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Improved observer-based $H\infty$ control for fuzzy interconnected systems*

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***Abstract*—In this paper, based on Lyapunov-Krasovskii functional approach and the decentralized control theory of interconnected systems, we consider the observer-based control design for a class of fuzzy descriptor interconnected systems. Firstly, the stability conditions of the T-S fuzzy model with a prescribed performance level are provided in theorem 1, but there is no efficient algorithm solving the inequalities. Theorem 2 proposes a single-step method, reducing the conservatism of the conditions. This method overcomes the drawback of the two-step approach in other methods. The stability conditions in theorem 1 are the sufficient and necessary conditions of the ones in theorem 2. The parameters of reliable $H\infty$ controllers can be found via LMI toolbox in MATLAB software. Finally a numerical example is given to show the validity of the proposed method.**

Keywords*—*interconnected systems; observer-based control; $H\infty$ control; a single-step method

I. INTRODUCTION

In modern times, interconnected systems are widely applied in industrial control and actual production. There are a lot of factors affecting their normal operation, for example, information structure constraints, multi-objective, multi-decision-makers, uncertainty, and nonlinear as well as human factors. Stability, observability and controllability are three basic nature of the system. The task of stabilization is typical control problem.

Takagi-Sugeno (T-S) fuzzy model has become a popular and effective approach to control complex systems, and a lot of significant results on stabilization and $H\infty$ control via linear matrix inequality (LMI) approach have been reported, see [1-4]. There are some works about stability and stabilization of fuzzy large-scale systems [5-8], quadratic performance or/and $H\infty$ performance [9–14]. The technology of descriptor model transformation is used in [15, 16]. T-S fuzzy descriptor tracking control design for nonlinear systems with a guaranteed $H\infty$ model reference tracking performance is discussed [17]. Up to now, many important issues have been studied for observer-based $H\infty$ control of nonlinear systems [18, 19].

This paper investigates the observer-based control issue for the interconnected systems. Firstly the stability conditions of the T-S fuzzy model with a prescribed $H\infty$ performance level are given. But there is no efficient algorithm solving the inequalities by one step. This paper improves the stepwise design method of observer-based $H\infty$ controller in the existing literature, and provides a single-step method, reducing the conservatism of conditions. Then we can design the observers and controllers in single-step.

Finally a numerical example is given to show the validity of the proposed method. Through theoretical derivation and simulation, the design method of the fuzzy controller presented in this article is proved to be feasible, effective and convenient.

II. SYSTEMS DESCRIPTION

Suppose the following interconnected systems consist of N interconnected subsystems S_k, $k=1,\cdots,N$. Each rule of the subsystem S_k is represented by a T-S fuzzy model as follows:

$$\begin{cases} \text{If } \xi_{k1} \text{ is } M_{1i} \; and \ldots and \; \xi_{kp} \text{ is } M_{pi} \\ \text{Then } \dot{x}_k(t) = A_{ik}x_k(t) + B_{1ik}w_k(t) + B_{2ik}u_k(t) + \sum\limits_{n=1,n\neq k}^{N} C_{ink}x_n(t) \\ z_k(t) = D_{ik}x_k(t) + E_{ik}u_k(t) \end{cases} \tag{1}$$

Where $x_k(t)\in R^{n'}$, $u_k(t)\in R^{m'}$, $z_k(t)\in R^{p'}$ and $w_k(t)$ denote the state, input, output and disturbance vector. A_{ik}, B_{1ik}, B_{2ik} D_{ik}, E_{ik} denote system matrices, C_{ink} denote the interconnection matrices between nth and kth subsystem of the ith rule. $\xi_{k1},\cdots,\xi_{kp}$ are the premise variables, M_{pi} is the fuzzy set.

If we utilize the singleton fuzzifier, product fuzzy inference and central-average defuzzifier, (1) can be inferred as:

$$\begin{cases} \dot{x}_k(t) = \sum\limits_{i=1}^{r_k} \mu_{ik}(x_k)\Big(A_{ik}x_k(t) + B_{1ik}w_k(t) + B_{2ik}u_k(t) \\ \qquad + \sum\limits_{n=1,n\neq k}^{N} C_{ink}x_n(t)\Big) \\ z_k(t) = \sum\limits_{i=1}^{r_k} \mu_{ik}(x_k)\big(D_{ik}x_k(t) + E_{ik}u_k(t)\big) \end{cases} \tag{2}$$

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Where,

$$\omega_{ik}(\xi_k) = \prod_{p=1}^{P} M_{pi}(\xi_p), \mu_{ik}(\xi_k) = \frac{\omega_{ik}(\xi_k)}{\sum_{i=1}^{r_k} \omega_{ik}(\xi_k)}$$ $\xi_{k1}, \cdots, \xi_{kp}$ are the premise variables, $M_{pi}(\xi_p)$ is the grade of membership of $\xi_{kp}(t)$ in M_{pi}. It can be seen that $\omega_{ik}(\xi_k) > 0$, $\mu_{ik}(x_k) \geq 0$, and $\sum_{i=1}^{r_k} \mu_{ik}(x_k) = 1$.

Observer Rule k:

$$\begin{cases} \text{If } \xi_{k1} \text{ is } M_{1i} \text{ and} \ldots \text{and } \xi_{kp} \text{ is } M_{pi} \\ \text{Then } \dot{\hat{x}}_k(t) = \sum_{i=1}^{r_k} \mu_{ik} \Big(A_{ik}\hat{x}_k + B_{1ik} w_k + B_{2ik} u_k \\ \qquad + \sum_{n=1, n\neq k}^{N} C_{ink} \hat{x}_n \Big) - \sum_{i=1}^{r_k} \mu_{ik} L_{ik} \left(y_k - \hat{y}_k \right) \end{cases} \tag{3}$$

Where L_k is the observer gain of the kth rule, and:

$$\hat{y}_k = \sum_{j=1}^{r_k} \mu_{jk} \left(D_{yjk}\hat{x}_k + E_{yjk} w_k \right) \tag{4}$$

According to the conventional parallel distributed compensation concept, the fuzzy controllers corresponding to S_k are used as follows:

$$\begin{cases} \text{If } \xi_{k1} \text{ is } M_{1i} \text{ and ... and } \xi_{kp} \text{ is } M_{pi} \\ \text{Then } u_k(t) = K_{ik}\hat{x}_k \end{cases} \tag{5}$$

The output of the fuzzy controller of subsystem S_k is:

$$u_k(t) = \sum_{i=1}^{r_k} \mu_{ik} K_{ik} \hat{x}_k \tag{6}$$

Then, the closed-loop fuzzy subsystem can be expressed as the following form:

$$\dot{x}_k = \sum_{i=1}^{r_k}\sum_{j=1}^{r_k} \mu_{ik}\mu_{jk} \left(A_{ik}x_k + B_{1ik}w_k + B_{2ik}K_{jk}\hat{x}_k + \sum_{n=1, n\neq k}^{N} C_{ink}x_n \right) \tag{7}$$

Therefore, the whole fuzzy observer system becomes:

$$\dot{\hat{x}}_k(t) = \sum_{i=1}^{r_k} \mu_{ik} \Big(A_{ik}\hat{x}_k + B_{1ik}w_k + B_{2ik}u_k + \sum_{n=1, n\neq k}^{N} C_{ink}\hat{x}_n - L_{ik}\left(y_k - \hat{y}_k \right) \Big) \tag{8}$$

Denote the error of the system:

$$e_k = \hat{x}_k - x_k \tag{9}$$

Then we have:

$$\dot{e}_k = \sum_{i=1}^{r_k}\sum_{j=1}^{r_k} \mu_{ik}\mu_{jk} \left(\left(A_{ik} + L_{ik}D_{yjk} \right) e_k + \sum_{n=1, n\neq k}^{N} C_{ink}e_n \right) \tag{10}$$

Then the augmented system can be described as:

$$\dot{\tilde{x}}_k = \begin{bmatrix} \dot{x}_k \\ \dot{e}_k \end{bmatrix} = \sum_{i=1}^{r_k}\sum_{j=1}^{r_k} \mu_{ik}\mu_{jk} \left(\tilde{A}_{ik}\tilde{x}_k + \tilde{B}_{ik}w_k + \tilde{C}_{ink}\tilde{x}_n \right) \tag{11}$$

$$z_k = \sum_{i=1}^{r_k}\sum_{j=1}^{r_k} \mu_{ik}\mu_{jk} \begin{bmatrix} D_{ik} + E_{ik}K_{jk} & E_{ik}K_{jk} \end{bmatrix} \tilde{x}_k \tag{12}$$

Where,

$$\tilde{A}_{ik} = \begin{bmatrix} A_{ik} + B_{2ik}K_{jk} & B_{2ik}K_{jk} \\ 0 & A_{ik} + L_{ik}D_{yjk} \end{bmatrix}, \tilde{B}_{ik} = \begin{bmatrix} B_{1ik} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{w}_k = \begin{bmatrix} w_k \\ w_k \end{bmatrix}, \tilde{C}_{ink} = \begin{bmatrix} \sum_{n=1, n\neq k}^{N} C_{ink} & 0 \\ 0 & \sum_{n=1, n\neq k}^{N} C_{ink} \end{bmatrix} \tilde{x}_n = \begin{bmatrix} x_n \\ e_n \end{bmatrix}.$$

Definition 1 [20]**.** For given prescribed level of disturbance attenuation $\gamma_k > 0, k = 1, \cdots, N$, the system (11) has the $H\infty$ performance index, if the following two conditions are satisfied:

1. When $w_i(t) = 0$, the whole interconnected closed-loop system is asymptotically stable.

2. For the zero initially condition $\left(\Phi^T(t) = 0, t \in [-\overline{\tau}, 0] \right)$, $\sum_{i=1}^{J} \|z_i\|_2 \leq \sum_{i=1}^{J} \gamma_i \|w_i\|_2$ .Where $\|z_i\|_2 = \left(\int_0^{\infty} z_i^T(t) z_i(t) dt \right)^{1/2}$.

Lemma 1 [21]. For any matrix C_{ij}, P_i, vector x_i, the following inequality holds:

$$\sum_{i=1}^{J} \left(x_i^T P_i \left(\sum_{j=1, j\neq i}^{J} C_{ij}x_j \right) + \left(\sum_{j=1, j\neq i}^{J} x_j^T C_{ij}^T \right) P_i x_i \right) \leq \sum_{i=1}^{J} \left(x_i^T \left(N_i P_i^2 + \sum_{j=1, j\neq i}^{J} C_{ij}^T C_{ij} \right) x_i \right)$$

Where N_i is the number of $C_{ij} \neq 0, j = 1, 2, ..., J$.

Theorem 1. For given prescribed level of disturbance attenuation $\gamma_k > 0, i = 1, \cdots, N$, if there exist matrices X_k, Y_k, symmetric matrix X_{ijk}, and $X_{jik} = X_{ijk}^T,\ j > i,\ i, j = 1, 2, ..., r_k$, satisfying the following inequalities (13)-(15), then the system (7) is asymptotically stable and achieves the $H\infty$ output tracking performance.

$$\Omega_{iik} < X_{iik} \tag{13}$$

$$\Omega_{ijk} + \Omega_{jik} \leq X_{ijk} + X_{jik}, i \neq j \tag{14}$$

$$\begin{bmatrix} X_{11k} & \cdots & X_{1rk} & U_{1jk}^T \\ \vdots & \ddots & \vdots & \vdots \\ X_{r1k} & \cdots & X_{rrk} & U_{rjk}^T \\ U_{1jk} & \cdots & U_{rjk} & -I \end{bmatrix} < 0 \tag{15}$$

Where

$$\Omega_{ijk}=\begin{bmatrix}\Omega_{ijk}^{11} & X_kB_{2ik}K_{jk}\\ * & \Omega_{ijk}^{22}\end{bmatrix}$$

$$\Omega_{ijk}^{11}=\left(A_{ik}^T+K_{jk}^TB_{2ik}^T\right)X_k+X_k\left(A_{ik}+B_{2ik}K_{jk}\right)$$
$$+\frac{1}{\gamma_k^2}X_kB_{1ik}B_{1jk}^TX_k+\left(N-1\right)X_k{}^2+\sum_{n=1,n\neq k}^{N}C_{ink}^TC_{ink},$$
$$\Omega_{ijk}^{22}=\left(A_{ik}^T+D_{yik}^TL_{ik}^T\right)Y_k+Y_k\left(A_{ik}+L_{ik}D_{yik}\right)$$
$$+\left(N-1\right)Y_k{}^2+\sum_{n=1,n\neq k}^{N}C_{ink}^TC_{ink}$$
$$U_{ijk}=\left[D_{ik}+E_{ik}K_{mk}\quad E_{ik}K_{mk}\right],m=1,2,...,r_k,$$

Proof:

We choose the following Lyapunov function for the neutral systems (7):

$$V=\sum_{k=1}^{N}\tilde{x}_k^TP_k\tilde{x}_k=\sum_{k=1}^{N}\begin{bmatrix}x_k\\ e_k\end{bmatrix}^T\begin{bmatrix}X_k & 0\\ 0 & Y_k\end{bmatrix}\begin{bmatrix}x_k\\ e_k\end{bmatrix} \tag{16}$$

Therefore, we have:

$$\dot{V}=\sum_{k=1}^{N}\left(\dot{\tilde{x}}_k^TP_k\tilde{x}_k+\tilde{x}_k^TP_k\dot{\tilde{x}}_k\right)$$
$$=\sum_{k=1}^{N}\sum_{i=1}^{r_k}\sum_{j=1}^{r_k}\mu_{ik}\mu_{jk}\left[\tilde{x}_k^T\left(\tilde{A}_{ik}^TP_k+P_k\tilde{A}_{ik}\right)\tilde{x}_k+w_k^T\tilde{B}_{ik}^TP_k\tilde{x}_k\right.$$
$$\left.+\tilde{x}_k^TP_k\tilde{B}_{ik}w_k+\tilde{x}_n^T\tilde{C}_{ink}^TP_k\tilde{x}_k+\tilde{x}_k^TP_k\tilde{C}_{ink}\tilde{x}_n\right]$$
$$\leq\sum_{k=1}^{N}\sum_{i=1}^{r_k}\sum_{j=1}^{r_k}\mu_{ik}\mu_{jk}\left[\dot{x}_k^T\quad \dot{e}_k^T\right]\Omega_{ijk}\begin{bmatrix}x_k\\ e_k\end{bmatrix}+\sum_{k=1}^{N}\gamma_k^2w_k^Tw_k^T$$

According to (13) and (14), we have:

$$\dot{V}\leq\sum_{k=1}^{N}\sum_{i=1}^{r_k}\sum_{j=1}^{r_k}\mu_{ik}\mu_{jk}\left[\dot{x}_k^T\quad \dot{e}_k^T\right]X_{ijk}\begin{bmatrix}x_k\\ e_k\end{bmatrix}+\sum_{k=1}^{N}\gamma_k^2w_k^Tw_k^T \tag{17}$$

According to (15), use Schur complement, we have:

$$\sum_{k=1}^{N}\sum_{i=1}^{r_k}\sum_{j=1}^{r_k}\mu_{ik}\mu_{jk}\left[x_k^T\quad e_k^T\right]X_{ijk}\begin{bmatrix}x_k\\ e_k\end{bmatrix}$$
$$<-\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}\gamma_i\gamma_j\gamma_u\gamma_v\left[x_k^T\quad e_k^T\right]U_{iu}^TU_{jv}\begin{bmatrix}x_k\\ e_k\end{bmatrix}=-z_k^Tz_k \tag{18}$$

When $w_k\equiv 0$, $\dot{V}\leq\sum_{k=1}^{N}\sum_{i=1}^{r_k}\sum_{j=1}^{r_k}\mu_{ik}\mu_{jk}\left[\dot{x}_k^T\quad \dot{e}_k^T\right]X_{ijk}\begin{bmatrix}x_k\\ e_k\end{bmatrix}<0$.

When $w_k\neq 0$, according to (18), $\dot{V}\leq -z_k^Tz_k+\gamma_k^2w_k^Tw_k<0$.

With zero initial condition $V\left(X\left(0\right)\right)=0$, integrating both sides of (13) from 0 to ∞, the $H\infty$ performance $\sum_{k=1}^{N}\left\|z_k\right\|_2\leq\sum_{k=1}^{N}\gamma_k\left\|w_k\right\|_2$ is satisfied.

There is no efficient algorithm solving the inequalities in therom1, because they are nonstandard. But the inequalities can be worked out by two steps. A single step LMI method for fuzzy tracking control system is given in the following theorem.

Theorem 2. For given prescribed level of disturbance attenuation $\gamma_k>0,i=1,\cdots,N$, if there exist matrices $\bar{M}_{ik}$, $\bar{J}_{ik}$, symmetric matrix P_{iik}, Q_{iik} $i,j=1,2,......r_k, j>i$, and symmetric positive definite matrices $\bar{X}_k,\bar{Y}_k$, satisfying the following LMIs (19)-(24), then system (7) is asymptotically stable and achieves the $H\infty$ output tracking performance.

$$\begin{bmatrix}\Pi_{iik}-P_{iik} & \bar{X}_kV_{ik}\\ * & -I\end{bmatrix}<0 \tag{19}$$

$$\begin{bmatrix}\Theta_{ijk}-Q_{iik} & \bar{Y}_k\\ * & -\left(N-1\right)^{-1}\end{bmatrix}<0 \tag{20}$$

$$\begin{bmatrix}\Pi_{ijk}+\Pi_{jik}-P_{ijk}-P_{jik} & \bar{X}_kV_{ik} & \bar{X}_kV_{jk}\\ * & -I & 0\\ * & * & -I\end{bmatrix}<0 \tag{21}$$

$$\begin{bmatrix}\Theta_{ijk}+\Theta_{jik}-Q_{ijk}-Q_{jik} & \bar{Y}_k\\ * & -2^{-1}\left(N-1\right)^{-1}\end{bmatrix}<0 \tag{22}$$

$$\begin{bmatrix}Q_{11k} & \cdots & Q_{1rk}\\ \vdots & \ddots & \vdots\\ Q_{1rk}^T & \cdots & Q_{rrk}\end{bmatrix}<0 \tag{23}$$

$$\begin{bmatrix}P_{11k} & \cdots & P_{1rk} & \bar{X}_kD_{1k}^T+M_{mk}^TE_{1k}^T\\ \vdots & \ddots & \vdots & \vdots\\ P_{r1k} & \cdots & P_{rrk} & \bar{X}_kD_{rk}^T+M_{mk}^TE_{rk}^T\\ D_{1k}\bar{X}_k+E_{1k}M_{mk} & \cdots & D_{rk}\bar{X}_k+E_{rk}M_{mk} & -I\end{bmatrix}<0 \tag{24}$$

Where

$$\Pi_{ijk}=\bar{X}_kA_{ik}^T+A_{ik}\bar{X}_k+\bar{M}_{ik}^TB_{2jk}^T+B_{2ik}\bar{M}_{ik}$$
$$+\frac{1}{\gamma_k^2}B_{1ik}B_{1ik}^T+\left(N-1\right)$$
$$\Theta_{ijk}=A_{ik}^T\bar{Y}_k+\bar{Y}_kA_{ik}+D_{yik}^T\bar{J}_{ik}^T+\bar{J}_{ik}D_{yik}+\sum_{n=1,n\neq k}^{N}C_{ink}^TC_{ink}$$
$$V_{ik}=\left[C_{i1k}^T\quad\cdots\quad C_{ink}^T\quad\cdots\quad C_{iNk}^T\right],n=1,2\cdots N,n\neq k.$$
$$K_i=\bar{M}_i\bar{X}^{-1},L_i=\bar{Y}^{-1}\bar{J}_i,m=1,2,\cdots\cdots r_k.$$

Proof :

Let $X_{ijk}=\begin{bmatrix}\tilde{X}_{ijk} & *\\ * & \tilde{Q}_{ijk}\end{bmatrix}$, $\bar{X}_k=X_k^{-1}$, $P_{iik}=\bar{X}_k\tilde{X}_{iik}\bar{X}_k$, plug them into (13), pre-multiply and post-multiply (13) by matrices $\left[I\quad 0\right]$ and $\left[I\quad 0\right]^T$, and we have (25).

Let $V_{ik}=\left[C_{i1k}^T\quad\cdots\quad C_{ink}^T\quad\cdots\quad C_{iNk}^T\right]n=1,2\cdots N,n\neq k,$ $\bar{M}_{ik}=K_{ik}\bar{X}_k$, use Schur complement, and we have (19).

$$\bar{X}_k A_{ik}^T + A_{ik}\bar{X}_k + \bar{X}_k K_{ik}^T B_{2ik}^T + B_{2ik} K_{ik}\bar{X}_k + \frac{1}{\gamma_k^2} B_{1ik} B_{1ik}^T + (N-1) - P_{iik} + \sum_{n=1,n\neq k}^{N} \bar{X}_k C_{ink}^T C_{ink}\bar{X}_k < 0 \tag{25}$$

Let $\bar{Y}_k = Y_k$, $Q_{iik} = \tilde{Q}_{iik}$, pre-multiply and post-multiply (13) by matrices $[I \quad 0]$ and $[I \quad 0]^T$, and we have:

$$A_{ik}^T\bar{Y}_k + D_{yik}^T L_{ik}^T \bar{Y}_k + \bar{Y}_k A_{ik} + \bar{Y}_k L_{ik} D_{yik} + (N-1)\bar{Y}_k^2 + \sum_{n=1,n\neq k}^{N} C_{ink}^T C_{ink} - Q_{iik} < 0 \tag{26}$$

Let $\bar{J}_{ik} = \bar{Y}_k L_{ik}$, use Schur complement, and we have (20).

According to (14), we can achieve (21) and (22) by the same method.

Where $X_{ijk} = \begin{bmatrix} \tilde{X}_{ijk} & * \\ * & \tilde{Q}_{ijk} \end{bmatrix}$, $X_{jik} = \begin{bmatrix} \tilde{X}_{jik} & * \\ * & \tilde{Q}_{jik} \end{bmatrix}$.

According to (15), (23) and (24) are obtained by elementary transformation and Schur complement.

(25) and (26) can be transformed to the following form:

$$\begin{bmatrix} \Phi_{iik} - \Phi_{iik}^{11} & 0 \\ 0 & \Upsilon_{iik} \end{bmatrix} < \begin{bmatrix} P_{iik} - P_{iik}^{11} & 0 \\ 0 & Q_{iik} \end{bmatrix} \tag{27}$$

Where

$$\Phi_{iik}^{11} = (B_{2ik}K_{uk})\Upsilon^{-1}{}_{iik}(B_{2ik}K_{ik})^T$$
$$P_{iik}^{11} = (B_{2ik}K_{ik})\Upsilon^{-1}{}_{iik}Q_{iik}\Upsilon^{-1}{}_{iik}(B_{2ik}K_{ik})^T$$
$$\Phi_{iik} = \bar{X}_k\Lambda_{iik}^T + \Lambda_{iik}\bar{X}_k + \frac{1}{\gamma_k^2}B_{1ik}B_{1ik}^T + (N-1) + \sum_{n=1,n\neq k}^{N}\bar{X}_k C_{ink}^T C_{ink}\bar{X}_k$$
$$\Upsilon_{iik} = \Gamma_{iik}^T\bar{Y}_k + \bar{Y}_k\Gamma_{iik} + (N-1)\bar{Y}_k^2 + \sum_{n=1,n\neq k}^{N} C_{ink}^T C_{ink}$$
$$\Lambda_{iik} = A_{ik} + B_{2ik}K_{ik}, \Gamma_{iik} = A_{ik} + L_{ik}D_{yik}$$

Pre-multiply and post-multiply (27) by matrices Z and its transposition, and we have:

$$\begin{bmatrix} \Phi_{iik} & B_{2ik}K_{ik} \\ (B_{2ik}K_{ik})^T & \Upsilon_{iik} \end{bmatrix} < \begin{bmatrix} P_{iik} & \Pi_{iik} \\ \Pi_{iik}^T & Q_{iik} \end{bmatrix} \tag{28}$$

Where $Z = \begin{bmatrix} I & (B_{2ik}K_{jk})\Upsilon^{-1}{}_{iik} \\ 0 & I \end{bmatrix}$, $\Pi_{iik} = (B_{2ik}K_{ik})\Upsilon^{-1}{}_{iik}Q_{iik}$.

Let $X_k = \bar{X}_k^{-1}, Y_k = \bar{Y}_k$, pre-multiply and post-multiply (28) by matrices $\begin{bmatrix} \bar{X}_k^{-1} & 0 \\ 0 & I \end{bmatrix}$ and $\begin{bmatrix} \bar{X}_k^{-1} & 0 \\ 0 & I \end{bmatrix}$, and we have (13).

By elementary transformation and Schur complement, (21) and (22) can be converted to (29):

$$\begin{bmatrix} \Phi_{11k} & 0 \\ 0 & \Upsilon_{ijk} + \Upsilon_{jik} \end{bmatrix} \le \begin{bmatrix} \Upsilon_{11k} & 0 \\ 0 & Q_{ijk} + Q_{jik} \end{bmatrix} \tag{29}$$

Where

$$P_{ijk} + \varepsilon_1\bar{X}_k = \bar{P}_{ijk}, Q_{ijk} + \varepsilon_1\bar{Y}_k = \bar{Q}_{ijk}$$
$$\Phi_{ijk} = \bar{X}_k\Lambda_{ijk}^T + \Lambda_{ijk}\bar{X}_k + \frac{1}{\gamma_k^2}B_{1ik}B_{1ik}^T + (N-1) + \sum_{n=1,n\neq k}^{N}\bar{X}_k C_{ink}^T C_{ink}\bar{X}_k$$
$$\Phi_{jik} = \bar{X}_k\Lambda_{jik}^T + \Lambda_{jik}\bar{X}_k + \frac{1}{\gamma_k^2}B_{1jk}B_{1jk}^T + (N-1) + \sum_{n=1,n\neq k}^{N}\bar{X}_k C_{jnk}^T C_{jnk}\bar{X}_k$$
$$\Upsilon_{ijk} = \Gamma_{ijk}^T\bar{Y}_k + \bar{Y}_k\Gamma_{ijk} + (N-1)\bar{Y}_k^2 + \sum_{n=1,n\neq k}^{N} C_{ink}^T C_{ink}$$
$$\Upsilon_{jik} = \Gamma_{jik}^T\bar{Y}_k + \bar{Y}_k\Gamma_{jik} + (N-1)\bar{Y}_k^2 + \sum_{n=1,n\neq k}^{N} C_{jnk}^T C_{jnk}$$
$$\Phi_{11k} = \Phi_{ijk} + \Phi_{jik} - (B_{2ik}K_{jk} + B_{2jk}K_{ik})(\Upsilon_{ijk} + \Upsilon_{jik})^{-1}(B_{2ik}K_{jk} + B_{2jk}K_{ik})^T$$
$$\Upsilon_{11k} = \bar{P}_{ijk} + \bar{P}_{jik} - (B_{2ik}K_{jk} + B_{2jk}K_{ik})(\Upsilon_{ijk} + \Upsilon_{jik})^{-1}(Q_{ijk} + Q_{jik})(\Upsilon_{ijk} + \Upsilon_{jik})^{-1}(B_{2ik}K_{jk} + B_{2jk}K_{ik})^T$$

By the same transformation as the one done to (27), we have (14).Where,

$$X_{ijk} = \begin{bmatrix} \bar{X}_k^{-1}\bar{P}_{ijk}\bar{X}_k^{-1} & \bar{X}_k^{-1}B_{2ik}K_{jk} \\ * & Q_{ijk} \end{bmatrix},$$
$$X_{jik} = \begin{bmatrix} \bar{X}_k^{-1}\bar{P}_{jik}\bar{X}_k^{-1} & \bar{X}_k^{-1}B_{2jk}K_{ik} \\ * & Q_{jik} \end{bmatrix}.$$

According to (23) and (24), (30) and (31) are obviously true.

$$\begin{bmatrix} P_{11k} & \cdots & P_{1rk} & \Phi_{1k}^T \\ \vdots & \ddots & \vdots & \vdots \\ P_{r1k} & \cdots & P_{rrk} & \Phi_{2k}^T \\ \Phi_{1k} & \cdots & \Phi_{rk} & -I \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} Q_{11k} & \cdots & Q_{1rk} \\ \vdots & \ddots & \vdots \\ Q_{r1k} & \cdots & Q_{rrk} \end{bmatrix} < 0 \tag{31}$$

Use Schur complement, pre-multiply and post-multiply the obtained inequality by $diag\left[\bar{X}_k^{-1} \ \dots \ \bar{X}_k^{-1} \ I \ I \ \dots \ I\right]$, then we have (15). Where $\Phi_{ik} = (D_{ik} + E_{ik}K_{jk})\bar{X}_k, i = 1,2,...r.$

$$X_{iik}=\begin{bmatrix} \bar{X}_k^{-1}P_{iik}\bar{X}_k^{-1} & \bar{X}_k^{-1}\left(B_{2ik}K_{jk}\right)\Upsilon^{-1}{}_{iik}Q_{iik} \\ Q_{iik}\Upsilon^{-1}{}_{iik}\left(B_{2ik}K_{jk}\right)^T\bar{X}_k^{-1} & Q_{iik} \end{bmatrix}.$$

So far, the proof of therom2 is completed.

III. Illustrative Examples

Consider the inverted pendulum system:

$$\begin{aligned} \dot{x}_{1k}(t) &= x_{2k}(t) \\ \dot{x}_{2k}(t) &= \left[\left(g/cl\right)-\left(ka(a-cl)/cml^2\right)\right]x_{1k}(t) \\ &\quad -m/M\sin\left(x_{1k}(t)\right)x_{2k}^2(t)+\left(1/cml^2\right)u_k(t) \\ &\quad +0.1\omega_{k2}(t)+\left(ka(a-cl)/cml^2\right)x_{1n}(t) \\ y_k(t) &= x_{1k}(t)+0.01v_k(t) \end{aligned}$$

Where $x_{1k}(t)$, $x_{2k}(t)$ represents the angle and velocity of the *kth* inverted pendulum respectively, $u_k(t)$ is the control of the *kth* inverted pendulum, $y_k(t)$ is the output of the *kth* inverted pendulum, $\omega_{k2}(t)$ refers to the external disturbance, $v_k(t)$ refers to measurement noise, $x_{1n}(t)$ is the transmit correlation coefficient between the *kth* machine and the *nth* machines.

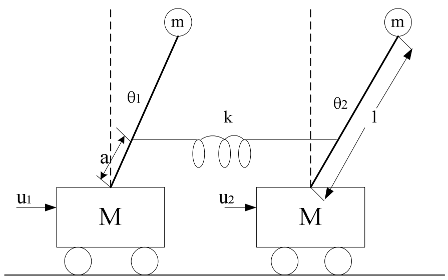

Figure 1 inverted pendulum device installed in two cars connected by a spring

Set the parameters of the double organ system as follows:

$c=m/(m+M)$;

$m=1$ kg (the weight of the inverted pendulum);

$M=5$ kg (the weight of the car);

$a=0.2$ m ;

$l=1$ m (the length of the inverted pendulum);

$k=1$ N/m (spring constant);

$g=9.8$ m/s^2 (gravity constant).

In order to reduce the complexity of the system, the systems are approximated by the fuzzy model with nine-rule which is similar as [22]. Other parameters are shown as follows:

$$A_{1k}=\begin{bmatrix}0 & 1\\ 58.6168 & -0.15\end{bmatrix}, A_{2k}=\begin{bmatrix}0 & 1\\ 58.7600 & 0\end{bmatrix},$$

$$A_{3k}=\begin{bmatrix}0 & 1\\ 58.6168 & 0.15\end{bmatrix}, \quad A_{4k}=\begin{bmatrix}0 & 1\\ 58.5350 & 0\end{bmatrix},$$

$$A_{5k}=\begin{bmatrix}0 & 1\\ 58.7600 & 0\end{bmatrix}, \quad A_{6k}=\begin{bmatrix}0 & 1\\ 58.5350 & 0\end{bmatrix},$$

$$A_{7k}=\begin{bmatrix}0 & 1\\ 58.6168 & 0.15\end{bmatrix}, \quad A_{8k}=\begin{bmatrix}0 & 1\\ 58.7600 & 0\end{bmatrix},$$

$$A_{9k}=\begin{bmatrix}0 & 1\\ 58.6168 & -0.15\end{bmatrix}, k=1,2.$$

$$w_k(t)=\begin{bmatrix}0\\ w_{2k}\end{bmatrix}, B_{2ik}=\begin{bmatrix}0\\ 6\end{bmatrix}, B_{1ik}=\begin{bmatrix}0 & 0\\ 0 & 0.1\end{bmatrix},$$

$$C_{ink}=\begin{bmatrix}0 & 0\\ 0.04 & 0\end{bmatrix}, D_{yik}=\begin{bmatrix}1 & 0\end{bmatrix}, E_{yik}=0.01$$

$$w_{21}=5\sin(10t)e^{-0.5t}\in L_2, w_{22}=\cos(10t)e^{-0.5t}\in L_2,$$

$$v_1(k)=10\sin\left(\sin(10t)\right)e^{-0.5t}\in L_2,$$

$$v_2(k)=10\sin\left(\cos(10t)\right)e^{-0.5t}\in L_2$$

The control parameters obtained from Matlab simulation are as follows:

$$\begin{aligned} &K_{i1}=[-2.302 \quad -1.933], \ K_{i2}=[-15.98 \quad -6.239] \\ &L_{1j}=[76.978 \quad 591.521], L_{2j}=[87.166 \quad 670.848] \\ &L_{3j}=[102.246 \quad 790.923], L_{4j}=[101.818 \quad 787.135] \\ &L_{5j}=[104.607 \quad 806.945], L_{6j}=[107.369 \quad 826.384] \\ &L_{7j}=[107.066 \quad 823.253], L_{8j}=[122.564 \quad 949.002] \\ &L_{9j}=[131.301 \quad 1019.372] \\ &i=1,2,\ldots,9. j=1,2. \end{aligned}$$

For the initial condition $x_1(0)=[-1 \quad 3]^T$, $x_2(0)=[1 \quad -2]^T$, The simulation curves are shown by figure 2 to figure 5.

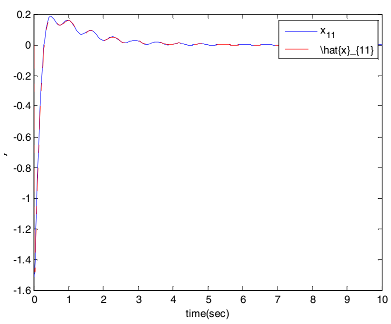

Fig. 2 The curves of x_{11} and $\hat{x}_{11}$

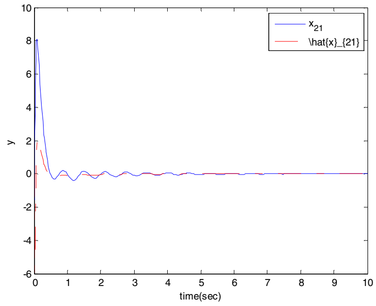

Fig. 3 The curves of x_{21} and $\hat{x}_{21}$

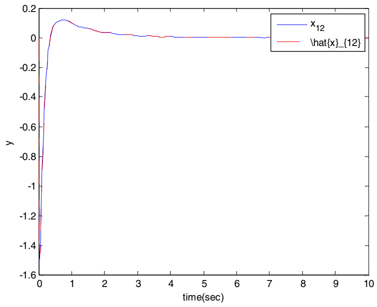

Fig. 4 The curves of x_{12} and $\hat{x}_{12}$

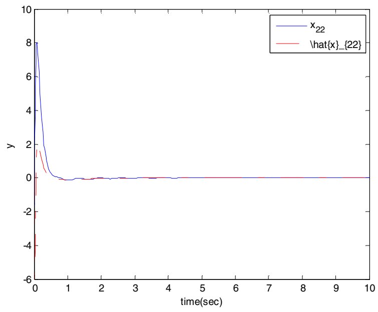

Fig. 5 The curves of x_{22} and $\hat{x}_{22}$

IV. CONCLUSION

The problem of observer-based $H\infty$ control design for a class of fuzzy descriptor interconnected systems has been addressed. The stability conditions of the T-S fuzzy model with a prescribed $H\infty$ performance level have been given. In order to improve the stepwise design method of observer-based $H\infty$ Controller , a single-step method reducing the conservatism of the conditions has been proposed. Then we can design the observers and controllers in single-step.

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