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A Method of Remote Sensing Image Auto Classification Based on Interval Type-2 Fuzzy C-Means

Xianchuan Yu, *Senior Member, IEEE,* Wei Zhou, and Hui He

***Abstract*—The pattern set of a remote sensing image contains many kinds of uncertainties. Uncertain information can create imperfect expressions for pattern sets in various pattern recognition algorithms, such as clustering algorithms. Methods based the fuzzy c-means algorithm can manage some uncertainties. As soft clustering methods, They are known to perform better on auto classification of remote sensing images than hard clustering methods. However, if the clusters in a pattern set are of different density and high order uncertainty, performance of FCM may significantly vary depending on the choice of fuzzifiers. Thus, we cannot obtain satisfactory results by using type-1 fuzzy set. Type-2 fuzzy sets permit us to model various uncertainties which cannot be appropriately managed by type-1 fuzzy sets. This paper introduces the theory of interval type-2 fuzzy set into the unsupervised classification of remote sensing images and proposes the automatic remote sensing image classification method based on the interval type-2 fuzzy c-means. Experimental results indicate that our method can obtain more coherent clusters and more accurate boundaries from the data with density difference. Our type-2 fuzzy model can manage the uncertainties of remote sensing images more appropriately and get a more desirable result.**

***Index Terms*—Type-2 fuzzy sets; Uncertainty; Fuzzy c-means; Fuzzy clustering; IT2FCM; Remote sensing classification**

I. Introduction

AUTOMATIC classification of remote sensing images is a classic study project. It has been widely used and plays an important role in map update, target recognition, disaster monitoring, resource application etc. [1]

At present, the automatic classification mainly utilize the traditional unsupervised classification methods such as ISODATA, K-Means and Fuzzy C-Means (FCM) [2]. These methods can realize conveniently and rapidly the spectral classification of remote sensing images without any prior knowledge [3], [4]. But due to the uncertainties contained by the characteristics of remote sensing images, the ISODATA and K-Means, as the hard clustering methods, produce unsatisfying processing results, while the Fuzzy C-Means as a soft clustering method could often get better results [1], [5]–[7]. When the pattern set has clusters of similar size and density with the hypersphere shape, the methods based on FCM are good. But if the clusters of the pattern set have significant different densities, the FCM will show quite different effects depending on the different fuzzifiers [8]. Therefore if we process the remote sensing images with great different densities and high order uncertainties by methods based on FCM, it is difficult to obtain satisfactory results.

The type-2 fuzzy sets can deal with various uncertainties more properly than type-1 [9]–[11]. Type-2 fuzzy set needs to calculate the secondary membership for each primary membership, so type-2 fuzzy set usually increases the computational complexity than type-1. Because the secondary memberships of the interval type-2 fuzzy set are all equal to 1, so its computation complexity gets significantly reduced [12], [13].

Based on the type-2 fuzzy set theory, Rhee and Hwang proposed a type-2 fuzzy c-means clustering algorithm in 2001 [14], which shows a good noise resistance, but needs improvement in its computational complexity and the constructor of secondary membership. In 2007, on the basis of their research in 2001, under the theoretical framework of interval type-2 fuzzy set, Hwang and Rhee put forward the interval type-2 fuzzy c-means clustering method. This method uses two different fuzzifiers to construct the membership, and carries on the classification experiment on a variety of simple data sets, but the hard partition method is too complex and the fuzzifier is too arbitrary [8]. In 2009, Choi and Rhee proposed various construction methods of interval type-2 fuzzy membership function in the field of pattern recognition [15]. In 2013, Linda and Manic extended the interval type-2 fuzzy c-means clustering algorithm to general type-2 fuzzy c-means clustering algorithm [16]. The theory of type-2 fuzzy set is a new hotspot in the research of the fuzzy set theory, but has not yet been applied to the automatic classification of remote sensing images.

This paper introduces the theory of interval type-2 fuzzy set into the unsupervised classification of remote sensing images and proposes the automatic classification method of remote sensing images based on interval type-2 fuzzy c-means. The rest of this paper is arranged as follows: The second part introduces the theoretical method, including the uncertainty of fuzzifier m, and the basic knowledge of the interval type-2 fuzzy set theory and the method of extending fuzzy c-means (FCM) to interval type-2 fuzzy c-means (IT2FCM). The third part is the experimental analysis. The fourth part is the conclusion.

II. Interval Type-2 Fuzzy C-means Method

This section first introduces fuzzy c-means clustering algorithm and the uncertainty of the fuzzifier m, then introduces the basic knowledge of the interval type-2 fuzzy set theory, and finally introduces the method of extending the fuzzy c-means (FCM) to the interval type-2 fuzzy c-means (IT2FCM).

X. Yu is with the College of Information Science and Technology, Beijing Normal University, Beijing, China (e-mail:chuan.yu@ieee.org, yuxianchuan@163.com). W. Zhou is with the Graduate School of Beijing Normal University, Beijing, China(e-mail:zhouwei@bnu.edu.cn). H. He is with Beijing Normal University, ZhuHai, China.

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A. Type-1 Fuzzy C-Means

Sample space

$$X = \{x_1, x_2, ..., x_n\}, x_k = (x_{k1}, x_{k2}, ..., x_{kp}), 1 \le k \le n$$

A matrix U of $c \times n$ defines a fuzzy C partition of the data set X, if $\sum_{i=1}^{c} u_{ij} = 1, 0 \le \sum_{j=1}^{n} u_{ij} \le n$. Each row of the matrix U defines a fuzzy clustering of X.

$$u_{ij} = u_i(x_j) = \frac{1}{\sum_{k=1}^{c} (d_{ij}/d_{kj})^{2/(m-1)}} \quad (1)$$

Eq. (1) shows the membership of sample x_j to center v_i, $d_{ij}(d_{ki})$ means the distance between the center $v_i(v_k)$ and the pattern x_j, $m > 1$ is a fuzzifier.

$$v_i = \frac{\sum_{j=1}^{n} u_i(x_j)^m x_j}{\sum_{j=1}^{n} u_i(x_j)^m} \quad (2)$$

Eq. (2) is the clustering center of the cluster i, and note functional

$$J(U, V) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m d_{ij}^2 \quad (3)$$

is the objective function of the fuzzy clustering. The most optimal fuzzy partition is to obtain the minimum value of the functional $J(U, V)$.

Bezdek presented the iterative algorithm for solving the optimal fuzzy partition, namely the FCM algorithm. References [5]–[7] and a lot of other literature have put it into application or made improvements based on it, which won't be elaborated on here. Discussions below focus on the uncertainty of fuzzifier m and the method of constructing interval type-2 fuzzy set.

B. Uncertainty of Fuzzifier

FCM clustering algorithm is an iterative algorithm, that mainly uses Eq. (1) and Eq. (2) to update the center iteratively after the initialization. The membership depends on the pattern, the centers and the fuzzifier. When the pattern set has been given, usually the clustering center will converge in the process of iteration, therefore m is a very important parameter. It directly affects the fuzzy degree of clustering.

As shown in Fig. 1, there is an uncertainty in the parameter m. Fig. 1a means the curve of the relative distance and membership corresponding to different fuzzifiers, when the pattern moves along the ligature between two centers with a relative distance of 1. Obviously the curve of membership is significantly impacted by m. When $m \to 1$, the fuzzy clustering degenerates into hard clustering. When $m \to \infty$, the fuzzy clustering is in the maximum fuzzification and loses the ability to partition. Experts provide experience value of the fuzzifier $m = 2$ [17]. When clusters of pattern set have difference on density, the effects of FCM significantly vary with m [8].

A single fuzzifier makes it difficult to achieve satisfactory results when the algorithm processes the remote sensing images with a large density difference and uncertainty. So it is necessary to utilize multiple fuzzifiers to build new FCM algorithm. As shown in Fig. 1b, $m_1 = 1.1, m_2 = 5$ constitute the uncertainty region, showing that every relative distance corresponds to a fuzzy membership interval. we can use this interval to define the uncertainty of membership and build the interval type-2 fuzzy set as in [13].

C. Interval Type-2 Fuzzy Set

An interval type-2 fuzzy set $\widetilde{A}$ is defined as follows:

$$\widetilde{A} = \{((x, u), u_{\widetilde{A}}) | \forall x \in X, \forall u \in J_x \subset [0, 1], u_{\widetilde{A}}(x, u) = 1\} \quad (4)$$

u is called primary membership of x, $u_{\widetilde{A}}(x, u) = 1$ is called the secondary membership, and interval type-2 fuzzy set's secondary membership is identically equal to 1.

As shown in figure Fig. 1b, for a pattern x, membership can be expressed by lower bound $\underline{u}(x)$ and upper bound $\overline{u}(x)$ respectively, that is to say, every x corresponds to a primary membership interval

$$J_x = [\underline{u}(x), \overline{u}(x)] \quad (5)$$

The calculation of fuzzy membership in FCM algorithm follows Eq. (1). With two fuzzifiers, the type-1 fuzzy set is extended to interval type-2 fuzzy set. Naturally Eq. (6) can be used to determine the upper and lower bounds of fuzzy membership to build J_x. Each corresponding line of the membership matrix $\underline{U}$ and $\overline{U}$ defines an interval type-2 fuzzy clustering of X. Here we consider the clustering center calculation method of this fuzzy clustering.

$$\begin{aligned} \overline{u}_j(x_i) &= \max\left(\frac{1}{\sum_{k=1}^{c} (\frac{d_{ji}}{d_{ki}})^{\frac{2}{m_1-1}}}, \frac{1}{\sum_{k=1}^{c} (\frac{d_{ji}}{d_{ki}})^{\frac{2}{m_2-1}}}\right) \\ \underline{u}_j(x_i) &= \min\left(\frac{1}{\sum_{k=1}^{c} (\frac{d_{ji}}{d_{ki}})^{\frac{2}{m_1-1}}}, \frac{1}{\sum_{k=1}^{c} (\frac{d_{ji}}{d_{ki}})^{\frac{2}{m_2-1}}}\right) \end{aligned} \quad (6)$$

Eq. (2) is the calculation formula of center of the type-1 fuzzy set. We extend Eq. (2) to the center of the interval type-2 fuzzy set [18], as shown in Eq. (7):

$$v_{\widetilde{x}} = [v_l, v_r] = \sum_{u(x_1) \in J_{x_1}} \cdots \sum_{u(x_n) \in J_{x_n}} 1 \Big/ \frac{\sum_{i=1}^{n} u(x_i)^m x_i}{\sum_{i=1}^{n} u(x_i)^m} \quad (7)$$

m is the fuzzifier, $u(x_i) = [\underline{u}(x_i), \overline{u}(x_i)]$ is the primary membership of x_i. To solve Eq. (7), if we replace $u(x_i)^m$ with w_i, we get the following problems:

$$v_{lq} = \min_{\forall w_i \in [\underline{w_i}, \overline{w_i}]} \frac{\sum_{i=1}^{N} x_{iq} w_i}{\sum_{i=1}^{N} w_i} \quad (8)$$

$$v_{rq} = \max_{\forall w_i \in [\underline{w_i}, \overline{w_i}]} \frac{\sum_{i=1}^{N} x_{iq} w_i}{\sum_{i=1}^{N} w_i} \quad (9)$$

$1 \le q \le p$,$v_l = (v_{l1}, ..., v_{lp})$,$v_r = (v_{r1}, ..., v_{rp})$. Make

$$f(w_1, ..., w_N) = \frac{\sum_{i=1}^{N} x_i w_i}{\sum_{i=1}^{N} w_i} \quad (10)$$

The above question is converted to the solution the extreme problem of multivariate function f of $w_1, ..., w_N$, then we can calculate the partial derivative of f for w_k.

$$\frac{\partial f(w_1, ..., w_N)}{\partial w_k} = \frac{x_k - f(w_1, ..., w_N)}{\sum_{i=1}^{N} w_i} \quad (11)$$

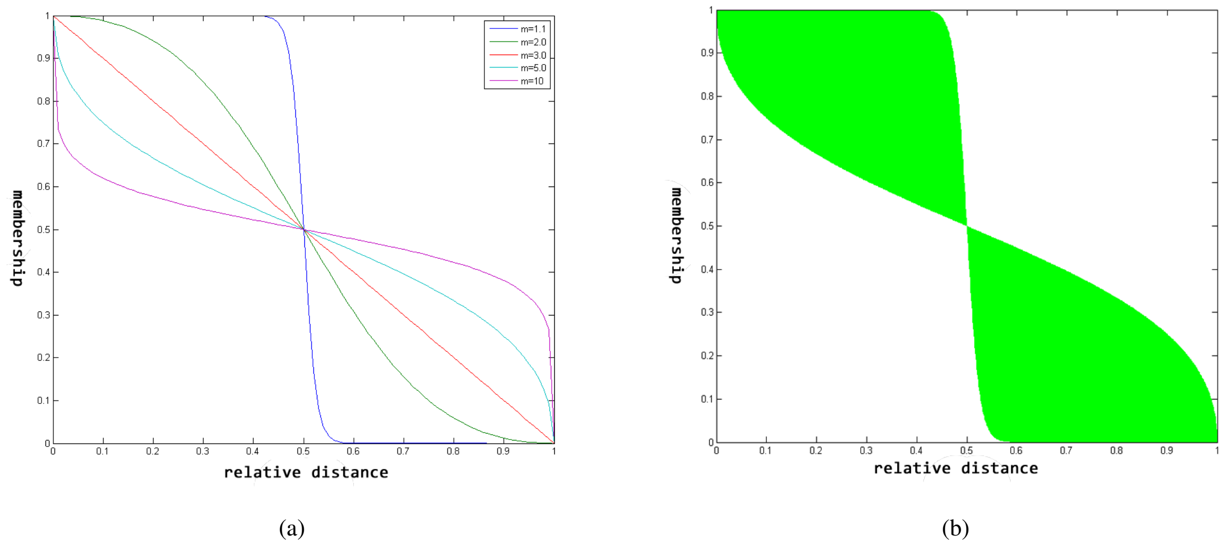

Fig. 1: Uncertainty of fuzzifier m. (a)Curve of the relative distance and membership corresponding to the different values of m (b) Uncertainty areas constructed by $m_1 = 1.1, m_2 = 5$

Since $\sum_{i=1}^{N} w_i > 0$, then

$$x_k > f, w_k \uparrow \Rightarrow f \uparrow \wedge w_k \downarrow \Rightarrow f \downarrow$$

$$x_k < f, w_k \uparrow \Rightarrow f \downarrow \wedge w_k \downarrow \Rightarrow f \uparrow$$

Among them, symbol $\uparrow$ means increase, $\downarrow$ means decrease, $\Rightarrow$ means contains, and $\wedge$ means logical AND. Therefore, we can use the iterative algorithm to solve Eq. (7), but only need to choose the upper and lower bounds of the interval membership for calculation according to the relationship of the pattern and center, i.e., the Karnik Mendel algorithm [19], [20] can be used to calculate the v_l, v_r.

The center of the interval type-2 fuzzy set obtained through the Karnik Mendel algorithm is

$$v_j = 1.0/[v_{l_j}, v_{r_j}] \tag{12}$$

This is still a fuzzy set. Mean value method can be used to defuzzy as follows

$$v_j = \frac{v_{l_j} + v_{r_j}}{2} \tag{13}$$

And then, We rebuild the interval type-2 fuzzy set for the pattern set using the new center and repeat the above steps to get the final cluster center and interval fuzzy partition matrix.

At last, we have to do a hard partition, which is to convert the fuzzy partition to the ordinary one. For the type-1 fuzzy partition matrix, the maximum membership principle can be used to convert it to ordinary partition matrix. However, the maximum membership principle cannot be directly applied to the interval fuzzy partition matrix, so the algorithm of reference [8] builds a type-1 fuzzy partition matrix by three steps. First, it determines the average value of the upper and lower boundary of the membership chosen when the dimensional characteristics compute the center with KM algorithm, and treat it as the interval membership of the whole characteristic. Second, it forms a type-1 partition matrix by using the interval center as a new membership. Last, it does hard partition with the principle of maximum membership.

This hard partition method needs to store the interval membership of each dimension's characteristics, so it doesn't obtain satisfactory time complexity and space complexity. Actually, from Fig. 1 and Eq. (7) we can figure out that when the center is determined, using different fuzzifiers to calculate the membership of all the samples will not change the ordering of each sample's membership to the center, and the average interval memberships calculated by two different fuzzifiers also have the same order. We can use any m to calculate a type-1 fuzzy partition matrix and convert it to an ordinary partition matrix with the maximum membership principle. Therefore it is not only easy to understand, but also has simple calculation and low space complexity if we use the closest to center principle to do the hard partition directly.

D. Steps of Algorithm

The previous section introduces in detail the establishment, the calculation method and the hard partition method of the interval type-2 fuzzy set. Both the interval type-2 fuzzy c-means and type-1 fuzzy c-means clustering algorithm are iterative algorithms. The difference is the interval type-2 fuzzy set center uses the Karnik-Mendel algorithm and needs defuzzification. Fig. 2 is the scheme of classification based on IT2FCM. Specific steps are as follows:

1) Determine the clustering number $c(2 \leq c \leq n)$ and the parameters $m, m_1, m_2, (1 < m, m_1, m_2 < \infty), \varepsilon > 0$, set the initial center matrix V^0, iterate step by step, $L = 0, 1, 2, ...$;

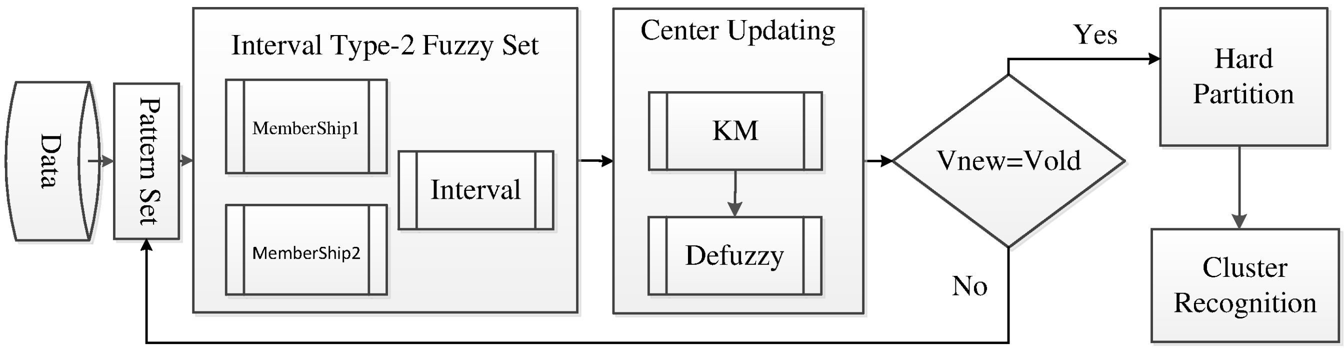

Fig. 2: The scheme of classification based on IT2FCM

2) Calculate the membership matrix $\underline{U}, \overline{U}$ by Eq. (6) with V^L as the center;
3) Use Karnik-Mendel algorithm to calculate the center matrix V_l, V_r, make the $V^{L+1} = \frac{V_l+V_r}{2}$;
4) If $\|V^{L+1} - V^L\| < \varepsilon$, turn to the next step, otherwise set $L = L + 1$ and turn to step 2;
5) Get the final cluster centers and membership matrix, do hard partition by the principle of the closest to center.

The clustering result of the interval type-2 fuzzy clustering can be obtained after the hard partition, and the final classification results are obtained after the clustering discriminant in the end.

III. Experimental Analysis

We select the TM multi-spectral data acquired on on November 15, 1999 in Hengqin Island, Zhuhai City, China. It is an 8-bit data with 30m spatial resolution, 795x452 pixels size. Features of the image include vegetation (grassland, forest land), clear water, turbid water, wetland, building land (housing, roads, airport), and farms (raise oysters, water flooded rice fields). We extract (4, 3, 2) bands as the RGB channel to build the standard false color image for classification as shown in Fig. 3a. The mountain vegetation data in the figure (red area), presents evident brightness changes, which means the vegetation data are distributed in a relatively large hypersphere with a reddish center. While the water data are mainly represented by cyan areas and the building data mainly are mainly represented by white areas. So this kind of data is distributed within a relatively small hypersphere respectively. In this case, the classification results will evidently vary with the fuzzifier m. Thus it is difficult to get a satisfactory result with the model of type-1 fuzzy c-means, while the interval type-2 fuzzy c-means may lead to a satisfactory result.

We choose K-MEANS, ISODATA and FCM with fuzzifiers $m = 2, m = 3, m = 5, m = 10$ to classify the experiment data by the Euclidean distance. We get the clustering results of the image, respectively, as shown in Fig. 3b, Fig. 3c and Fig. 3d, Fig. 3e. Then our IT2FCM method with clustering number $c = 6$, fuzzifier $m = 3, m_1 = 2, m_2 = 10$, $\varepsilon = 10^{-5}$ classify the experimental data by Euclidean distance. The classification result is shown in Fig. 3f.

Area 1 is one part of a wetland. The spectral characteristics are similar to those of the building land. K-MEANS method and FCM methods with different fuzzifiers all misclassified this area as building land, while ISODATA method and our IT2FCM method have successfully identified this part of wetland, and our IT2FCM method had better boundaries.

Area 2 is one part of vegetation. K-MEANS method and FCM methods with smaller fuzzifiers misclassified much water as farms, and ISODATA method got a result of crossly distributed vegetation and farms, it misclassified a lot of vegetation and water to farms, while our method only misclassified a small amount of water to farms.

Area 3 is one part of vegetation on the mountain. ISODATA method misclassify a lot as vegetation and farms. The results of K-MEANS method and FCM methods with small fuzzifiers are similar. While we compare FCM methods with different fuzzifiers, we can find differences between each result. When fuzzifier m is smaller, the fuzziness of classification is lower. So when the spectrum of vegetation obvious vary in brightness, it lead to a lot misclassification. When fuzzifier m is larger, the fuzziness of classification is higher. The misclassification of mountain vegetation was significantly reduced. But it was too fuzziness to got a clear clustering boundary. The FCM methods are difficult to obtain satisfactory results. Our interval type-2 fuzzy c-means(IT2FCM) method not only succeeded in identifying the main body of the vegetation with brightness changes but also got a very clear boundary.

Area 4 is Macao airport. our method got significantly less isolated points, much better classification continuity, and clearer boundaries than other methods.

In order to verify the results by objective evaluation, we selected 59 points randomly for ground validation. The classification accuracy and Kappa coefficient are shown in Table I. According to objective evaluation results, clustering accuracy and Kappa coefficient of FCM are greatly affected by fuzzifier. The clustering accuracy and Kappa coefficient aren't monotonic relationship with the fuzzifier. The results indicate that our interval type-2 fuzzy c-means method (IT2FCM) increases the classification accuracy and Kappa coefficient, and can obviously improve the clustering result. The objective result is also consistent with the visual interpretation.

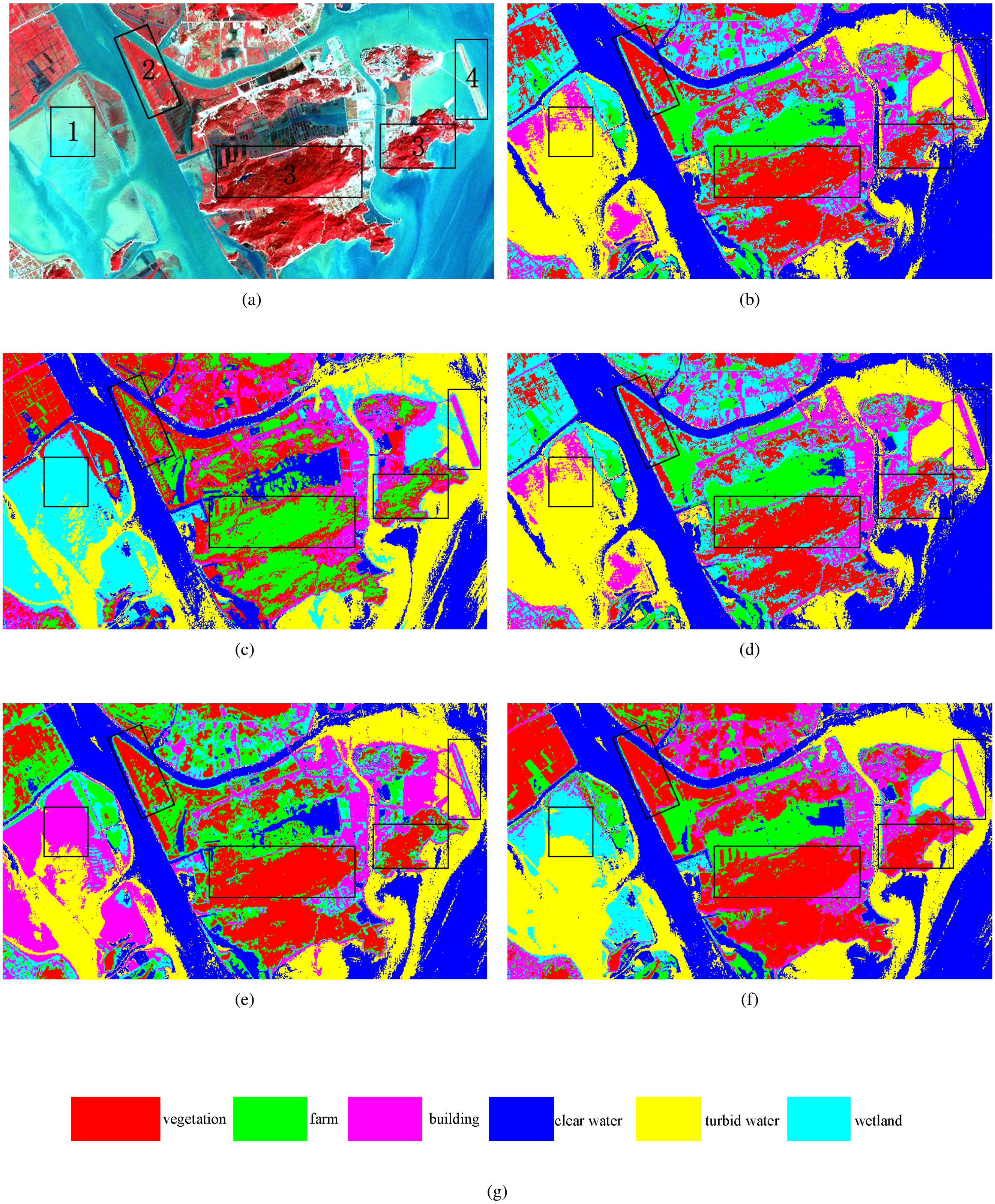

Fig. 3: Region of Hengqin Island, Zhuhai City, China. (a) Combined RGB image (795x452) (b)Resulting image of K-Means (c)Resulting image of ISODATA (d)Resulting image of FCM($m = 2$) (e)Resulting image of FCM($m = 10$) (f)Resulting image of our method (g)Legend.

TABLE I: Objective evaluation

Methods	Overall	Kappa coefficient
K-Means	54.24%	0.4381
ISODATA	69.49%	0.6170
FCM($m = 2$)	57.63%	0.4760
FCM($m = 3$)	57.63%	0.4764
FCM($m = 5$)	67.80%	0.6006
FCM($m = 10$)	61.02%	0.5166
IT2FCM	81.36%	0.7670

From the analysis above, compared with K-MEANS, ISODATA and type-1 FCM, our IT2FCM method has obvious improvement in both subjective evaluation and objective evaluation. It is more suitable for solving misclassification caused by same object with the different spectra characteristics. The method has good anti-interference ability. When the target is affected by the neighborhood spectrum, our method can get more continuous areas and more accurate boundaries.

Our method did not distinguish between the data of grassland and forest, it is difficult to solve the phenomenon of same spectrum with different objects, but it can be work out by our hierarchical classification method [21] or hyperspectral data. In addition, the iteration process is more complicated than type-1 fuzzy c-means. So for some real time systems, the efficiency of the algorithm needs to be improved.

IV. Conclusions

In this paper, we introduce the interval type-2 fuzzy set theory into the automatic classification of remote sensing images. It is the first time that the interval type-2 fuzzy c-means is applied to the unsupervised classification of remote sensing images whose pattern sets contain many kinds of uncertainties. Traditional fuzzy c-means method uses a certain fuzzifier to calculate the membership matrix, so it is hard to handle the uncertainties of remote sensing images, especially for the ones with density difference. Type-2 fuzzy sets permit us to model various uncertainties which cannot be appropriately managed by type-1 fuzzy sets.

The experimental results showed obvious improvement in both subjective evaluation and objective evaluation. The results indicate that our IT2FCM method has the following advantages over traditional methods such as K-MEANS, ISODATA and FCM. Our IT2FCM method has a stronger ability to manage the uncertainties of remote sensing images, especially those with density difference. It is more suitable and has a better anti-interference ability when solving the misclassification caused by the same object with different spectra characteristics. It can get more coherent clusters and more accurate boundaries.

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