

# An Approach based on Computing with Words to Manage Experts Behavior in Consensus Reaching Processes with Large Groups

Iván Palomares, Francisco J. Quesada and Luis Martínez

**Abstract**—Group decision making problems are characterized by the participation of multiple experts with different points of view, who attempt to find a common solution to a problem composed by a set of alternatives. Such problems are often defined in environments of uncertainty caused by the imprecision and vagueness of information, therefore experts must utilize appropriate information domains to deal with such uncertainty when expressing their preferences, e.g. linguistic information. Usually, in group decision making problems it is necessary to apply a consensus reaching process, in which experts discuss and make their opinions closer to each other, in order to achieve a high level of agreement before making the decision. Nevertheless, in large-scale group decision making problems, where a large group of individuals take part, it is more frequent the existence of certain subgroups with a non-cooperative behavior towards consensus reaching. For this reason, it would be convenient to identify such subgroups and deal with them, so that their behavior does not affect the consensus reaching process negatively. In this contribution, we present an approach based on computing with words and fuzzy set theory, to study the behavior of experts in consensus reaching processes, with the aim of identifying and penalizing the importance weights of those experts whose behavior does not contribute to reach a collective agreement.

## I. INTRODUCTION

Group Decision Making (GDM) problems are decision situations in which a group of individuals or experts, try to find a common solution to a problem consisting of a set of alternatives [1], [2]. To do so, experts assess the different alternatives that might provide a solution to such a problem. Many real-life GDM problems are often defined under an environment of uncertainty, so that experts should provide the information about their preferences by using an information domain closer to human natural language, which is suitable to deal with such uncertainty [3], [4]. Fuzzy set theory [5] and the fuzzy linguistic approach [6], [7], [8], have been some of the most fundamentally utilized approaches in decision problems under uncertainty [9].

---

Iván Palomares, Francisco J. Quesada and Luis Martínez are with the Department of Computer Science, University of Jaén, Jaén (Spain) (email: {ivanp.fgreal,martin}@ujaen.es).

This work was supported by Research Project TIN-2012-31263 and ERDF

Classically, GDM problems have been solved by applying just an alternative selection process [10]. However, sometimes it is possible that, as a result of such a process, not all preferences of experts are taken into account properly. This may lead to situations in which some experts do not feel satisfied with the decision made and they do not accept it, because they consider that their individual concerns have not been considered sufficiently. In order to overcome this drawback, Consensus Reaching Processes (CRPs) were introduced as an additional phase in the resolution process for GDM problems [11]. In a CRP, experts try to achieve a high level of group agreement before making a decision, by discussing and modifying their individual preferences, making them closer to each other [12]. Many consensus models have been proposed by different authors to support and guide groups in CRPs conducted in different GDM frameworks [13], [14], [15], [16], [17], [18].

GDM problems have been traditionally carried out by a small number of experts in organizational and enterprise environments. Nevertheless, the appearance of new technological environments and paradigms to make group decisions, such as group e-marketplaces or social media, have caused that decision problems in which large groups of experts can take part attain greater importance in the last few years [18]. In many CRPs, especially those ones in which many experts are involved, it may occur that some participating experts or subgroups of them seek their own interests rather than the collective interest, therefore they do not cooperate to move their opinions closer to the rest of the group [19]. Consequently, it would be convenient to identify and deal with such individuals or subgroups, in order to prevent that their non-cooperative behavior deviates the group solution to the GDM problem in their favor, thus affecting the normal development of the CRP.

In this contribution, we present an approach to analyze the behavior of experts involved in CRPs, according to the type of behavior they present, with the aim of identifying and managing non-cooperative behaviors of experts who do not collaborate to reach a collective agreement. Such an approach is based

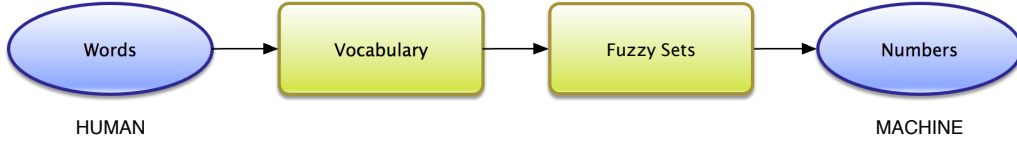


Fig. 1. Paradigm of man-machine understanding

on fuzzy set theory, and it also utilizes reasoning processes under the paradigm Computing with Words (CW) [4] to evaluate the type of behavior adopted by each expert across the CRP. A weighting scheme is then applied to assign different importance weights to experts, according to their behavior. As a result, the approach rewards experts when they cooperate to reach consensus by assigning them higher importance weights, and penalizes them otherwise.

The contribution is set out as follows: Section 2 introduces some preliminaries about reasoning processes based on CW paradigm, linguistic GDM and CRPs. Section 3 presents the proposal for managing experts' behaviors in CRPs and its integration with a consensus model for GDM problems in a linguistic framework. An application example to illustrate the use of the proposal during the resolution of a large-scale GDM problem, is shown in Section 4. Finally, in Section 5 some concluding remarks are pointed out.

## II. PRELIMINARIES

In this section, we firstly revise the methodology of CW for reasoning processes, which will be taken into account in the proposal presented in this paper. Some basic concepts about linguistic GDM problems and CRPs are then revised.

### A. Computing with Words for Reasoning Processes

Human beings utilize linguistic terms to communicate, reason and understand the environment around them. Machines, on the other hand, require much more formal symbols [20]. One of the most widely considered proposals to establish a comprehensive link of communication between human beings and machines, is the so-called paradigm of CW [4], proposed by L.A. Zadeh and based on fuzzy sets theory [5]. The methodology of CW provides a framework in which concepts belonging to a vocabulary can be modeled by means of fuzzy sets, so that they can be easily understood by both human beings and machines, e.g. computers (see Figure 1).

Linguistic terms are a key concept in CW. A linguistic term is a word or phrase, utilized to express the value of an attribute. For example, if we consider an attribute called *distance*, some possible linguistic terms

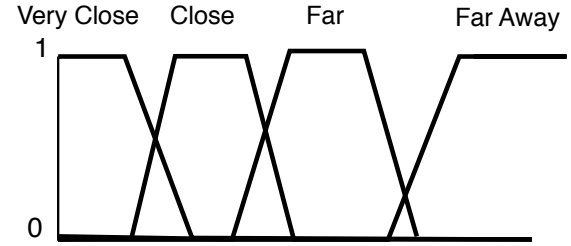


Fig. 2. Different linguistic terms for the attribute *distance*

to express the value of such an attribute could be: “*very close*”, “*close*”, “*far*” and “*far away*”. Thus, with the aid of linguistic terms, humans can better understand and reason about the different features of their environment.

Given the inherent vagueness and imprecision that the values of linguistic terms present, fuzzy sets constitute a useful tool to formalize the concepts associated to them (see Figure 2), thus allowing computers to understand and carry out computational processes over such concepts. Let  $\mathbf{P}$  be a linguistic term (e.g. “close”) belonging to a vocabulary associated to an attribute  $\mathbf{A}$  (e.g. *distance*). We can then express  $\mathbf{P}$  as a fuzzy subset in the domain  $Y \in \mathbb{R}$  of  $\mathbf{A}$ . Given a value  $y \in Y$ , its membership degree to  $\mathbf{P}$ ,  $\mu_{\mathbf{P}}(y) \in [0, 1]$  indicates the compatibility degree of the value  $y$  with the linguistic term  $\mathbf{P}$ .

The choice of a vocabulary of linguistic terms for describing an attribute, and the definition of the semantics associated to such terms (given by their corresponding fuzzy sets), must be carried out by human beings, who are responsible for providing computers with the linguistic terms that will be utilized, and the membership functions of their associated fuzzy sets.

### B. Linguistic Group Decision Making (GDM)

GDM implies the participation of several experts who must make a collective decision to find a common solution to a problem. A decision making process in which several experts take part, having each one his/her own knowledge and experience, may often lead to better decisions than those made by a single expert only [1].

Formally, a GDM problem is characterized by [2]:

- The existence of a common problem to be solved.
- A set  $X$  of *alternatives* or possible solutions to the problem.

$$X = \{x_1, \dots, x_n\} \quad (n \geq 2) \quad (1)$$

- A set  $E$  of individuals or *experts*, who express their opinions or preferences over the set of alternatives  $X$ .

$$E = \{e_1, \dots, e_m\} \quad (m \geq 2) \quad (2)$$

Experts normally utilize a preference structure to express their opinions over alternatives. Some of the most widely utilized preference structures in GDM problems under uncertainty are the ones based on linguistic information, for instance the so-called *linguistic preference relations* [9]. A linguistic preference relation  $P_i$  associated to expert  $e_i$ , can be represented for  $X$  finite as a  $n \times n$  matrix, as follows:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

being each linguistic *assessment*  $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in S$ , the degree of preference of the alternative  $x_l$  over  $x_k$ ,  $l, k \in \{1, \dots, n\}$ ,  $l \neq k$ , according to the expert  $e_i$ . An assessment  $p_i^{lk}$  is expressed as a linguistic term  $s_u$  (e.g. “slightly worse”, “absolutely better”) belonging to a linguistic term set  $S = \{s_0, \dots, s_g\}$  with granularity  $g$ . Without loss of generality, in this paper we consider that  $S$  is composed by linguistic terms  $s_u$ ,  $u \in \{0, \dots, g\}$ , that are symmetrically distributed in an ordered scale around a central term, therefore  $S$  has odd cardinality,  $|S| = g + 1$ .

The solution for a GDM problem can be determined by applying either a direct approach or an indirect approach [10]. In a *direct approach*, the solution is directly obtained from the individual preferences of experts, without constructing a social opinion first [21], whereas in an *indirect approach*, a social opinion or *collective preference* is determined a priori from individual opinions, and then it is utilized to find a solution for the problem. Regardless of the approach considered, the classical selection process for reaching a solution to GDM problems is composed by two phases [22], as illustrated in Figure 3:

- (i) *Aggregation phase*: Experts’ preferences are combined.

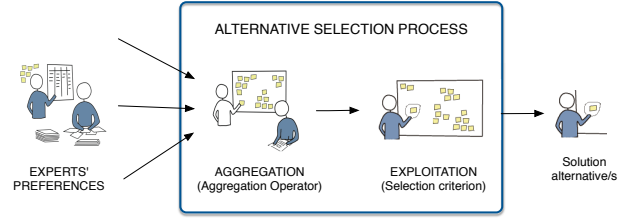


Fig. 3. Classical resolution process for GDM problems

- (ii) *Exploitation phase*: It consists in obtaining an alternative or subset of alternatives as the solution to the problem.

In the specific case of linguistic GDM problems [9], classical resolution processes show the necessity of using models that not only operate with linguistic information accurately, but also allow to obtain understandable results [23]. The methodology of CW proposed by L. Zadeh in [11] not only facilitates reasoning processes (see Section II-A), but also computational and decision making processes on linguistic information. Several linguistic computational models have been proposed in the field of CW, defining each one of them different operations on linguistic information (e.g. aggregation or comparison between linguistic terms). One of the most utilized models in decision making is the so-called 2-tuple linguistic model [24], which provides accurate and understandable results while avoiding loss of information.

### C. Consensus Reaching Processes (CRPs)

When an alternative selection process is applied solely to solve a GDM problem, it may occur that one or several experts feel that their opinions have not been heard to find the solution, therefore they might not accept such a solution. A high level of agreement amongst all experts becomes crucial in many real-life situations, therefore it is necessary to apply a CRP, thus introducing an additional phase in the resolution process for GDM problems. CRPs aim at obtaining a high level of agreement between experts before making a group decision [25].

The term *consensus* can be defined as the agreement produced by mutual consent between all members in a group or between several groups [11], [12]. The process to reach consensus is a dynamic and iterative process, consisting of several rounds of discussion, and frequently coordinated by a human figure: the moderator. The moderator is a key figure in CRPs, being in charge of supervising and guiding experts across the discussion process [25]. Figure 4 shows a general CRP scheme followed in several consensus models for linguistic GDM [15], [16], [17], [26]. Its main phases are described below:

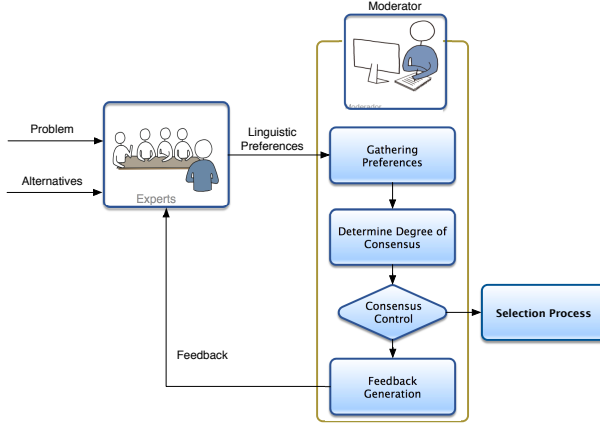


Fig. 4. General scheme for CRPs

- 1) *Gathering preferences*: Each expert  $e_i$  provides the moderator with his/her preferences over alternatives in  $X$ , e.g. by means of a linguistic preference relation.
- 2) *Determine degree of consensus*: The moderator computes the current degree of consensus in the group,  $cr$ , usually expressed as a value in the  $[0,1]$  interval (where a value of 1 indicates full or unanimous agreement between all experts on all the alternatives). To do so, different consensus measures can be utilized. Such measures are normally based on the use of metrics to calculate degrees of similarity between preferences of experts, and aggregation operators that obtain the degree of consensus in the group from such similarity values. Notice that computational processes based on CW can be utilized here to carry out computations on linguistic information.
- 3) *Consensus Control*: The consensus degree  $cr$  obtained in the previous phase, is compared with a consensus threshold  $\mu \in [0, 1]$  fixed a priori, that indicates the minimum level of agreement required by the group. If  $cr > \mu$  then consensus has been achieved and the group proceeds to the selection process; otherwise, it is necessary another discussion round. Another parameter,  $Maxround \in \mathbb{N}$  could be used to limit the number of discussion rounds allowed.
- 4) *Feedback Generation*: The moderator computes a collective preference of the group,  $P_c$ , by aggregating the individual preferences of all experts. Based on  $P_c$ , the moderator identifies those experts' assessments  $p_i^{lk}$  which are farthest from consensus, and advises them to modify such assessments with the aim of increasing the consensus degree in the

following ground. Experts are responsible for modifying their assessments, by assigning a higher or lower value to them, so that the new value is closer to  $P_c$ . Each piece of advice consists in a triplet  $(e_i, (x_l, x_k), Direction)$  which indicates that the expert  $e_i$  must modify his/her assessment  $p_i^{lk}$  in the direction given by  $Direction \in \{increase, decrease\}$ .

### III. MANAGEMENT OF EXPERTS' BEHAVIOR IN CRPs

In this section, we present a methodology to deal with different behaviors of experts in CRPs carried out to the resolution of GDM problems in a linguistic framework. Firstly, we define the scheme of the approach proposed, describing the necessary steps to manage experts' behaviors at each round of the CRP. The approach is then integrated with a consensus model for GDM problems with linguistic preference relations, that follows the CRP scheme revised in the previous section.

#### A. Scheme of the Methodology to Manage Experts' Behaviors

Yager proposed in [19] a weight-based approach to penalize experts who try to strategically manipulate the solution to a GDM problem, deviating the collective opinion in their favor based on their own preference values. Such an approach is based on assigning importance weights to experts, so that when some importance weights are penalized, the opinions of experts associated to such weights attain a lower importance than the opinions of the remaining experts. Importance weights are taken into account when obtaining the collective opinion used as a solution for the GDM problem.

The approach presented in this work is inspired by the ideas exposed by Yager, and it consists in assigning an importance weight  $w_i^t \in [0, 1]$  to each expert  $e_i \in E$ , being  $t \in \mathbb{N}$  the current round of discussion in the CRP. Similarly to Yager's proposal, weights are utilized to compute the collective preference,  $P_c$ , by applying a weighted aggregation operator over individual preferences  $P_i$ ,  $i \in \{1, \dots, m\}$ . As a result, the preferences of most cooperating experts (who would have associated higher importance weights) are taken into account to a higher degree in the computation of  $P_c$ .

The scheme of the approach presented to manage behaviors of experts across the CRP, is composed by three phases, as shown in Figure 5:

- (1) *Compute behavior metrics*
- (2) *Experts weighting*
- (3) *Weights normalizing*

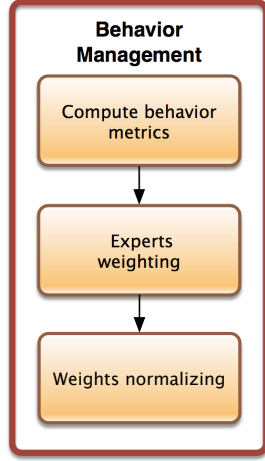


Fig. 5. Scheme of the methodology to manage behaviors of experts in CRPs

Such phases are described with detail in the following subsections.

1) *Compute behavior metrics*: The objective in this phase is to analyze the behavior adopted by each expert at a given moment during the CRP, and evaluate it by means of a behavior metric that indicates how good such a behavior is. Different behavior metrics can be considered to compute the degree of cooperation of experts in the CRP, based on different aspects, such as: (i) the amount of generated advices that an expert accepts and applies on his/her assessments (see Section II-C), or (ii) the degree of change that an expert applies when modifying his/her assessments.

Although we suggest the definition and combined use of different behavior metrics to evaluate the behavior of experts, in this paper we will define and consider (without loss of generality) a behavior metric so-called *cooperation coefficient*.

**Definition 1:** Let  $\#ADV_i^t$  be the number of advices provided to  $e_i$  to modify some of his/her assessments  $p_i^{lk}$  before beginning the CRP round  $t$ , and let  $\#ACPT_i^t$  be the number of advices that  $e_i$  accepts to modify in accordance with the feedback received. Then, the cooperation coefficient  $CC_i^t \in [0, 1]$  of  $e_i$  at round  $t$  is defined as follows:

$$CC_i^t = \begin{cases} 1 & \text{if } \#ADV_i^t = 0, \\ \frac{\#ACPT_i^t}{\#ADV_i^t} & \text{otherwise.} \end{cases} \quad (3)$$

The value of the cooperation coefficient represents the degree to which an expert modifies his/her opinions moving them closer to consensus, as suggested in the

advices he/she received. Notice that if an expert does not receive any advice at a given round, this means that all his/her assessment values are close to consensus, therefore we consider that  $CC_i^t = 1$  in this case.

2) *Experts Weighting*: In this phase, the previously reviewed concepts about fuzzy sets and reasoning processes in CW, are utilized to assign each expert an importance weight, based on the value of the behavior metric computed in the previous phase (in our case, the cooperation coefficient). To do so, a linguistic term “cooperative” is defined, being its semantics given by a fuzzy subset  $COOP$  in the unit interval, according to the following membership function:

$$\mu_{COOP}(y) = \begin{cases} 0 & \text{if } y < \alpha, \\ \frac{y-\alpha}{\beta-\alpha} & \text{if } \alpha \leq y < \beta, \\ 1 & \text{if } y \geq \beta. \end{cases} \quad (4)$$

being  $\alpha, \beta, y \in [0, 1]$ ,  $\alpha < \beta$ . The importance weight of expert  $e_i$  at round  $t$ ,  $w_i^t$ , is computed as the membership degree of his/her cooperation coefficient  $CC_i^t$  to  $COOP$ :

$$w_i^t = \mu_{COOP}(CC_i^t) \quad (5)$$

Furthermore, let us consider that the fact of no cooperating when the CRP is at an advanced stage (i.e. after several discussion rounds), may suppose a greater penalization than no cooperating at the first rounds of the process, thus adopting a more permissive attitude towards the behavior of experts at the earliest stage of the CRP. In order to reflect this circumstance in our proposal, we propose the flexible use of different membership functions to define the semantics of the linguistic term “cooperative” at each round, by increasing the values of  $\alpha, \beta$  gradually, so that the support of the fuzzy set [5] becomes narrower as the CRP goes on. Figure 6 illustrates this process.

3) *Weights Normalizing*: Given that different values for experts’ weights  $w_i^t$  will be obtained at each consensus round, in this phase a normalization of such weights is applied, as follows:

$$\hat{w}_i^t = \frac{w_i^t}{\sum_{i=1}^m w_i^t} \quad (6)$$

where  $\hat{w}_i^t \in [0, 1]$  and  $\sum \hat{w}_i^t = 1$ . Once weights have been normalized, they will be taken into account for computing the collective preference in the current round of discussion.

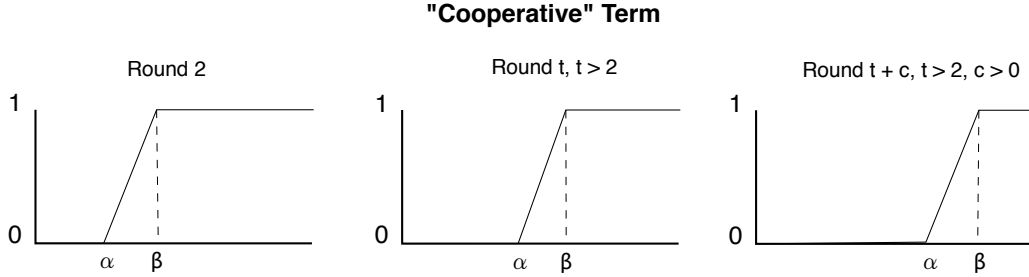


Fig. 6. Evolution of the fuzzy membership function associated to the linguistic term “cooperative” across the CRP

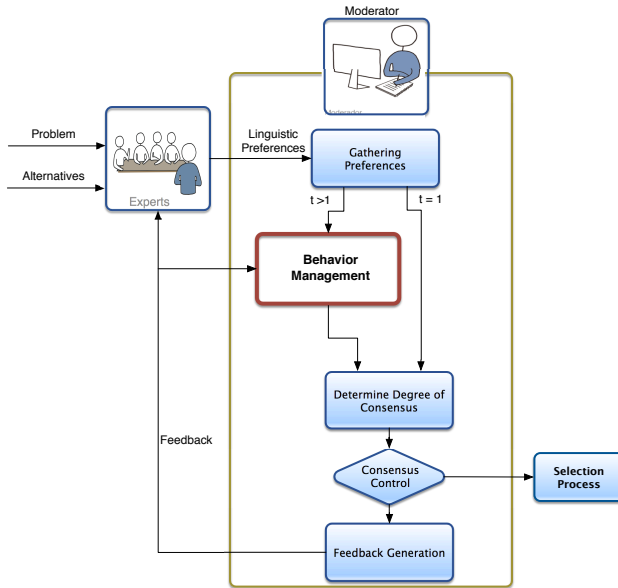


Fig. 7. Scheme of a CRP with behavior management

### B. Integration in a Consensus Model

Once the proposal for managing behaviors in CRPs has been presented, here we show its integration in a consensus model for linguistic GDM problems that extends the general scheme for CRPs shown in Figure 4. Figure 7 shows an scheme of CRPs that incorporates the behavior management approach. As can be seen, the approach is first applied at the beginning of the second consensus round ( $t > 2$ ), because the behavior of each expert is evaluated by analyzing the following information:

- The advices received by each expert at the end of the previous round,  $t - 1$ .
- The updated preferences of each expert, after having received his/her corresponding advice.

Notice that such information does not exist yet at the beginning of the CRP, hence the approach is applied after the first round of discussion ends. Moreover, it is assumed that when the CRP begins ( $t = 1$ ), all experts have equal importance weights. As the CRP goes on, the weight of each expert might vary based on his/her behavior at each round. For example, the weight of an expert who does not cooperate during the first two consensus rounds should decrease after such rounds. On the other hand, if the expert decides to cooperate from the third round onwards, making his/her opinions closer to the rest of the group, then the corresponding weight should be increased again.

## IV. APPLICATION EXAMPLE

In this section we show an example to illustrate the use of our proposal to manage non-cooperative behaviors, by simulating a CRP for the resolution of a large-scale linguistic GDM problem.

An enterprise committee formed by 40 experts,  $E = \{e_1, \dots, e_{40}\}$  must reach an agreement about the choice of the annual supportive actions to be carried out this year. There are four possible proposals,  $X = \{x_1: \text{Hurricane victims}, x_2: \text{Hospitalized children}, x_3: \text{Endangered species}, x_4: \text{Reforestation plans}\}$ .

Experts utilize the following linguistic term set to express their assessments,  $p_i^{lk}$ , over pairs of alternatives:  $S = \{s_0 : \text{absolutely worse}, s_1 : \text{much worse}, s_2 : \text{slightly worse}, s_3 : \text{indifferent}, s_4 : \text{slightly better}, s_5 : \text{much better}, s_6 : \text{absolutely better}\}$ .

The minimum level of agreement required is  $\mu = 0.85$ . Regarding the membership function of the fuzzy set *COOP*, its initial parameters are  $\alpha = 0.2$  and  $\beta = 0.5$ . After the fourth consensus round, the value of both parameters is increased in 0.1 per round, until each one of them reaches a value of 1. Thus, the approach becomes less permissive with the notion of cooperative behavior as the CRP goes on (see Figure 6).



TABLE I. NUMBER OF ADVICES RECEIVED AND ACCEPTED BY EXPERTS  $e_{31} - e_{40}$  AT EACH ROUND

$t$	$e_{31}$	$e_{32}$	$e_{33}$	$e_{34}$	$e_{35}$	$e_{36}$	$e_{37}$	$e_{38}$	$e_{39}$	$e_{40}$
1	6/8	0/0	0/0	2/2	5/6	0/0	0/0	0/0	0/0	6/8
2	5/9	0/0	0/0	0/0	4/5	1/1	0/0	0/0	1/1	9/9
3	6/8	0/0	0/0	0/2	2/2	0/0	0/0	0/0	0/0	5/8
4	<b>3/8</b>	0/0	0/0	1/2	1/3	0/0	0/0	0/0	0/0	4/8
5	3/7	0/1	0/0	2/2	2/3	0/0	0/0	0/0	0/1	3/7
6	4/5	0/0	0/0	0/0	0/0	0/0	0/0	0/0	2/5	1/5
7	<b>2/4</b>	0/0	0/0	0/0	0/0	0/0	0/0	0/0	2/3	2/4
8	3/4	0/0	0/0	0/0	0/0	0/0	0/0	0/0	3/4	4/4
9	2/4	0/0	0/0	0/0	0/0	0/0	0/0	0/0	2/3	3/4

TABLE II. NORMALIZED WEIGHTS OF EXPERTS,  $\hat{w}_i^t$ , THROUGHOUT THE CRP

$t$	$e_1 - e_{30}$	$e_{31}$	$e_{32}$	$e_{33}$	$e_{34}$	$e_{35}$	$e_{36}$	$e_{37}$	$e_{38}$	$e_{39}$	$e_{40}$
2	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
3	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
4	0.0256	0.0256	0.0256	0.0256	0	0.0256	0.0256	0.0256	0.0256	0.0256	0.0256
5	0.0265	<b>0.0066</b>	0.0265	0.0265	0.0176	0.0029	0.0265	0.0265	0.0265	0.0265	0.0176
6	0.0277	0.0026	0	0.0277	0.0277	0.0246	0.0277	0.0277	0.0277	0	0.0026
7	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0.0263	0	0
8	0.0261	<b>0</b>	0.0261	0.0261	0.0261	0.0261	0.0261	0.0261	0.0261	0.0058	0.0261
9	0.0267	0.0044	0.0267	0.02678	0.0267	0.0267	0.0267	0.0267	0.0267	0.0044	0

Experts present different patterns of behavior:

- *Cooperative* (experts  $e_1 - e_{30}$ ): experts apply all changes suggested on their assessments, as indicated in the feedback they receive throughout the whole CRP.
- *Undefined* (experts  $e_{31} - e_{40}$ ): The behavior of these experts varies across the CRP, so that they may either apply changes suggested or ignore them.

Table I shows the number of advices that experts in  $e_{31} - e_{40}$  received to modify some of their assessments at each round,  $\#ADV_i^t$ , and the number assessments modified as indicated in such advices,  $\#ACPT_i^t$ . Both data are depicted together in the table, under the format  $\#ACPT_i^t/\#ADV_i^t$ . Notice that in Table I, some experts in  $e_{31} - e_{40}$  do not receive any advices across the CRP, hence  $CC_i^t = 1$  in these cases, and they will be assigned the highest weight value. This occurs because their opinions are close to the collective opinion since the beginning of the CRP, therefore they do not need to apply changes on their preferences.

Table II shows the evolution of experts' normalized weights at each CRP round ( $t \geq 2$ ). As can be observed, all cooperating experts ( $e_1 - e_{30}$ ) have associated higher importance weights. However, experts with an undefined behavior may present different weights, depending on their degree of cooperation at each round (measured by means of the cooperation coefficient  $CC_i^t$ ), not exceeding in any case the weight of full cooperating experts in such a round. The main effect of normalizing weights is the compensation between weights of full cooperating experts (and experts with opinions closer to consensus), and weights of experts who have been penalized because of not having coop-

erated enough.

In order to illustrate with more detail the computation of weights based on different behaviors, in the following we show the procedure to compute  $e_{31}$ 's weights at rounds  $t = 5$  and  $t = 8$ :

- $t = 5$ : In this round, the parameters of the fuzzy set *COOP* have the following values:  $\alpha = 0.3$  and  $\beta = 0.6$ .  $e_{31}$  received  $\#ADV_{31}^5 = 8$  advices for assessments at the end of the previous round ( $t = 4$ , see Table I), and she modified  $\#ACPT_{31}^5 = 3$  out of these assessments, according to the advice received. The cooperation coefficient is  $CC_{31}^5 = 3/8 = 0.375$ , and the weight is computed as follows:

$$w_{31}^5 = \mu_{COOP(5)}(0.375) = \frac{0.375 - 0.3}{0.6 - 0.3} = 0.25$$

After normalizing,  $e_{31}$ 's weight is computed as:

$$\hat{w}_{31}^5 = \frac{w_{31}^5}{\sum_{i=1}^{40} w_i^5} = 0.0066$$

- $t = 8$ :  $e_{31}$  received four advices on assessments, and she applied two of them, therefore the cooperation coefficient is higher in this case:  $CC_{31}^8 = 2/4 = 0.5$ . However, in this round we have  $\alpha = 0.6$  y  $\beta = 0.9$ , i.e. after a high number of discussion rounds, the concept of cooperative behavior (given by the semantics of the fuzzy set *COOP*) becomes more restrictive:

$$w_{31}^8 = \mu_{COOP(8)}(0.5) = 0$$

And,  $\hat{w}_{31}^8 = 0$ .

The desired degree of consensus is achieved after nine rounds of discussion. Once consensus has been reached, an alternative selection process is applied to choose the best alternative [10], [22], based on computing the collective preference of the group by means of a linguistic weighted aggregation operator (e.g. 2-tuple weighted mean [24]) that takes into account to a higher degree the opinions of experts with a higher importance weight.

## V. CONCLUDING REMARKS

In consensus reaching processes where a large number of experts are involved, it is common that some individuals or subgroups of them adopt different types of behavior during discussion, regarding the way they cooperate with the rest of the group to reach an agreement. In this contribution, we have presented a proposal based on computing with words to manage the behavior of experts in consensus reaching processes, for the resolution of large-scale group decision making problems under a linguistic framework. The approach is aimed at rewarding experts who cooperate with each other to achieve consensus, and penalizing experts whose behavior is not cooperative enough, by using a weighting-based scheme. Reasoning processes based on computing with words methodology and fuzzy set theory, are applied to compute weights of experts. Finally, the proposal has been integrated in a consensus model and put in practice to show an application example that illustrates its performance.

## REFERENCES

- [1] J. Lu, G. Zhang, D. Ruan, and F. Wu, *Multi-Objective Group Decision Making*. Imperial College Press, 2006.
- [2] J. Kacprzyk, "Group decision making with a fuzzy linguistic majority," *Fuzzy Sets and Systems*, vol. 18, no. 2, pp. 105–118, 1986.
- [3] R. Bellman and L. Zadeh, "Decision-making in a fuzzy environment," *Management Science*, vol. 17, no. 4, pp. 141–164, 1970.
- [4] L. Zadeh, "Fuzzy logic = computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 2, pp. 103–111, 1996.
- [5] —, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
- [6] —, "The concept of a linguistic variable and its application to approximate reasoning (i)," *Information Sciences*, vol. 8, no. 3, pp. 199 – 249, 1975.
- [7] —, "The concept of a linguistic variable and its application to approximate reasoning (ii)," *Information Sciences*, vol. 8, no. 4, pp. 301 – 357, 1975.
- [8] —, "The concept of a linguistic variable and its application to approximate reasoning (iii)," *Information Sciences*, vol. 9, no. 1, pp. 43 – 80, 1975.
- [9] F. Herrera and E. Herrera-Viedma, "Linguistic decision analysis: Steps for solving decision problems under linguistic information," *Fuzzy Sets and Systems*, vol. 115, no. 1, pp. 67–82, 2000.
- [10] F. Herrera, E. Herrera-Viedma, and J. Verdegay, "A sequential selection process in group decision making with linguistic assessments," *Information Sciences*, vol. 85, no. 4, pp. 223–239, 1995.
- [11] C. Butler and A. Rothstein, *On Conflict and Consensus: A Handbook on Formal Consensus Decision Making*. Food Not Bombs Publishing, 2006.
- [12] S. Saint and J. R. Lawson, *Rules for Reaching Consensus. A Modern Approach to Decision Making*. Jossey-Bass, 1994.
- [13] N. Bryson, "Group decision-making and the analytic hierarchy process. exploring the consensus-relevant information content," *Computers and Operations Research*, vol. 23, no. 1, pp. 27–35, 1996.
- [14] R. Parreiras, P. Ekel, J. Martini, and R. Palhares, "A flexible consensus scheme for multicriteria group decision making under linguistic assessments," *Information Sciences*, vol. 180, no. 7, pp. 1075–1089, 2010.
- [15] Z. Wu and J. Xu, "Consensus reaching models of linguistic preference relations based on distance functions," *Soft Computing*, vol. 16, no. 4, pp. 577–589, 2012.
- [16] E. Herrera-Viedma, L. Martínez, F. Mata, and F. Chiclana, "A consensus support system model for group decision making problems with multigranular linguistic preference relations," *IEEE Transactions on Fuzzy Systems*, vol. 13, no. 5, pp. 644–658, 2005.
- [17] F. Mata, L. Martínez, and E. Herrera-Viedma, "An adaptive consensus support model for group decision-making problems in a multigranular fuzzy linguistic context," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 2, pp. 279–290, 2009.
- [18] I. Palomares, L. Martínez, and F. Herrera, "A consensus model to detect and manage non-cooperative behaviors in large-scale group decision making," *IEEE Transactions on Fuzzy Systems*, vol. Inpress, DOI: 10.1109/TFUZZ.2013.2262769, 2014.
- [19] R. Yager, "Penalizing strategic preference manipulation in multi-agent decision making," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 3, pp. 393–403, 2001.
- [20] —, "Concept representation and database structures in fuzzy social relational networks," *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, vol. 40, no. 2, pp. 413–419, 2010.
- [21] F. Herrera, E. Herrera-Viedma, and J. Verdegay, "Direct approach processes in group decision making using linguistic OWA operators," *Fuzzy Sets and Systems*, vol. 79, no. 2, pp. 175–190, 1996.
- [22] M. Roubens, "Fuzzy sets and decision analysis," *Fuzzy Sets and Systems*, vol. 90, no. 2, pp. 199–206, 1997.
- [23] R. Rodríguez and L. Martínez, "An analysis of symbolic linguistic computing models in decision making," *International Journal of General Systems*, vol. 42, no. 1, pp. 121–136, 2013.
- [24] F. Herrera and L. Martínez, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 6, pp. 746–752, 2000.
- [25] L. Martínez and J. Montero, "Challenges for improving consensus reaching process in collective decisions," *New Mathematics and Natural Computation*, vol. 3, no. 2, pp. 203–217, 2007.
- [26] F. Herrera, E. Herrera-Viedma, and J. Verdegay, "Linguistic measures based on fuzzy coincidence for reaching consensus in group decision making," *International Journal of Approximate Reasoning*, vol. 16, no. 3-4, pp. 309–334, 1997.