2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

A Fuzzy Logic Based Bargaining Model in Discrete Domains: Axiom, Elicitation and Property

Jieyu Zhan, Xudong Luo* and Cong Feng
Institute of Logic and Cognition
Sun Yat-sen University
Guangzhou, China 510275

Wenjun Ma
School of Electronics, Electrical Engineering and Computer Science
Queen's University Belfast
Belfast, UK BT7 1NN

***Abstract*—This paper builds a multi-demand bargaining model based on fuzzy rules, and introduces its agreement concept, which satisfies four intuitive properties of consistency, collective rationality, disagreement and minimum concession. In the model, the fuzzy rules are used to calculate how much bargainers should change their preference during a bargaining. Moreover, the psychological experiment are used to elicit the fuzzy rules. In addition, we analyse how bargainers' risk attitude, patience and regret degree influence agreement of our bargaining game, and identify the existence conditions of bargaining agreement.**

I. Introduction

Bargaining problem is about how agents should allocate profit, goods, resources and so on among a number of agents. It is one of the most common phenomena in our daily lives. So, since Nash built the first bargaining model [1], various models have been proposed in different areas, such as economics, management science, sociology, and especially computer science.

Most of the work about bargaining focus on handling one demand with one or multiple attributes in continuous domains. In contrast, there is relatively little work that deals with multi-demand in discrete domains. However, in real life, this kind of bargaining problems is very common. For example, in congress, different parties often bargain many political demands in discrete domains, and in collective design problems, agreements must be reached by a group of stake holders with different discrete demands.

To address this problem, in this paper, we develop a fuzzy logic based model that can reflect bargainers' psychological characteristics about regret, risk and patience. Moreover, we carry out a psychological experiment to elicit the fuzzy rules that are used to dynamically change bargainers' preferences during a bargaining. We also analyse: (i) how bargainers' psychological characteristics about regret, risk and patience influence their preference changing during a bargaining, and (ii) under which conditions an agreement can be reached.

The rest of the paper is organised as follows. Section II introduces our bargaining model and its agreement concept. Section III presents our fuzzy logic system and psychological experiment that elicits the rules. Section IV reveals some properties of our model. Section V discusses the related work. Finally, Section VI concludes the paper with future work.

*The corresponding author: luoxd3@mail.sysu.edu.cn.

II. Bargaining model

This section introduces our bargaining model.

Definition 1: The input of a bargaining is a tuple of $(N, \{D_i, \succcurlyeq_i^{(0)}, \succcurlyeq_i^{(1)}\}_{i \in N})$*, where:*

- $N = \{1, \cdots, n\}$ *is the set of all the bargainers;*
- D_i *is the demand set of bargainer i, in which each demand is expressed in a propositional language, denoted as* $\mathcal{L}$*, consisting of a finite set of propositional variables and standard propositional connectives* $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$*;*
- $\succcurlyeq_i^{(0)}$ *is bargainer i's original demand preference ordering, which is a total pre-order on* D_i *(i.e., satisfying totality, reflexivity, and transitivity); and*
- $\succcurlyeq_i^{(1)}$ *is bargainer i's initial dynamic demand preference ordering, which is a total pre-order on* D_i *(i.e., satisfying totality, reflexivity, and transitivity).*

In the above definition, the bargainers' demands are represented by logical propositions, and before the bargaining, each bargainer has two preference orderings over his demands. The original one just reflects his own favorites in his mind without considering whether or not an agreement can be reached. The initial dynamic one considers not only their own taste but also his thinking about which demand should be given up earlier or insisted on during the bargaining.

A bargainer aims to reach an agreement consisting of logically consistent statements. In the following, we will define the process of a bargaining. Firstly, we introduce the concept of a bargainer's demand preference hierarchy:

Definition 2: Let $(D_i^{(\lambda)}, \succcurlyeq_i^{(\lambda)})$ *be bargainer i's dynamic preference structure in the* λ*-th round of bargaining. Then* $\{D_i^{(1,\lambda)}, \cdots, D_i^{(H_i(\lambda),\lambda)}\}$ *is called bargainer i's demand preference hierarchy if* $\forall j, k \in \{1, \cdots, H_i(\lambda)\}$,

(i) $D_i^{(\lambda)} \neq \emptyset$;
(ii) $D_i^{(\lambda)} = D_i^{(1,\lambda)} \cup \cdots \cup D_i^{(H_i(\lambda),\lambda)}$;
(iii) $D_i^{(j,\lambda)} \cap D_i^{(k,\lambda)} = \emptyset$ *if* $j \neq k$;
(iv) $\forall d_i, d_i' \in D_i^{(j,\lambda)}, d_i \succcurlyeq_i^{(\lambda)} d_i'$ *and* $d_i' \succcurlyeq_i^{(\lambda)} d_i$*; and*
(v) $\forall d_i \in D_i^{(j,\lambda)}, d_i' \in D_i^{(k,\lambda)}, d_i \succ_i^{(\lambda)} d_i'$ *if* $j < k$.

Here $D_i^{(j,\lambda)}$ *is called the j-th level of bargainer i's demand preference hierarchy in round* λ *of bargaining, and* $H_i(\lambda)$ *is*

978-1-4799-2072-3/14/$31.00 ©2014 IEEE

called the height of demand preference hierarchy of bargainer i in the λ-th round of bargaining.

Clearly, in the above definition, the highest level is $D_i^{(1,\lambda)}$, and the lowest level is $D_i^{(H_i(\lambda),\lambda)}$. $\forall d \in D_i$, let $l_i^{(\lambda)}(d)$ denote the level of d in the dynamic preference hierarchy in the λ-th round. In the following definition, in round λ, "move demand $d^{\pm}$ down one or two levels" means to move $d^{\pm}$ from its current level in $\{D_i^{(1,\lambda)}, \cdots, D_i^{(H_i(\lambda),\lambda)}\}$ down one or two levels.

Definition 3: The process of a bargaining is a tuple of $(FLS, \mathcal{A}, \mathcal{U})$, where:

- *FLS is a fuzzy logic system for calculating the preference change degree ζ.*
- *$\mathcal{A}$ is bargainers' action function defined as follows:*

$$\mathcal{A}(\zeta, d^{\pm}, \lambda) = \begin{cases} \text{move } d^{\pm} \text{ down two levels from its} \\ \text{current level in round } \lambda, \\ \quad \text{if } \zeta \geqslant 0.7 \wedge l_i^{(\lambda)}(d^{\pm}) \leqslant H_i(1) - 2, \\ \text{move } d^{\pm} \text{ down one level from its} \\ \text{current level in round } \lambda, \\ \quad \text{if } (0.7 > \zeta \geqslant 0.3 \wedge \\ \quad\; l_i^{(\lambda)}(d^{\pm}) \leqslant H_i(1) - 1) \vee \\ \quad\; (\zeta \geqslant 0.7 \wedge l_i^{(\lambda)}(d^{\pm}) = H_i(1) - 1), \\ \text{do nothing} \\ \quad \text{otherwise,} \end{cases} \tag{1}$$

where ζ is the change degree, $d_i^{\pm}$ belongs to the set of the bargainer i's conflicting demand set $D_i^{\pm}$ (in which each element $d_i^{\pm}$ is inconsistent with one demand d_j of at least another bargainer, i.e., $d_i^{\pm} \wedge d_j \rightarrow \perp$), and λ means the λ-th round of the bargaining game.

- *$\mathcal{U}$ is bargainers' update function. Let the dynamic preference structures of bargainer i at the λ-th and (λ+1)-th rounds be $(D_i^{(\lambda)}, \succcurlyeq_i^{(\lambda)})$ and $(D_i^{(\lambda+1)}, \succcurlyeq_i^{(\lambda+1)})$, respectively. Then update function $\mathcal{U}$ is given by:*

$$(D_i^{(\lambda+1)}, \succcurlyeq_i^{(\lambda+1)}) = \mathcal{U}(D_i^{(\lambda)}, \succcurlyeq_i^{(\lambda)}), \tag{2}$$

where:

$$D_i^{(\lambda+1)} = D_i^{(\lambda)} - \{\underline{d_i}\} \tag{3}$$

where $\underline{d_i} \in D_i^{(H_i(\lambda),\lambda)} \bigcap D_i^{\pm}$ if $\exists d_i \in D_i^{(H_i(\lambda),\lambda)} \bigcap D_i^{\pm}$ or $\underline{d_i} \in D_i^{(H_i(\lambda),\lambda)} \backslash D_i^{\pm}$ if $\nexists d_i \in D_i^{(H_i(\lambda),\lambda)} \bigcap D_i^{\pm}$; and $\succcurlyeq_i^{(\lambda+1)}$ is defined as follow:

(i) $\forall d_i, d_i' \in D_i^{(j,\lambda+1)}, d_i \succcurlyeq_i^{(\lambda+1)} d_i'$ and $d_i' \succcurlyeq_i^{(\lambda+1)} d_i$, and

(ii) $\forall d_i \in D_i^{(j,\lambda+1)}, d_i' \in D_i^{(k,\lambda+1)}, d_i \succ_i^{(\lambda+1)} d_i'$ if $j < k$,

where $D_i^{(j,\lambda+1)}$ and $D_i^{(k,\lambda+1)}$ are in $\{D_i^{(1,\lambda+1)}, \cdots, D_i^{(H_i(\lambda+1),\lambda+1)}\}$, which is obtained by applying action function (1) to $\{D_i^{(1,\lambda)}, \cdots, D_i^{(H_i(\lambda),\lambda)}\}$.

Accordingly, after the λ-th round, dynamic demand preference structure $(D_i^{(\lambda)}, \succcurlyeq_i^{(\lambda)})$ of bargainer i will be updated to a new one, $(D_i^{(\lambda+1)}, \succcurlyeq_i^{(\lambda+1)})$, by a certain action chosen using action function (1), where its input (i.e., change degree ζ) is determined by fuzzy logic system FLS (see Section III for the detailed discussion).

A bargaining game consists of the bargaining input and process. Formally, we have:

Definition 4: A bargaining game is a tuple of (I, P), where $I = (N, \{D_i, \succcurlyeq_i^{(0)} \succcurlyeq_i^{(1)}\}_{i \in N})$ is the input of the bargaining, and $P = (FLS, \mathcal{A}, \mathcal{U})$ is the process of the bargaining.

Generally speaking, a bargain agreement should satisfy the intuitive properties: (i) there are no conflicting logical formulas in an agreement; (ii) all bargainers should accept all of each others' demands when they have no conflicting demands with each other; (iii) there are no agreements when one of the bargainers cannot bargain any more because he gave up all his demands; and (iv) if after the λ-th round of bargaining all the demands of all bargainers become consistent logically, it is unnecessary to carry out concession. Formally, we have:

Definition 5: For bargaining game $G = (I, P)$, suppose $A(G) \subseteq \bigcup_{i \in N} D_i$. Then $A(G)$ is an agreement among all the bargainers of game G if:

(i) Consistency: $A(G) \vdash \top$;

(ii) Collective-rationality: if $\bigcup_{i \in N} D_i \vdash \top$, then $\forall i \in N, A(G) = \bigcup_{i \in N} D_i$;

(iii) Disagreement: if $\exists k \in N$, $D_k^{(\lambda)} = \emptyset$ then $A(G) = \emptyset$; and

(iv) Minimum-concession: let $\underline{\lambda}$ be the last round of a bargaining game , $\forall i \in N$, $\underline{d_i}$ is the demand of bargainer i given up after the $(\underline{\lambda} - 1)$-th round, then $A(G) \cup \{d_i, \cdots, d_{|N|}\}$ is inconsistent.

Now, we can define our agreement concept as follows:

Definition 6: The agreement of a bargaining game G is given by:

$$A(G) = \begin{cases} D_1^{(\underline{\lambda})} \bigcup \cdots \bigcup D_{|N|}^{(\underline{\lambda})} & \text{if } \forall i \in N, D_i^{(\underline{\lambda})} \neq \emptyset, \underline{\lambda} < |D|_{min}, \\ \emptyset & \text{otherwise,} \end{cases} \tag{4}$$

where $\underline{\lambda} = \min\{\lambda \mid \bigcup_{i=1}^{|N|} D_i^{(\lambda)}$ is consistent$\}$ is the minimal rounds of concessions of the game, $D_i^{(\underline{\lambda})}$ is the set of demands of bargainer i after $\underline{\lambda}$ rounds of the bargaining and $|D|_{min} = \min\{|D_i| \mid i \in N\}$.

The following theorem confirms that our agreement concept indeed satisfies all the properties listed in Definition 5.

Theorem 1: For game G=((N, $\{D_i, \succcurlyeq_i^{(0)}, \succcurlyeq_i^{(1)}\}_{i \in N}$), (FLS, $\mathcal{A}$, $\mathcal{U}$)), its agreement $A(G)$ satisfies the properties of consistency, collective-rationality, disagreement and minimum-concession listed in Definition 5.

Proof: (i) Consistency. If $\underline{\lambda} < |D|_{min}$ where $|D|_{min}$ is the minimum of demand amount among all bargainers' demand sets, then by the definition of $\underline{\lambda}$ in Definition 6, $\bigcup_{i=1}^{|N|} D_i^{(\lambda)} \vdash \top$; otherwise, $A(G) = \emptyset$ (in this case, $\bigcup_{i=1}^{|N|} D_i^{(\lambda)} \vdash \top$). (ii) Collective-rationality. Since G is non-conflictive, $\bigcup_{i \in N} D_i$ is logically consistent. Then by Definition 6, none of the demands will be given up. So, by formula (4), $A(G) = \bigcup_{i \in N} D_i$. (iii) Disagreement. If $\exists k \in N$, $D_k = \emptyset$, then by formula (4), $A(G) = \emptyset$. (iv) Minimum-concession. By the definition of $\underline{\lambda}$ in Definition 6, before the $\underline{\lambda}$-th round, $\exists d_i^{\pm} \in D_i^{\pm}$ and $d_j^{\pm} \in D_j^{\pm}$ such that $d_i^{\pm} \wedge d_j^{\pm} \rightarrow \perp$, so if $\forall i \in N, d_i$ is the demand that bargainer i given up after the $(\underline{\lambda} - 1)$-th round, then $A(G) \cup \{d_i, \cdots, d_{|N|}\} \vdash \perp$. □

III. Fuzzy logic system

This section will discuss our fuzzy logic system FLS.

A. Input parameters

The change degree of preference mainly depends on three human cognitive factors:

1) Regret degree (ϑ): In Longman English Dictionary, *regret* is defined as "sadness that you feel about something, especially because you wish it had not happened". So, a bargainer may regret because he gives up some preferred or consistent demands at the expense of bargaining with each other. However, by our bargaining process, the effect of the first reason is less obvious than the second one because bargainers give up the less preferred demands in the beginning. Thus, we depict a bargainer's regret degree through the second character. That is, (i) the more consistent demands have been given up the more the bargainer regrets; (ii) if no consistent demands have been given up then the regret degree is the lowest; and (iii) if all consistent demands have been given up during bargaining then the regret degree is the highest. Formally, we have:

Definition 7: Let $n_{c,i}$ be the number of consistent demands of bargainer i in D_i and $n_{r,i}^{(\lambda)}$ be the number of remaining consistent demands of bargainer i after the λ-th round of bargaining. A function $f_i^{(\lambda)}$ is the regret degree function of bargainer i after λ-th round of bargaining if it satisfies:

(i) if $n_{r,i}^{(\lambda)} \geqslant n_{r,i}^{(\lambda')}$ then $f_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \leqslant f_i^{(\lambda)}(n_{r,i}^{(\lambda')})$;
(ii) $\forall n_{r,i}^{(\lambda)}$, $f_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \geqslant f_i^{(\lambda)}(n_{c,i})$; and
(iii) $\forall n_{r,i}^{(\lambda)}$, $f_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \leqslant f_i^{(\lambda)}(0)$.

Theorem 2: The following formula

$$\vartheta_i^{(\lambda)}(n_{r,i}^{(\lambda)}) = \frac{n_{c,i} - n_{r,i}^{(\lambda)}}{n_{c,i}}, \tag{5}$$

is a regret degree of bargainer i after the λ-th round.

Proof: (i) If $n_{r,i}^{(\lambda)} \geqslant n_{r,i}^{(\lambda')}$ then $n_{c,i} - n_{r,i}^{(\lambda)} \leqslant n_{c,i} - n_{r,i}^{(\lambda')}$. Hence, $\vartheta_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \leqslant \vartheta_i^{(\lambda)}(n_{r,i}^{(\lambda')})$. (ii) Because $n_{r,i}^{(\lambda)} \leqslant n_{c,i}$, according to (i), $\vartheta_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \geqslant \vartheta_i^{(\lambda)}(n_{c,i})$. (iii) Because $n_{r,i}^{(\lambda)} \geqslant 0$, according to (i), $\vartheta_i^{(\lambda)}(n_{r,i}^{(\lambda)}) \leqslant \vartheta_i^{(\lambda)}(0)$. □

2) Patience descent degree (ρ): It can be calculated in the following three ways:

$$\rho_i(\lambda) = \frac{\lambda}{|D_i|}, \tag{6}$$

$$\rho_i(\lambda) = \frac{\sqrt{\lambda(2|D_i| - \lambda)}}{|D_i|}, \tag{7}$$

$$\rho_i(\lambda) = 1 - \frac{\sqrt{|D_i|^2 - \lambda^2}}{|D_i|}, \tag{8}$$

where λ is the number of completed rounds of bargaining and D_i is bargainer i's demand set. The difference among formulas (6)-(8) is the descent rate of patience: formula (6) reflects that a bargainer's patience declines in a constant speed during a bargaining; formula (7) reflects that a bargainer's patience declines swiftly first and then slows down; during a bargaining; and formula (8) reflects the reverse situation that a bargainer's patience declines slowly and speeds up during a bargaining.

Formulas (6)-(8) are reasonable to calculate patience descent degrees of bargainers. In Longman English Dictionary, *patience* is defined as "(1) the ability to continue waiting or doing something for a long time without becoming angry or anxious; and (2) the ability to accept trouble and other people's annoying behaviour without complaining or becoming angry". So, if we use *patience descent degree (ρ)* to represent how much patience of a bargainer will be affected after every round of a bargaining game, it should reflect: (i) the more rounds completed, the less patience a bargainer has; (ii) at the beginning of a bargaining, a bargainer is the most patient; and (iii) at the end of a bargaining, a bargainer is the most impatient. Formally, we have:

Definition 8: A function f_i is the patience descent degree function of bargainer i if it satisfies:

(i) $\forall \lambda, \omega \leqslant |D_i|$, if $\lambda \leqslant \omega$ then $f_i(\lambda) \leqslant f_i(\omega)$;
(ii) $\forall \lambda \leqslant |D_i|$, $f_i(\lambda) \geqslant f_i(0)$; and
(iii) $\forall \lambda \leqslant |D_i|$, $f_i(\lambda) \leqslant f_i(|D_i|)$,

where $|D_i|$ is the number of bargainer i's demands.

The following theorem shows that all the three axioms in the above definition are indeed met by formulas (6)-(8).

Theorem 3: Formulas (6)-(8) are appropriate for using as patience descent degree functions for bargainer i.

Proof: If $\lambda \leqslant \omega$ then $\frac{\lambda}{|D_i|} \leqslant \frac{\omega}{|D_i|}$, i.e., $\rho_i(\lambda) \leqslant \rho_i(\omega)$. Because for all λ, $0 \leqslant \lambda \leqslant |D_i|$, hence $\rho_i(0) \leqslant \rho_i(\lambda) \leqslant \rho_i(|D_i|)$. Similarly, the other two formulas can be proved. □

3) Initial risk degree (γ): In Longman English Dictionary, *risk* is defined as "the possibility that something bad, unpleasant, or dangerous may happen". So, intuitively we know: (i) if a bargainer has the highest risk attitude, he will put all the conflicting demands at the top level of his preference hierarchy because by the simultaneous concession in our bargaining process, he may get most of his conflicting demands if his opponent is a risk-averser, but he may get the bargaining broken if his opponent is risk-seeking; (ii) on the contrary, he can show his lowest risk attitude when he puts all his conflicting demands in the lowest level of his initial dynamic preference hierarchy; (iii) if he does not change the preference, it means he is risk neutral; (iv) if a bargainer moves up one of his conflicting demands but keeps others unchanged, then he shows a higher risk degree; and (v) if a bargainer moves down one of his conflicting demands but keeps others unchanged, then he shows a lower risk degree. Formally we have:

Definition 9: $\forall d \in D_i$, let $l_i(d)$ and $l_i^{(1)}(d)$ denote the level of d in the original preference hierarchy and the initial dynamic preference hierarchy, respectively. A function f_i is the initial risk degree function of bargainer i if it satisfies:

(i) if $\forall d_{i,j}^{\pm} \in D_i^{\pm}$, $l_i^{(1)'}(d_{i,j}^{\pm}) = 1$, then $\forall l_i^{(1)} \neq l_i^{(1)'}$, $f_i(l_i^{(1)'}) \geqslant f_i(l_i^{(1)})$;
(ii) if $\forall d_{i,j}^{\pm} \in D_i^{\pm}$, $l_i^{(1)'}(d_{i,j}^{\pm}) = H_i$, then $\forall l_i^{(1)} \neq l_i^{(1)'}$ $f_i(l_i^{(1)'}) \leqslant f_i(l_i^{(1)})$;
(iii) if $\forall d_{i,j}^{\pm} \in D_i^{\pm}$, $l_i^{(1)}(d_{i,j}^{\pm}) = l_i(d_{i,j}^{\pm})$, $\max\{f_i\} \neq 0$ and $\min\{f_i\} \neq 0$, then $f_i(l_i^{(1)}) = \frac{\max\{f_i\}+\min\{f_i\}}{2}$;
(iv) if $\exists d_{i,j}'^{\pm} \in D_i^{\pm}$, $l_i^{(1)}(d_{i,j}'^{\pm}) < l_i^{(1)'}(d_{i,j}'^{\pm})$, $\forall d_{i,j}^{\pm} \in D_i^{\pm}$, $d_{i,j}^{\pm} \neq d_{i,j}'^{\pm}$, $l_i^{(1)}(d_{i,j}^{\pm}) = l_i^{(1)'}(d_{i,j}^{\pm})$, then $f_i(l_i^{(1)}) > f_i(l_i^{(1)'})$; and

(v) if $\exists d'^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)}_i(d'^{\pm}_{i,j}) > l^{(1)'}_i(d'^{\pm}_{i,j})$, *and* $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $d^{\pm}_{i,j} \neq d'^{\pm}_{i,j}$, $l^{(1)}_i(d^{\pm}_{i,j}) = l^{(1)'}_i(d^{\pm}_{i,j})$, *then* $f_i(l^{(1)}_i) < f_i(l^{(1)'}_i)$.

Theorem 4: The following formula

$$\gamma_i(l^{(1)}_i) = \begin{cases} \frac{\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j}))}{\left|\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} l_i(d^{\pm}_{i,j}) - \left|D^{\pm}_i\right|\right|} \\ \quad \text{if } \sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j})) > 0, \\ \frac{\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j}))}{\left|\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} l_i(d^{\pm}_{i,j}) - \left|D^{\pm}_i\right| H_i\right|} \\ \quad \text{if } \sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j})) < 0, \\ 0 \quad \text{otherwise,} \end{cases} \tag{9}$$

where $D^{\pm}_i$ is the conflicting demand set of bargainer i in D_i. is an initial risk degree function of bargainer i.

Proof: Let $D^{\pm}_i = \{d^{\pm}_{i,1}, \ldots, d^{\pm}_{i,|D^{\pm}_i|}\}$.

(i) If $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l_i(d^{\pm}_{i,j}) = 1$, then $\max\{\gamma_i\} = 0$ and if $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)'}_i(d^{\pm}_{i,j}) = 1$, then $\gamma_i(l^{(1)'}_i) = 0$, so $\forall l^{(1)}_i \neq l^{(1)'}_i$, $\gamma_i(l^{(1)'}_i) \geqslant \gamma_i(l^{(1)}_i)$; otherwise, $\max\{\gamma_i\} = 1$. If $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)'}_i(d^{\pm}_{i,j}) = 1$, then

$$\gamma_i(l^{(1)'}_i) = \frac{(l_i(d^{\pm}_{i,1}) - 1) + \cdots + (l_i(d^{\pm}_{i,|D^{\pm}_i|}) - 1)}{\left|\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} l_i(d^{\pm}_{i,j}) - \left|D^{\pm}_i\right|\right|} = 1.$$

So, $\forall l^{(1)}_i \neq l^{(1)'}_i$, $\gamma_i(l^{(1)'}_i) \geqslant \gamma_i(l^{(1)}_i)$.

(ii) If $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l_i(d^{\pm}_{i,j}) = H_i$, then $\min\{\gamma_i\} = 0$ and if $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)'}_i(d^{\pm}_{i,j}) = H_i$, then $\gamma_i(l^{(1)'}_i) = 0$, so $\forall l^{(1)}_i \neq l^{(1)'}_i$, $\gamma_i(l^{(1)'}_i) \leqslant \gamma_i(l^{(1)}_i)$; otherwise, $\min\{\gamma_i\} = -1$. If $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)'}_i(d^{\pm}_{i,j}) = H_i$, then

$$\gamma_i(l^{(1)'}_i) = \frac{(l_i(d^{\pm}_{i,1}) - H_i) + \cdots + (l_i(d^{\pm}_{i,|D^{\pm}_i|}) - H_i)}{\left|\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} l_i(d^{\pm}_{i,j}) - \left|D^{\pm}_i\right| H_i\right|} = -1.$$

So, $\forall l^{(1)}_i \neq l^{(1)'}_i$, $\gamma_i(l^{(1)'}_i) \leqslant \gamma_i(l^{(1)}_i)$.

(iii) By formular (9), if $\max\{\gamma\} \neq 0$ and $\min\{\gamma\} \neq 0$, then $\max\{\gamma\} = 1$ and $\min\{\gamma\} = -1$, then $\frac{\max\{\gamma_i\} + \min\{\gamma_i\}}{2} = 0$, and if $\forall d^{\pm}_{i,j} \in D^{\pm}_i$, $l^{(1)}_i(d^{\pm}_{i,j}) = l_i(d^{\pm}_{i,j})$, then $\gamma(l^{(1)}_i) = 0$. So, $\gamma(l^{(1)}_i) = \frac{\max\{\gamma_i\} + \min\{\gamma_i\}}{2}$.

(iv) If $\exists d'^{\pm}_{i,j} \in D^{\pm}_i$ such that $l^{(1)}_i(d'^{\pm}_{i,j}) < l^{(1)'}_i(d'^{\pm}_{i,j})$ and $\forall d^{\pm}_{i,j} \in D^{\pm}_i$ such that $d^{\pm}_{i,j} \neq d'^{\pm}_{i,j}$, $l^{(1)}_i(d^{\pm}_{i,j}) = l^{(1)'}_i(d^{\pm}_{i,j})$, then $\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j})) > \sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)'}_i(d^{\pm}_{i,j}))$, by formular (9), we have $\gamma_i(l^{(1)}_i) > \gamma_i(l^{(1)'}_i)$.

(v) If $\exists d'^{\pm}_{i,j} \in D^{\pm}_i$ such that $l^{(1)}_i(d^{\pm'}_{i,j}) > l^{(1)'}_i(d^{\pm'}_{i,j})$ and $\forall d^{\pm}_{i,j} \in D^{\pm}_i$ such that $d^{\pm}_{i,j} \neq d'^{\pm}_{i,j}$, $l^{(1)}_i(d^{\pm}_{i,j}) = l^{(1)'}_i(d^{\pm}_{i,j})$, then $\sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)}_i(d^{\pm}_{i,j})) < \sum_{d^{\pm}_{i,j} \in D^{\pm}_i} (l_i(d^{\pm}_{i,j}) - l^{(1)'}_i(d^{\pm}_{i,j}))$, by formular (9), we have $\gamma_i(l^{(1)}_i) < \gamma_i(l^{(1)'}_i)$. □

TABLE I. FUZZY RULES

If *regret degree* is *Low* then *change degree* is *Low*.
If *regret degree* is *Medium* then *change degree* is *Medium*.
If *regret degree* is *High* then *change degree* is *High*.
If *patience descent degree* is *Low* then *change degree* is *Low*.
If *patience descent degree* is *Medium* then *change degree* is *Medium*.
If *patience descent degree* is *High* then *change degree* is *High*.
If *initial risk degree* is *Low* then *change degree* is *High*.
If *initial risk degree* is *Medium* then *change degree* is *Medium*.
If *initial risk degree* is *High* then *change degree* is *Low*.

B. Fuzzy linguistic terms of fuzzy variables

The meanings of these parameters' linguistic terms are as follows. The *low* regret degree indicates that a bargainer just regrets a little for the demands given up in the previous round. The *medium* regret degree means that a bargainer regrets for the demands given up in the previous round. And the *high* regret degree means that a bargainer regrets very much for the demands given up in the previous round and more likely changes the preference ordering because it causes many consistent demands lost. Similarly, we can understand the linguistic terms of the other two parameters.

These linguistic terms can be modelled by fuzzy membership function:

$$\mu(x) = \begin{cases} 0 & \text{if } x \leqslant a, \\ \frac{x-a}{b-a} & \text{if } a \leqslant x \leqslant b, \\ 1 & \text{if } b \leqslant x \leqslant c, \\ \frac{d-x}{d-c} & \text{if } c \leqslant x \leqslant d, \\ 0 & \text{if } x \geqslant d. \end{cases} \tag{10}$$

For convenience, we denote formula (10) as $\mu(x)$=(a, b, c, d). Thus, the linguistic terms of regret degrees can be represented as $\mu_{low\ \vartheta}(\vartheta)$=(−0.2, 0, 0.2, 0.4), $\mu_{medium\ \vartheta}(\vartheta)$=(0.2, 0.4, 0.6, 0.8), and $\mu_{high\ \vartheta}(\vartheta)$=(0.6, 0.8, 1, 1.2). Similarly, we can have $\mu_{low\ \rho}(\rho)$=(−0.2, 0, 0.2, 0.4), $\mu_{medium\ \rho}(\rho)$ = (0.2, 0.4, 0.6, 0.8), and $\mu_{high\ \rho}(\rho)$=(0.6, 0.8, 1, 1.2); $\mu_{low\ \gamma}(\gamma)$=(−1.4, −1, −0.6, −0.2), $\mu_{medium\ \gamma}(\gamma)$=(−0.6, −0.2, 0.2, 0.6), and $\mu_{high\ \gamma}(\gamma)$ = (0.2, 0.6, 1, 1.4); and $\mu_{low\ \zeta}(\zeta)$ = (−0.2, 0, 0.2, 0.4), $\mu_{medium\ \zeta}(\zeta)$ = (0.2, 0.4, 0.6, 0.8), and $\mu_{high\ \zeta}(\zeta)$ = (0.6, 0.8, 1, 1.2).

C. Psychological experiment

We calculate a change degree from a bargainer's *regret degree*, *patience descent degree*, and *initial risk degree* by the fuzzy rules as shown in Table I. Rule 1 means that if a bargainer does not lose too many consistent demands, which makes him regret just a little, then his desire to change his preference ordering is low. Similarly, we can understand other rules. The relations between the rules' inputs and output are shown in the left column of Figure 1.

These fuzzy rules are established by a psychological survey study with 40 human subjects. Empirically, 30 is the minimal sample size required to conduct such a statistical analysis, while more than 50 is pointless [2]. So, it is reasonable to choose 40 (18 females and 22 males). They ranged in age from 19 to 40, and varied in careers and educational levels. All the subjects volunteered to participate and completed the questionnaires, which consists of the following four parts:

(i) *Risk Orientation Questionnaire*. It uses 12 items to assess individuals' risk propensity and cautiousness [3]. That

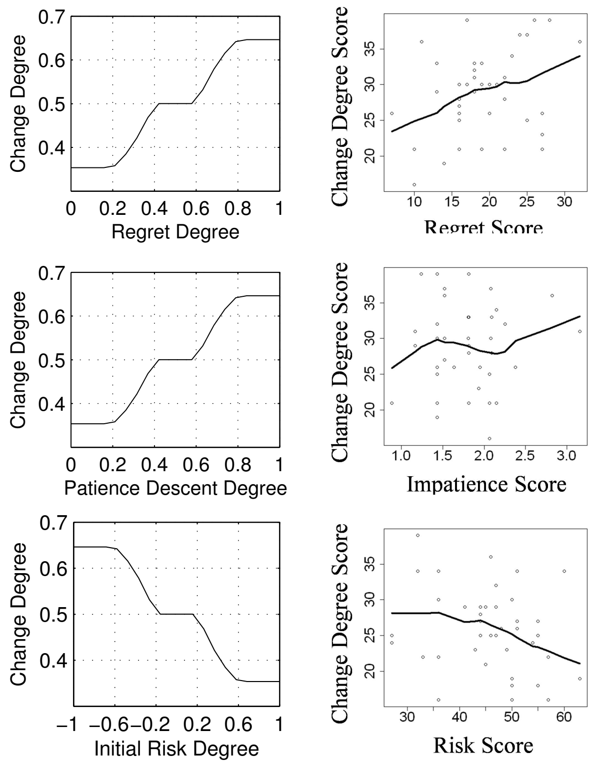

Fig. 1. The relations between the preference change degree and the three parameters in our fuzzy logic system (the first column) and in psychological experiments (the second column)

is, to choose an appropriate number, in-between 1 and 7 (1 means totally disagree and 7 means totally agree), to express how much a subject agrees with the following sentences: 1) I am very careful when making and implementing a plan. 2) My motto is "Nothing ventured, nothing gained". 3) I do not like making a risky decision. 4) As long as a task is very interesting, regardless of whether or not I am able to conduct it very well, I will try it. 5) I do not like to take a risk at the cost of what I have, I would rather sit safely in everything. 6) Even though I know it was not a full grasp, I still decided to gamble. 7) I often give myself smaller goals at work, so I can more easily achieve them. 8) Even though most people disagree with me, I will still air my own ideas. 9) I make decisions always after careful thinking. 10) I sometimes like to do things for others to show my ability although there will be the risk of error. 11) I often imagine the negative consequences of my actions. 12) I would rather take a great risk in order to succeed.

(ii) *Regret Scale*. It consists of 5 items designed to assess how the subjects deal with decision situations after the decision has been made, specifically the extent to which they experience regret [4]. That is, to choose a number, in-between 1 and 7 (1 means totally disagree and 7 means totally agree), to express how much a subject agrees with the following sentences: 1) Once I made a decision, I would not regret it. 2) Whenever after making a decision, I would like to know what would happen if I choose another. 3) When I find that other options can bring better results, I still feel very frustrated although the outcomes brought by the current selection is also good. 4) I will always think of the opportunities missed when I am thinking how well I live now. 5) I always gather the information of other options when I have to make a decision.

TABLE II. Regression analysis results. Here β is the standardised regression coefficient; $S.E.$ is the standard error of the estimate; and p is the significant level of the t- test.

	β	S.E.	t value	p
Intercept	-12.38	6.59	-1.88	0.07
Regret degree	0.36	0.17	2.12	0.04
Impatience	1.18	2.16	0.55	0.59
Risk degree	-0.17	0.10	1.66	0.10

(iii) *Delay-discounting rate*. It assesses a subject's patience level by offering a subject a series of choices between immediate but less rewards and larger but delayed rewards as follows [5]: 1) \$30 now vs. \$85 14 days later; 2) \$40 now vs. \$55 25 days later; 3) \$67 now vs. \$85 35 days later; 4) \$34 now vs. \$35 43 days later; 5) \$15 now vs. \$35 10 days later; 6) \$32 now vs. \$55 20 days later; 7) \$83 now vs. \$85 35 days later; 8) \$21 now vs. \$30 75 days later; 9) \$48 now vs. \$55 45 days later; 10) \$40 now vs. \$65 70 days later; 11) \$25 now vs. \$35 25 days later; 12) \$65 now vs. \$75 50 days later; 13) \$24 now vs. \$55 10 days later; 14) \$30 now vs. \$35 20 days later; 15) \$53 now vs. \$55 50 days later; 16) \$47 now vs. \$60 50 days later; 17) \$40 now vs. \$70 20 days later; 18) \$50 now vs. \$80 70 days later; 19) \$45 now vs. \$70 35 days later; 20) \$27 now vs. \$30 35 days later; 21) \$16 now vs. \$30 35 days later.

(iv) *Maximisation Scale Short* [6]. It uses 6 items to assess individuals' tendency to optimise decisions. And people with more tendencies to optimise their decision would less likely change their original decisions in our bargaining game scenario. That is, to choose an appropriate number, in-between 1 and 7 (1 means totally disagree and 7 means totally agree), to express how much a subject agrees with the following sentences: 1) No matter how satisfied I am with my current job, I am always looking for better opportunities. 2) No matter what I do, I would finish it in the highest standard. 3) When I am watching TV, even though I am now quite satisfied with the current programme, I am still searching for other channels to see whether or not there is a better one. 4) Shopping is very difficult for me because I always try to find the most appropriate things for me. 5) I am never satisfied with the second best choice. 6) I always think it is very difficult for me to help a friend pick a gift in a shop.

Multiple regression analysis is conducted to test the effect of the risk attitude, the regret degree and the patience level on how individuals approach their decision. The analysis results are reported in Table II. The regret degree is significantly relevant to the tendency to change their decisions (i.e., β =0.36 and p=0.04). Those, who experience more regret after the decision has been made, are more likely to change their decisions. Risk attitude is marginally related to the change degree (i.e., β =-0.17 and p=0.10). Those, who prefer a higher level of risk, tend to insist on their original decisions. The patience level is also positively relevant to the change degree (i.e., β =1.18 and p=0.59).

As shown in the right column of Figure 1, according to experiment results, we draw three scatter plots for ζ's change with the regret degree, the patience descent level, and the risk attitude, respectively. The curve was superimposed on each scatter plot using the scatter smoother function $lowess()$ of MASS package in the R system for statistical analysis.

D. Fuzzy inference method

We employ standard fuzzy inference method [7], [8].

The following definition is about the implication of the Mamdani method [8].

Definition 10: Let A_i be a Boolean combination of fuzzy sets $A_{i,1}, \cdots, A_{i,m}$, where $A_{i,j}$ is a fuzzy set defined on $U_{i,j}$ ($i = 1, \cdots, n; j = 1, \cdots, m$), and B_i be a fuzzy set on U' ($i = 1, \cdots, n$). Then when the inputs are $\mu_{A_{i,1}}(u_{i,1}), \cdots, \mu_{A_{i,m}}(u_{i,m})$, the output of such fuzzy rule $A_i \to B_i$ is fuzzy set B'_i defined as follows: $\forall u' \in U'$,

$$\mu_i(u') = \min\{f(\mu_{A_{i,1}}(u_{i,1}), \cdots, \mu_{A_{i,m}}(u_{i,m})), \mu_{B_i}(u')\}, \qquad (11)$$

where f is obtained through replacing $A_{i,j}$ in A_i by $\mu_{i,j}(u_{i,j})$ and replacing "and", "or", "not" in A_i by "min", "max", "$1-\mu$", respectively. And the output of all rules $A_1 \to B_1, \cdots, A_n \to B_n$, is fuzzy set M, which is defined as:

$$\forall u' \in U', \mu_M(u') = \max\{\mu_1(u'), \cdots, \mu_n(u')\}. \qquad (12)$$

The result that we get is still a fuzzy set. To defuzzify the fuzzy set, we need the following centroid method [8]:

Definition 11: The centroid point u_{cen} of fuzzy set M given by formula (12) is:

$$u_{cen} = \frac{\int_{U'} u' \mu_M(u') du'}{\int_{U'} \mu_M(u') du'}. \qquad (13)$$

Actually, u_{cen} is the centroid of the area covered by the curve of membership function μ_M and the horizontal ordinate.

IV. Properties

This section will reveal some properties of our model.

A. The influence of regret, patience and risk

In this subsection, we will discuss how a bargainer's psychological factors of regret, patience and risk influence the preference change degrees according to the fuzzy rules.

Theorem 5: Suppose after a bargaining round, a bargainer has regret degree ϑ, patience descent degree ρ, and initial risk degree γ, and thus gets the corresponding change degree ζ through our FLS. Then:

(i) If $\vartheta \geqslant 0.8$ then $\forall \rho \in [0, 1], \gamma \in [-1, 1]$, $\zeta \geqslant 0.5$; and if $\vartheta \leqslant 0.2$ then $\forall \rho \in [0, 1], \gamma \in [-1, 1]$, $\zeta \leqslant 0.5$.

(ii) If $\rho \geqslant 0.8$ then $\forall r_1 \in [0, 1], \gamma \in [-1, 1]$, $\zeta \geqslant 0.5$; and if $\rho \leqslant 0.2$ then $\forall \vartheta \in [0, 1], \gamma \in [-1, 1]$, $\zeta \leqslant 0.5$.

(iii) If $\gamma \geqslant 0.6$ then $\forall \vartheta \in [0, 1], \rho \in [0, 1]$, $\zeta \leqslant 0.5$; and if $\gamma \leqslant -0.6$ then $\forall \vartheta \in [0, 1], \rho \in [0, 1]$, $\zeta \geqslant 0.5$.

Proof: (i) If $\vartheta \in [0.8, 1]$, by the definitions of $\mu_{low\,\vartheta}$, $\mu_{medium\,\vartheta}$, and $\mu_{high\,\vartheta}$, we can get $\mu_{low\,\vartheta}(\vartheta) = \mu_{medium\,\vartheta}(\vartheta) = 0$ and $\mu_{high\,\vartheta}(\vartheta) = 1$. By formula (11), the outputs of 1st-3rd rules are $\mu_1(\zeta) = 0$, $\mu_2(\zeta) = 0$, and $\mu_3(\zeta) = \mu_{high\,\zeta}(\zeta)$, respectively. Now we want to find out the minimum of μ_{cen}. Because of $\rho \in [0, 1]$ and $\gamma \in [-1, 1]$, when the assignment of ρ or γ changes, the shape of $\mu_M(\zeta)$ may change. More specifically, by formulas (12) and (13) we have the situations as follows: In the case of $\rho \in [0, 0.2]$ and $\gamma \in [0.6, 1]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.4, \\ 0 & \text{if } 0.4 \leqslant \zeta \leqslant 0.6, \\ 5\zeta - 3 & \text{if } 0.6 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[0.6,1]} = 0.5.$$

In the case of $\rho \in [0, 0.2]$ and $\gamma \in [0.4, 0.6]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.1 + 0.5\gamma, \\ 1.5 - 2.5\gamma & \text{if } 0.1 + 0.5\gamma \leqslant \zeta \leqslant 0.9 - 0.5\gamma, \\ 5\zeta - 3 & \text{if } 0.9 - 0.5\gamma \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[0.4,0.6]} = 0.5.$$

In the case of $\rho \in [0, 0.2]$ and $\gamma \in [0.2, 0.4]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.3, \\ 5\zeta - 1 & \text{if } 0.3 \leqslant \zeta \leqslant 0.5 - 0.5\gamma, \\ 1.5 - 2.5\gamma & \text{if } 0.5 - 0.5\gamma \leqslant \zeta \leqslant 0.5 + 0.5\gamma, \\ 4 - 5\zeta & \text{if } 0.5 + 0.5\gamma \leqslant \zeta \leqslant 0.7, \\ 5\zeta - 3 & \text{if } 0.7 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[0.2,0.4]} = 0.5.$$

In the case of $\rho \in [0, 0.2]$ and $\gamma \in [-0.2, 0.2]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.3, \\ 5\zeta - 1 & \text{if } 0.3 \leqslant \zeta \leqslant 0.4, \\ 1 & \text{if } 0.4 \leqslant \zeta \leqslant 0.6, \\ 4 - 5\zeta & \text{if } 0.6 \leqslant \zeta \leqslant 0.7, \\ 5\zeta - 3 & \text{if } 0.7 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[-0.2,0.2]} = 0.5.$$

In the case $\rho \in [0, 0.2]$ and $\gamma \in [-0.4, -0.2]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.3, \\ 5\zeta - 1 & \text{if } 0.3 \leqslant \zeta \leqslant 0.5 + 0.5\gamma, \\ 1.5 + 2.5\gamma & \text{if } 0.5 + 0.5\gamma \leqslant \zeta \leqslant 0.5 - 0.5\gamma, \\ 4 - 5\zeta & \text{if } 0.5 - 0.5\gamma \leqslant \zeta \leqslant 0.7, \\ 5\zeta - 3 & \text{if } 0.7 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[-0.4,-0.2]} = 0.5.$$

In the case $\rho \in [0, 0.2]$ and $\gamma \in [-0.6, -0.4]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.1 - 0.5\gamma, \\ 1.5 + 2.5\gamma & \text{if } 0.1 - 0.5\gamma \leqslant \zeta \leqslant 0.9 + 0.5\gamma, \\ 5\zeta - 3 & \text{if } 0.9 + 0.5\gamma \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[-0.6,-0.4]} = 0.5.$$

In the case of $\rho \in [0, 0.2]$ and $\gamma \in [-1, -0.6]$, we have

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.4, \\ 0 & \text{if } 0.4 \leqslant \zeta \leqslant 0.6, \\ 5 - 3\zeta & \text{if } 0.6 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } 0.8 \leqslant \zeta \leqslant 1. \end{cases}$$
$$u_{cen}^{\vartheta\in[0.8,1],\rho\in[0,0.2],\gamma\in[-1,-0.6]} = 0.5.$$

Similarly, we discuss the other situations when ρ is in [0.2,0.3], [0.3,0.4], [0.4,0.6], [0.6,0.7], [0.7,0.8], and [0.8,1], respectively. And finally we can get a conclusion that when $\rho \in [0, 0.2]$ or $\gamma \in [0.6, 1]$, $\mu_{cen} = 0.5$ is the maximum. So, if $\vartheta \geqslant 0.8$ then $\forall \rho \in [0, 1], \gamma \in [-1, 1]$, $\zeta \geqslant 0.5$.

If $\vartheta \in [0, 0.2]$, by the definitions of $\mu_{low,\vartheta}$, $\mu_{medium\,\vartheta}$, and $\mu_{high\,\vartheta}$, we can get $\mu_{medium\,\vartheta}(\vartheta) = \mu_{high\,\vartheta}(\vartheta) = 0$ and $\mu_{low\,\vartheta}(\vartheta) = 1$. By formula (11), the outputs of the 1st-3rd rules are $\mu_1(\zeta) = \mu_{low\,\zeta}(\zeta)$, $\mu_2(\zeta) = 0$, and $\mu_3(\zeta) = 0$, respectively. By formulas (12) and (13) as well as the other 6 rules, similar to the above discussion, we know when $\rho \in [0.8, 1]$ or $\gamma \in [-1, -0.6]$, μ_{cen} is the maximum. We choose an appropriate situation where $\rho = 1$ and $\gamma = -1$ to calculate the maximum value. In this situation, we have:

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.4, \\ 0 & \text{if } 0.4 \leqslant \zeta \leqslant 0.6, \\ 5\zeta - 3 & \text{if } 0.6 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } \zeta \geqslant 0.8. \end{cases}$$

And by formula (13), we have:

$$u_{cen,max}^{\vartheta\in[0,0.2]} = 0.5.$$

So, if $\vartheta \leqslant 0.2$ then $\forall \rho \in [0, 1], \gamma \in [-1, 1]$, $\zeta \leqslant 0.5$.

Similarly, we can prove properties (ii) and (iii) of this theorem. □

This theorem states that when a parameter is higher or lower than a certain threshold, the change degree can be controlled in a certain range (higher or lower than a mid-value, i.e., 0.5 in our fuzzy system). This is in accord with our intuitions, i.e., when a bargainer regrets his preference changing extremely, even though he is patient and risk-seeking, very likely he is unwilling to insist on his original preference.

B. Agreement Existence

Now we discuss the agreement existence of bargaining games. In the discussion of this subsection, we use formulas (5), (6), and (9) as the regret degree, patience descent degree and initial risk degree functions, respectively.

Firstly, the following theorem states that no matter how different personalities the bargainers possess, if they have at least two demands in common, they can reach an agreement.

Theorem 6: In a bargaining game G, if $\forall i \in N_i$, $\exists\ d_{i,1}, d_{i,2} \notin D_i^{\pm}$ such that $l^{(d)}(d_{i,1}) \neq l^{(d)}(d_{i,2})$ then $A(G) \neq \emptyset$.

Proof: Firstly, similar to the discussion in the proof of Theorem 5, we can prove that when $\vartheta = 0.3$, $\rho \in [0, 0.2]$ and $\gamma \in [0.6, 1]$, μ_{cen} is the minimum. We choose an appropriate situation where $\vartheta = 0.3$, $\rho = 0$ and $\gamma = 1$ to calculate the minimum value. In this situation, by formulas (12) and (13), we have:

$$\mu_M(\zeta) = \begin{cases} 1 & \text{if } 0 \leqslant \zeta \leqslant 0.2, \\ 2 - 5\zeta & \text{if } 0.2 \leqslant \zeta \leqslant 0.3, \\ 0.5 & \text{if } 0.3 \leqslant \zeta \leqslant 0.7, \\ 4 - 5\zeta & \text{if } 0.7 \leqslant \zeta \leqslant 0.8, \\ 1 & \text{if } \zeta \geqslant 0.8; \end{cases}$$
$$u_{cen,min}^{\vartheta\in[0.3,1]} = 0.31 > 0.3.$$

So, if regret degree $\vartheta \geqslant 0.3$, no matter what the patience descent degree and the initial risk degree are, the corresponding preference change degree is not less than 0.3.

Secondly, we prove the theorem by using the above conclusion. Suppose the bargaining game has no agreements. Then by Definition 6, there does not exist a $\underline{\lambda}$ such that $\forall i \in N, D_i^{(\underline{\lambda})} \neq \emptyset, \underline{\lambda} < |D|_{min}$, where $|D|_{min}$ is the minimum of demand amount among all bargainers' demand sets. That is, before the end of the bargaining process, there is at least a bargainer who has at least one demand inconsistent with each other. However, this situation is impossible in our assumption because when the bargaining game continues to the above situation, at least one bargainer has to give up all his consistent demands. Nevertheless, let us consider the situation where a bargainer has given up $(m - 1)$ consistent demands (m is the total number of his consistent demands). By the formula of calculating regret degree (i.e., formula (5)), we know regret degree ϑ of the bargainer in this round is $\frac{m-1}{m}$. Since $\min\{\frac{m-1}{m} \mid m \in \mathbb{N}\} = \frac{1}{2} \geqslant 0.3$, we have $\vartheta \geqslant 0.3$. Hence, we know the corresponding change degree $\zeta \geqslant 0.3$. So, by action function (1), $\forall i \in N_i$, if $\exists d_i^{\pm} \in D_i^{\pm}$ such that $l^{(d)}(d_i^{\pm}) = 1$, demand $d_i^{\pm}$ will be downgraded and the left consistent demand will take the place after preference updating. So, it is impossible that in the end of the bargaining process there is at least a bargainer who has at least a demand inconsistent with others'. Hence, $A(G) \neq \emptyset$. □

The following theorem states that no matter how different personalities the bargainers own, if they have at least one demand in common and one of them is not at their low levels of preference hierarchies, but in the middle or high levels, then an agreement can be reached finally.

Theorem 7: In a bargaining game G, if $\forall i \in N_i$, $\exists\ d_i \notin D_i^{\pm}$ such that $\left|\{d_j \in D_i^{\pm} \mid l^{(d)}(d_j) < l^{(d)}(d_i)\}\right| > \lceil\frac{|D|_i}{3}\rceil$, then $A(G) \neq \emptyset$.

Proof: Similarly to Theorem 6, we can prove that if $\rho \geqslant 0.3$ then $\forall r_1 \in [0, 1], \gamma \in [-1, 1]$, $\zeta \geqslant 0.3$. Suppose the bargaining game has no agreements. Then by Definition 6, there does not exist a $\underline{\lambda}$ such that $\forall i \in N, D_i^{(\underline{\lambda})} \neq \emptyset, \underline{\lambda} < |D|_{min}$, where $|D|_{min}$ is the minimum of demand amount among all bargainers' demand sets. That is, before the end of the bargaining process, there is at least a bargainer who has at least one demand inconsistent with each other. However, this situation is impossible in our assumption because when the bargaining game continues to round $\lceil\frac{|D|_i}{3}\rceil$, $\rho_i(\lceil\frac{|D|_i}{3}\rceil) = \frac{\lceil\frac{|D|_i}{3}\rceil}{|D|_i} \geqslant 0.3$. Thus, by the above inference the corresponding change degree ζ will be not less than 0.3. So, by action function (1), $\forall i \in N_i$, if $\exists d_i^{\pm} \in D_i^{\pm}$ such that $l^{(d)}(d_i^{\pm}) = 1$, the demand $d_i^{\pm}$ will be downgraded and the left consistent demands will take the place after preference updating. So, it is impossible that in the last round of the bargaining game there is at least a bargainer who has at least one demand inconsistent with others', i.e., $\nexists i \in N, \exists d_i \in D_i^{(\lambda)}, \exists j \neq i, d_i \wedge D_j^{(\lambda)} \vdash \perp$. Hence, $A(G) \neq \emptyset$. □

V. Related work

In some bargaining or negotiation systems, fuzzy rules are also used to, for example, evaluate offers [9], [10] and analyse opponent's bargaining strategies [11]. However, in these systems fuzzy rules are not used to change the bargainers' preferences during bargaining like what we do in this paper.

Zhang [12] introduces a bargaining model, in which the preference ordering of a bargainer is defined on his demand set. Jing et al. [13] further extend the model by putting integrity constraints into consideration, and thus bargainers' preference orderings are restricted by integrity constraints. However, in both models, the problem of risk attitude modelling is similar: when a bargainer ranks a conflicting demand very high in his preference, it does not always mean that he is very risky, but maybe he just prefers the demand less.

Zhan et al [14] address the problem of risk attitude modelling in [12], [13]. However, its bargaining process has a serious problem. Like that in [12], in [14], when a bargainer makes concession, he gives up all the demands in the same level. For example, if a bargainer has 100 demands in a same preference hierarchy while another just has one in same hierarchy, then when making concession of this hierarchy, the first one has to give up 100 demands, but the second just needs to give up one demand. It is unfair and unrealistic. Rather, in our model, every bargainer just gives up one demand in a bargaining round, which is more practical. In addition, [14] does not reveal the details of the psychological experiment of eliciting the fuzzy rules, while we present the detail in this paper. Moreover, [14] does not give the axioms that their agreement concept and their fuzzy rules' input parameters calculation should satisfy, while we do all these. Finally, we reveal some insights into our model, but [14] does not.

Vo and Li [15] also build an axiomatic bargaining model, in which the bargaining situation is described in logic language and the preference over outcomes is ordinal. Their solution satisfies the axioms of fairness, un-biasedness and unanimously efficient (stronger than Pareto Efficiency). However, unlike our model, their model does not reflect the bargainers' risk attitudes and patience, which are very important factors for bargaining in real life; and their preference cannot change during a bargaining process, either.

VI. Conclusion

This paper introduces a new fuzzy rule based bargaining model and its agreement concept to deal with the problems of bargaining multi-demand in discrete domains. Moreover, we axiomatically characterise our agreement concept as well as the calculation of our fuzzy rules' input parameters. We also detail the psychological experiment that is used to establish our fuzzy rules. In addition, by theoretic analyses, we reveal: (i) our model and its agreement conception reflect well how human psychological characteristics about risk, patience and regret influence their preference change during a bargaining; and (ii) under which conditions the agreement of such a bargaining problem can be reached. Many could be done in the future. For example, it is interesting to integrate more concession strategies in continuous domains (e.g., those proposed in [16]) into bargaining models in discrete domains.

Acknowledgment

This paper is supported by MOE Project of Key Research Institute of Humanities and Social Sciences at Universities (No. 13JJD720017) China, Bairen Plan, Major Project Raising Program of Sun Yat-sen University (No. 1309089), China National Natural Science Foundation (No. 61173019), Major Projects of the Ministry of Education (No. 10JZD0006) China, and China National Social Science Fund of Major Projects (13&ZD186).

References

[1] J. F. Nash Jr, "The bargaining problem," *Econometrica: Journal of the Econometric Society*, pp. 155–162, 1950.

[2] X. Luo, N. R. Jennings, and N. Shadbolt, "Acquiring user tradeoff strategies and preferences for negotiating agents: A default-then-adjust method," *International Journal of Man-Machine Studies*, vol. 64, no. 4, pp. 304–321, 2006.

[3] B. Rohrmann, "Risk attitude scales: Concepts and questionnaires," University of Melbourne, Project Report, 2004.

[4] B. Schwartz, A. Ward, J. Monterosso, S. Lyubomirsky, K. White, and D. R. Lehman, "Maximizing versus satisficing: Happiness is a matter of choice," *Journal of Personality and Social Psychology*, vol. 83, no. 5, pp. 1178–1197, 2002.

[5] K. N. Kirby and N. N. Maraković, "Delay-discounting probabilistic rewards: Rates decrease as amounts increase," *Psychonomic Bulletin and Review*, vol. 3, no. 1, pp. 100–104, 1996.

[6] G. Y. Nenkov, M. Morrin, B. Schwartz, A. Ward, and J. Hulland, "A short form of the maximization scale: Factor structure, reliability and validity studies," *Judgment and Decision Making*, vol. 3, no. 5, pp. 371–388, 2008.

[7] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[8] E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller," *International Journal of Man-Machine Studies*, vol. 7, no. 1, pp. 1–13, 1975.

[9] K. Kolomvatsos, C. Anagnostopoulos, and S. Hadjiefthymiades, "A fuzzy logic system for bargaining in information markets," *ACM Transactions on Intelligent Systems and Technology*, vol. 3, no. 2, pp. 32:1–32:26, 2012.

[10] B. Zuo and Y. Sun, "Fuzzy logic to support bilateral agent negotiation in e-commerce," in *2009 International Conference on Artificial Intelligence and Computational Intelligence*, vol. 4, 2009, pp. 179–183.

[11] F. Costantino and G. D. Gravio, "Multistage bilateral bargaining model with incomplete information—A fuzzy approach," *International Journal of Production Economics*, vol. 117, no. 2, pp. 235–243, 2009.

[12] D. Zhang, "A logic-based axiomatic model of bargaining," *Artificial Intelligence*, vol. 174, no. 16-17, pp. 1307–1322, 2010.

[13] X. Jing, D. Zhang, and X. Luo, "A logical framework of bargaining with integrity constraints," in *AI 2013: Advances in Artificial Intelligence*, ser. Lecture Notes in Computer Science. Springer, 2013, vol. 8272, pp. 1–13.

[14] J. Zhan, X. Luo, K. M. Sim, C. Feng, and Y. Zhang, "A fuzzy logic based model of a bargaining game," in *Knowledge Science, Engineering and Management*, ser. Lecture Notes in Artificial Intelligence, 2013, vol. 8041, pp. 387–403.

[15] V. N. Q. Bao and M. Li, "From axiomatic to strategic models of bargaining with logical beliefs and goals," in *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems*, vol. 1, 2012, pp. 525–532.

[16] L. Pan, X. Luo, M. Meng, C. Maio, M. He, and X. Guo, "A two-stage win-win multiattribute negotiation model: Optimazation and then concession," *Computational Intelligence*, vol. 29, no. 3, pp. 577– 626, 2013.