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Globally Fuzzy Model Based Adaptive Variable Structure Control for a Class of Nonlinear Time-Varying Systems

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Abstract --- **In this paper, a nonlinear time-varying dynamic system is first approximated by *N* fuzzy-based linear state-space subsystems. To track a trajectory dominant by a specific frequency, the reference models with desired amplitude and phase features are established by the same fuzzy sets of the system rule. It is known that linear state feedback control for each fuzzy subsystem is inferior to that using nonlinear feedback control. It is also known that most of the fuzzy adaptive controls must be in a specific domain for the function approximation. To overcome the above shortcomings, we propose a globally fuzzy model based adaptive variable structure control with a switching function to determine when the learning law should be used. As the norm of the switching surface is inside of a defined set, the learning law starts; simultaneously, as it is outside of the other set which is larger than the previous defined set, the learning law stops. In this situation, the proposed control is verified to converge into a convex set, which is smaller than the set for the function approximation. For the purpose of smoothing the discontinuity of control input, a transition between outside and inside of approximated set is also assigned. Under these circumstances, the proposed control can automatically tune as a control without or with the learning compensation of uncertainties. Finally, the stability of the overall system is verified by Lyapunov stability theory.**
Keywords: Global adaptive control, Takagi-Sugeno fuzzy linear model, Variable structure control, Reference Model, Fuzzy basis function model, Approximation theory, Learning law.

I. INTRODUCTION

It is known that practical control systems are often nonlinear and time-variant. The major advantage of heuristics-based fuzzy control or modeling is that a mathematical model for the system is not required. Only some input/output data are employed to obtain an effective control or model. However, it is lack of systematic design or stability analysis or stability proof. Furthermore, its performance is usually not more excellent than that of the other controllers. Due to these shortcomings, a model-based fuzzy control or modeling (e.g., [1-3]), is a promising method to reinforce the performance of fuzzy system. In addition, the above-mentioned approaches only use a linear state-feedback control for every subsystem. Its robustness is often poorer than that using a nonlinear control for every subsystem [1, 4]. Furthermore, most of the fuzzy adaptive controls (e.g., [5-7]) must be in a specific domain for the function approximation. To overcome the above disadvantages, we propose a globally fuzzy model based adaptive variable structure control with a switching function to determine when the learning law should be used. On the other hand, without the compensation of learning uncertainties for the proposed control can force the system state outside of approximated set into the approximated domain.

At beginning, a class of nonlinear time-varying dynamic systems is approximated by *N* fuzzy-based linear state-space subsystems. That is, a local linearization of nonlinear nominal dynamic system is achieved by *N* fuzzy-based linear state-space subsystems. For tracking a trajectory dominant by a specific frequency, the reference models with desired amplitude and phase features by using the same fuzzy sets of the system rule are constructed. Then the same fuzzy sets of the system rule are applied to design the globally fuzzy model based adaptive variable structure control (GFMBAVSC), which contains a switching function to determine whether the learning uncertainties are executed or not.

The uncertainties in this paper include the approximation error of fuzzy-model, time-varying uncertain vector functions, and the interaction dynamics resulting from the other subsystems. There have five possible approaches to deal with uncertainties. The first approach is to assume that it is bounded by a known function to design an effective controller. This is the methodology for the deterministic robust control (e.g., [8], [9]). The disturbance observer also can be employed to estimate the external disturbance which is then compensated by the controller (e.g., [10]-[11]). If the uncertainties possess the strong randomness, a probabilistic approach is one of suitable choice ([12]-[13]). The fourth approach is to apply the neural network modeling of uncertainties, which are applied to attenuate their effect ([14]-[15]). The fifth approach is to use fuzzy set theory to designate the membership function for the description of uncertainties, which are on-line learned for compensation ([1], [16]). In this paper, fuzzy model for the approximation of these non-autonomous uncertainties, which are respectively assumed to be absolutely bounded for time variable and relatively bounded for the other variables.

If uncertainties are excess, the learning law is progressed to learn the uncertainties for compensation, and then to improve system performance and stability. The learning law with a suitable learning rate and *e*-modification rate can effectively learn system uncertainties without the risk of unbounded learning weight [1]. The main contributions (or features) of the proposed GFMBAVSC are as follows: (i) The global trajectory tracking for different initial system states outside of approximated set is obtained. (ii) The transient response caused by different initial learning weight is reduced because the learning law executes only after the convergence to a smaller convex set. (iii) It not only improves steady state performance as compared with GFMBAVSC without the learning compensation (or other robust controls), but also enhances the system stability in the face of excess uncertainties.

II. MATHEMATICAL PRELIMINARIES

Throughout this paper, a continuous-time signal at t is represented by $x(t)$. The notation $\Re^{n\times m}$ denotes the sets of real matrices with dimension $n\times m$. The symbol M_j denotes a fuzzy set of $x_j(t)$. The notation M_j^i denotes a fuzzy term of M_j selected for *rule i*. The symbol $\prod_{j=1}^{N} f_j = f_1 f_2 ... f_N$ represents a scalar multiplication. The symbol $\|x\|$ represents an Euclidean norm of vector x. The notation $\lambda[A]$ denotes the eigenvalues of matrix A. Define the trace operator as $tr[\cdot]$ and $tr[A]=\sum_{i=1}^{n} a_{ii}$, where $A\in\Re^{n\times n}$ The property $tr[ABC]=tr[CAB]=tr[BCA]$ exists, where A, B and C are three compatible square (or non-square) matrices. The notation $\|\cdot\|_F$ denotes the Frobenius norm, i.e., $\|W\|_F^2 = tr[W^T W] = tr[WW^T]$, where $W\in\Re^{n\times m}$. The symbol I_n denotes a unit matrix of dimension n.

Definition 1 [16]: The solutions of a dynamic system $\dot{x}(t) = A(x,t),\ x(t)\in\Re^n$ are said to be UUB if there exist positive constants υ and κ, and for every $\Delta\in(0,\kappa)$ there is a positive constant $T=T(\Delta)$, such that $\|x(t_0)\|<\Delta \rightarrow \|x(t)\|\le\upsilon,\ \forall t\ge t_0+T.$ If the initial state $x(t_0)\in\Re^n$, they are said to be GUUB.

III. PROBLEM FORMULATION

Consider a class of nonlinear time-varying dynamic systems:

$$\dot{x}(t) = F(x,t) + G(x,t)u(t) \tag{1a}$$

where $x(t)\in\Re^n$ is the system state which is available, $u(t)\in\Re^m$ is the control input, and the continuous mappings $F(x,t):\Re^n\times\Re^+\rightarrow\Re^n, G(x,t):\Re^n\times\Re^+\rightarrow\Re^{n\times m}$. It is assumed that

$$F(x,t)=\bar{F}(x)+\Delta F(x,t), G(x,t)=\bar{G}(x)+\Delta G(x,t) \tag{1b}$$

where $\bar{F}(x)$ and $\bar{G}(x)$ denote the nominal system vector functions, $\Delta F(x,t)$ and $\Delta G(x,t)$ are the uncertain system vector functions. The system (1) is assumed to be reachable and observable at $x_0\in\Re^n$. Based on the linearizing around some suitable operating point, e.g., equilibrium point (x_0,u_0) of the controlled system (1), a fuzzy dynamic system using Takagi and Sugeno model to represent local linear input/output relations of nonlinear dynamic systems is described by the following fuzzy IF-THEN rules:

System rule i: IF $\phi_1(z)$ is M_1^i... and $\phi_{\bar{n}}(z)$ is $M_{\bar{n}}^i$

THEN $\dot{x}(t) = A^i x(t) + B^i u(t)$, for $i=1,2,...,N,$ (2)

where $z^T(t)=[x^T(t)\ u^T(t)],\quad \bar{n}\le n, A^i = \partial\bar{F}(x)/\partial x\big|_{z=z_0}\in\Re^{n\times n}$, $B^i = \partial\bar{G}(x)/\partial x\big|_{z=z_0}\in\Re^{n\times m}$, N is the number of IF-THEN rules, $x(t)$ denotes the output from the *ith* IF-THEN rules, and $\phi_1(z),\phi_2(z),...,\phi_{\bar{n}}(z)$ are the premise variables. Assume that (A^i,B^i) for $i=1,2,...,N$ are known and controllable. The output of the overall fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{N}\mu^i(z)\left[A^i x(t) + B^i u(t)\right] \tag{3a}$$

where

$$\mu^i(z) = \rho^i(z)\Big/\sum_{i=1}^{N}\rho^i(z)\ge 0,$$
$$\rho^i(z) = \prod_{j=1}^{\bar{n}} M_j^i(\phi_j(z)) \text{ and } \sum_{i=1}^{N}\mu^i(z)=1. \tag{3b}$$

Then the following uncertain system vector functions $\Delta A(z,t)$ and $\Delta B(z,t)$, those are denoted as nonlinear time-varying uncertainties caused by the approximation error of fuzzy-model and uncertain system vector functions:

$$\Delta A(z,t) = F(x,t) - \sum_{i=1}^{N}\mu^i(z)A^i x(t)$$
$$\Delta B(z,t) = G(x,t) - \sum_{i=1}^{N}\mu^i(z)B^i\ . \tag{4}$$

Similarly, a fuzzy dynamic system using Takagi and Sugeno model to represent local linear input/output relations of reference model is described by the following fuzzy IF-THEN rules:

Reference Model Rule i: IF $\phi_1(z)$ is M_1^i... and $\phi_{\bar{n}}(z)$ is $M_{\bar{n}}^i$

THEN $\dot{x}_m(t) = A_m^i x_m(t) + B_m^i r(t+\theta)$, for $i=1,2,...,N,$ (5)

where $A_m^i\in\Re^{n\times n}, \Re e\{\lambda[A_m^i]\}<0, B_m^i\in\Re^{n\times m},\ r(t)\in\Re^m$ is a known reference input, and θ denotes a phase-lead angle for compensating the phase-lag of the reference model. The output of the overall reference model system is inferred as follows:

$$\dot{x}_m(t) = \sum_{i=1}^{N}\mu^i(z)\left[A_m^i x_m(t) + B_m^i r(t+\theta)\right]. \tag{6}$$

Define the following switching surface:

$$S(t) = D\tilde{x}(t) \tag{7}$$

where $\tilde{x}(t) = x(t) - x_m(t)$ and $D\in\Re^{m\times n}$ is chosen such that the dynamics of $S(t)=0$ is Hurwitz. Suppose that the proposed GFMBAVSC share the same fuzzy sets with the fuzzy system (2).

Controller Rule i: IF $\phi_1(z)$ is M_1^i... and $\phi_{\bar{n}}(z)$ is $M_{\bar{n}}^i$

THEN $u(t) = u_{eq}^i(t) + u_{sw}^i(t)$. (8)

Let

$$u_{eq}^i(t) = -\left(DB^i\right)^{-1}\Big\{DA^i x(t) - DA_m^i x_m(t) - DB_m^i r(t+\theta) + \sigma(\|S(t)\|)\left[\hat{W}^i(t)\right]^T\Psi(z)\Big\} \tag{9a}$$

$$u_{sw}^i(t) = -\left(DB^i\right)^{-1}\left\{\eta_1^i S(t) + \eta_2^i S(t)\Big/\left[\|S(t)\|+\varepsilon^i\right]\right\}\Big/\left(\sqrt{m}-\alpha^i\right) \tag{9b}$$

where $DB^i\in\Re^{m\times m}$ is nonsingular, $\eta_1^i = diag\{\eta_{11}^i,...,\eta_{1n}^i\}$, $\eta_2^i = diag\{\eta_{21}^i,...,\eta_{2n}^i\}, \varepsilon^i>0,\ \alpha^i$ satisfies the following inequality:

$$\left\| D\Delta B(z,t)\left(DB^i\right)^{-1} \right\|_F \le \alpha^i < \sqrt{m},\ \forall i,t,z(t) \qquad (9c)$$

and the scalar function $\sigma(\|S(t)\|)$ is designed so that globally fuzzy model based adaptive variable structure control is achieved:

$$\sigma(\|S(t)\|) = \begin{cases} 0, & \text{as } \|S(t)\| > n_{s1} \\ 1, & \text{as } \|S(t)\| < n_{s2} \\ \left(\|S(t)\| - n_{s2}\right)/\left(n_{s1} - n_{s2}\right), & \text{otherwise} \end{cases} \qquad (9d)$$

where $n_{s1} > h_m > n_{s2} > p_m > 0$. The details of these parameters h_m and p_m are discussed in the next section. The following equation describes the system uncertainties caused by the approximation error of fuzzy-model, uncertain system vector functions, and the interaction dynamics resulting from the other subsystems.

$$\begin{aligned} \Omega^i(z,t) = D\Big\{ & \Delta A(z,t) + B^i \sum_{j=1, j\ne i}^{N} \mu^j(z) u_{eq}^j(t) \\ & + \Delta B(z,t) u_{eq}(t) + \left(B^i + \Delta B(z,t)\right) \sum_{j=1, j\ne i}^{N} \mu^j(z) u_{sw}^j(t) \Big\} \end{aligned} \qquad (10)$$

where $z(t)$ is the same as (2). If the system uncertainties are mild, then $\sigma(\|S(t)\|) = 0$ is set. Otherwise, the following learning law (e.g., [1]) is considered.

$$\dot{\hat{W}}^i(t) = \beta^i \Psi(z) S^T(t) - \gamma^i \hat{W}^i(t) \qquad (11a)$$

where $\beta^i > 0 \in \Re$ denotes a learning rate, $\gamma^i > 0 \in \Re$ denotes an e-modification rate to ensure the boundedness of learning weight, $\hat{W}^i(t) \in \Re^{L\times m}$ stands for the learning weight, $\Psi(z) \in \Re^L$ represents the fuzzy basis function:

$$\Psi(z) = \begin{bmatrix} 1 & \varphi_2(z) & \dots & \varphi_L(z) \end{bmatrix}^T \qquad (11b)$$

where $\varphi_j(z) = \exp\left[-\|z(t) - c_j\|^2 / \sigma_j^2 \right]$, L, c_j, σ_j for j = 2, 3,…, L are known, and the centers c_j for $j = 2,3,...,L$ are chosen as the normal distribution in the corresponding domain. Hence, the overall control law is described as follows:

$$u(t) = \sum_{i=1}^{N} \mu^i(z)\left[u_{eq}^i(t) + u_{sw}^i(t) \right]. \qquad (12)$$

The objectives of the paper are expressed as follows (cf. Fig. 1).
(i) If the system uncertainties (10) are not enormous, based on a fuzzy model (2) a GFMBAVSC (9) with $\sigma(\|S(t)\|) = 0$ or $\hat{W}^i(t) = 0$ is constructed to stabilize the nonlinear time-varying dynamic system (1). Then the system state $x(t)$ will asymptotically track the output of reference model, i.e., $x_m(t)$. In addition, an eligible selection of reference model makes the system state track a trajectory dominant by a specific frequency. (ii) Similarly, if the system uncertainties (10) are huge, the proposed control with learning law (11) and $\sigma(\|S(t)\|)$=1 is considered to enhance the system performance and stability. (iii) According to the designed $\sigma(\|S(t)\|)$, the proposed control is also achieved to improve the robust performance, e.g., accuracy of trajectory tracking, smoothness of control input. (iv) Based on the designed scalar function (9d), the globally fuzzy model based adaptive variable structure control is achieved.

IV. CONTROLLER DESIGN AND STABILITY ANALYSIS

Before verifying the stability, the derivative of switching surface is given as follows:

$$\begin{aligned} \dot{S}(t) = \sum_{i=1}^{N} \mu^i(z) \Big\{ & \left[I + D\Delta B(z,t)\left(DB^i\right)^{-1} \right] \\ & \bullet \left(DB^i\right) u_{sw}^i(t) + \Omega^i(z,t) \Big\}. \end{aligned} \qquad (13)$$

If the system uncertainties in (10) are mild, their upper bounds are estimated as follows:

$$\left\| \Omega^i(z,t) \right\| \le h_\Omega^i(z), \forall z(t), t \qquad (14)$$

where $h_\Omega^i(z)$ is a known scalar function. Then the proposed control becomes a robust control, i.e., without the compensation of learning uncertainties (i.e., $\sigma(\|S(t)\|) = 0$ in (9a)). The corresponding convergent set of $\|S(t)\|$ will be discussed later.

Similarly, if system uncertainties are excess, the learning law (11) is employed to learn the system uncertainties $\Omega^i(z,t)$. It is supposed that the unknown signals $\Omega^i(z,t)$ can be smoothly truncated outside of $z(t) \in \bar{D}(z)$ (a compact subset in $\Re^{n+m}$) for $\forall t \in \Re^+$. Based on the learning model, the proposed GFMBAVSC is then constructed to improve system performance. The extension of universal approximation theory is stated as follows.

Theorem 1: Suppose $z(t) \in \bar{D}(z)$ (a compact subset of $\Re^{n+m}$), $f(z,t): \bar{D} \times \Re^+ \to \Re^m$ is a continuous vector function, which is absolutely and relatively bounded with respect to arguments t and $z(t)$, respectively. For an arbitrary constant $\varepsilon > 0$, there exists an integer L (the number of hidden neurons) and real constant matrix $\bar{W} \in \Re^{m\times L}$, where $\|\bar{W}\|_F^2 \le W_m$, such that

$$f(z,t) = \bar{W}^T \Theta(z) + \varepsilon_f(z,t)$$

where $\Theta(z)$ is the fuzzy basis function, $\|\varepsilon_f(z,t)\| \le \varepsilon, \forall t$ and $z(t) \in \bar{D}(z)$.

Based on the result of *Theorem 1*, the system uncertainties in a compact subset $\bar{D}(z)$ are assumed to be continuous and approximated by the following fuzzy basis function model:

$$\Omega^i(z,t) = \left(\bar{W}^i\right)^T \Psi(z) + \upsilon^i(z,t) \qquad (15)$$

where $\bar{W}^i \in \Re^{L\times m}$ is constant matrix which is not necessarily unique, $\|\upsilon^i(z)\| \le p_\upsilon^i\ \forall z(t) \in \bar{D}(z)$. In addition, the upper bound of $\bar{W}^i$ is known, i.e., $\|\bar{W}^i\|_F \le W_m^i$.

The compact subsets $\bar{D}(z)$ can be achieved because the result of the proposed control without learning compensation in (9a) with $\sigma(\|S(t)\|)=0$ can guarantee the boundedness of $z(t)$. The estimated weight error of the *ith* fuzzy subsystem defined as $\tilde{W}^i(t)=\bar{W}^i-\hat{W}^i(t)$. Then the properties of the closed-loop system are addressed as follows.

Theorem 2: Consider the nonlinear time-varying dynamic system (1) with $x(0)\in\Re^n$ and the proposed GFMBAVSC (9) with the switching gains $\eta_1^i>\delta I_n/2>0,\ \eta_2^i>p_\upsilon^i I_n$ and the learning law (11). Then $u(t)$, $\hat{W}^1(t),\ldots,$and $\hat{W}^N(t)$ are UUB, $S(t)$ and $x(t)$ GUUB. The system performances are achieved as follows:

(i) As $\|S(t)\|>n_{s1}$,

$$\Psi_1=\left\{S(t)\in\Re^m \mid 0\le\|S(t)\|\le h_m\right\} \tag{16a}$$

where

$$h_1^i(z)=\left\{\varepsilon^i+\left[\|\eta_2^i\|_F-h_\Omega^i(z)\right]\Big/\left(\|\eta_1^i\|_F-\delta/2\right)\right\}\Big/2 \tag{16b}$$

$$h_2^i(z)=\varepsilon^i h_\Omega^i(z)\Big/\left(\|\eta_1^i\|_F-\delta/2\right) \tag{16c}$$

$$h^i(z)=\sqrt{\left[h_1^i(z)\right]^2+h_2^i(z)}-h_1^i(z) \tag{16d}$$

$$h_m=\max_{1\le i\le N}\left[h^i(z)\right]. \tag{16e}$$

(ii) As $\|S(t)\|<n_{s2}$, where $n_{s2}<h_m$,

$$\Psi_2=\left\{Z(t)\in\Re^{N+1} \mid 0\le\|S(t)\|\le p_m,\ 0\le\|\tilde{W}^i(t)\|_F\le q^i,\text{for } 1\le i\le N\right\} \tag{17a}$$

where

$$Z^T(t)=\left[\ \|S(t)\|\ \ \|\tilde{W}^1(t)\|_F\ \ \ldots\ \ \|\tilde{W}^N(t)\|_F\ \right] \tag{17b}$$

$$p_m=\max_{1\le i\le N}\left(p^i\right),\quad q^i=\gamma^i W_m^i\big/\left(\gamma^i-\delta/2\right) \tag{17c}$$

$$p_1^i=\left\{\varepsilon^i+\left(\|\eta_2^i\|_F-p_\upsilon^i\right)\Big/\left(\|\eta_1^i\|_F-\delta/2\right)\right\}\Big/2 \tag{17d}$$

$$p_2^i=\varepsilon^i p_\upsilon^i\Big/\left(\|\eta_1^i\|_F-\delta/2\right) \tag{17e}$$

$$p^i=\sqrt{\left(p_1^i\right)^2+p_2^i}-p_1^i. \tag{17f}$$

(iii) As $n_{s2}\le\|S(t)\|\le n_{s1}$, the corresponding result can be obtained by the mean value of (16) and (17).

Proof: For simplicity, the arguments of variable are omitted. As $\|S\|>n_{s1}$, the following Lyapunov function is defined.

$$V_1=S^TS/2>0,\ \text{as } S\ne 0. \tag{18}$$

Taking the time derivative of (18) and assuming that $\dot{V}_1\le-\delta V_1$, where $\delta>0$, gives

$$\dot{\bar{V}}_1=S^T\dot{S}+\delta\, S^TS/2 \tag{19}$$

where $\dot{\bar{V}}_1=\dot{V}_1+\delta V_1$. Substituting (13), (14), and (9) with $\sigma(\|S\|)=0$ into (19) yields

$$\begin{aligned}\dot{\bar{V}}_1&\le S^T\left\{\sum_{i=1}^N\mu^i\left[\frac{-\left(I+D\Delta B\left(DB^i\right)^{-1}\right)}{\sqrt{m}-\alpha^i}\left(\eta_1^iS+\frac{\eta_2^iS}{\|S\|+\varepsilon^i}\right)+\Omega^i\right]+\frac{\delta S}{2}\right\}\\&\le-\sum_{i=1}^N\frac{\mu^i\left(\|\eta_1^i\|_F-\delta/2\right)\|S\|}{\|S\|+\varepsilon^i}\Bigg\{\|S\|\left(\|S\|+\varepsilon^i\right)\\&\quad+\frac{\|\eta_2^i\|_F}{\left(\|\eta_1^i\|_F-\delta/2\right)}\|S\|-\frac{h_\Omega^i}{\left(\|\eta_1^i\|_F-\delta/2\right)}\left(\|S\|+\varepsilon^i\right)\Bigg\}\\&\le-\sum_{i=1}^N\mu^i\left(\|\eta_1^i\|_F-\delta/2\right)\|S\|H^i\left(\|S\|\right)\Big/\left(\|S\|+\varepsilon^i\right)\end{aligned} \tag{20}$$

where $H^i(\|S\|)=\|S\|^2+2h_1^i\|S\|-h_2^i$. When $\|S\|\ge h_m$, the inequalities $H^i(\|S\|)\ge 0$ for $i=1,2,\ldots,N$, are satisfied. Then, outside of the domain Ψ_1 in (16a) making $\dot{\bar{V}}\le 0$ (or $\dot{V}\le-\delta\ V$) is achieved. Hence, the signal S exponentially converges into the domain Ψ_1.

Similarly, as $\|S\|<n_{s2}<h_m$, the following Lyapunov function is defined.

$$\begin{aligned}V_2&=S^TS/2+\sum_{i=1}^N\mu^i tr\left\{\left(\tilde{W}^i\right)^T\tilde{W}^i\right\}\Big/\left(2\beta^i\right)\\&=Z^TRZ>0,\ \text{as } S\ne 0 \text{ or } \tilde{W}^i\ne 0\end{aligned} \tag{21}$$

where $R=Diag\left[1/2\ \ \mu^1/(2\beta^1)\ \ \ldots\ \ \mu^N/(2\beta^N)\right]>0$ $\in\Re^{(N+1)\times(N+1)}$. Similarly, taking the time derivative of (21) and assuming that $\dot{V}_2\le-\delta V_2$, where $\delta>0$, and using (9), (11), (12) yields

$$\begin{aligned}\dot{\bar{V}}_2&=\dot{V}_2+\delta\ V_2\\&\le\left\{\sum_{i=1}^N\mu^i\left[\frac{-\left(I+D\Delta B\left(DB^i\right)^{-1}\right)}{\sqrt{m}-\alpha^i}\left(\eta_1^iS+\frac{\eta_2^iS}{\|S\|+\varepsilon^i}\right)+\left(\tilde{W}^i\right)^T\Psi+\upsilon^i\right]+\frac{\delta S}{2}\right\}\\&\quad-\sum_{i=1}^N\mu^i tr\left\{\left(\tilde{W}^i\right)^T\left(\beta^i\Psi S^T-\gamma^i\hat{W}^i\right)\right\}\Big/\beta^i\\&\quad+\delta\sum_{i=1}^N\mu^i tr\left\{\left(\tilde{W}^i\right)^T\tilde{W}^i\right\}\Big/\left(2\beta^i\right)\\&=\left\{\sum_{i=1}^N\mu^i\left[\frac{-\left(I+D\Delta B\left(DB^i\right)^{-1}\right)}{\sqrt{m}-\alpha^i}\left(\eta_1^iS+\frac{\eta_2^iS}{\|S\|+\varepsilon^i}\right)+\upsilon^i\right]+\frac{\delta S}{2}\right\}\\&\quad+\sum_{i=1}^N\mu^i tr\left\{\left(\tilde{W}^i\right)^T\gamma^i\left(\bar{W}^i-\tilde{W}^i\right)+\delta\left(\tilde{W}^i\right)^T\tilde{W}^i/2\right\}\Big/\beta^i\end{aligned}$$

$$\leq -\sum_{i=1}^{N} \frac{\mu^i\left(\|\eta_1^i\|_F - \delta/2\right)\|S\|}{\|S\|+\varepsilon^i}\left\{\|S\|\left(\|S\|+\varepsilon^i\right)\right.$$

$$\left. + \frac{\|\eta_2^i\|_F}{\left(\|\eta_1^i\|_F - \delta/2\right)}\|S\| - \frac{p_\upsilon^i}{\left(\|\eta_1^i\|_F - \delta/2\right)}\left(\|S\|+\varepsilon^i\right)\right\}$$

$$-\sum_{i=1}^{N}\mu^i\left\{\frac{\left(\gamma^i-\delta/2\right)\|\tilde{W}^i\|_F^2 - \gamma^i W_m^i\|\tilde{W}^i\|_F}{\beta^i}\right\} \quad (22)$$

$$\leq -\sum_{i=1}^{N}\mu^i\left\{\frac{\left(\|\eta_1^i\|_F - \delta/2\right)\|S\| P^i\left(\|S\|\right)}{\|S\|+\varepsilon^i}\right.$$

$$\left. + \frac{\left(\gamma^i-\delta/2\right)\|\tilde{W}^i\|_F Q^i\left(\|\tilde{W}^i\|_F\right)}{\beta^i}\right\}$$

where $P^i\left(\|S\|\right) = \|S\|^2 + 2p_1^i\|S\| - p_2^i$, $Q^i\left(\|\tilde{W}^i\|_F\right) = \|\tilde{W}^i\|_F - q^i$. When $\|S\| \geq p_m$ and $\|\tilde{W}^i\|_F \geq q^i$ for $i = 1,2,...,N$, the inequalities $P^i\left(\|S\|\right) \geq 0$ and $Q^i\left(\|\tilde{W}^i\|_F\right) \geq 0$ for $i = 1,2,...,N$, are obtained. Then outside of the domain Ψ_2 in (17a) makes $\dot{\bar{V}}_2 \leq 0$ (or $\dot{V}_2 \leq -\delta V_2$). Hence, the signal Z exponentially converges into the domain Ψ_2. Similarly, as $n_{s2} \leq \|S\| \leq n_{s1}$, the corresponding result can be obtained by the mean value of (16) and (17). Finally, from (7)-(8)) $\left\{u, \hat{W}^1, \hat{W}^2, \ldots, \hat{W}^N\right\}$ are UUB, and $\{S, x\}$ are GUUB because $x(0) \in \Re^n$..

Q.E.D.

Remark 1: Because $p_\upsilon^i << h_\Omega^i(z)$, the comparison between p_m in (17a) and h_m in (16a) gives the tracking performance, i.e., $\|S(t)\|$, of GFMBAVSC with $\sigma\left(\|S(t)\|\right)=1$ is much smaller than that with $\sigma\left(\|S(t)\|\right)=0$.

Remark 2: To obtain the result $n_{s1} > h_m > n_{s2} > p_m > 0$, a suitable selection of n_{s1} and n_{s2} is dependent on the system. However, the information of h_m in (16e) can help to assign them. A thumb rule of their selection can be described as follows:

(i) The simulation (or experiment) using GFMBAVSC with $\sigma\left(\|S(t)\|\right)=0$ is first progressed. Then the steady state response of $\|S(t)\|$ (i.e., S_{ss}) is obtained.

(ii) The value of $h_m > S_{ss}$ is assigned.

(iii) The values of $n_{s1} \approx h_m + \varepsilon$ and $n_{s2} \approx h_m - \varepsilon$, where ε is a suitably small positive constant (e.g., $\varepsilon = 0.1h_m$), are accomplished.

Remark 3: If the system uncertainties are small enough, then the value of h_m is also small enough. If the value of h_m (or the system performance) satisfies the specification, then the value of n_{s1} approaches zero, i.e., the learning law (11) is shut down.

Remark 4: If $\varepsilon^i = 0$ (i.e., no boundary layer for the *ith* subsystem), then $h_m \to 0$ as $t \to \infty$, for $i = 1,2,...,N$. Hence, $\|S(t)\| \to 0$ (or $\|\tilde{x}(t)\| \to 0$) as $t \to \infty$. However, the control input is probably in a chattering way and the amplitude of control input is larger if the upper bound of uncertainties is larger. Although a larger value of ε^i will make the control input smooth, the tracking accuracy is generally deteriorated. Hence, a compromise must be made. This is one of important motivation that the system uncertainties must be learned to design an extra compensation so that the tracking accuracy and the degree of chattering control input are improved.

Remark 5: The coefficients of the reference model of the *ith* subsystem, i.e., A_m^i, B_m^i, for $i = 1,2,...,N$, are chosen so that the corresponding transfer function matrix, i.e., $G_m^i(s) = C_m^i\left[sI_n - A_m^i\right]^{-1}B_m^i = diag\left\{g_1^i(s), g_2^i(s), ..., g_n^i(s)\right\}$, has the properties: $\left|g_j^i(iw_j)\right| = 1$, $\angle g_j^i(iw_j) = \theta_j$, where w_j for denotes the specific frequency of the *jth* channel reference input. The matrices C_m^i, $i = 1,2,...,N$, are chosen so that the desired output of the fuzzy reference model is attained. For example, a sinusoidal (or triangular) trajectory with frequency w_j is set, the reference input of the *jth* channel should add an extra lead phase to compensate the phase lag of the reference model, i.e., $r_j(t) = \bar{r}_j \sin(w_j t + \theta_j)$, $j = 1,2,...,n$ where $\bar{r}_j$ is a assigned constant.

V. CONCLUSIONS

In the beginning, a nonlinear time-varying dynamic system is approximated by *N* fuzzy-based linear state-space subsystems. Then the same fuzzy sets of the system rule are employed to design GFMBAVSC. Learning has the capability of reducing the system uncertainties affecting the performance of a dynamic system. Under suitable conditions, an asymptotical tracking result of the controlled system with mild uncertainties by the proposed GFMBAVSC without the compensation of learning uncertainties (i.e., $\sigma\left(\|S(t)\|\right)=0$) is obtained. As the uncertainties are huge, the tracking performances become only acceptable or even unstable. Under these circumstances, the proposed GFMBAVSC with the compensation of learning uncertainties (i.e., $\sigma\left(\|S(t)\|\right)=1$) can improve the tracking performance and the stability of the closed-loop system. In addition, the global tracking for initial states outside of approximated domain is obtained as compared with previous studies, which are only semi-global ultimate bounded. It is believed that the proposed control scheme can be extended to a class of highly nonlinear time-varying dynamic systems.

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REFERENCES

[1] C. L. Hwang, "A novel Takagi-Sugeno-based robust adaptive fuzzy sliding-mode controller," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 5, pp. 676-687, Oct. 2004.

[2] Y. W. Liang, S. D. Xu, D. C. Liaw and C. C. Chen, "A study of T–S model-based SMC scheme with application to robot control," *IEEE Trans. Ind. Electron.*, vol.55, no. 11, pp. 3964-3970, Nov. 2008.

[3] J. A. Meda-Campa˜na, J. C. G´omez-Mancilla and B. Castillo-Toledo, "Exact output regulation for nonlinear systems described by Takagi–Sugeno fuzzy models," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 2, pp. 235-245, Apr. 2012.

[4] H. K. Khalil, *Nonlinear Systems.* Prentice-Hall, 2nd Ed. 1996.

[5] F. L. Lewis, K. Liu and A. Yesildirek, "Neural net robot controller with guaranteed tracking performance," *IEEE Trans. Neural Networks*, vol. 6, no. 3, pp. 703-715, May 1995.

[6] F. J. Lin, P. H. Chou, P. H. Shieh and S. Y. Chen, "Robust control of an LUSM-based *X–Y –θ* motion control stage using an adaptive interval type-2 fuzzy neural network," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 24-38, Feb. 2009.

[7] J. I. Mulero-Martínez, "Robust GRBF static neurocontroller with switch logic for control of robot manipulators," *IEEE Trans. Neural Networks and Learning Systems*, vol. 23, no. 7, pp. 1053-1064, Jul. 2012.

[8] B. S. Chen, C. S. Tsen and H. J. Uang, "Robustness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 5, pp. 571-585, Oct. 1999.

[9] Q. Gao, X. J. Zeng, G. Feng, Y. Wang and J. Qiu, "T–S-fuzzy-model-based approximation and controller design for general nonlinear systems," *IEEE Trans. SYst. Man & Cybern., Pt. B*, vol. 42, no. 4, pp. 1143- 1154, Aug. 2012.

[10] K. Natori and K. Ohnishi, "A design method of communication disturbance observer for time-delay compensation, taking the dynamic property of network disturbance into account," *IEEE Trans. Ind. Electron.*, vol. 55, no. 5, pp. 2152-2168, May 2008.

[10] W. S. Huang, C. W. Liu, P. L. Hsu and S. S. Yeh, "Precision control and compensation of servomotors and machine tools via disturbance observer," *IEEE Trans. Ind. Electronics*, vol. 57, no. 1, pp. 420-429, Jan. 2010.

[12] B. S. Chen and C. H. Wu, "Robust optimal reference- tracking design method for stochastic synthetic biology systems: T–S fuzzy approach," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 6, pp. 1144-1154, Dec. 2010.

[13] Q. Gao, G. Feng, Y. Wang and J. Qiu, "Universal fuzzy models and universal fuzzy controllers for stochastic non-affine nonlinear systems," *IEEE Trans. Fuzzy Syst.*, to be appeared, 2014.

[14] C. M. Lin and C. F. Hsu, "Neural-network hybrid control for antilock braking systems," *IEEE Trans. Neural Networks*, vol. 14, no. 2, pp. 351-359, Mar. 2003.

[15] Y. H. Chang and W. S. Chan, "Adaptive dynamic surface control for uncertain nonlinear systems with interval type-2 fuzzy neural networks," *IEEE Trans. Cybern.*, vol. 44, no. 2, pp. 293-304, Feb. 2014.

[16] Y. H. Chen, Q. Huang and A. Cheng, "Adaptive robust control for fuzzy mechanical systems: constraint-following and redundancy in constraints," *IEEE Trans. Fuzzy Syst.*, to be appeared, 2014.

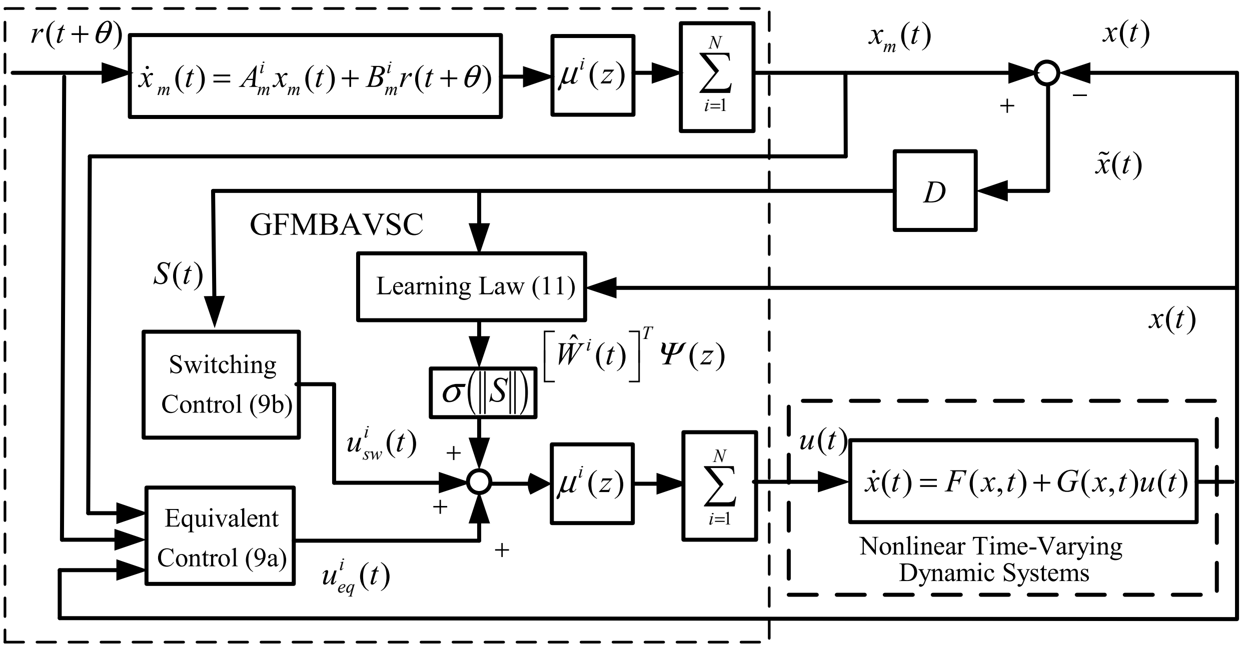

Fig. 1. Control block diagram.