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A Novel Adaptive Fuzzy Control for a Class of Discrete-Time Nonlinear Systems in Strict-Feedback Form

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Abstract—**In this paper, a backstepping based adaptive fuzzy control algorithem is presented for a class of uncertain nonlinear discrete-time systems in the strict-feedback form. By introducing the "minimal learning parameter (MLP)" technique, the proposed scheme is able to circumvent the problem of "curse of dimension" for high-dimensional systems. Meanwhile, all the virtual control laws and actual control law in the system are updated by a novel actual adaptive update law, thus the number of parameters updated online for whole system is only by one. Takagi-Sugeno (T-S) fuzzy systems are used to approximate the unknown system functions. It is shown via Lyapunov theory that all signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB). Finally, a simulation example is employed to illustrate the effectiveness and advantages of the proposed scheme.**

Keywords—**discrete-time nonlinear systems; adaptive fuzzy control; minimal learning parameter (MLP); strict-feedback system;**

I. Introduction

In the past decades, adaptive fuzzy control schemes have been found to be particularly useful for the control of nonlinear uncertain systems with unknown nonlinear functions since the excellent universal approximation ability of the fuzzy logic systems (FLS) is proven [1]. Recently, with the help of FLS approximation, much significant development of adaptive fuzzy control algorithm has been achieved in [2-5] for uncertain nonlinear systems.

However, the problem of "explosion of learning param -eters" which is the well-known "curse of dimensionality" exists in aforementioned control design methods. That is, the number of parameters to be tuned online in the adaptive fuzzy control schemes is very large, especially for high dimensional systems. Led to the online learning time tends to become unacceptably large when implemented, which is not acceptable in real application. This problem has been pointed out in [6] and first solved by Yang et al. in their pioneering works [7-10], where some kinds of so called "minimal learning parameter (MLP)" algorithms containing much less online adaptive parameters were constructed. In this years, combine with the backstepping technique, the idea of MLP algorithms was extended to adaptive-fuzzy-control schemes for both SISO time-delay systems with unknown virtual control coefficients in [11], and MIMO uncertain systems in [12-13]. More recently, by incorporating the "dynamic surface control (DSC)" and "MLP" techniques, the robust adaptive tracking control schemes is developed for a class of uncertain nonlinear systems [14-16], which both problem of "curse of dimension" and "explosion of complexity" inherent in conventional backstepping methods are circumvented.

It is common knowledge that the discrete-time systems rather than the continuous-time systems are the closest for decribing a real plant. However, in contrast to above continuous-time systems, control design for discrete-time systems is more difficult due to lack of mathematical tools. For instance, noncausal problem will be encountered if we directly apply backstepping design to discrete-time systems in lower triangular form, since discrete-time systems are described by difference equations, which involve state variables at different instants. To solve the noncausal problem, for discrete-time systems transformable to the parametric-strict-feedback and the parametric-pure-feedback form, the noncausal problem was elegantly solved in backstepping using a time-varying mapping [17], which was further extended to cases with time-varying parameters and nonparametric uncertainties in [18]. But it is't simpler to extend this technique to more general systems. a class of discrete-time nonlinear system transformation using prediction functions of future states was studied in [19-20], in which adaptive NN backstepping design has been applied to the transformed strict-feedback discrete-time systems without noncausal problem. Subsequently, many elegant adaptive control schemes are studied in [21-30] for discrete-time nonlinear systems based on the approximation property of the neural network.

Similarly to continuous-time systems, it is obvious that the problem of too many adaptive parameters are needed to be tuned for discrete-time nonlinear systems is also exist in the above adaptive control approachs [17-30]. More recently, several certain results have been achieved to reduce the number of the adjustable parameters and lighten the online computation burden are studied in [31-36]. However, for above control approaches, these works are still suffer from the problem of "curse of dimensionality", eg: the number of parameters updated online is needed at last one parameter for each subsystems.

In this paper, motivated by aforementioned works in literature, an adaptive fuzzy control scheme, in which the "minimal learning parameters (MLP)" technique is introduced, is proposed for a class of uncertain discrete-time

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nonlinear systems in strict-feedback form. The problem of "curse of dimensionality" for high-dimensional systems is avoided by proposed scheme, i.e., the adaptive mechanism with minimal learning parameterization is achieved, no matter how many rules are used in fuzzy systems and how many input variables exist in the system. In the meantime, all the virtual control laws and actual control law in the system are updated by only a novel actual adaptive update law, which is proposed by merging all virtual update laws with actual update law. By this approach, the number of parameters updated online for proposed adaptive fuzzy controller is reduced to only one. Takagi-Sugeno (T-S) fuzzy systems are usd to approximate the un known system functions. By using the Lyapunov analysis method, all the signals in the closed-loop system are guaranteed to be SGUUB, and the tracking error converges to a small neighborhood of the origin. One simulation example is utilized to illustrate the effectiveness and advantages of the proposed scheme.

II. Problem Formulation and Proliminaries

A. Problem Formulation

Consider the following single-input and single-output (SISO) discrete-time nonlinear system in strict-feedback form

$$\begin{cases} x_i(k+1) = f_i(\bar{x}_i(k)) + g_i(\bar{x}_i(k))x_{i+1}(k), \\ i = 1,2,\dots,n-1 \\ x_n(k+1) = f_n(\bar{x}_n(k)) + g_n(\bar{x}_n(k))u(k) \\ y_k = x_1(k) \end{cases} \tag{1}$$

where $\bar{x}_i(k) = [x_1(k), x_2(k), \dots, x_i(k)]^T \in R^i$, $i = 1,2 \dots n$, $u(k) \in R$ and $y_k \in R$ are the state variables, system input and output respectively; $f_i(\bar{x}_i(k))$ and $g_i(\bar{x}_i(k))$, $i = 1,2,\dots,n$ are unknow smooth functions.

The control objective is to design an adaptive NN controller for system (1) such that: (i) all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) and (ii) the system output follows the desired reference signal $y_d(k)$.

In the following , it needs to make the following assumptions based on the systems (1).

Assumption 1. the desired reference signal $y_d(k) \in \Omega_y$, $\forall k > 0$ is smooth and known, where $\Omega_y := \{\chi | \chi = x_1\}$.

Assumption 2. the signal of $g_i(\bar{x}_i(k))$, $i = 1,2,\dots,n$ are known and there exist constants $\underline{g_i} > 0$ and $\bar{g}_i > 0$ such that $\underline{g_i} \le |g_i(\bar{x}_i(k))| \le \bar{g}_i, \forall \bar{x}_n(k) \in \Omega \subset R^n$.

Without losing generality, we shall assume that $g_i(\bar{x}_i(k))$ and $g_n(\bar{x}_n(k))$ are positive in this paper. i.e., it is assumed that $\underline{g_i} \le g_i(\bar{x}_i(k)) \le \bar{g}_i$.

Definition 1. [19] the solution of (1) is semi-globally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of R^n and all $\bar{x}_n(k_0) \in \Omega$, there exist an $\varepsilon > 0$ and a number $N(\varepsilon, \bar{x}_n(k_0))$ such that $\|\bar{x}_n(k)\| < \varepsilon$ for all $k \ge k_0 + N$.

B. Takagi-Sugeno (T-S) Fuzzy Logic Systems

Here, we birefly describe the structure of T-S type fuzzy logic systems (FLS). Generally, the fuzzy system can be constructed by the following K ($K > 1$) fuzzy rules:

R_i: IF x_1 is $\Psi^i_{h_1}$ AND x_2 is $\Psi^i_{h_2}$ … AND x_n is $\Psi^i_{h_n}$

THEN y_i is $\Omega^i_{h_1,h_2\dots h_n}$, $i = 1,2,\dots,K$,

where $\Omega^i_{h_1,h_2\dots h_n}$ denotes an output fuzzy set. If $\Omega^i_{h_1,h_2\dots h_n}$ is a singleton fuzzy set, its membership function is 1 only at $y_i = \sigma_i$ (an arbitrary unknown constant) and 0 at other position, then it is called Mamdani-type fuzzy system. If $\Omega^i_{h_1,h_2\dots h_n}$ is a function of $a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n$ with a_{ij}, $i = 1,2,\dots,K$ and $j = 1,2,\dots,n$ being constants, then it is called Takagi-Sugeno (T-S) type fuzzy system. The product fuzzy inference is employed to evaluate the ANDs in the fuzzy rules. After being defuzzified by a typical center-average defuzzifier, the output of T-S fuzzy system is in the vector form

$$\hat{f}(x, A_x) = \xi(x) A_x x \tag{2}$$

where $\xi(x) = [\xi_1(x), \xi_2(x), \dots, \xi_K(x)]$ and $\xi_i(x) = \prod_{j=1}^{n} \mu^i_{h_j}(x_j) \Big/ \sum_{i=1}^{K} [\prod_{j=1}^{n} \mu^i_{h_j}(x_j)]$, are called fuzzy basis functions, $\mu^i_{h_j}$ are the membership functions corresponding to the antecedents $\Psi^i_{h_j}$, $i = 1,2,\dots,K$, and

$$A_x = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{K1} & a_{K2} & \cdots & a_{Kn} \end{bmatrix}$$

It has been proven that a T-S fuzzy model is a universal approximator of any smooth nonlinear system on a compact set [37],[38], which is stated as follows.

Lemma 1: For any given real continuous function $f(x)$ on a compact set $U \in R^n$ and $\forall \varepsilon > 0$, there exists a fuzzy system $\hat{f}(x, A_x)$ in the form (2) such that

$$\sup_{x \in U} \left\| f(x) - \hat{f}(x, A_x) \right\| \le \varepsilon \tag{3}$$

where ε is called the approximation error and has an upper bound ε^*.

Remark 1: As pointed out in Remark 2 of [16], for any

n-dimensional continuous function $f(x)$, if N_i+1 input fuzzy sets for each variable x_i are used, these will be $\prod_{i=1}^{n}(N_i+1)$ IF-THEN fuzzy rules in the fuzzy system. It implies that there will be a total of $\prod_{i=1}^{n}(N_i+1)$ parameters to be updated online in the Mamdani-type fuzzy system, and $n\prod_{i=1}^{n}(N_i+1)$ parameters to be updated in the T-S type fuzzy system. On the contrary, only one parameter needs to be updated online in the scheme to be proposed in this paper.

III. Adaptive Control Design and Stability Analysis

Consider the strict-feedback SISO nonlinear discrete-time system described in (1). The control objective of this paper is formulated as follows. For a known and bounded signal $y_d(k)$, design an adaptive control $u(k)$ for system (1) which makes system output y_k follow the desired signal $y_d(k)$, while maintaining all closed-loop signals SGUUB.

The causality contradiction is one of the major problems that we will encounter in discrete-time domain when we construct a controller for the general strict-feedback nonlinear system through backstepping. However, the above problem can be avoied if we transform the system equation into a special form which is suitable for backstepping design. If we consider the original system (1) as a one-step ahead predictor, we can transform it into an equivalent maximum n-step ahead predictor, which can predict the future states $x_1(k+n)$, $x_2(k+n-1)$, ..., $x_n(k+1)$, then the causality contradiction is avoid when controller is constructed based on the maximum n-step ahead prediction [19]. Similarly, with the help of the transformation process in [19], system (1) is equivalent to

$$\begin{aligned} &x_1(k+n)=F_1(\bar{x}_n(k))+G_1(\bar{x}_n(k))x_2(k+n-1),\\ &\quad\vdots\\ &x_{n-1}(k+2)=F_{n-1}(\bar{x}_n(k))+G_{n-1}(\bar{x}_n(k))x_n(k+1),\\ &x_n(k+1)=f_n(\bar{x}_n(k))+g_n(\bar{x}_n(k))u(k),\\ &y_k=x_1(k)\end{aligned} \tag{4}$$

where $F_i(\bar{x}_n(k))$ and $G_i(\bar{x}_n(k))$ are unknown functions. The above equations show that the functions $F_i(\bar{x}_n(k))$, $i=1,\ldots,n-1$ are highly nonlinear. The problem of causality contradiction does not appear in system (4). It is obvious that $G_i(\bar{x}_n(k))$ satisfies $\underline{g}_i\le G_i(\bar{x}_n(k))\le\bar{g}_i$, $\forall\bar{x}_n(k)\in\Omega$ under Assumption 1.

For convenience of analysis and discussion, for $i=1,\ldots,n-1$, let

$$F_i(k)=F_i(\bar{x}_n(k)),\quad G_i(k)=G_i(\bar{x}_n(k)),$$
$$f_n(k)=f_n(\bar{x}_n(k)),\quad g_n(k)=g_n(\bar{x}_n(k)),$$

for $i=1,2,\ldots,n$, let $\xi_i(k)=\xi_i(\bar{x}_n(k))$,

$$\psi_i(k)=\psi_i(\bar{x}_n(k)),\ \Phi_i(k)=\Phi_i(\bar{x}_n(k)),$$

they are functions of system states $\bar{x}_n(k)$ at the k th step.

Step 1: For $\eta_1(k)=x_1(k)-y_d(k)$, its n th difference is given by

$$\begin{aligned}\eta_1(k+n)&=x_1(k+n)-y_d(k+n)\\ &=F_1(k)+G_1(k)x_2(k+n-1)-y_d(k+n)\end{aligned} \tag{5}$$

By viewing $\alpha_2^*(k)=x_2(k+n-1)$ as a virtual control input for (1), it is obvious that $\eta_1(k+n)=0$ if we choose

$$x_2(k+n-1)=\alpha_2^*(k)=-\frac{1}{G_1(k)}\left[F_1(k)-y_d(k+n)\right] \tag{6}$$

since $F_1(k)$ and $G_1(k)$ are unknown function, they are not available for constructing a virtual control $\alpha_2^*(k)$. However, according to Lemma 1, a suitable T-S fuzzy system $\hat{f}(x,A_x)$ with input vector $\bar{x}_n\in U_{\bar{x}_n}$, where $U_{\bar{x}_n}$ is a compact set, is proposed here to approximate unknown function with A_n being a matrix containing unknown contants. Then, we can use T-S fuzzy to approximate $\alpha_2^*(k)$ as follows:

$$\begin{aligned}\alpha_2^*(k)&=b_1\xi_1(k)\omega_1+\xi_1(k)A_1y_d(k+n)+\varepsilon_1\\ &=b_1\xi_1(k)\omega_1+v_1\end{aligned} \tag{7}$$

where ε_1 is the approximation error. Let $b_1=\|A_1\|$, $A_1^m=A_1/b_1$ and $\omega_1=A_1^m\eta_1(k+n)$. here, $v_1=\xi_1(k)A_1y_d(k+n)+\varepsilon_1$, by noticing the bound of ε_1, one has

$$\|v_1\|\le\|\xi_1(k)A_1y_d(k+n)+\varepsilon_1\|\le g_{\min}\theta_1^T\psi_1(k) \tag{8}$$

where $\theta_1^T=g_{\min}^{-1}\max\left(\|A_1y_d(k+n)\|,\|\varepsilon_1\|\right)$, and $\psi_1(k)=1+\|\xi_1(k)\|$. It is clear that $\|v_1\|$ is bounded because θ_1 is bounded due to the boundedness of $y_d(k+n)$ and ε_1.

Therefore, $\alpha_2^*(k)$ can be expressed as

$$\alpha_2^*(k)=\lambda(k)\Phi_1(k) \tag{9}$$

where $\Phi_1(k)=\frac{1}{4r_1^2}\|\xi_1(k)\|^2+\frac{1}{4l_1^2}\psi_1^2(k)$, r_1 and l_1 are positive design constants. the parameter $\lambda(k)$ will be given at the last step.

Letting $\hat{\lambda}$ be the estimate of λ, choose the virtual controller

$$\begin{aligned}x_2(k+n-1)&=\alpha_2(k)+\eta_2(k+n-1)\\ &=\hat{\lambda}(k)\Phi_1(k)+\eta_2(k+n-1)\end{aligned} \tag{10}$$

Substituting virtual control (10) into (5), the error equation (5) is re-written as

$$\eta_1(k+n) = F_1(k) + G_1(k)[\hat{\lambda}(k)\Phi_1(k) + \eta_2(k+n-1)] - y_d(k+n) \tag{11}$$

Adding and subtracting $G_1(k)\alpha_2^*(k)$ on the right-hand side of (11), we have

$$\eta_1(k+n) = F_1(k) - y_d(k+n) + G_1(k)[\hat{\lambda}(k)\Phi_1(k) + \eta_2(k+n-1) - \lambda(k)\Phi_1(k)] + G_1(k)\alpha_2^*(k) \tag{12}$$

Substituting (6) into (12) leads to

$$\eta_1(k+n) = G_1(k)[\tilde{\lambda}(k)\Phi_1(k) + \eta_2(k+n-1)] \tag{13}$$

Step i *:* For $\eta_i(k) = x_i(k) - \alpha_i(k_{i-1})$. Its $(n-i+1)$ th difference is given by

$$\begin{aligned}\eta_i(k+n-i+1) &= x_i(k+n-i+1) - \alpha_i(k) \\ &= F_i(k) + G_i(k)x_{i+1}(k+n-i) \\ &\quad - \alpha_i(k)\end{aligned} \tag{14}$$

Similarly, consider $\alpha_{i+1}^*(k) = x_{i+1}(k+n-i)$ as a virtual control for (14). It is obvious that $\eta_i(k+n-i+1) = 0$ is ture when we choose

$$x_{i+1}(k+n-i) = \alpha_{i+1}^*(k) = -\frac{1}{G_i(k)}[F_i(k) - \alpha_i(k)] \tag{15}$$

Similarly, $\alpha_{i+1}^*(k)$ can be approximated by T-S fuzzy system

$$\alpha_i^*(k+n-i) = b_i\xi_i(k)\omega_i + v_i \tag{16}$$

where ε_i is the approximation error. Let $b_i = \|A_i\|$, $A_i^m = A_i/b_i$ and $\omega_i = A_i^m\overline{\eta}_i(k)$. Here, $v_i = \xi_i(k)A_i^1 y_d(k+n) + \sum_{h=2}^{i}\xi_i A_i^h \alpha_h(k) + \varepsilon_i$, by noticing the bound of ε_i, one has

$$\|v_i\| \le g_{\min}\theta_i^T\psi_i(k) \tag{17}$$

where $\theta_i^T = g_{\min}^{-1}\max\left(\|A_i^1 y_d(k+n)\|, \ldots, \|A_i^i\alpha_i(k)\|, \|\varepsilon_i\|\right)$, and $\psi_i(k) = 1 + i\|\xi_i(k)\|$. It is clear that $\|v_i\|$ is bounded because θ_i is bounded due to the boundedness of $y_d(k+n)$, $\cdots, \alpha_i(k)$ and ε_i.

Therefore, $\alpha_{i+1}^*(k)$ can be expressed as

$$\alpha_{i+1}^*(k) = \lambda(k)\Phi_i(k) \tag{18}$$

where $\Phi_i(k) = \frac{1}{4r_i^2}\|\xi_i(k)\|^2 + \frac{1}{4l_i^2}\psi_i^2(k)$. r_i and l_i are positive design constants.

choose the virtual controller as follows:

$$\begin{aligned}x_{i+1}(k+n-i) &= \alpha_{i+1}(k) + \eta_{i+1}(k+n-i) \\ &= \hat{\lambda}(k)\Phi_i(k) + \eta_{i+1}(k+n-i)\end{aligned} \tag{19}$$

then we obtain the i th step error equation

$$\eta_i(k+n-i+1) = G_i(k)[\tilde{\lambda}(k)\Phi_i(k) + \eta_{i+1}(k+n-i)] \tag{20}$$

Step n *:* For $\eta_n(k) = x_n(k) - \alpha_n(k-1)$, its first difference is given by

$$\begin{aligned}\eta_n(k+1) &= x_n(k+1) - \alpha_n(k) \\ &= f_n(k) + g_n(k)u(k) - \alpha_n(k)\end{aligned} \tag{21}$$

It is obvious that $\eta_n(k+1) = 0$, if we choose

$$u(k) = u^*(k) = -\frac{1}{g_n(k)}[f_n(k) - \alpha_n(k)] \tag{22}$$

Similarly, $u^*(k)$ can be approximated by T-S fuzzy system

$$\begin{aligned}u^*(k) &= b_n\xi_n(k)\omega_n + \xi_n(k)A_n^1 y_d(k+n) \\ &\quad + \sum_{h=2}^{n}\xi_n(k)A_n^h\alpha_n(k) + \varepsilon_n \\ &= b_n\xi_n(k)\omega_n + v_n\end{aligned} \tag{23}$$

where ε_n is the approximation error. Let $b_n = \|A_n\|$, $A_n^m = A_n/b_n$ and $\omega_n = A_n^m\overline{\eta}_n(k)$. Here, $v_n = \xi_n(k)A_n^1 y_d(k+n) + \sum_{h=2}^{n}\xi_n(k)A_n^h\alpha_n(k) + \varepsilon_n$, by noticing the bound of ε_n, one has

$$\|v_n\| \le g_{\min}\theta_n^T\psi_n(k) \tag{24}$$

where $\theta_n^T = g_{\min}^{-1}\max\left(\|A_n^1 y_d(k+n)\|, \ldots, \|A_n^n\alpha_n(k)\|, \|\varepsilon_n\|\right)$, and $\psi_n(k) = 1 + n\|\xi_n(k)\|$. It is clear that $\|v_n\|$ is bounded because θ_n is bounded due to the boundedness of $y_d(k+n)$, $\ldots, \alpha_n(k)$ and ε_n.

Therefore, $u^*(k)$ can be expressed as

$$u^*(k) = \lambda(k)\Phi_n(k) \tag{25}$$

where $\lambda = g_{\min}^{-1}\max(b^2, \theta^2)$, $b = \max(b_1, b_2, \ldots, b_n)$, $\Phi_n(k) = \frac{1}{4r_n^2}\|\xi_n(k)\|^2 + \frac{1}{4l_n^2}\psi_n^2(k)$ and $\theta = \max(\theta_1, \theta_2, \ldots, \theta_n)$. r_n and l_n are positive design constants.

choose the virtual controller and the update law as follows:

$$u(k) = \hat{\lambda}(k)\Phi_n(k) \tag{26}$$

$$\hat{\lambda}(k+1) = \hat{\lambda}(k) - \Gamma[\sum_{i=1}^{n}\Phi_i(k)\eta_i(k+1) + \sigma\hat{\lambda}(k)] \tag{27}$$

for the n th step error equation

$$\eta_n(k+1) = g_n(k)\tilde{\lambda}(k)\Phi_n(k) \tag{28}$$

Theorem 1: The closed-loop adaptive system consisting of plant (1), controller (26) and update law (27) is SGUUB and has an equilibrium at $\eta = [\eta_1, \eta_2, \ldots, \eta_n]^T = 0$, if $\overline{x}_n(0)$ is initialized in Ω. This guarantees that all the signals include the states $\bar{x}_n = [x_1, x_2, \ldots, x_n]$, the control u and the design parameter $\hat{\lambda}_i$, $i = 1,2,\ldots,n$ are SGUUB, subsequently,

$$\lim_{k\to\infty} |y_k - y_d(k)| \le \varepsilon$$

where ε is a small positive number.

Consider the Lyapunov function candidate as follows:

$$V(k) = \sum_{i=1}^{n} \frac{1}{\bar{g}_i} \eta_i^2(k) + \tilde{\lambda}^2(k)\Gamma^{-1} \tag{29}$$

Noting $\tilde{\lambda}(k)\Phi_i(k) = \dfrac{\eta_i(k+1)}{G_i(k)} - \eta_{i+1}(k), i = 1,2,\ldots,$ $n-1$ and $\tilde{\lambda}(k)\Phi_n(k) = \dfrac{\eta_n(k+1)}{g_n(k)}$. The first difference of (39) along (27) and (28) is given by

$$\begin{aligned}
\Delta V &= \sum_{i=1}^{n} \frac{1}{\bar{g}_i}[\eta_i^2(k+1) - \eta_i^2(k)] \\
&\quad + \tilde{\lambda}^2(k+1)\Gamma^{-1} - \tilde{\lambda}^2(k)\Gamma^{-1} \\
&= \sum_{i=1}^{n} \frac{1}{\bar{g}_i}[\eta_i^2(k+1) - \eta_i^2(k)] \\
&\quad - 2\tilde{\lambda}(k)[\sum_{i=1}^{n} \Phi_i(k)\eta_i(k+1)\sigma\hat{\lambda}(k)] \\
&\quad + \Gamma[\sum_{i=1}^{n} \Phi_i(k)\eta_i(k+1) + \sigma\hat{\lambda}(k)]^2 \\
&= \sum_{i=1}^{n} -\frac{1}{\bar{g}_i}[\eta_i^2(k+1) + \eta_i^2(k)] \\
&\quad + \sum_{i=1}^{n-1} 2\eta_i(k+1)\eta_{i+1}(k) \\
&\quad - 2\delta\bar{\lambda}(k)\hat{\lambda}(k) + \sum_{i=1}^{n} \Gamma\Phi_i^2(k)\eta_i^2(k+1) \\
&\quad + \sum_{i=1}^{n-1} 2\Phi_i(k)\Phi_{i+1}(k)\eta_i(k+1)\eta_{i+1}(k+1) \\
&\quad + \sum_{i=1}^{n-2} 2\Phi_i(k)\Phi_{i+2}(k)\eta_i(k+1)\eta_{i+2}(k+1) \\
&\quad + \ldots + 2\Phi_1(k)\Phi_n(k) \times \eta_1(k+1)\eta_n(k+1) \\
&\quad + \sum_{i=1}^{n} 2\sigma\hat{\lambda}(k)\Gamma\Phi_i(k)\eta_i(k+1) + \sigma^2\Gamma\hat{\lambda}^2(k)
\end{aligned}$$

Using the facts that

$$\begin{aligned}
&\Phi^2(k) < K, \\
&\Gamma\Phi^2(k) < \bar{\gamma}K, \\
&2\sigma\hat{\lambda}(k)\Gamma\Phi(k)\eta_i(k+1) \le \frac{\bar{\gamma}K\eta_i^2(k+1)}{\bar{g}_i} + \bar{g}_i\sigma^2\bar{\gamma}\hat{\lambda}^2(k), \\
&2\tilde{\lambda}(k)\hat{\lambda}(k) = \tilde{\lambda}^2(k) + \hat{\lambda}^2(k) - \lambda^2, \quad i = 1,2,\ldots,n \\
&2\eta_i(k+1)\eta_{i+1}(k) \le \frac{\bar{\gamma}\eta_i^2(k+1)}{\bar{g}_i} + \frac{\bar{g}_i\eta_{i+1}^2(k)}{\bar{\gamma}}, i = 1,2,\ldots,n-1 \\
&2\eta_i(k+1)\eta_{i+1}(k+1) \le \eta_i^2(k+1) + \eta_{i+1}^2(k+1)
\end{aligned}$$

we obtain

$$\begin{aligned}
\Delta V \le &\sum_{i=1}^{n}[-\frac{\rho}{\bar{g}_i}\eta_i^2(k+1) - \frac{1}{\bar{g}_i}\eta_i^2(k)] + \beta + \sum_{i=1}^{n-1}\frac{\bar{g}_i\eta_{i+1}^2(k)}{\bar{\gamma}} \\
&- \sigma(1 - \sigma\bar{\gamma} - \sum_{i=1}^{n}\bar{g}_i\sigma\bar{\gamma})\hat{\lambda}^2(k)
\end{aligned}$$

where $\rho = 1 - \bar{\gamma} - \bar{\gamma}K - n\bar{g}_i\bar{\gamma}K$, $\beta = \sigma\lambda^2$.

If we choose the design parameters as follows:

$$\bar{\gamma} < \frac{1}{1 + K + \sum_{i=1}^{n} n\bar{g}_i K}, \ \sigma < \frac{1}{(1 + \sum_{i=1}^{n}\bar{g}_i)\bar{\gamma}} \tag{31}$$

It is obvious that $\Delta V \le 0$ once $|\eta_n(k)| > \sqrt{\bar{g}_n\beta}$. This implies the boundedness of $V(k)$ for all $k \ge 0$, which leads to the boundedness of the tracking error $\eta_n(k)$ and will converge to the compact set denoted by $\Omega_\eta \subset R$, where $\Omega_\eta := \left\{\chi \middle| \chi \le \sqrt{\bar{g}_n\beta}\right\}$. Form the boundedness of $\eta_n(k)$, the boundedness of the extra term $\dfrac{\bar{g}_i\eta_{i+1}^2(k)}{\bar{\gamma}}, i = 1,2,\ldots\ n-1$ is readily obtaine. Then it can be seen from the above design procedures that $\eta_i(k)$ and $x_i(k)$ are bounded, $i = 1,2,\ldots,n-1$.

The adaptation dynamics (27) can be written as

$$\begin{aligned}
\tilde{\lambda}(k+1) &= \tilde{\lambda}(k) - \Gamma[\textstyle\sum_{i=1}^{n}\Phi_i(k)\eta_i(k+1) \\
&\qquad + \sigma\tilde{\lambda}(k) + \sigma\lambda] \\
&= B(k)\tilde{\lambda}(k) - \textstyle\sum_{i=1}^{n-1}\Gamma\Phi_i(k)G_i(k)\eta_{i+1}(k) \\
&\qquad - \sigma\Gamma\lambda
\end{aligned} \tag{32}$$

where

$B = 1 - \sigma\Gamma - \sum_{i=1}^{n-1}\Gamma\Phi_i^2(k)G_i(k) - \Gamma g_n(k)\Phi_n^2(k)$,

function $G_i(k)$ and $g_n(k)$ are bounded form Assumption 2, and the boundedness of $\eta_n(k)$ is proved in above. Similar to the proof in [22], $\tilde{\lambda}(k)$ is bounded in a compact set denoted by $\Omega_{\lambda 1}$, and hence the boundedness of $\hat{\lambda}(k)$ is assured.

Based on the procedure above, we can conclude that $\bar{x}_n(k+1) \in \Omega$ and $u(k)$ are bounded if $\bar{x}_n(k) \in \Omega$. Finally, if we initialize $\bar{x}_n(0) \in \Omega$, and choose the design parameters according to (31), there exsits a k^*, such that all errors asymptotically converge to Ω_n. This implies that the closed-loop system is SGUUB. Then $\bar{x}_n(k) \in \Omega$ and $\hat{\lambda}$ will hold for all $k > 0$.

IV. Simulation Examples

In this section, the effectiveness and merits of the proposed scheme are demonstrated by considering the following two second-order uncertain strict-feedback nonlinear systems:

In simulation, define five fuzzy sets, which are characterized by the following membership functions:

$\mu_{A_{hi}^1} = \exp[-(x+1)^2]$, $\mu_{A_{hi}^2} = \exp[-(x+0.5)^2]$,

$\mu_{A_{hi}^3} = \exp[-x^2]$, $\mu_{A_{hi}^4} = \exp[-(x-0.5)^2]$,

$\mu_{A_{hi}^5} = \exp[-(x-1)^2]$.

The discrete-time SISO plant described by

$$\begin{aligned} x_1(k+1) &= f_1(x_1(k)) + 0.3x_2(k), \\ x_2(k+1) &= f_2(\bar{x}_2(k)) + u(k), \\ y_k &= x_1(k), \end{aligned} \tag{33}$$

where $f_1(x_1(k)) = \dfrac{x_1^2(k)}{1+x_1^2(k)}$,

$$f_2(\bar{x}_2(k)) = \frac{x_1(k)}{1+x_1^2(k)+x_2^2(k)}.$$

It can be checked that Assumption 1 and 2 are satisfied. The tracking objective is to make the output y_k following a desired reference signal:

$$y_d(k) = \sin(k\pi/30)/2 + \sin(k\pi/20)/2.$$

The initial condition for system state is $x(0) = [0.4\ 0]^T$ and the adaptive laws are $\hat{\lambda}(0) = 0.1$. Other controller parameters are $\Gamma = 0.1$, $\sigma = 0.01$, $r_1 = r_2 = 0.25$, $l_1 = l_2 = 0.25$. The simulation results are presented in Fig.1, 2 and 3.

From Fig.1, we can see that the better tracking performance is obtained. Fig.2 illustrates the trajectories of the systems' actual control and the virtual control. The estimation of parameter is shown in the Fig.3. it can be observed from the simulation results that they are bounded.

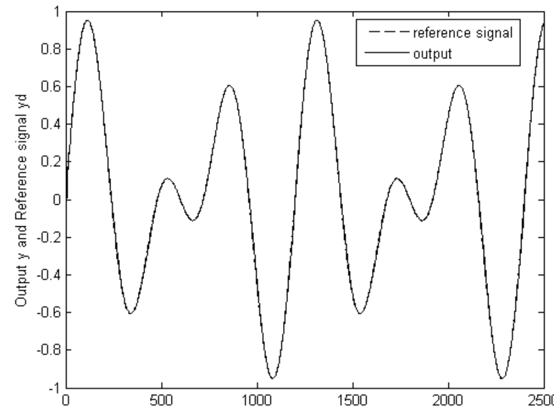

Fig. 1 Tracking performance of Example 1

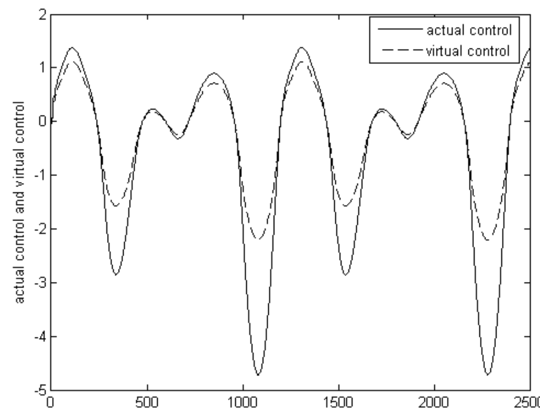

Fig. 2 Actual and virtual control of Example 1

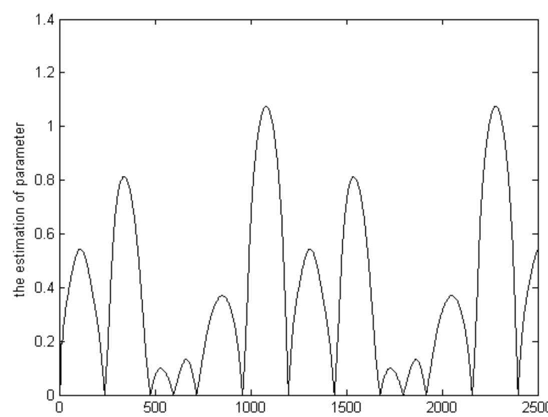

Fig. 3 The parameter estimation of Example 1

V. Conclusion

In this paper, the adaptive fuzzy control problem has considered for a class of strict-feedback uncertain discrete-time nonlinear systems. by incorporating the MLP technique into the controller design procedures, a fuzzy logic systems based adaptive control algorithm has been developed. The main feature of the proposed scheme is that the adaptive mechanism with minimal learning parameterizations is achieved, i.e., the number of parameters updated online for whole system is reduced to only one. The computation load of proposed adaptive fuzzy controller is reduced and the learning time tends to much shorter, thus, this algorithm is much easier to be implemented in applications. It is shown that the closed-loop system is SGUUB via the Lyapunov theory. One simulation example has been presented to demonstrate the performance and the effectiveness of the proposed algorithm.

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