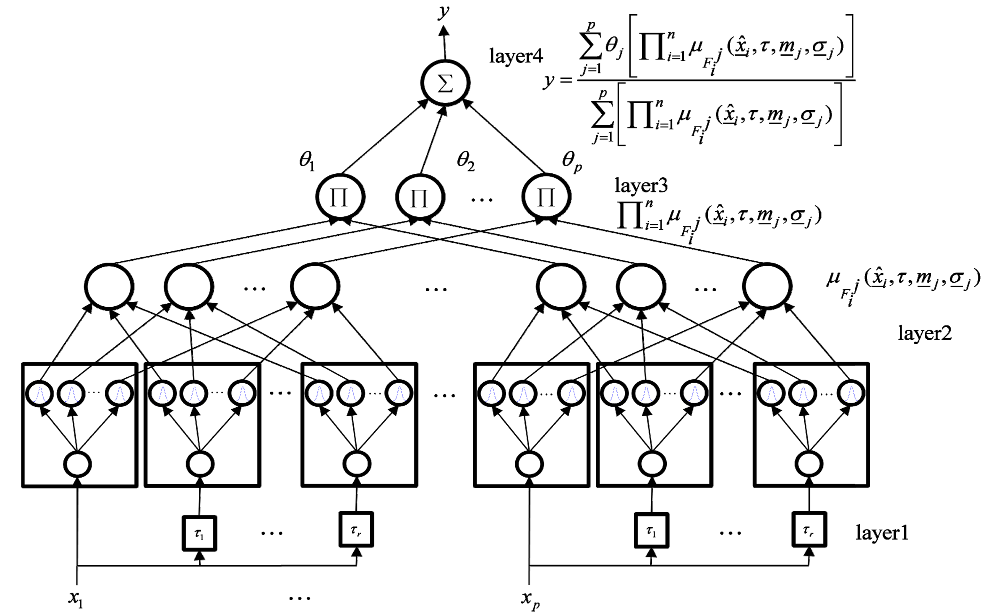

Fig.1 The adaptive time delay FNN structure.

Layer 1 is the input layer. The nodes in this layer transmit input values and delay input values to the next layer.

Layer 2 is the membership layer. In this layer, each node expresses the terms of respective linguistic variables and the Gaussian function is adopted as the membership function.

Layer 3 is the rule base and inference layer. The structure of this layer incorporates the fuzzy rules and the fuzzy inference. By fuzzy rules and inference to obtain fuzzy output, each node is corresponding to a fuzzy rule. The input links and the output links of each node express the preconditions of the corresponding rule and the firing strength of the corresponding rule, respectively.

Layer 4 is the output layer. Due to fuzzy inference output is a fuzzy value. We must use defuzzification to get a crisp value.

FNN was proven that has the characteristic of approaching a nonlinear function. In this paper, we construct the adaptive time delay fuzzy logic system to approach control input $u_j(\underline{x}_i,\tau)$. The direct adaptive fuzzy controller can be defined as follow

$$u_{dj}(\underline{\hat{x}}_i,\tau|\theta_i,m_i,s_i)=$$

$$\sum_{j=1}^{p}\theta_j\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,m_i,s_i)\right]\Big/\sum_{j=1}^{p}\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,m_i,s_i)\right]$$

where

$$\prod_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,m_i,s_i)/\sum_{j=1}^{p}\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,m_i,s_i)\right]$$

$=\xi^T(\underline{\hat{x}}_i,\tau,m_i,s_i)$ and $\mu_{F_i^j}(\underline{\hat{x}}_i,\tau,m_i,s_i)=$

$$\mu_{F_i^j}(\underline{\hat{x}}_i,m_i,s_i)\mu_{F_i^j}(\underline{\hat{x}}_i(t-\tau_1),m_i,s_i)...\mu_{F_i^j}(\underline{\hat{x}}_i(t-\tau_r),m_i,s_i)$$

The direct adaptive fuzzy controller can be rewritten as

$$u_{dj}(\underline{\hat{x}}_i,\tau|\theta_i,m_i,s_i)=\xi^T(\hat{x}_i,\hat{x}_i(t-\tau_1)\cdots\hat{x}_i(t-\tau_r),m_i,s_i)\theta_i \tag{13}$$

By tuning the parameters of weight θ_i, membership center m_i and membership width s_i, we can have the optimal direct adaptive fuzzy controller.

IV. Direct adaptive control law design based on time delay FNN controller with observer and supervisory controller of unknown nonlinear MIMO dynamical systems

The FNN controller with observer and supervisory controller is developed as follow.

Consider the error dynamics (11), the Lyapunov function can be defined as

$$V_{\underline{\tilde{e}}}=V_{\underline{\tilde{e}}1}+V_{\underline{\tilde{e}}2}+\cdots+V_{\underline{\tilde{e}}p} \tag{14}$$

Consider separately ith Lyapunov function

$$V_{\underline{\tilde{e}}i}=\frac{1}{2}\underline{\tilde{e}}_i^T P_i\underline{\tilde{e}}_i \tag{15}$$

Taking the derivative of the (15) with respect to time, we have

$$\dot{V}_{\underline{\tilde{e}}i}=\frac{1}{2}\dot{\underline{\tilde{e}}}_i^T P_i\underline{\tilde{e}}_i+\frac{1}{2}\underline{\tilde{e}}_i^T P_i\dot{\underline{\tilde{e}}}_i \tag{16}$$

By substituting Lyapunov equation (11) and (12) into (16),we obtain

$$\dot{V}_{\underline{\tilde{e}}i}=-\frac{1}{2}\underline{\tilde{e}}_i^T Q_i\underline{\tilde{e}}_i$$

$$+\underline{\tilde{e}}_i^T P_i B_i\left\{\sum_{j=1}^{p}\left(g_{ij}(\underline{x}_i,\tau)\left(u_j^*-u_{dj}(\underline{x}_i,\tau|\underline{\theta}_i,m_i,s_i)\right)\right)-u_{si}-d_i\right\} \tag{17}$$

By the triangle inequality, we can have (18).

$$\dot{V}_{\underline{\tilde{e}}i}\le-\frac{1}{2}\underline{\tilde{e}}_i^T Q_i\underline{\tilde{e}}_i+\left|\underline{\tilde{e}}_i^T P_i B_i\right|$$

$$\left\{\sum_{j=1}^{p}\left(\left|g_{ij}(\underline{x}_i,\tau)u_j^*\right|\right)+\sum_{j=1}^{p}\left(\left|g_{ij}(\underline{x}_i,\tau)u_{dj}(\underline{x}_i,\tau|\underline{\theta}_i,m_i,s_i)\right|\right)\right\}$$

$$+\left|\underline{\tilde{e}}_i^T P_i B_i d_i\right|-\underline{\tilde{e}}_i^T P_i B_i u_{si} \tag{18}$$

In order to design supervisory controller, we need the following assumption.

Assumption:

We can determine function $f_i^U(\hat{\underline{x}},\tau)$ and $g_{ij}{}^U(\hat{\underline{x}}_i,\tau)$ such that $|f_i(\underline{x}_i,\tau)| \le f_i^U(\underline{x}_i,\tau) \approx f_i^U(\hat{\underline{x}}_i,\tau)$, $g_{ij}(\underline{x}_i,\tau) \le g_{ij}{}^U(\underline{x}_i,\tau) \approx g_{ij}{}^U(\hat{\underline{x}}_i,\tau)$, where $f_i^U(\underline{x}_i,\tau) \approx f_i^U(\hat{\underline{x}}_i,\tau) < \infty$, $g_{ij}{}^U(\underline{x}_i,\tau) \approx g_{ij}{}^U(\hat{\underline{x}}_i,\tau) < \infty$, Also external disturbance is bounded $|d_i| \le d_{im}$, where d_{im} is the upper bound of noise d_i, $|G^{-1}(\hat{\underline{x}},\tau)| \le G_m$ where $|G^{-1}(\hat{\underline{x}},\tau)|$ is Euclidean norm (max singular value).

By observing (18), the supervisory controller is chosen as

$$u_{si} = I^* \operatorname{sgn}(\tilde{\underline{e}}_i^T P_i B_i)\left\{\sum_{j=1}^{p}\left(G_m g_{ij}^U(\hat{\underline{x}},\tau)\left(f_i^U(\hat{\underline{x}},\tau)+\left|y_{mi}^{(n_i)}\right|+\left|k_{c_i}^T\hat{e}_i\right|\right)\right)\right.$$

$$\left.+\sum_{j=1}^{p}\left(\left|g_{ij}(\hat{\underline{x}},\tau)u_{dj}(\hat{\underline{x}},\tau\,|\,\underline{\theta}_i,m_i,s_i)\right|\right)+d_{mi}\right\} \tag{19}$$

If $V_{\underline{e}i} > \bar{V}_i$ ($\bar{V}_i$ is a constant chosen by designer) then $I^* = 1$, if $V_{\underline{e}i} \le \bar{V}_i$ then $I^* = 0$.

Inserting (19) into (18) we can be obtain as

$$\dot{V}_{\underline{e}i} \le -\frac{1}{2}\tilde{\underline{e}}_i^T Q_i \tilde{\underline{e}}_i + \left|\tilde{\underline{e}}_i^T P_i B_i\right|$$

$$\left\{\sum_{j=1}^{p}\left(G_m g_{ij}(\underline{x}_i,\tau)(\left|f_i(\underline{x}_i,\tau)\right|+\left|y_{mi}^{(n_i)}\right|+\left|k_{ci}^T\hat{e}_i\right|)\right)\right.$$

$$\left.+\sum_{j=1}^{p}\left(g_{ij}(\underline{x}_i,\tau)\left|u_{dj}(\underline{x}_i,\tau\,|\,\underline{\theta}_i,m_i,s_i)\right|\right)+|d_i|\right\}$$

$$-\left|\tilde{\underline{e}}_i^T P_i B_i\right|\left\{\sum_{j=1}^{p}\left(G_m g_{ij}^U(\hat{\underline{x}}_i,\tau)(f_i^U(\hat{\underline{x}}_i,\tau)+\left|y_{mi}^{(n_i)}\right|+\left|k_{ci}^T\hat{e}_i\right|)\right)\right.$$

$$\left.+\sum_{j=1}^{p}\left(g_{ij}^U(\hat{\underline{x}}_i,\tau)\left|u_{dj}(\hat{\underline{x}}_i,\tau\,|\,\underline{\theta}_i,m_i,s_i)\right|\right)+d_{mi}\right\} \tag{20}$$

with the assumptions, we have

$$\dot{V}_{\underline{e}i} \le -\frac{1}{2}\tilde{\underline{e}}_i^T Q_i \tilde{\underline{e}}_i \le 0 \tag{21}$$

Define approximation error $\tilde{\theta}_i = \theta_i - \theta_i^*$, $\tilde{m}_i = m_i - m_i^*$, $\tilde{s}_i = s_i - s_i^*$, we can obtain as

$$u_{dj}(\underline{x}_i,\tau|\theta_i,m_i,s_i) - u_j^* =$$

$$(\xi(\underline{x}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s)\tilde{\theta}_i + (\tilde{m}_i\xi_m - \tilde{s}_i\xi_s)\theta_i$$

$$+(m_i^*\xi_m - s_i^*\xi_s)\tilde{\theta}_i \tag{22}$$

The optimal parameter vector is defined as

$$\underline{\theta}_i^* = \arg\min_{\theta_i\in M_\theta}\left[\sup_{\hat{\underline{x}}_i\in M_\theta}\left|u(\hat{\underline{x}}_i,\tau|\theta_i,m_i,s_i) - u^*\right|\right] \tag{23}$$

where M_θ is the bound of θ_i.

The minimum approximation error is defined as

$$\omega_i = \sum_{j=1}^{p}\left(g_{ij}(\underline{x}_i,\tau)(m_i^*\xi_m - s_i^*\xi_s)\tilde{\theta}_i\right) - d_i \tag{24}$$

The error dynamics (11) can be rewritten as

$$\dot{\tilde{\underline{e}}} = A_i\tilde{\underline{e}}_i - B_i\sum_{j=1}^{p}(g_{ij}(\underline{x}_i,\tau)((\xi(\underline{x}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s)\tilde{\theta}_i)$$

$$+(\tilde{m}_i\xi_m - \tilde{s}_i\xi_s)\theta_i) - B_i u_{si} + B_i\omega_i \tag{25}$$

Therefore, the main result is summarized in the following theorem.

Theorem:

Consider the MIMO nonlinear dynamic system with time delay (1) and control input vector (6), if the fuzzy-based adaptive laws are chosen as

$$\dot{\theta}_i = r_{i1}\sum_{j=1}^{p}(g_{ij}(\underline{x}_i,\tau)(\xi(\underline{x}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s))B_i^T P_i\tilde{\underline{e}}_i \tag{26}$$

$$\dot{m}_i = r_{i2}\theta_i\xi_m B_i^T P_i\tilde{\underline{e}}_i \tag{27}$$

$$\dot{s}_i = r_{i3}\theta_i\xi_s B_i^T P_i\tilde{\underline{e}}_i \tag{28}$$

by the adaptive laws, we can make the system gradually stabilized.

Proof:

In order to analyze the closed-loop stability, the Lyaounov function candidate is chosen as

$$V = V_1 + V_2 + \cdots + V_p \tag{29}$$

Considering separately ith Lyapunov function

$$V_i = \frac{1}{2}\tilde{\underline{e}}_i^T P_i\tilde{\underline{e}}_i + \frac{1}{2r_{i1}}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2r_{i2}}\tilde{m}_i^T\tilde{m}_i + \frac{1}{2r_{i3}}\tilde{s}_i^T\tilde{s}_i$$

$$+\frac{1}{2}\sum_{i=1}^{r}\int_{t-\tau_i}^{t}\tilde{e}^T(v)\tilde{e}(v)dv \tag{30}$$

Taking the derivative of the (30) with respect to time, we have

$$\dot{V}_i = \frac{1}{2}\dot{\tilde{\underline{e}}}_i^T P_i\tilde{\underline{e}}_i + \frac{1}{2}\tilde{\underline{e}}_i^T P_i\dot{\tilde{\underline{e}}}_i + \frac{1}{r_{i1}}\dot{\tilde{\theta}}_i^T\tilde{\theta}_i + \frac{1}{r_{i2}}\dot{\tilde{m}}_i^T\tilde{m}_i + \frac{1}{r_{i3}}\dot{\tilde{s}}_i^T\tilde{s}_i$$

$$+\frac{1}{2}\sum_{i=1}^{r}\tilde{e}^T(t)\tilde{e}(t) - \frac{1}{2}\sum_{i=1}^{r}\tilde{e}^T(t-\tau_i)\tilde{e}(t-\tau_i) \tag{31}$$

Substituting (25) into (31) yields

$$\dot{V}_i = \frac{1}{2}(\Lambda_i\tilde{\underline{e}}_i - B_i\sum_{j=1}^{p}(g_{ij}(\underline{x}_i,\tau)((\xi(\hat{\underline{x}}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s)\tilde{\theta}_i)$$

$$+(\tilde{m}_i\xi_m - \tilde{s}_i\xi_s)\theta_i) - B_i u_{si} + B_i\omega_i)^T P_i\tilde{\underline{e}}_i$$

$$+\frac{1}{2}\tilde{\underline{e}}_i^T P_i(\Lambda_i\tilde{\underline{e}}_i - B_i\sum_{j=1}^{p}(g_{ij}(\underline{x}_i,\tau)((\xi(\hat{\underline{x}}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s)\tilde{\theta}_i)$$

$$+(\tilde{m}_i\xi_m - \tilde{s}_i\xi_s)\theta_i) - B_i u_{si} + B_i\omega_i) + \frac{1}{r_{i1}}\dot{\tilde{\theta}}_j^T\tilde{\theta}_j + \frac{1}{r_{i2}}\dot{\tilde{m}}_i^T\tilde{m}_i + \frac{1}{r_{i3}}\dot{\tilde{s}}_i^T\tilde{s}_i$$

$$+\frac{1}{2}\sum_{i=1}^{r}\tilde{e}_i^T(t)\tilde{e}_i(t) - \frac{1}{2}\sum_{i=1}^{r}\tilde{e}_i^T(t-\tau_i)\tilde{e}_i(t-\tau_i) \tag{32}$$

Substituting Lyaounov equation (12) and adaptive laws (26) to (28) into (32)

$$\dot{V}_i = -\frac{1}{2}\tilde{\underline{e}}^T(Q_i - rI)\tilde{\underline{e}}_i + \frac{1}{r_{i1}}$$

$$(-r_1\tilde{\underline{e}}_i^T P_i B_i\sum_{j=1}^{p}(g_{ij}(\underline{x}_i,\tau)(\xi(\hat{\underline{x}}_i,\tau,m_i,s_i) - m_i\xi_m - s_i\xi_s)) + \dot{\tilde{\theta}}_i^T)\tilde{\theta}_i$$

$$+\frac{1}{r_{i2}}(-r_2\tilde{\underline{e}}_i^T P_i B_i\xi_m\theta_i + \dot{\tilde{m}}_i^T)\tilde{m}_i + \frac{1}{r_{i3}}(-r_3\tilde{\underline{e}}_i^T P_i B_i\xi_s\theta_i + \dot{\tilde{s}}_i^T)\tilde{s}_i$$

$$-\tilde{\underline{e}}_i^T P_i B_i u_{si} + \tilde{\underline{e}}_i^T P_i B_i\omega_i \tag{33}$$

Observing from (19), we know that $\tilde{\underline{e}}_i^T P_i B_i u_{si} \ge 0$, hence $\dot{V}_i \le 0$. The proof is completed.

Remark:

The adaptive law (26) needs to know $g_{ij}(\underline{x}_i,\tau)$ beforehand, i.e., theorem is only favorable for which those nonlinear system functions $g_{ij}(\underline{x}_i,\tau)$ are well known. If $g_{ij}(\underline{x}_i,\tau)$ is expressed as $g_{ij}(\underline{x}_i,\tau) = g_{oij}(\underline{x}_i,\tau) + \tilde{g}_{ij}(\underline{x}_i,\tau)$, where $g_{oij}(\underline{x}_i,\tau)$ is the well known portion and $\tilde{g}_{ij}(\underline{x}_i,\tau)$ is the uncertain portion, $\tilde{g}_{ij}(\underline{x}_i,\tau)u_j$ can be considered as a part of external disturbance.

V. Simulation example

In this section, we will apply our direct adaptive fuzzy H^{∞} tracking controller to force the MIMO master time delay system to track the trajectory of the MIMO slave time delay system.

Master system:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = 1.8y_1 - 0.5y_2 - 0.4y_3 + 1.3y_4 - y_1^3 - 1.5\cos(0.4t) \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -1.5y_1 - 1.1y_2 - 1.6y_3 - 1.0y_4 - y_3^3 - 1.9\cos(1.8t) \end{cases}$$

Slave system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 0.25x_1 - 0.45x_2 + 1.2x_3 - 1.6x_4 - x_1^3 \\ \qquad + 0.002x_1(t-0.002) + 0.001x_2(t-0.002) \\ \qquad - 1.1\cos(0.4t) + 1.7u_1 + 0.5u_2 + d(t) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -1.3x_1 + 1.8x_2 - 1.35x_3 - 1.5x_4 - x_3^3 \\ \qquad + 0.002x_3(t-0.002) + 0.001x_4(t-0.002) \\ \qquad - 0.75\cos(1.8t) + 0.35u_1 + 1.5u_2 + d(t) \end{cases}$$

where external disturbance $d(t) = d_1(t) = d_2(t) = \sin(t)$, The main objective is to control the trajectories of the slave system to track the reference trajectories which is obtained from the master system. The initial conditions of master and slave systems are chosen as $x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0$, $y_1(0) = y_2(0) = y_3(0) = y_4(0) = 0.23$. The initial membership functions for x_i are shown in Fig. 2 and Fig. 3, chosen as $x_1 = x_2,\ \mu_{F_1^j} = \mu_{F_2^j}$, $x_3 = x_4,\ \mu_{F_3^j} = \mu_{F_4^j}$.

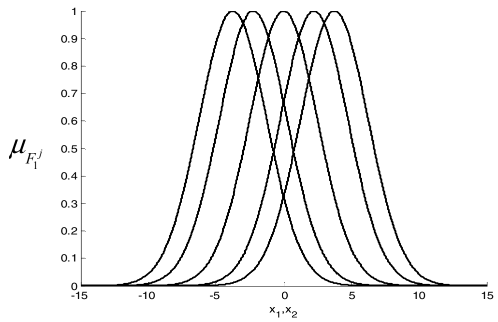

Fig. 2 The membership function for x_i, $i = 1,2$.

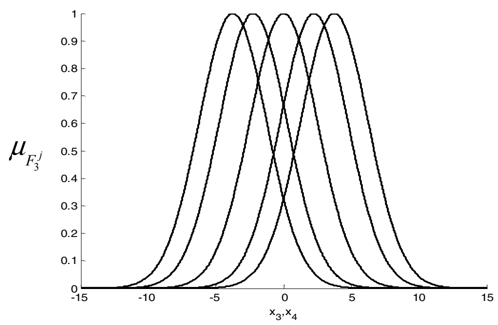

Fig. 3 The membership function for x_i, $i = 3,4$.

Design constants are selected as $k_{11} = 1$, $k_{12} = 2$, $k_{c1} = 1, k_{c2} = 2$, $r_{12} = r_{22} = r_{32} = r_{42} = r_{13} = r_{23} = r_{33} = r_{43} = 0.4$, $r_{11} = r_{21} = 2.7$, $r_{31} = r_{41} = 3$ and step size h=0.001. Fig.s 4 and 5 show the trajectories of the states x_1, x_2, x_3, x_4 and $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, $\hat{x}_4$, respectively.

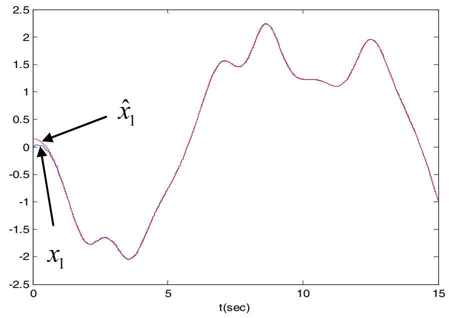

Fig. 4 The trajectories of the states x_1 and $\hat{x}_1$.

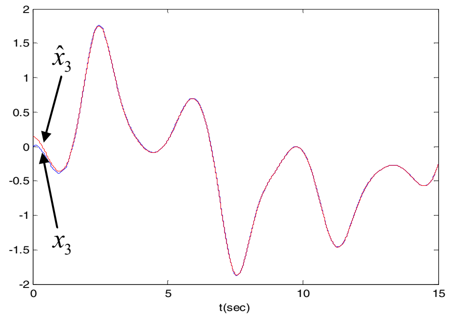

Fig. 5 The trajectories of the states x_3 and $\hat{x}_3$.

The final membership functions are given in Fig.s 6 and 7.

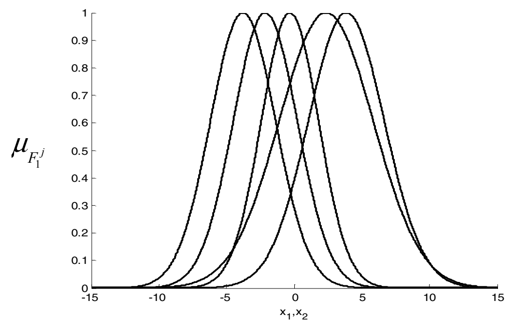

Fig. 6 The final membership function for x_i, $i = 1,2$.

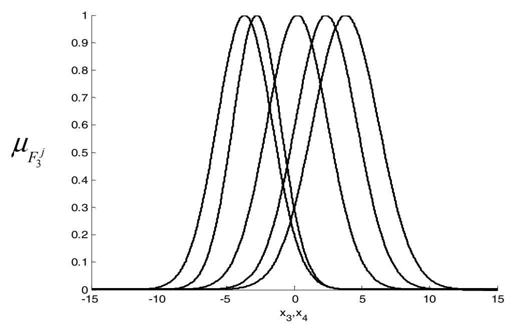

Fig. 7 The final membership function for x_i, $i = 3,4$.

Fig.s 8 and 9 show the trajectories of the master system output y_1, y_3 and slave system output x_1, x_3, respectively.

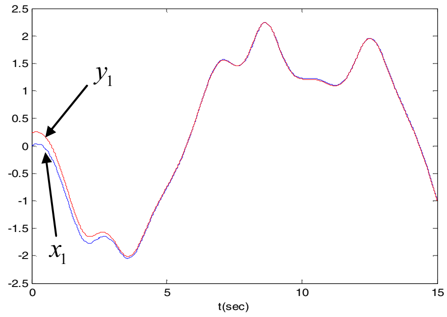

Fig. 8 The trajectories of the states x_1 and y_1.

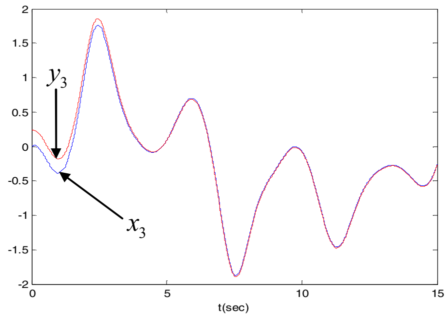

Fig. 9 The trajectories of the states x_3 and y_3.

The overall control efforts are given in Fig.s 10 and 11 which include both direct adaptive controller u_d and supervisory controller u_s. We can see that the supervisory controller only works in the starting period and it will be de-activated when system is stabilized.

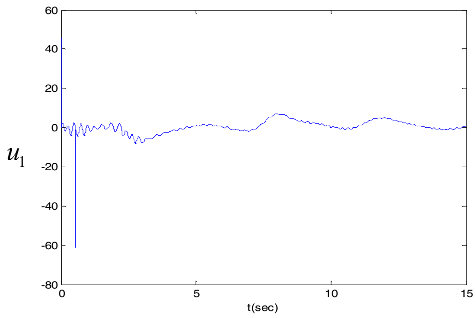

Fig. 10 Trajectory of the control effort u_1.

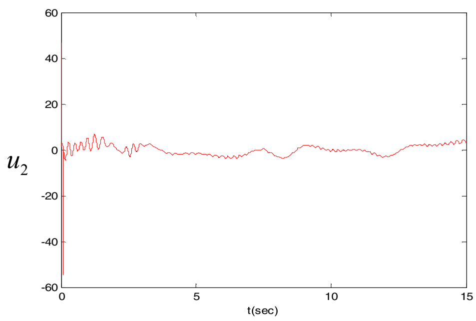

Fig. 11 Trajectory of the control effort u_2.

Fig.s 12 and 13 show the graph of $\dot{V}_i(t),\ i=1,\ 2$ which are always negative defined and consequently is stable, where $\dot{V}_{1\max}(t) = -1.102\mathrm{e}-6$ and $\dot{V}_{2\max}(t) = -2.231\mathrm{e}-4$.

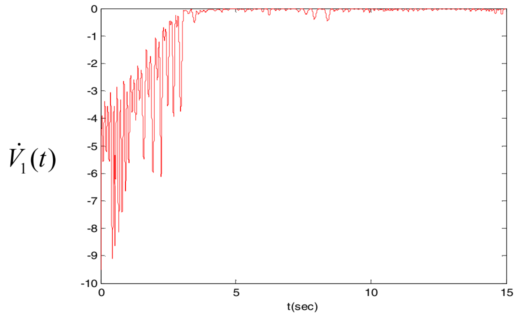

Fig. 12 The graph of $\dot{V}_1(t)$.

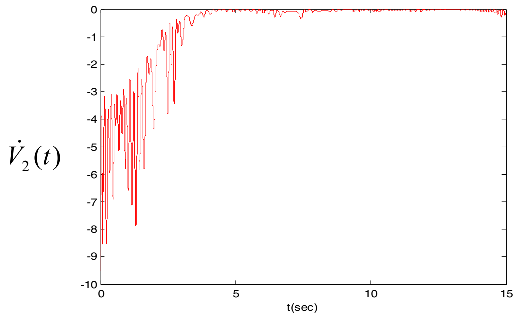

Fig. 13 The graph of $\dot{V}_2(t)$.

VI. Conclusion

This paper proposes a direct adaptive fuzzy H^∞ control with observer and supervisory controller for uncertain MIMO nonlinear dynamical systems. Based on the Lyapunov synthesis approach, free parameters of the adaptive fuzzy controller can be tuned on line by output feedback control effort vector and adaptive laws. The simulation example, synchronization of two different MIMO with time delay chaotic systems, is given to demonstrate the effectiveness of the proposed methodology. Simulation results show that the fast synchronization between master system and drive system can be achieved.

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