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Uncertain Nonlinear Time Delay Systems Fast and Large Disturbance Rejection Based on Adaptive Interval Type-2 Fuzzy PI control

Tsung-Chih Lin[1] Chien-Liang Chen[1]
[1]Department of Electronic Engineering, Feng-Chia University, Taichung, Taiwan

ABSTRACT

In this paper, adaptive interval type-2 fuzzy proportional integral (PI) control scheme to attenuate fast and large disturbance for a class of uncertain nonlinear time delay systems is proposed. By incorporating adaptive interval type-2 time delay fuzzy logic controller (AT2DFLC) with PI controller, not only the typical switching law chattering can be significantly attenuated but also the instability resulting from system time delay can be overcome. Based on the Lyapunov theory of stability, the free parameters of the AT2DFLC and PI controller coefficients can be tuned on-line by output feedback adaptive laws derived from Lyapunov function with time delays. Simulation results show that the chattering phenomena can be attenuated and the prescribed tracking performance can be preserved simultaneously by the advocated control scheme.

Keywords: Time delay system, PI control, interval type-2 FLS, Lyapunov theory.

I. INTRODUCTION

Owing to time delays are the main source of the instability and lead to unsatisfactory performances, control system design with uncertain time delays has been an active area of research. Over the past years, a number of different researches have been invested in the stability analysis and robust controller design of uncertain systems with delay [19]-[24]. Moreover, robust H^{∞} control methods for linear systems with time delay [18] and a class of nonlinear time delay systems control [19]-[23] have been proposed for many years. Unfortunately, in reality, system uncertainties and external disturbance input are unpredicted, i.e., may be both large and fast. A PI adaptive fuzzy control scheme for a class of uncertain nonlinear systems is introduced in [6]-[8], [12] to handle large and fast but bounded external disturbances and uncertainties.

In the past several decades, based on universal approximation theorem [1]-[5], a significant adaptive fuzzy neural network (FNN) control structure [6], [7], [9] has been proposed to incorporate with the expert information systematically and the stability can be guaranteed. For the systems with high degree of nonlinear uncertainty are very difficult to control using the conventional control theory, such as chemical process, aircraft, and so on. But human operators can often successfully control them. A globally stable adaptive FNN controller is defined as an FNN logic system equipped with an adaptation algorithm thanks to the fact that FNN logic systems are capable of uniformly approximating a nonlinear function over a compact set to any degree of accuracy. On the other hand, interval type-2 fuzzy logic system (FLS) [16], [17] which is an extension of type-1 FLS is introduced to overcome the limitations thanks to type-1 FLS cannot fully handle the linguistic and high level uncertainties [10], [13]-[15].

In this paper, a PI controller incorporated into an adaptive interval type-2 TDFLC constructed by adaptive time delay FNN is proposed to deal with large and fast external disturbance, system uncertainties and system time delay which is a source of instability.

This paper is organized as follows. Problem formulation is given is Section II. A brief description of adaptive interval type-2 time delay fuzzy neural network (TDFNN) is described in Section III. Section IV provides adaptive interval type-2 fuzzy proportional integral (PI) control scheme. Simulation example to illustrate the performance of the proposed control structure is shown in Section V. Section VI concludes the effectiveness of the advocated design methodology.

II. PROBLEM FORMULATION

Consider the nth-order nonlinear dynamical time delay system of the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_n &= f(\underline{x}, \underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r)) \\ y &= x_1 \end{aligned} \tag{1}$$

or equivalently the form

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \cdots, x^{(n-1)}) \\ &+ g(x, \dot{x}, \cdots, x^{(n-1)})u + d, \quad y = x \\ x(t) &= \Xi(t), t \in [-\varsigma_x, 0] \end{aligned} \tag{2}$$

where f and g are unknown but bounded functions, $\Xi(t)$ is the continuous function, $\tau_i (i = 1, 2, ..., r)$ is the time delay, and $\varsigma_x = \max\{\tau_i | 1 \le i \le r\}$. Moreover $u \in R$ and $y \in R$ are the control input and output of the system, respectively and d is the external bounded disturbance, $|d(t)| \le D$, D is a positive constant. Let $f(\underline{x}, \tau) = f(\underline{x}, \underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r))$ and $g(\underline{x}, \tau) = g(\underline{x}(t-\tau_1)\cdots\underline{x}(t-\tau_r))$, (2) can be rewritten in state space representation as

$$\begin{aligned} \dot{\underline{x}} &= \mathrm{A}\underline{x} + \mathrm{B}(f(\underline{x}, \tau) + g(\underline{x}, \tau)u + d) \\ y &= \mathrm{C}^T \underline{x} \end{aligned} \tag{3}$$

where

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$C=\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

and $\underline{x}=\left[x_1,x_2,\cdots,x_n\right]^T=\left[x,\dot{x},\cdots,x^{(n-1)}\right]^T \in R^n$ is the state vector. In order for (2) to be controllable, it is required that $g(\underline{x},\tau)\neq 0$ for x in certain controllability region $U_c \subset R^n$. Without loss of generality, we assume that $g(\underline{x},\tau)>0$ for $x\in U_c$. The control object is to force the system output y to follow a given bounded reference signal y_r, under the constraint that all signals involved must be bounded.

To begin with, the reference signal vector $\underline{y}_r$ and the tracking error vector $\underline{e}$ will be defined as

$$\underline{y}_r=\left[y_r,\dot{y}_r,\cdots,y_r^{(n-1)}\right]^T \in R^n,$$

$$\underline{e}=\underline{y}_r-\underline{x}=\left[e,\dot{e},\cdots,e^{(n-1)}\right]^T \in R^n$$

Let $\underline{k}_c=\left[k_1^c,k_2^c,\cdots,k_n^c\right]^T \in R^n$ to be chosen such that all roots of the polynomial $p(s)=s^n+k_n^c s^{n-1}+\cdots+k_1^c$ are in the open left half-plane. If the functions $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$ are known and the system is free of external disturbance d, then control law of the certainty equivalent controller is obtained as [13]-[15]

$$u^*=\frac{1}{g(\underline{x},\tau)}\left[-f(\underline{x},\tau)+y_r^{(n)}+\underline{k}_c^T\underline{e}\right] \tag{5}$$

Substituting (5) into (2), we have [9]

$$e^{(n)}+k_n^c e^{(n-1)}+\cdots+k_1^c e=0$$

which is the main objective of control, $\lim_{t\to\infty} e(t)=0$. Therefore, there exists a positive definite symmetric $n\times n$ matrix P which satisfies the Lyapunov equation

$$(A-B\underline{k}_c^T)^T P+P(A-B\underline{k}_c^T)=-Q \tag{6}$$

where Q is an arbitrary positive definite symmetric $n\times n$ matrix. However, $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$ are unknown, the equivalent controller (5) is unavailable. The interval type-2 adaptive time delay FNN system structure described in next section will be developed to approximate $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$.

III. DESCRIPTION OF INTERVAL TYPE-2 TIME DELAY FNN

The adaptive time delay FNN system structure is shown in Fig.1.

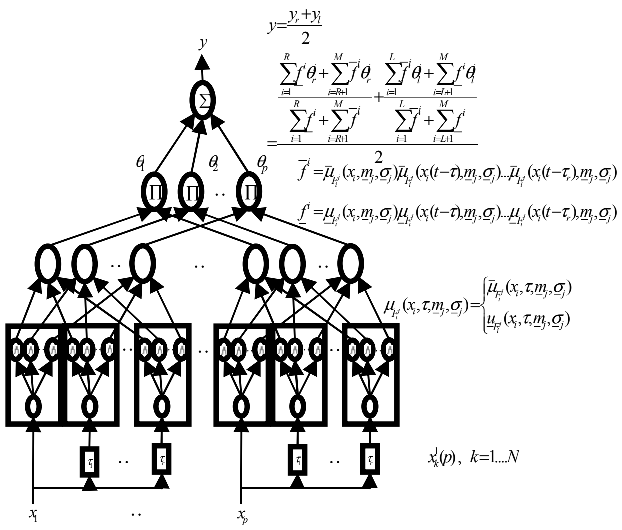

Fig. 1. The interval type-2 adaptive time delay FNN system structure.

Layer 1 is the input layer. The nodes in this layer transmit input values and delay input values to the next layer.

Layer 2 is the membership layer. In this layer, each node expresses the terms of respective linguistic variables and the Gaussian function is adopted as the membership function.

Layer 3 is the rule base and inference layer. The structure of this layer incorporates the fuzzy rules and the fuzzy inference. By fuzzy rules and inference to obtain fuzzy output, each node is corresponding to a fuzzy rule. The input links and the output links of each node express the preconditions of the corresponding rule and the firing strength of the corresponding rule, respectively.

Layer 4 is the output layer. Due to fuzzy inference output is a fuzzy value. We must use defuzzification to get a crisp value.

FNN was proven that has the characteristic of approaching a nonlinear function. In this paper, we construct the adaptive time delay fuzzy logic system to approach fuzzy system function $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$ as

$$f(\underline{x},\tau\,|\,\theta_f,m_f,\sigma_f)=\sum_{j=1}^{p}\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{x},\tau,m_{fi},\sigma_{fi})\right]\theta_{fi} \Bigg/ \sum_{j=1}^{p}\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{x},\tau,m_{fi},\sigma_{fi})\right] \tag{7}$$

where $\mu_{F_i^j}(\underline{x},\tau,m_{fi},\sigma_{fi})=\mu_{F_i^j}(\underline{x},m_{fi},\sigma_{fi})$ $\mu_{F_i^j}(\underline{x}(t-\tau_1),m_{fi},\sigma_{fi})\ldots\mu_{F_i^j}(\underline{x}(t-\tau_r),m_{fi},\sigma_{fi})$.

Let $\prod_{i=1}^{n}\mu_{F_i^j}(\underline{x},\tau,m_{fi},\sigma_{fi})\Big/\sum_{j=1}^{p}\left[\prod_{i=1}^{n}\mu_{F_i^j}(\underline{x},\tau,m_{fi},\sigma_{fi})\right]$ $=\xi_f^T(\underline{x},\tau,m_f,\sigma_f)$,

Hence, System function $f(\underline{x},\tau)$ (7) can be rewritten as

$$f\left(\underline{x},\tau\,\middle|\,\theta_f,m_f,\sigma_f\right)=$$

$$\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{fr}+\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{fl} \tag{8}$$

Similarly, system function $g(\underline{x},\tau)$ can be expressed as

$$g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)=$$

$$\frac{1}{2}\xi_g\left(\underline{x},\tau,m_g,\sigma_g\right)\theta_{gr}+\frac{1}{2}\xi_g\left(\underline{x},\tau,m_g,\sigma_g\right)\theta_{gl} \tag{9}$$

IV. ADAPTIVE INTERVAL TYPE-2 FUZZY PI CONTROLLER DESIGN

An indirect adaptive FNN controller uses FNN system to model the plant and constructs the controller assuming that the FNN systems represent the true plant. However, in reality, the knowledge used to construct FLS is often uncertain such as linguistic uncertainty and noisy training data. In the meantime, time delays are always the main source of the instability and lead to unsatisfactory performances. In order to overcome the limitations from type-1 FLS and the instability resulting from system time delay, the interval type-2 time delay FNN is constructed to approximate system functions $f(\underline{x},\tau)$ and $g(\underline{x},\tau)$. Furthermore, PI control structure is developed to handle fast and large but bounded external disturbances. The PI error feedback structure is defined as

$$\rho(\underline{e}^T P B|\theta_p)=\begin{cases}[K_P,K_I][\underline{e}^T PB,\int \underline{e}^T PBdt]^T & \left|\underline{e}^T PB\right|\le\psi\\ \hat{D}_m\,\mathrm{sgn}(\underline{e}^T PB) & \left|\underline{e}^T PB\right|>\psi\end{cases} \tag{10}$$

where P is the matrix given in (6), $[K_P,K_I]=\theta_D^T$ is the PI parameter vector to be adapted, $[\underline{e}^T PB,\int \underline{e}^T PBdt]^T=\varsigma(\underline{e}^T PB)$. $\hat{D}_m=\hat{D}+\hat{\Omega}$ is an estimate of $D_m=D+\Omega$, D and Ω are the disturbance bound and the minimum approximation error of the interval yupe-2 time delay FNN, respectively. The thickness of the boundary layer ψ is a compromise between chattering attenuation and accelerating the speed of convergence. Therefore, the control effort (5) can be rewritten as

$$u=\frac{1}{g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)}\left[-f\left(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f\right)-\rho(\underline{e}^T PB|\theta_p)+x_d^{(n)}-\underline{k}_c^T\underline{e}\right] \tag{11}$$

where $f\left(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f\right)$ and $g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)$ are the interval type-2 time delay FNN outputs which are described in (8) and (9) as follows:

$$f\left(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f\right)=$$

$$\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{fr}+\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{fl}$$

$$g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)=$$

$$\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{gr}+\frac{1}{2}\xi\left(\underline{x},\tau,m_f,\sigma_f\right)\theta_{gl}$$

By using (11) into (3), the error dynamic equation can be expressed as

$$\dot{\underline{e}}=\left(A-Bk_c^T\right)\underline{e}+B\left(f\left(\underline{x},\tau\right)-f\left(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f\right)\right)$$
$$+B\left(g\left(\underline{x},\tau\right)-g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)\right)u$$
$$+B\left(d\left(x,t\right)-\rho\left(\underline{e}^T PB\middle|\theta_p\right)\right) \tag{12}$$

Applying double Taylor series expansion and let $\tilde{\theta}_{fr}=\theta_{fr}^*-\theta_{fr}$, $\tilde{m}_{fr}=m_{fr}^*-m_{fr}$, $\tilde{\sigma}_{fr}=\sigma_{fr}^*-\sigma_{fr}$, $\tilde{\theta}_{fl}=\theta_{fl}^*-\theta_{fl}$, $\tilde{m}_{fl}=m_{fl}^*-m_{fl}$, $\tilde{\sigma}_{fl}=\sigma_{fl}^*-\sigma_{fl}$, $\tilde{\theta}_{gr}=\theta_{gr}^*-\theta_{gr}$, $\tilde{m}_{gr}=m_{gr}^*-m_{gr}$, $\tilde{\sigma}_{gr}=\sigma_{gr}^*-\sigma_{gr}$, $\tilde{\theta}_{gl}=\theta_{gl}^*-\theta_{gl}$, $\tilde{m}_{gl}=m_{gl}^*-m_{gl}$, $\tilde{\sigma}_{gl}=\sigma_{gl}^*-\sigma_{gl}$, $\tilde{\theta}_p=\theta_p^*-\theta_p$, $\tilde{D}_m=D_m-\hat{D}_m$, we have

$$f(\underline{x},\tau)-f\left(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f\right)$$
$$=-\frac{1}{2}\{[\xi_{fr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)-m_{fr}\xi_{mfr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)$$
$$-\sigma_{fr}\xi_{\sigma fr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)]\tilde{\theta}_{fr}+(\tilde{m}_{fr}\xi_{mfr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)$$
$$+\tilde{\sigma}_{fr}\xi_{\sigma fr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right))\theta_{fr}+m_{fr}^*\xi_{mfr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)\tilde{\theta}_{fr}$$
$$+\sigma_{fr}^*\xi_{sfr}\left(\underline{x},\tau,m_{fr},\sigma_{fr}\right)\tilde{\theta}_{fr}+[\xi_{fl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)$$
$$-m_{fl}\xi_{mfl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)-\sigma_{fl}\xi_{\sigma fl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)]\tilde{\theta}_{fl}$$
$$+(\tilde{m}_{fl}\xi_{mfl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)+\tilde{\sigma}_{fl}\xi_{\sigma fl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right))\theta_{fl}$$
$$+m_{fl}^*\xi_{mfl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)\tilde{\theta}_{fl}+\sigma_{fl}^*\xi_{\sigma fl}\left(\underline{x},\tau,m_{fl},\sigma_{fl}\right)\tilde{\theta}_{fl}\} \tag{13}$$

and

$$g(\underline{x},\tau)-g\left(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g\right)$$
$$=-\frac{1}{2}\{[\xi_{gr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)-m_{gr}\xi_{mgr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)$$
$$-\sigma_{gr}\xi_{\sigma gr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)]\tilde{\theta}_{gr}+(\tilde{m}_{gr}\xi_{mgr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)$$
$$+\tilde{\sigma}_{gr}\xi_{\sigma gr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right))\theta_{gr}+m_{gr}^*\xi_{mgr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)\tilde{\theta}_{gr}$$
$$+\sigma_{gr}^*\xi_{\sigma gr}\left(\underline{x},\tau,m_{gr},\sigma_{gr}\right)\tilde{\theta}_{gr}+[\xi_{gl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)$$
$$-m_{gl}\xi_{mgl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)-\sigma_{gl}\xi_{\sigma gl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)]\tilde{\theta}_{gl}$$
$$+(\tilde{m}_{gl}\xi_{mgl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)+\tilde{\sigma}_{gl}\xi_{\sigma gl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right))\theta_{gl}$$
$$+m_{gl}^*\xi_{mgl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)\tilde{\theta}_{gl}+\sigma_{gl}^*\xi_{\sigma gl}\left(\underline{x},\tau,m_{gl},\sigma_{gl}\right)\tilde{\theta}_{gl}\} \tag{14}$$

where

$$\theta_f^*=\arg\min_{\theta_f\in R^M}\left[\sup_{x\in R^n}\left|f(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f)-f(x,\tau)\right|\right] \tag{15}$$

$$\theta_g^*=\arg\min_{\theta_g\in R^M}\left[\sup_{x\in R^n}\left|g(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g)-g(x,\tau)\right|\right] \tag{16}$$

$$m_f^*=\arg\min_{\theta_f\in R^M}\left[\sup_{x\in R^n}\left|f(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f)-f(x,\tau)\right|\right] \tag{17}$$

$$m_g^*=\arg\min_{\theta_g\in R^M}\left[\sup_{x\in R^n}\left|g(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g)-g(x,\tau)\right|\right] \tag{18}$$

$$\sigma_f^*=\arg\min_{\theta_f\in R^M}\left[\sup_{x\in R^n}\left|f(\underline{x},\tau\middle|\theta_f,m_f,\sigma_f)-f(x,\tau)\right|\right] \tag{19}$$

$$\sigma_g^*=\arg\min_{\theta_g\in R^M}\left[\sup_{x\in R^n}\left|g(\underline{x},\tau\middle|\theta_g,m_g,\sigma_g)-g(x,\tau)\right|\right] \tag{20}$$

In order to simplify all formulas, the following notations are defined as

$$T_{fr1}(\underline{x},\tau,m_{fr},\sigma_{fr})=\left[\xi_{fr}(\underline{x},\tau,m_{fr},\sigma_{fr})\right.$$

$$-m_{fr}\xi_{m_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})-\sigma_{fr}\xi_{\sigma_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})\Big] \quad (21)$$

$$T_{fr2}(\underline{x},\tau,m_{fr},\sigma_{fr})=$$
$$\Big[\tilde{m}_{fr}\xi_{m_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})+\tilde{\sigma}_{fr}\xi_{\sigma_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})\Big] \quad (22)$$

$$T_{fl1}(\underline{x},\tau,m_{fl},\sigma_{fl})=\Big[\xi_{fl}(\underline{x},\tau,m_{fl},\sigma_{fl})$$
$$-m_{fl}\xi_{m_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})-\sigma_{fl}\xi_{\sigma_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})\Big] \quad (23)$$

$$T_{fl2}(\underline{x},\tau,m_{fl},\sigma_{fl})=$$
$$\Big[\tilde{m}_{fl}\xi_{m_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})+\tilde{\sigma}_{fl}\xi_{\sigma_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})\Big] \quad (24)$$

$$T_{gr1}(\underline{x},\tau,m_{gr},\sigma_{gr})=\Big[\xi_{gr}(\underline{x},\tau,m_{gr},\sigma_{gr})$$
$$-m_{gr}\xi_{m_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})-\sigma_{gr}\xi_{\sigma_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})\Big] \quad (25)$$

$$T_{gr2}(\underline{x},\tau,m_{gr},\sigma_{gr})=$$
$$\Big[\tilde{m}_{gr}\xi_{m_{gr}}(x\underline{x},\tau,m_{gr},\sigma_{gr})+\tilde{\sigma}_{gr}\xi_{\sigma_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})\Big] \quad (26)$$

$$T_{gl1}(\underline{x},\tau,m_{gl},\sigma_{gl})=\Big[\xi_{gl}(\underline{x},\tau,m_{gl},\sigma_{gl})$$
$$-m_{gl}\xi_{m_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})-\sigma_{gl}\xi_{\sigma_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})\Big] \quad (27)$$

$$T_{gl2}(\underline{x},\tau,m_{gl},\sigma_{gl})=$$
$$\Big[\tilde{m}_{gl}\xi_{m_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})+\tilde{\sigma}_{gl}\xi_{\sigma_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})\Big] \quad (28)$$

By using (13) ~ (28), the error dynamics (12) can be re-expressed as

$$\dot{\underline{e}}=\left(A-Bk_c^T\right)\underline{e}+B\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,m_{fr},\sigma_{fr})\tilde{\theta}_{fr}$$
$$+T_{fr2}(\underline{x},\tau,m_{fr},\sigma_{fr})\theta_{fr}+T_{fl1}(\underline{x},\tau,m_{fl},\sigma_{fl})\tilde{\theta}_{fr}$$
$$+T_{fr2}(\underline{x},\tau,m_{fr},\sigma_{fr})\tilde{\theta}_{fr}]\}+B\{-\frac{1}{2}[T_{gr1}(\underline{x},\tau,m_{gr},\sigma_{gr})\tilde{\theta}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,m_{gr},\sigma_{gr})\tilde{\theta}_{gr}+T_{gl1}(\underline{x},\tau,m_{gl},\sigma_{gl})\tilde{\theta}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,m_{gr},\sigma_{gr})\tilde{\theta}_{gr}]\}u+B\left(\rho\left(\underline{e}^TPB\middle|\theta_p^*\right)-\rho\left(\underline{e}^TPB\middle|\theta_p\right)\right)$$
$$+B\left(d\left(x,t\right)-\rho\left(\underline{e}^TPB\middle|\theta_p^*\right)\right)+B\omega \quad (29)$$

where ω is the bounded minimum approximation error of the interval type-2 time delay FNN, i.e., $|\omega|\le\Omega$

$$\omega=-\frac{1}{2}\{(m_{fr}^*(\xi_{m_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})+\sigma_{fr}^*\xi_{\sigma_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr}))\tilde{\theta}_{fr}$$
$$+(m_{fl}^*(\xi_{m_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})+\sigma_{fl}^*\xi_{\sigma_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl}))\tilde{\theta}_{fl}$$
$$+(m_{gr}^*(\xi_{m_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})+\sigma_{gr}^*\xi_{\sigma_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr}))\tilde{\theta}_{gr}$$
$$+(m_{gl}^*(\xi_{m_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})+\sigma_{gl}^*\xi_{\sigma_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl}))\tilde{\theta}_{gl}\} \quad (30)$$

Following the preceding consideration, the following theorem is declared to show that the proposed overall control scheme is asymptotically stable.

Theorem: Consider the *n*th-order nonlinear dynamical time delay system in the form of (1) with the control law in (11), the all design parameters are adjusted by the adaptive laws (31)-(39)

$$\dot{\theta}_{fr}=-r_1\Big[\xi_{fr}(\underline{x},\tau,m_{fr},\sigma_{fr})-m_{fr}\xi_{m_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})$$
$$-\sigma_{fr}\xi_{\sigma_{fr}}(\underline{x},\tau,m_{fr},\sigma_{fr})\Big]^T\left(B^TP\underline{e}\right) \quad (31)$$

$$\dot{\theta}_{fl}=-r_2\Big[\xi_{fl}(\underline{x},\tau,m_{fl},\sigma_{fl})-m_{fl}\xi_{m_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})$$
$$-\sigma_{fl}\xi_{\sigma_{fl}}(\underline{x},\tau,m_{fl},\sigma_{fl})\Big]^T\left(B^TP\underline{e}\right) \quad (32)$$

$$\dot{\theta}_{gr}=-r_3\Big[\xi_{gr}(\underline{x},\tau,m_{gr},\sigma_{gr})-m_{gr}\xi_{m_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})$$
$$-\sigma_{gr}\xi_{\sigma_{gr}}(\underline{x},\tau,m_{gr},\sigma_{gr})\Big]^T\left(B^TPe\right)u \quad (33)$$

$$\dot{\theta}_{gl}=-r_4\Big[\xi_{gl}(\underline{x},\tau,m_{gl},\sigma_{gl})-m_{gl}\xi_{m_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})$$
$$-\sigma_{gl}\xi_{\sigma_{gl}}(\underline{x},\tau,m_{gl},\sigma_{gl})\Big]^T\left(B^TPe\right)u \quad (34)$$

$$\dot{m}_{fr}=-r_5\theta_{fr}^T\xi_{m_{fr}}^T(\underline{x},\tau,m_{fr},\sigma_{fr})\left(B^TPe\right) \quad (35)$$
$$\dot{m}_{fl}=-r_6\theta_{fl}^T\xi_{m_{fl}}^T(\underline{x},\tau,m_{fl},\sigma_{fl})\left(B^TP\underline{e}\right) \quad (36)$$
$$\dot{m}_{gr}=-r_7\theta_{gr}^T\xi_{m_{gr}}^T(\underline{x},\tau,m_{gr},\sigma_{gr})\left(B^TP\underline{e}\right)u \quad (37)$$
$$\dot{m}_{gl}=-r_8\theta_{gl}^T\xi_{m_{gl}}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\left(B^TP\underline{e}\right)u \quad (38)$$
$$\dot{\sigma}_{fr}=-r_9\theta_{fr}^T\xi_{\sigma_{fr}}^T(\underline{x},\tau,m_{fr},\sigma_{fr})\left(B^TP\underline{e}\right) \quad (39)$$
$$\dot{\sigma}_{fl}=-r_{10}\theta_{fl}^T\xi_{\sigma_{fl}}^T(\underline{x},\tau,m_{fl},\sigma_{fl})\left(B^TP\underline{e}\right) \quad (40)$$
$$\dot{\sigma}_{gr}=-r_{11}\theta_{gr}^T\xi_{\sigma_{gr}}^T(\underline{x},\tau,m_{gr},\sigma_{gr})\left(B^TP\underline{e}\right)u \quad (41)$$
$$\dot{\sigma}_{gl}=-r_{12}\theta_{gl}^T\xi_{\sigma_{gl}}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\left(B^TP\underline{e}\right)u \quad (42)$$
$$\dot{\sigma}_{gl}=-r_{12}\theta_{gl}^T\xi_{\sigma_{gl}}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\left(B^TP\underline{e}\right)u \quad (43)$$
$$\dot{\theta}_p=\gamma_{13}\underline{e}^TPB\zeta\left(\underline{e}^TPB\right),\ \dot{D}_m=\gamma_{14}\left|\underline{e}^TPB\right| \quad (44)$$

where $r_i>0, i=1\sim 14$, based on the Barbalat's lemma [11], the tracking error $e(t)$ will be asymptotically approaches to zero, i.e., $\lim_{t\to\infty}e(t)=0$.

Proof:

To begin with, the Lyapunov function candidate is defined as

$$V=\frac{1}{2}\underline{e}^TP\underline{e}+\frac{1}{2}\sum_{i=1}^{r}\int_{t-\tau_i}^{t}\underline{e}^T(v)\underline{e}(v)dv+\frac{1}{4\gamma_1}\tilde{\theta}_{fr}^T\tilde{\theta}_{fr}$$
$$+\frac{1}{4\gamma_2}\tilde{\theta}_{fl}^T\tilde{\theta}_{fl}+\frac{1}{4\gamma_3}\tilde{\theta}_{gr}^T\tilde{\theta}_{gr}+\frac{1}{4\gamma_4}\tilde{\theta}_{gl}^T\tilde{\theta}_{gl}+\frac{1}{4\gamma_5}\tilde{m}_{fr}\tilde{m}_{fr}^T$$
$$+\frac{1}{4\gamma_6}\tilde{m}_{fl}\tilde{m}_{fl}^T+\frac{1}{4\gamma_7}\tilde{m}_{gr}\tilde{m}_{gr}^T+\frac{1}{4\gamma_8}\tilde{m}_{gl}\tilde{m}_{gl}^T+\frac{1}{4\gamma_9}\tilde{\sigma}_{fr}\tilde{\sigma}_{fr}^T$$
$$+\frac{1}{4\gamma_{10}}\tilde{\sigma}_{fl}\tilde{\sigma}_{fl}^T+\frac{1}{4\gamma_{11}}\tilde{\sigma}_{gr}\tilde{\sigma}_{gr}^T+\frac{1}{4\gamma_{12}}\tilde{\sigma}_{gl}\tilde{\sigma}_{gl}^T+\frac{1}{2\gamma_{13}}\tilde{\theta}_p^T\tilde{\theta}_p$$
$$+\frac{1}{2\gamma_{14}}\tilde{D}_\omega^2 \quad (45)$$

Differentiating (45) with respect to time *t* along the trajectory (29) we obtain

$$\dot{V}=\frac{1}{2}\dot{\underline{e}}^TP\underline{e}+\frac{1}{2}\underline{e}^TP\dot{\underline{e}}+\frac{1}{2\gamma_1}\tilde{\theta}_{fr}^T\dot{\tilde{\theta}}_{fr}+\frac{1}{2\gamma_2}\tilde{\theta}_{fl}^T\dot{\tilde{\theta}}_{fl}$$
$$+\frac{1}{2\gamma_3}\tilde{\theta}_{gr}^T\dot{\tilde{\theta}}_{gr}+\frac{1}{2\gamma_4}\tilde{\theta}_{gl}^T\dot{\tilde{\theta}}_{gl}+\frac{1}{2\gamma_5}\dot{\tilde{m}}_{fr}\tilde{m}_{fr}^T+\frac{1}{2\gamma_6}\dot{\tilde{m}}_{fl}\tilde{m}_{fl}^T$$
$$+\frac{1}{2\gamma_7}\dot{\tilde{m}}_{gr}\tilde{m}_{gr}^T+\frac{1}{2\gamma_8}\dot{\tilde{m}}_{gl}\tilde{m}_{gl}^T+\frac{1}{2\gamma_9}\dot{\tilde{\sigma}}_{fr}\tilde{\sigma}_{fr}^T+\frac{1}{2\gamma_{10}}\dot{\tilde{\sigma}}_{fl}\tilde{\sigma}_{fl}^T$$
$$+\frac{1}{2\gamma_{11}}\dot{\tilde{\sigma}}_{gr}\tilde{\sigma}_{gr}^T+\frac{1}{2\gamma_{12}}\dot{\tilde{\sigma}}_{gl}\tilde{\sigma}_{gl}^T+\frac{1}{\gamma_{13}}\tilde{\theta}_S^T\dot{\tilde{\theta}}_S+\frac{1}{\gamma_{14}}\dot{\tilde{D}}_\omega$$
$$+\frac{1}{2}\sum_{i=1}^{r}e^T(t)e(t)-\frac{1}{2}\sum_{i=1}^{r}e^T(t-\tau_i)e(t-\tau_i) \quad (46)$$

Substituting (29) into (46), $\dot{V}$ can rewritten as

$$\dot{V}=\frac{1}{2}\Big\{\left(A-Bk_c^T\right)\underline{e}+B\Big\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,m_{fr},\sigma_{fr})\tilde{\theta}_{fr}$$
$$+T_{fr2}(\underline{x},\tau,m_{fr},\sigma_{fr})\theta+T_{fl1}(\underline{x},\tau,m_{fl},\sigma_{fl})\tilde{\theta}_{fl}$$
$$+T_{fl2}(\underline{x},\tau,m_{fl},\sigma_{fl})\theta_{fl}]-\frac{1}{2}\Big[T_{gr1}(\underline{x},\tau,m_{gr},\sigma_{gr})\tilde{\theta}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,m_{gr},\sigma_{gr})\theta_{gr}+T_{gl1}(\underline{x},\tau,m_{gl},\sigma_{gl})\tilde{\theta}_{gl}$$
$$+T_{gl2}(\underline{x},\tau,m_{gl},\sigma_{gl})\theta_{gl}\Big]u\Big\}+B\left(\rho\left(\underline{e}^TPB\middle|\theta_s^*\right)-\rho\left(\underline{e}^TPB\middle|\theta_s\right)\right)$$
$$+B\omega+B\left(d\left(X,t\right)-\rho\left(\underline{e}^TPB\middle|\theta_s^*\right)\right)\Big\}^T P\underline{e}$$
$$+\frac{1}{2}\underline{e}^TP\Big\{\left(A-Bk_c^T\right)\underline{e}+\frac{1}{2}B\Big\{-\frac{1}{2}[T_{fr1}(\underline{x},\tau,m_{fr},\sigma_{fr})\tilde{\theta}_{fr}$$
$$+T_{fr2}(\underline{x},\tau,m_{fr},\sigma_{fr})\theta_{fr}+T_{fl1}(\underline{x},\tau,m_{fl},\sigma_{fl})\tilde{\theta}_{fl}$$
$$+T_{fl2}(\underline{x},\tau,m_{fl},\sigma_{fl})\theta_{fl}]-\frac{1}{2}[T_{gr1}(\underline{x},\tau,m_{gr},\sigma_{gr})\tilde{\theta}_{gr}$$
$$+T_{gr2}(\underline{x},\tau,m_{gr},\sigma_{gr})\theta_{gr}+T_{gl1}(\underline{x},\tau,m_{gl},\sigma_{gl})\tilde{\theta}_{gl}$$
$$+T_{gl2}(\underline{x},\tau,m_{gl},\sigma_{gl})\theta_{gl}]u\}+B\left(\rho\left(\underline{e}^TPB\middle|\theta_p^*\right)-\rho\left(\underline{e}^TPB\middle|\theta_p\right)\right)$$
$$+B\omega+B\left(d\left(x,t\right)-\rho\left(\underline{e}^TPB\middle|\theta_p^*\right)\right)\Big\}$$
$$+\frac{1}{2\gamma_1}\tilde{\theta}_{fr}{}^T\dot{\tilde{\theta}}_{fr}+\frac{1}{2\gamma_2}\tilde{\theta}_{fl}{}^T\dot{\tilde{\theta}}_{fl}+\frac{1}{2\gamma_3}\tilde{\theta}_{gr}{}^T\dot{\tilde{\theta}}_{gr}+\frac{1}{2\gamma_4}\tilde{\theta}_{gl}{}^T\dot{\tilde{\theta}}_{gl}$$
$$+\frac{1}{2\gamma_5}\dot{\tilde{m}}_{fr}\tilde{m}_{fr}^T+\frac{1}{2\gamma_6}\dot{\tilde{m}}_{fl}\tilde{m}_{fl}^T+\frac{1}{2\gamma_7}\dot{\tilde{m}}_{gr}\tilde{m}_{gr}^T+\frac{1}{2\gamma_8}\dot{\tilde{m}}_{gl}\tilde{m}_{gl}^T$$
$$+\frac{1}{2\gamma_9}\dot{\tilde{\sigma}}_{fr}\tilde{\sigma}_{fr}^T+\frac{1}{2\gamma_{10}}\dot{\tilde{\sigma}}_{fl}\tilde{\sigma}_{fl}^T+\frac{1}{2\gamma_{11}}\dot{\tilde{\sigma}}_{gr}\tilde{\sigma}_{gr}^T+\frac{1}{2\gamma_{12}}\dot{\tilde{\sigma}}_{gl}\tilde{\sigma}_{gl}^T$$
$$+\frac{1}{2r_{fl3}}\underline{\tilde{\sigma}}_{fl}^T\dot{\underline{\tilde{\sigma}}}_{fl}+\frac{1}{2r_{gl3}}\underline{\tilde{\sigma}}_{gl}^T\dot{\underline{\tilde{\sigma}}}_{gl}+\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t)\underline{e}(t)$$
$$-\frac{1}{2}\sum_{i=1}^{r}\underline{e}^T(t-\tau_i)\underline{e}(t-\tau_i) \tag{47}$$
$$\le-\frac{1}{2}\{\underline{e}^T(Q-rI)\underline{e}$$
$$+\tilde{\theta}_{fr}^T\left[-T_{fr1}^T(\underline{x},\tau,m_{fr},\sigma_{fr})\left(B^TP\underline{e}\right)-\frac{1}{r_1}\dot{\theta}_{fr}\right]$$
$$+\tilde{\theta}_{fl}^T\left[-T_{fl1}^T(\underline{x},\tau,m_{fl},\sigma_{fl})\left(B^TP\underline{e}\right)-\frac{1}{r_2}\dot{\theta}_{fl}\right]$$
$$+\tilde{\theta}_{gr}^T\left[-T_{gr1}^T(\underline{x},\tau,m_{gr},\sigma_{gr})\left(B^TP\underline{e}\right)u-\frac{1}{r_3}\dot{\theta}_{gr}\right]$$
$$+\tilde{\theta}_{gl}^T\left[-T_{gl1}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\left(B^TP\underline{e}\right)u-\frac{1}{r_4}\dot{\theta}_{gl}\right]$$
$$+\left[-\xi_{m_{fr}}^T(\underline{x},\tau,m_{fr},\sigma_{fr})\theta_{fr}\left(B^TP\underline{e}\right)-\frac{1}{r_5}\dot{m}_{fr}\right]\tilde{m}_{fr}^T$$
$$+\left[-\xi_{m_{fl}}^T(\underline{x},\tau,m_{fl},\sigma_{fl})\theta_{fl}\left(B^TP\underline{e}\right)-\frac{1}{r_6}\dot{m}_{fl}\right]\tilde{m}_{fl}^T$$
$$+\left[-\xi_{m_{gr}}^T(\underline{x},\tau,m_{gr},\sigma_{gr})\theta_{gr}\left(B^TP\underline{e}\right)u-\frac{1}{r_7}\dot{m}_{gr}\right]\tilde{m}_{gr}^T$$
$$+\left[-\xi_{m_{gl}}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\theta_{gl}\left(B^TP\underline{e}\right)u-\frac{1}{r_8}\dot{m}_{gl}\right]\tilde{m}_{gl}^T$$
$$+\left[-\xi_{\sigma_{fr}}^T(\underline{x},\tau,m_{fr},\sigma_{fr})\theta_{fr}\left(B^TP\underline{e}\right)-\frac{1}{r9}\dot{\sigma}_{fr}\right]\tilde{\sigma}_{fr}^T$$
$$+\left[-\xi_{\sigma_{fl}}^T(\underline{x},\tau,m_{fl},\sigma_{fl})\theta_{fl}\left(B^TP\underline{e}\right)-\frac{1}{r_{10}}\dot{\sigma}_{fl}\right]\tilde{\sigma}_{fl}^T$$
$$+\left[-\xi_{\sigma_{gr}}^T(\underline{x},\tau,m_{gr},\sigma_{gr})\theta_{gr}\left(B^TP\underline{e}\right)u-\frac{1}{r_1}\dot{\sigma}_{gr}\right]\tilde{\sigma}_{gr}^T$$
$$+\left[-\xi_{\sigma_{gl}}^T(\underline{x},\tau,m_{gl},\sigma_{gl})\theta_{gl}\left(B^TP\underline{e}\right)u-\frac{1}{r_{12}}\dot{\sigma}_{gl}\right]\tilde{\sigma}_{gl}^T$$
$$+\tilde{\theta}_p{}^T\left[\underline{e}^TPB\zeta\left(\underline{e}^TPB\right)-\frac{1}{\gamma_{13}}\dot{\theta}_p\right]+\left|\underline{e}^TPB\right|D_m$$
$$+\left|\underline{e}^TPB\right|\hat{D}_m-\frac{1}{\gamma_{14}}\tilde{D}_m\dot{D}_m+\underline{e}^TPB\omega \tag{48}$$

Substituting adaptive laws (31)-(44) into (48) we have

$$\dot{V}\le-\frac{1}{2}e^T(Q-rI)e+\underline{e}^TPB\omega \tag{49}$$

Since ω is the bounded minimum approximation error, Q and r can be determined such that $\dot{V}\le-\frac{1}{2}e^T(Q-rI)e+\underline{e}^TPB\omega<0$. Therefore, by the Barbalat's lemma [11], the tracking error $e(t)$ will be asymptotically approaches to zero, i.e., $\lim_{t\to\infty}e(t)=0$. The proof is completed.

V. SIMULATION EXAMPLE

In this section, we will apply our adaptive interval type-2 fuzzy PI controller for a single-machine-infinite-bus (SMIB) power system described by delay differential equations (DDE). The equation governing the motion of this DDE is given as follows.

$$\dot{x}_1=x_2$$
$$\dot{x}_2=-2x_2-2\sin x_1+5\sin 5t+3\sin(5(x_1(t-\tau)))+u(t)+d(t)$$

where $\tau=0.002\sec$ is delay time, external disturbance $d=4\cos(5\pi t)$ and training data are corrupted by white Gaussian noise with signal-to-noise ratio (SNR) 20 dB. The reference trajectory is $y_r=\frac{\pi}{30}\sin(t)$.

The output and reference trajectories are given in Fig.2 and the control effort obtained by (11) is shown in Fig. 3. We can see that the chattering phenomena can be attenuated and the prescribed tracking performance can be preserved simultaneously. Also, the 3D tracking performance is shown in Fig. 4 and the PI parameters adaptation K_P and K_I are described in Fig. 5.

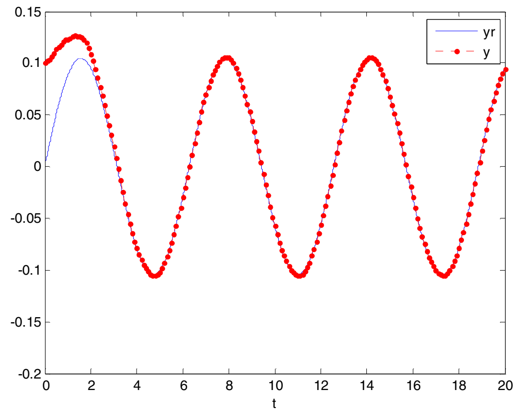

Fig. 2.1 The output and reference trajectories.(type2)

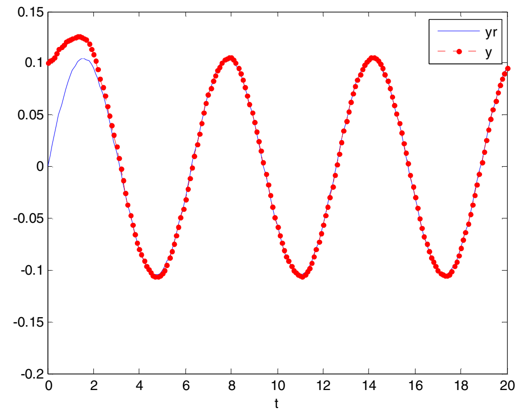

Fig. 2.2 The output and reference trajectories. (type1)

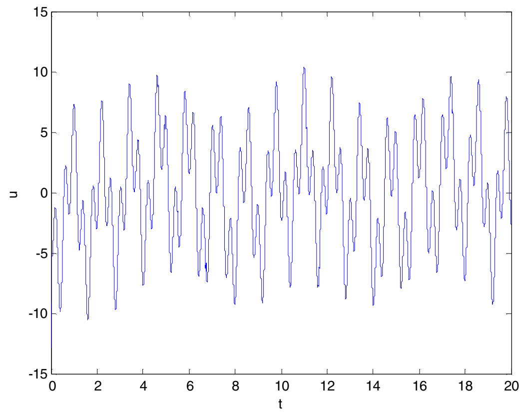

Fig. 3.1 The control effort. (type2)

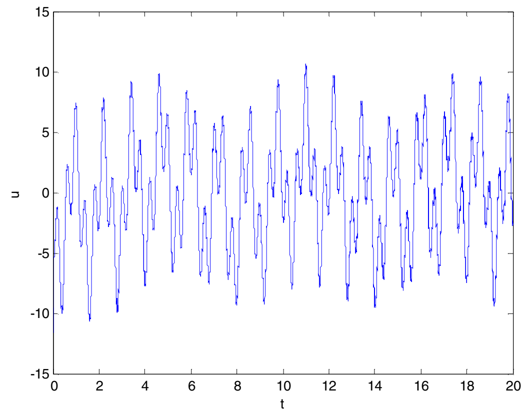

Fig. 3.2 The control effort. (type1)

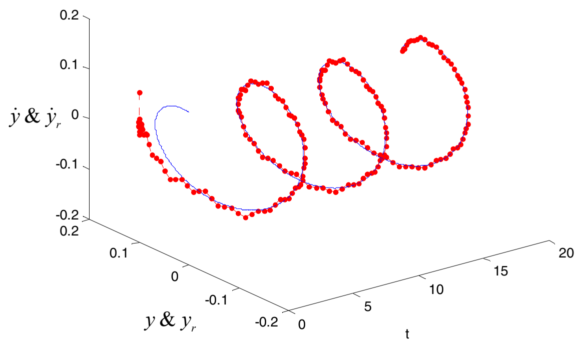

Fig. 4.1 The 3D tracking performance. (type2)

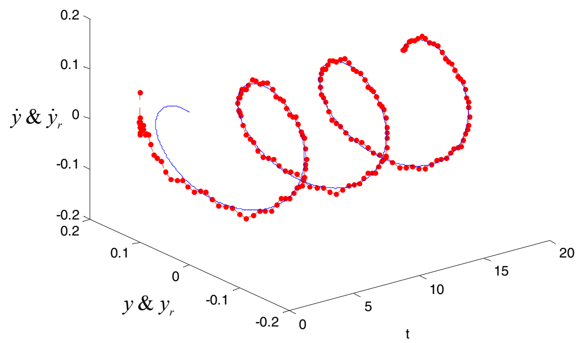

Fig. 4.2 The 3D tracking performance. (type1)

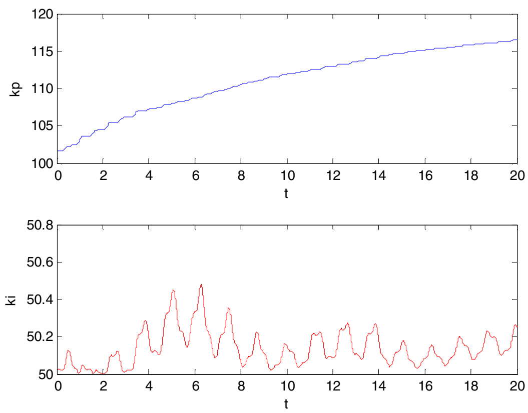

Fig. 5.1 The PI parameters adaption(type2)

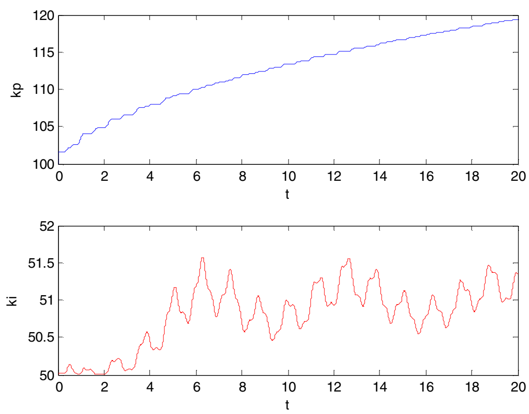

Fig. 5.2 The PI parameters adaption(type1)

VI. CONCLUSIONS

In order to provide robustness in the presence of fast and large disturbance and to eliminate the instability resulting from system time delay, adaptive interval type-2 fuzzy PI control scheme is proposed by incorporating AT2DFLC with PI controller. Interval type-2 time delay FNN is constructed so as to fully handle the linguistic and high level uncertainties and to estimate the behaviors of the system functions. Simulation results show that not only the prescribed tracking performance can be preserved by the advocated control scheme bust also the controller of type2 is more stable in critical point than type1.

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