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Building Fuzzy Inference Systems with Similarity Reasoning: NSGA II-based Fuzzy Rule Selection and Evidential Functions

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***Abstract*—In our previous investigations, two Similarity Reasoning (SR)-based frameworks for tackling real-world problems have been proposed. In both frameworks, SR is used to deduce unknown fuzzy rules based on similarity of the given and unknown fuzzy rules for building a Fuzzy Inference System (FIS). In this paper, we further extend our previous findings by developing (1) a multi-objective evolutionary model for fuzzy rule selection; and (2) an evidential function to facilitate the use of both frameworks. The Non-Dominated Sorting Genetic Algorithms-II (NSGA-II) is adopted for fuzzy rule selection, in accordance with the Pareto optimal criterion. Besides that, two new evidential functions are developed, whereby given fuzzy rules are considered as evidence. Simulated and benchmark examples are included to demonstrate the applicability of these suggestions. Positive results were obtained.**

Keywords-Fuzzy Inference System, Non-Dominated Sorting Genetic Algorithms-II, Similarity Reasoning, evidential functions, fuzzy rule selection

I. INTRODUCTION

Two interesting research areas in fuzzy modeling are considered, i.e., fuzzy rule selection and Similarity Reasoning (SR). On one hand, fuzzy rule section is a solution to the design of a fuzzy inference system (FIS) for pattern recognition, usually from numerical data, e.g. see [1-4]. Besides that, methods for fuzzy rule selection with the use of multi-objective evolutionary algorithms are available [1,4]. On the other hand, a variety of SR schemes, e.g., Approximate Analogical Reasoning Schema (AARS) [5], Fuzzy Rule Interpolation (FRI) [6], and qualitative reasoning [7], have been developed. These schemes are useful to deduce unknown fuzzy rules based on similarity of the given and unknown fuzzy rules. An example from Zadeh [7] is as follows:

R1: If pressure is high, Then volume is small
R2: If pressure is low, Then volume is large

Therefore, *If pressure is medium*, *Then volume is* $(w1 \cap small + w2 \cap large)$, where $w1 = sup\ (high \cap medium)$, and $w2 = sup\ (low \cap medium)$.

It is worth noting that traditional SR schemes usually focus on reasoning and/or interpolation of two neighboring fuzzy rules within a relatively small local region [8]. In our previous investigation [8], we have argued that SR may not be efficient for the whole domain, or even a relatively large region, in real-world applications. Furthermore, an optimization-based SR scheme for the monotonicity-preserving FIS was proposed, with a number of real world applications demonstrated [8-11]. The importance of the monotonicity property as an additional piece of qualitative information for modeling the FIS has been pointed out [12]. Our proposed optimization-based SR scheme [8-11] attempts to exploit the monotonicity property as additional qualitative information to increase reasoning accuracy for a relatively large range of operating region.

We have previously proposed two application frameworks that comprise SR-based schemes for tackling real-world problems, i.e., a two-stage framework [8] and an online updating framework [13]. The main aim of the two-stage framework [8] is to search for a set of *stage-1 fuzzy rules* in the whole domain such that reasoning and/or interpolation for a relatively large region can be avoided or minimized. *Stage-1 fuzzy rules* are solicited from experts. An SR scheme is then used to deduce the remaining fuzzy rules, which are denoted as *stage-2 fuzzy rules*. Applications of the two-stage framework [8] to two real-world problems, i.e., education assessment [10] and failure mode and effect analysis [8], have also been demonstrated.

In [8, 10], a genetic algorithm (GA) has been used to identify the *stage-1 fuzzy rules*. The GA objective is to minimize the number of *stage-1 fuzzy rules*, subject to a constraint. The constraint is a pre-defined minimal similarity measure between a set of *stage-1 fuzzy rules* and its *stage-2 fuzzy rules*. It is worth noting that various multi-objective evolutionary algorithms [14-17] have been widely used recently, owing to their ability to obtain pareto-optimal solutions. As a result, the main motivation of this paper is on the use of multi-objective evolutionary algorithm for selecting the *stage-1 fuzzy rules*. Specifically, the Non-Dominated Sorting in Genetic Algorithm II (NSGA-II) [14, 17] is used in this study to select the *stage-1 fuzzy rules*. Two objectives are considered: (1) minimize the number of *stage-1 fuzzy rules*; and (2) maximize the similarity measure between a set of *stage-1 fuzzy rules* and its *stage-2 fuzzy rules*. With the use of NSGA-II, a Pareto optimal selection of *stage-1 fuzzy rules* is obtained. The Pareto optimal selection provides a set of solutions that contain trade-offs between the number of *stage-1 fuzzy rules* required and the similarity measure between *stage-1 fuzzy rules* and its *stage-2 fuzzy rules*.

Besides that, the use of evidential measures to facilitate the modeling process of the FIS has been suggested in [13]. Three measures inspired from the Dempster-Shafer Theory (DST) of evidence, i.e., belief, plausibility, and evidential measures, have been introduced for modeling a monotone Sugeno FIS model [13]. These measures are based on the fuzzy membership values. In this paper, we further introduce two generalized evidential functions that are based on fuzzy membership values and distance.

This paper is organized as follows. In Section II, our previous findings are explained. In Section III, the use of NSGA-II to select the *stage-1 fuzzy rules* is presented. Simulated and benchmark [18] examples are included too. In Section IV, the idea of distance-based evidential measure is presented. Finally, concluding remarks are presented in Section V.

II. BACKGROUND

A. A two-stage framework

A two-stage framework for constructing an FIS model proposed in [8] is outlined in Figure 1. It consists of five steps (i.e., A to E). Each step is explained, as follows.

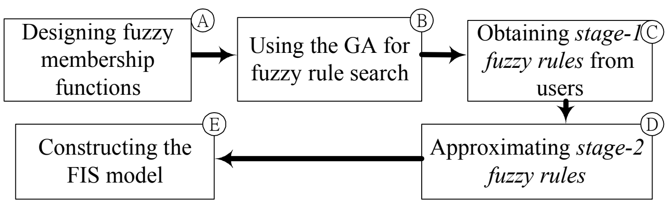

Fig.1. The two-stage framework

Step (A): Designing fuzzy membership functions

The fuzzy membership functions are designed.

Step (B): Using the GA for fuzzy rule search

The GA is adopted to search for an optimal set of *stage-1 fuzzy rules* (denoted as $R_{stage-1}^{n_{stage-1}}: A_{stage-1}^{n_{stage-1}} \rightarrow b_{stage-1}^{n_{stage-1}}$, $n_{stage-1} = 1,2,3,\ldots$) with a predefined constraint. The similarity measure between *stage-1 fuzzy rules* and its *stage-2 fuzzy rules* (denoted as $R_{stage-2}^{n_{stage-2}}: A_{stage-2}^{n_{stage-2}} \rightarrow b_{stage-2}^{n_{stage-2}}$, $n_{stage-2} = 1,2,3,\ldots$) is $\cap_{n_{stage-2}=1}^{m_{stage-2}} \left(A_{stage-2}^{n_{stage-2}}, \cup_{n_{stage-1}=1}^{m_{stage-1}} A_{stage-1}^{n_{stage-1}} \right)$. The constraint is $\cap_{n_{stage-2}=1}^{m_{stage-2}} \left(A_{stage-2}^{n_{stage-2}}, \cup_{n_{stage-1}=1}^{m_{stage-1}} A_{stage-1}^{n_{stage-1}} \right) \geq \tau$, where τ is predefined threshold.

Step (C): Obtaining stage-1 fuzzy rules from users

The *stage-1 fuzzy rules* are solicited from human.

Step (D): Approximating stage-2 fuzzy rules

The *stage-2 fuzzy rules* are approximated using an SR scheme.

Step (E) Constructing the FIS model

The FIS model is constructed using the *stage-1* and *stage-2 fuzzy rules*.

B. Evidential Measures

Consider an FIS model in the form of $y = f(x;\ \theta)$, θ is the parameters of the model, $x \in X$, and $y \in Y$. The evidential measure gives an indication whether x falls in the regions that are not supported by any evidence, i.e., fuzzy rules from domain experts. An example as depicted in Figure 2 is considered. There are six fuzzy membership functions in domain X, i.e., A_1 , A_2^*, A_3^*, A_4 , A_5^*, and A_6 . There are three *stage-1 fuzzy rules*, i.e., $R_{stage-1}: A_1 \rightarrow B_1$, $R_{stage-1}: A_4 \rightarrow B_4$ and $R_{stage-1}: A_6 \rightarrow B_6$. There are three *stage-2 fuzzy rules*, i.e., $R_{stage-2}: A_2^* \rightarrow B_2^*$, $R_{stage-2}: A_3^* \rightarrow B_3^*$ and $R_{stage-2}: A_5^* \rightarrow B_5^*$. Using an SR scheme, B_2^*, B_3^*, and B_5^* are approximated. Fuzzy membership functions of A_j is denoted as $\mu_j\ (x)$. A zero-order Sugeno FIS model is used. The evidential function [13] for this example can be obtained as follows.

$$Evi(x) = 1 - \frac{\mu_2\ (x) + \mu_3\ (x) + \mu_5\ (x)}{\sum_{j=1}^{j=6} \left(\mu_j\ (x) \right)} \tag{1}$$

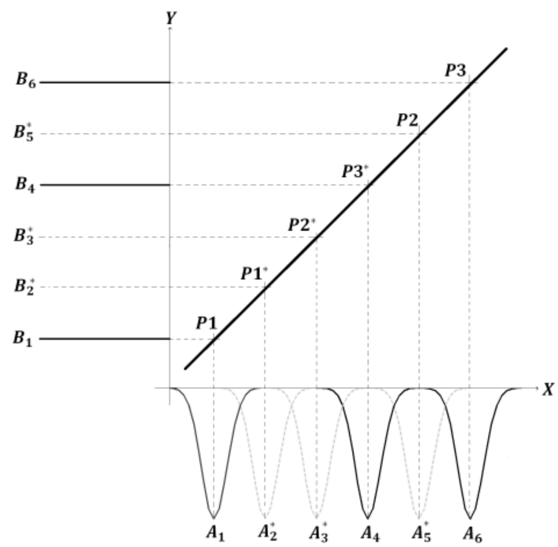

Fig. 2. An example of stage-1 fuzzy rules and stage-2 fuzzy rules

III. USE OF NSGA-II FOR FUZZY RULE SELECTION

A. The Proposed Formulation

In this section, the focus is on *Step (B)* of the two-stage framework shown in Figure 1 and explained in Section II(A). The NSGA-II is adopted. An FIS model, i.e., $y = f(\bar{x};\ \theta)$,is considered, in which θ contains the parameters of the model, and $\bar{x} = [x_1, x_2, \ldots, x_n] \in X_1 \times X_2 \times \ldots \times X_n$. Each input, denoted as x_k, where $x_k \in [x_1, x_2, \ldots, x_n]$, $k = 1,2,3,\ldots,n$, is defined with a range of $[\underline{x_k}, \overline{x_k}]$ and with m_k partitions. Each partition is represented by a fuzzy membership function, denoted as $\mu_k^{n_k}(x_k)$, and is associated with a linguistic term, i.e., $A_k^{n_k}$, where $n_k = 1,2,3,\dots,m_k$. Each fuzzy rule is associated with a binary chromosome, $S_i = 0,1$, where 0 represents the *stage-2 fuzzy rule* and 1 represents the *stage-1 fuzzy rule*, $i = 1,2,3,\dots,\prod_{k=1}^{n} m_k$.

Consider the *stage-1 fuzzy rules*, i.e., $R_{stage-1}^{n_{stage-1}}: A_{stage-1}^{n_{stage-1}} \rightarrow b_{stage-1}^{n_{stage-1}}$, where $n_{stage-1} = 1,\dots,m_{stage-1}$, and the *stage-2 fuzzy rules*, i.e., $R_{stage-2}^{n_{stage-2}}: A_{stage-2}^{n_{stage-2}} \rightarrow b_{stage-2}^{n_{stage-2}}$, where $n_{stage-2} = 1,\dots,m_{stage-2}$. $m_{stage-1} + m_{stage-2} = \prod_{k=1}^{n} m_k$. Note that $b_{stage-1}^{n_{stage-1}}$ is obtained from users, while $b_{stage-2}^{n_{stage-2}}$ is unknown, and needs to be deduced by an SR scheme. NSGA-II is used to select a set of *stage-1 fuzzy rules* with the following two objectives:

$Minimize$

$$obj1 = -\bigcap_{n_{stage-2}=1}^{m_{stage-2}} \left(A_{stage-2}^{n_{stage-2}}, \bigcup_{n_{stage-1}=1}^{m_{stage-1}} A_{stage-1}^{n_{stage-1}}\right) \quad (2)$$

$$obj2 = \sum_{i=1}^{i=M_1 \times M_2 \times \dots \times M_n} S_i \quad (3)$$

The NSGA-II procedure for fuzzy rule selection is summarized in Figure 3. It comprises several user-defined parameters, i.e., the number of generations (t_{max}), number of population ($n_{individual}$), crossover rate ($P_{crossover}$), and mutation rate ($P_{mutation}$). The output is a set of Pareto optimal solutions, i.e., P_{Best}.

NSGA-II_fuzzy_rule_selection ($t_{max}, n_{individual}, P_{crossover}, P_{mutation}$)
1. $t = 1$
2. Initiate population $P(t)$ with $n_{individual}$ individual
3. While $t \leq t_{max}$
   4. Sort each individual $P(t)$ based on its non-dominant fronts
   5. Perform Fitness (Crowding distance) and Ranking
   6. Select $parents$ with the binary tournament selection
   7. Perform Crossover and Mutation
   8. Create new generation, $P(t+1)$
   9. $t = t + 1$
10. End While
11. Compute objective values for each individual $P(t)$
12. Identify the Pareto optimal solutions, i.e., P_{Best}

Return (P_{Best})

Fig.3. The NSGA-II procedure for fuzzy rule selection

B. A simulated example

A two-input FIS, i.e., $y = f(x_1, x_2)$, is considered. Each input has five fuzzy membership functions. Fuzzy membership functions for x_1 and x_2 are shown in Figures 4 and 5, respectively.

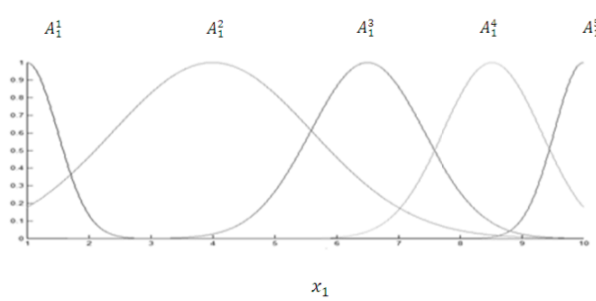

Fig. 4 Fuzzy membership functions for x_1

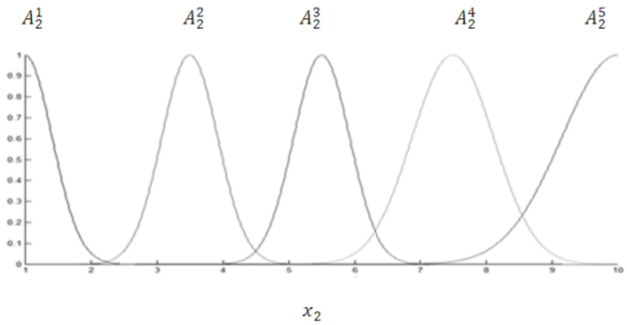

Fig. 5 Fuzzy membership functions for x_2

The parameter settings used in this study are: $n_{individual} = 2000$, $t_{max} = 1000$, $P_{crossover} = 0.9$ and $P_{mutation} = 0.1$. The Pareto optimal solutions are depicted in Figure 6. The Pareto optimal solutions suggest that for 13 *stage-1 fuzzy rules*, the first objective score (Eq. (2)) for the best solution is -0.4578. Besides that, for 6 *stage-1 fuzzy rules*, the first objective score (Eq. (2)) for the best solution is -0.1007.

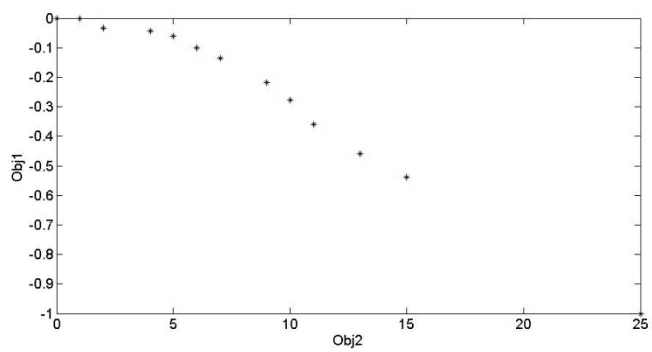

Fig. 6. Pareto optimal fuzzy rules selection

Figures 7 and 8 depict the best solutions for 13 and 6 *stage-1 fuzzy rules*, respectively. The *Stage-2 fuzzy rules* are shaded.

x_1					
A_1^5	$b_{stage-1}^{5,1}$	$b_{stage-1}^{5,2}$	$b_{stage-2}^{5,3}$	$b_{stage-1}^{5,4}$	$b_{stage-2}^{5,5}$
A_1^4	$b_{stage-2}^{4,1}$	$b_{stage-2}^{4,2}$	$b_{stage-2}^{4,3}$	$b_{stage-1}^{4,4}$	$b_{stage-2}^{4,5}$
A_1^3	$b_{stage-1}^{3,1}$	$b_{stage-1}^{3,2}$	$b_{stage-2}^{3,3}$	$b_{stage-1}^{3,4}$	$b_{stage-2}^{3,5}$
A_1^2	$b_{stage-1}^{2,1}$	$b_{stage-1}^{2,2}$	$b_{stage-2}^{2,3}$	$b_{stage-1}^{2,4}$	$b_{stage-2}^{2,5}$
A_1^1	$b_{stage-1}^{1,1}$	$b_{stage-1}^{1,2}$	$b_{stage-2}^{1,3}$	$b_{stage-1}^{1,4}$	$b_{stage-2}^{1,5}$
	A_2^1	A_2^2	A_2^3	A_2^4	A_2^5 x_2

Fig.7. A rule matrix with the simulated example for the best *13 stage-1 fuzzy rules*. The *Stage-2 fuzzy rules* are shaded.

x_1					
A_1^5	$b^{5,1}_{stage-2}$	$b^{5,2}_{stage-1}$	$b^{5,3}_{stage-2}$	$b^{5,4}_{stage-2}$	$b^{5,5}_{stage-2}$
A_1^4	$b^{4,1}_{stage-2}$	$b^{4,2}_{stage-2}$	$b^{4,3}_{stage-2}$	$b^{4,4}_{stage-1}$	$b^{4,5}_{stage-2}$
A_1^3	$b^{3,1}_{stage-1}$	$b^{3,2}_{stage-2}$	$b^{3,3}_{stage-2}$	$b^{3,4}_{stage-1}$	$b^{3,5}_{stage-2}$
A_1^2	$b^{2,1}_{stage-2}$	$b^{2,2}_{stage-2}$	$b^{2,3}_{stage-2}$	$b^{2,4}_{stage-2}$	$b^{2,5}_{stage-2}$
A_1^1	$b^{1,1}_{stage-2}$	$b^{1,2}_{stage-1}$	$b^{1,3}_{stage-2}$	$b^{1,4}_{stage-1}$	$b^{1,5}_{stage-2}$
	A_2^1	A_2^2	A_2^3	A_2^4	A_2^5 x_2

Fig.8. A rule matrix with the simulated example, for the best 6 *stage-1 fuzzy rules*. The *Stage-2 fuzzy rules* are shaded.

C. Simulation with benchmark data/information [18]

A benchmark example from [18] is considered. A fuzzy model is developed to predict soil erosion. The FIS model [18] considers two input parameters, i.e., land use ratio and slope angle, denoted as x_1 and x_2 in this section, respectively. Membership functions for these two parameters are depicted in Figure 9. There are two and five fuzzy membership functions, for slope angle and land use ratio, respectively. The two membership functions for the land use ratio are low and high. The five fuzzy membership functions for slope angle are very small (*VSM*), small (*SM*), moderate (*MOD*), high (*H*), and very high (*VH*). Fuzzy membership functions for the two inputs is explained as a fuzzy partition. The complete fuzzy model consists of ten fuzzy rules.

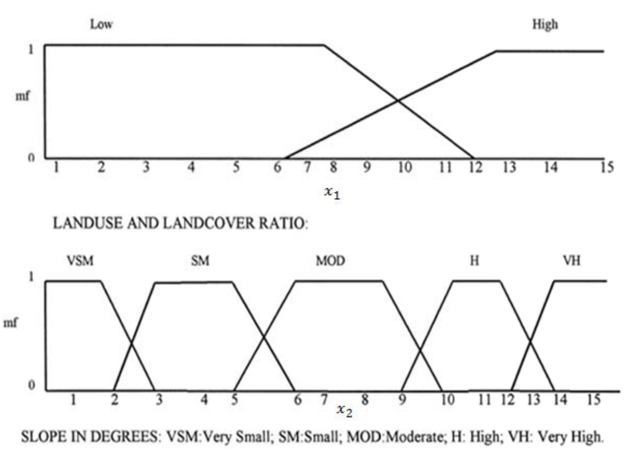

Fig. 9. Fuzzy sets used in [18]

In this section, the fuzzy partition [18] is considered. The proposed approach in Section III (A) is implemented in order to select the *stage-1 fuzzy rules*. The parameter settings are: $n_{individual} = 250$, $t_{max} = 50$, $P_{crossover} = 0.9$ and $P_{mutation} = 0.1$. The Pareto optimal solutions are depicted in Figure 10. The Pareto optimal solutions suggest that for 3 *stage-1 fuzzy rules*, the first objective value (Eq. (2)) for the best solution is -0.5. Besides that, for 2 *stage-1 fuzzy rules*, the first objective score (Eq. (2)) for the best solution is -0.25.

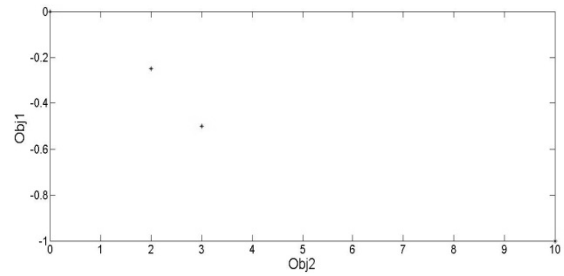

Fig.10. Pareto optimal fuzzy rules selection for benchmark data [18]

Figures 11 and 12 depict the best solutions for 3 *stage-1 fuzzy rules* and 2 *stage-1 fuzzy rules,* respectively. The *Stage-2 fuzzy rules* are shaded. A_1^1 and A_1^2 represent low and high in land use ratio, respectively. $A_2^1, A_2^2, A_2^3,$ A_2^4, and A_2^5 represent *VSM, SM, MOD, H* and *VH* in slope angle, respectively.

x_1					
A_1^2	$b^{2,1}_{stage-2}$	$b^{2,2}_{stage-2}$	$b^{2,3}_{stage-1}$	$b^{2,4}_{stage-2}$	$b^{2,5}_{stage-2}$
A_1^1	$b^{1,1}_{stage-1}$	$b^{1,2}_{stage-2}$	$b^{1,3}_{stage-2}$	$b^{1,4}_{stage-2}$	$b^{1,5}_{stage-1}$
	A_2^1	A_2^2	A_2^3	A_2^4	A_2^5 x_2

Fig.11. A rule matrix with benchmark data [18] for the best *3 stage-1 fuzzy rules*. The *Stage-2 fuzzy rules* are shaded.

x_1					
A_1^2	$b^{2,1}_{stage-2}$	$b^{2,2}_{stage-2}$	$b^{2,3}_{stage-2}$	$b^{2,4}_{stage-1}$	$b^{2,5}_{stage-2}$
A_1^1	$b^{1,1}_{stage-1}$	$b^{1,2}_{stage-2}$	$b^{1,3}_{stage-2}$	$b^{1,4}_{stage-2}$	$b^{1,5}_{stage-2}$
	A_2^1	A_2^2	A_2^3	A_2^4	A_2^5 x_2

Fig.12. A rule matrix with benchmark data [18] for the best *2 stage-1 fuzzy rules*. The *Stage-2 fuzzy rules* are shaded

Figure 11 suggests that the 3 *stage-1 fuzzy rules* are as follows:

i. If land use ratio is *Low* and slope angle is *VSM*
ii. If land use ratio is *Low* and slope angle is *VH*
iii. If land use ratio is *High* and slope angle is *MOD*

Figure 12 suggests that the 2 *stage-1 fuzzy rules* are as follows:

i. If land use ratio is *Low* and slope angle is *VSM*
ii. If land use ratio is *High* and slope angle is *H*

It is worth noting that the two sets of optimal *stage-1 fuzzy rules* selected are different. The *stage-1 fuzzy rules* are selected in a way such that reasoning of the *stage-2 fuzzy rules,* for a relatively large region can be avoided or minimized.

D. Remarks

In this section, NSGA-II is used for fuzzy rule section. Rather than considering Eq (2) as a constraint (as in [8]), Eq (2) is considered as an objective to be optimized. In addition, NSGA-II offers a set of Pareto optimal solutions, which contains trade-offs between the number of *stage-1 fuzzy rules* selected and the similarity measure between the *stage-1 fuzzy rules* and its *stage-2 fuzzy rules.*

IV. EVIDENTIAL FUNCTIONS

A. The Proposed Formulation

The definition of the input space of a target function, i.e., $g: X \rightarrow Y$ is presented as Definition 1, as follows.

Definition 1: Consider an input space, X. Variables x and $A_k(x)$ are the precise and imprecise elements of X, respectively, i.e., $x \in X$, and $A_k(x) \in X$, where $k = 1,2,3,\ldots,n_k$. $Rep(A_k)$ is the representative value of A_k. In this paper, $Rep(A_k)$ is the point in the X domain, in which the membership value of A_k is 1.

Suppose a set of fuzzy rules is given, as in Definition 2.

Definition 2: A set of original sparse fuzzy rules, i.e., $R_k: A_k \rightarrow B_k$, is provided. Note that B_k is a fuzzy set of the output space, i.e., Y.

Considering R_k as evidence. Two evidential functions based on overlapping of fuzzy sets and a distance measure of fuzzy sets are formulated, as in Eq (4) and (5), respectively.

$$evi\_over(x) = \vee_{k=1}^{n_k}(A_k(x)) \quad (4)$$

$$evi\_dist(x) = e^{-\left(\wedge_{k=1}^{n_k}|x-Rep(A_k)|\right)} \quad (5)$$

B. A simulated example

An example with $k = 1$ is shown in Figure 13. With $k = 1$, there is only one fuzzy rule. A Gaussian fuzzy set in the X domain is considered. Its evidential measure with Eqs (4) and (5) are further shown in Figure 13. With Eq (4), its evidential measure is the same as the fuzzy set in the X domain. With Eq (5), the distance between x and $Rep(A_k)$ is considered. The evidential measure reduces exponentially with the distance between x and $Rep(A_k)$.

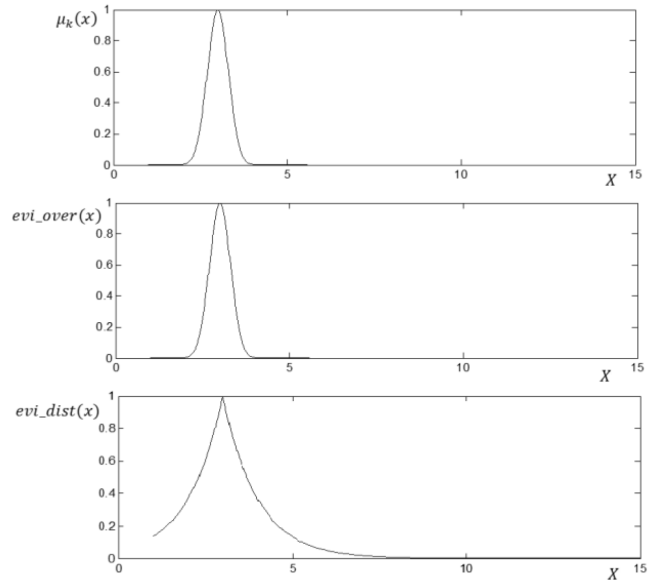

Fig.13. An example with $k = 1$for a Gaussian fuzzy set in the X domain

An example with $k = 2$ is shown in Figure 14. With $k = 2$, there are two fuzzy rules. Two Gaussian fuzzy sets in the X domain are considered. Its evidential measure with Eqs (4) and (5) are further shown in Figure 14. With Eq (4), its evidential measure in the X domain is obtained. The evidential measure at some areas of X is close to zero. Again, with Eq (5), the distance between x and $Rep(A_k)$ are considered. The evidential measure reduces exponentially with the distance between x and $Rep(A_k)$, where $k = 1,2$.

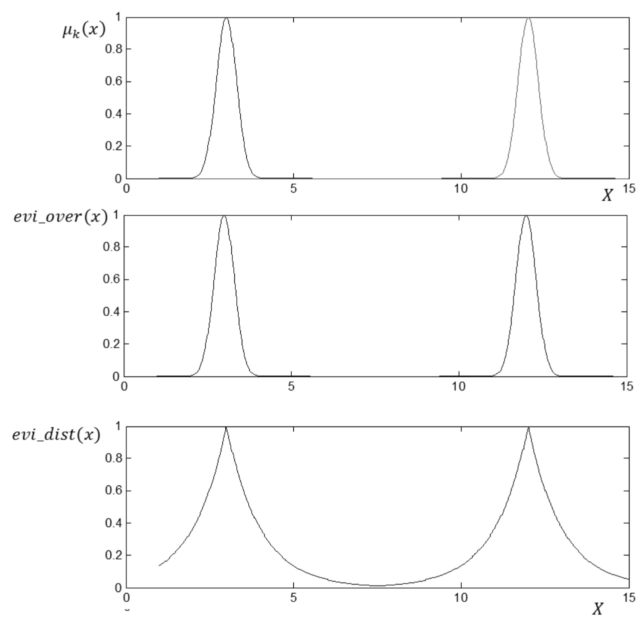

Fig.14. An example with $k = 2$ for two Gaussian fuzzy sets in the X domain

C. Remarks

The overlapping-based evidential function gives an evidential measure of zero at a point in the X domain, i.e., $x \in X$, if there is no fuzzy set coverage at x. The use of the distance-based evidential function (i.e., Eq (5)) provides a non-zero evidential measure. Note that the use of the distance-based evidential function may be possible if partial ordering [19] among fuzzy variables in X exists.

A number of evidential functions have been proposed [13], i.e., evidential mass, belief, and plausibility functions. The evidential mass function provides an indication if $x \in X$ is covered by fuzzy rules approximated by SR. The belief function provides an indication if $x \in X$ of the resulting FIS model is supported by fuzzy rules from experts. The plausibility function gives an indication if $x \in X$ of the resulting FIS model is against by fuzzy rules provided by experts. The formulations in Section IV of this paper can be used as the belief and plausibility measures, by considering R_k as fuzzy rules from experts that support the resulting FIS, and fuzzy rules from experts that against the resulting FIS, respectively. The evidential mass measure can be obtained by considering R_k as the SR approximated fuzzy rules.

V. Conclusions

In this paper, we have extended our previous findings in [8, 13]. Firstly, NSGA-II is used for fuzzy rule selection. As a result, a set of Pareto optimal solutions that consider the trade-offs between the number of selected *stage-1 fuzzy rules* and the similarity measure between a set of *stage-1 fuzzy rules* and its *stage-2 fuzzy rules*, is obtained. Besides that, two evidential functions (i.e., based on overlapping of fuzzy sets and distance of fuzzy sets) for supporting the practical implementation of SR have been formulated.

For future work, other multi-objective meta-heuristic techniques, e.g., particle swarm optimization, harmony search, can be applied to generate the *stage-1 fuzzy rules*. The effect of different parameter settings for NSGA-II can also be studied. The use of (distance-based) evidential function as part of fuzzy rule selection to reduce computation complexity can be studied. Furthermore, evaluation of the proposed framework can be studied by implementing to real world case studies.

Acknowledgment

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