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An Approach of Decision Making With Linguistic Weight

Li Zou, Yunxia Zhang, Zhiyan Chang, Yong Zhang

Abstract—**In the reality, people use linguistic term rather than numerical information to express their evaluations or preferences in decision making problems. To deal with the qualitative information, we propose a linguistic decision making approach based on the ten-element linguistic-valued lattice implication algebra. In this paper, we discuss the properties of two important operations, i.e. $\otimes$ and $\oplus$, in ten-element linguistic lattice implication algebra. In the decision making approach proposed in this paper, we use the operation $\otimes$ to calculate the weighted criteria in view of its properties. The illustration example shows that the proposed approach seems more effective for decision making under a fuzzy environment with both comparable and incomparable linguistic truth values.**

I. INTRODUCTION

In many qualitative decision making problems, people prefer to use linguistic information to express their preferences or opinions. Following the idea of Computing with Words (CWW) [1], many linguistic methods are proposed and applied in various fields [2-4].

There are always comparable and incomparable information in the linguistic decision making problem. In order to deal with the comparable and incomparable information, Y. Xu has proposed lattice implication algebra [5]. Linguistic lattice implication algebra can process the qualitative information with linguistic term, at the same time, consider both comparable and incomparable information. It can reflect not only the fuzziness of the information, but also the properties of the human's natural language [6]. The lattice structure has been a very useful and widely applied branch [7]. Many other researchers have studied the lattice implication algebra deeply [8-12], and applied it to decision making, pattern recognition, cybernetics and risk analysis etc. [13-16].

In this paper, we analyze the properties of the operations $\otimes$ and $\oplus$ in ten-element lattice implication algebra. We propose a decision making approach based on ten-element linguistic-valued lattice implication algebra, where we use the operation $\otimes$ to calculate the weighted evaluation values. The paper is organized as follows: In Section 2, we briefly review some concepts of lattice implication algebra and introduce the ten-element linguistic-valued lattice implication algebra (10LV_LIA). In Section 3, we discuss the properties of operations $\otimes$ and $\oplus$ in 10LV_LIA. Then we present the phases of the linguistic decision making approach based on linguistic lattice implication algebra. In Section 4, we give an example to illustrate the effectiveness of this approach. In Section 5, we make some concluding marks and suggest further research.

Li Zou is with the School of Computer and Information Technology, Liaoning Normal University, Dalian, CO 116081 China and State Key Laboratory for Novel Software Technology, Nanjing University, Nanjing 210093, China (corresponding author, phone: 13795189212; e-mail: zoulicn@163.com).

Yunxia Zhang and Yong Zhang is with the School of Computer and Information Technology, Liaoning Normal University, Dalian, CO 116081 China.

Zhiyan Chang is with the College of Science, Tianjin University of Technology, Tianjin, CO 300384, China.

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II. THE CONCEPTS OF TEN-ELEMENT LINGUISTIC-VALUED LATTICE IMPLICATION ALGERIA

We briefly review some concepts of lattice implication algebra, the operations in $\mathcal{L}_5 \times \mathcal{L}_2$ and the relationship between $\mathcal{L}_5 \times \mathcal{L}_2$ and $\mathcal{L}_{V(5\times 2)}$. We refer to the related reference [5].

Definition 1[5]. Let (L, $\vee$, $\wedge$, O, I) be a bounded lattice with universal boundaries *O* (the least element) and *I* (the greatest element) respectively, and " ' " be an order-reversing involution. For any $x, y, z \in L$, if mapping $\rightarrow: L \times L \rightarrow L$ satisfies:

$$(I1): x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z);$$
$$(I2): x \rightarrow x = I;$$
$$(I3): x \rightarrow y = y' \rightarrow x';$$
$$(I4): x \rightarrow y = y \rightarrow x = I \text{ implies } x = y;$$
$$(I5): (x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x;$$
$$(I6): (x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z);$$
$$(I7): (x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$$

Then (L, $\vee$, $\wedge$, ' , $\rightarrow$, O, I) is a lattice implication algebra (LIA for short).

Let $L_5 = \{h_1, h_2, h_3, h_4, h_5\}$, $h_1 < h_2 < h_3 < h_4 < h_5$, and for any $i, j \in \{1,2,3,4,5\}$, we define that

$$h_i \vee h_j = h_{\max\{i,j\}} ,$$
$$h_i \wedge h_j = h_{\min\{i,j\}} ,$$
$$h_i \rightarrow h_j = h_{\min\{5-i+j,5\}} ,$$
$$h_i' = h_i \rightarrow h_1 = h_{6-i} .$$

Then $\mathcal{L}_5 = (L_5, \vee, \wedge, \rightarrow, h_1, h_5)$ is a five-element LIA.

Similarly, let $L_2 = \{c_1, c_2\}$, $c_1 < c_2$, and for any $i, j \in \{1,2\}$, we define that

$$c_1 \vee c_2 = c_2 ,$$
$$c_1 \wedge c_2 = c_1 ,$$
$$c_1' = c_2 , c_1' = c_2 ,$$
$$c_i \rightarrow c_j = c_{\min\{2-i+j,2\}} .$$

Then $\mathcal{L}_2 = (L_2, \vee, \wedge, \rightarrow, c_1, c_2)$ is a two-element LIA. It can represent two opposite sides, the positive side and the negative side.

We construct a ten-element LIA with $\mathcal{L}_2$ and $\mathcal{L}_5$.

Definition 2 For any $(h_i, c_j), (h_k, c_l) \in L_5 \times L_2$, we define as follows

$$(h_i, c_j) \vee (h_k, c_l) = (h_i \vee h_j, c_j \vee c_l),$$

$$(h_i, c_j) \wedge (h_k, c_l) = (h_i \wedge h_j, c_j \wedge c_l), \qquad (1)$$

$$(h_i, c_j)' = (h_i', c_j'),$$

$$(h_i, c_j) \rightarrow (h_k, c_l) = (h_i \rightarrow h_j, c_j \rightarrow c_l).$$

Then $(L_5 \times L_2, \vee, \wedge, \rightarrow, (h_1, c_1), (h_5, c_2))$ is a ten-element LIA, denoted by $\mathcal{L}_5 \times \mathcal{L}_2$. The implication operation in $\mathcal{L}_5 \times \mathcal{L}_2$ is obtained in table 1.

TABLE I

THE IMPLICATION OPERATION IN $\mathcal{L}_5 \times \mathcal{L}_2$

$\rightarrow$	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)
(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)
(h_4,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_5,c_1)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)
(h_3,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)
(h_2,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)	(h_4,c_1)
(h_1,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)	(h_5,c_1)
(h_5,c_1)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)
(h_4,c_1)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)
(h_3,c_1)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)
(h_2,c_1)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_4,c_2)
(h_1,c_1)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)	(h_5,c_2)

The Hasse diagram of $\mathcal{L}_5 \times \mathcal{L}_2$ is depicted in Fig. 1.

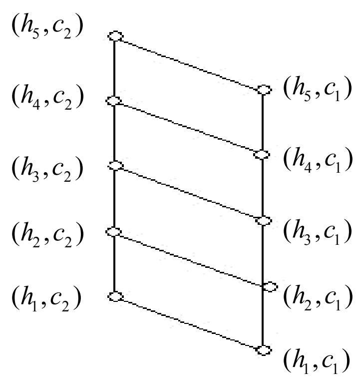

Fig.1 The Hasse diagram of $\mathcal{L}_5 \times \mathcal{L}_2$

Define a five-element linguistic hedges set as AD_5={*Sl*, *So*, *Ex*, *Ve*, *Ab*}, where "*Sl*" means slightly, "*So*" means somewhat, "*Ex*" means exactly, "*Ve*" means very and "*Ab*" means absolutely and an evaluating values set as *EV*={*Y*, *N*}, where "*Y*" means the positive evaluation such as "good" "satisfied" "true" and "*N*" means the negative evaluation such as "bad" "dissatisfied" "false".

Denote $L_{V(5\times 2)} = AD_5 \times EV$. The mapping g: $AD_5 \times EV \rightarrow \mathcal{L}_5 \times \mathcal{L}_2$, is defined as follows.

g(*Ab*, *Y*)= (h_5, c_2), g(*Ve*, *Y*)= (h_4, c_2), g(*Ex*, *Y*)= (h_3, c_2), g(*So*, *Y*)= (h_2, c_2), g(*Sl*, *Y*)= (h_1, c_2), g(*Sl*, *N*)= (h_5, c_1), g(*So*, *N*)= (h_4, c_1), g(*Ex*, *N*)= (h_3, c_1), g(*Ve*, *N*)= (h_2, c_1), g(*Ab*, *N*)= (h_1, c_1).

Then g is a bijection. Denote its inverse mapping as g^{-1}. In addition, for any $x, y \in L_{V(5\times 2)}$,

$$x \vee y = g^{-1}(g(x) \vee g(y)),$$

$$x \wedge y = g^{-1}(g(x) \wedge g(y)),$$

$$x' = g^{-1}(g(x)'),$$

$$x \rightarrow y = g^{-1}(g(x) \rightarrow g(y)).$$

It is easy to prove that $\mathcal{L}_{V(5\times 2)} = (L_{V(5\times 2)}, \vee, \wedge, ', \rightarrow, (Ab, N), (Ab, Y))$ is a LIA, which is called ten-element linguistic-valued lattice implication algebra (10LV-LIA), where *g* is an isomorphic mapping from $\mathcal{L}_{V(5\times 2)}$ onto $\mathcal{L}_5 \times \mathcal{L}_2$.

III. THE APPROACH BASED ON 10LV-LIA

There are some important operations in a lattice implication algebra. We will discuss their special properties in ten-element LIA. In the linguistic decision making approach, we represent the weighted criteria by this operation.

A. The operations in 10LV_LIA.

In a lattice implication algebra *L*, two binary operations $\otimes$ and $\oplus$ were defined as follows.

For any $x, y \in L$,

$$x \otimes y = (x \rightarrow y')';$$

$$x \oplus y = x' \rightarrow y.$$

Similarly, we define the two binary operations $\otimes$ and $\oplus$ in 10LV_LIA.

Definition 3 For any $(h_i, c_j), (h_k, c_l) \in \mathcal{L}_5 \times \mathcal{L}_2$,

$$(h_i, c_j) \otimes (h_k, c_l) = ((h_i, c_j) \rightarrow (h_k, c_l)')'$$
$$= (h_{\max\{i+k-5,1\}}, c_{\max\{j+l-2,1\}}), \qquad (2)$$

$$(h_i,c_j)\oplus(h_k,c_l)=(h_i,c_j)'\to(h_k,c_l)$$
$$=(h_{\min\{i+k-1,5\}},c_{\min\{j+l-1,2\}})\ .$$

The operation $\otimes$ plays a key role when we deal with the multi-criteria decision making problems with linguistic weight in this paper.

As $\mathcal{L}_5\times\mathcal{L}_2$ is a lattice implication algebra, the properties of the operations $\otimes$ and $\oplus$ in 10LV_LIA are hold.

Theorem 1 For any $(h_i,c_j),(h_k,c_l)\in\mathcal{L}_5\times\mathcal{L}_2$,

$$(h_i,c_j)\otimes(h_k,c_l)=(h_i\otimes h_k,c_j\otimes c_l)\ ,$$
$$(h_i,c_j)\oplus(h_k,c_l)=(h_i\oplus h_k,c_j\oplus c_l)\ .$$

Proof. For any $(h_i,c_j),(h_k,c_l)\in\mathcal{L}_5\times\mathcal{L}_2$,

$$(h_i,c_j)\otimes(h_k,c_l)=((h_i,c_j)\to(h_k,c_l)')'$$
$$=((h_i\to h_k'),(c_j\to c_l'))'$$
$$=((h_i\to h_k')',(c_j\to c_l')')$$
$$=(h_i\otimes h_k,c_j\otimes c_l)\ .$$

In the same way, we can prove that $(h_i,c_j)\oplus(h_k,c_l)=(h_i\oplus h_k,c_j\oplus c_l)$.

Theorem 2 For any $(h_i,c_j),(h_k,c_l)\in\mathcal{L}_5\times\mathcal{L}_2$,

$$(h_i,c_j)\otimes(h_k,c_l)\le(h_i,c_j)\ ,$$
$$(h_i,c_j)\otimes(h_k,c_l)\le(h_k,c_l)\ .$$

Proof. Because $k\le 5$, $l\le 2$, then

$$(h_i,c_j)\otimes(h_k,c_l)=(h_{\max\{i+k-5,1\}},c_{\max\{j+l-2,1\}})$$
$$=(h_{\max\{i+(k-5),1\}},c_{\max\{j+(l-2),1\}})$$
$$\le(h_i,c_j)$$

In the same way, we can prove $(h_i,c_j)\otimes(h_k,c_l)\le(h_k,c_l)$.

Corollary 1. Support that x_i is the evaluation of a certain criteria, its ω_i weight is a linguistic value. Then

$$\omega_i\otimes x_i<x_i$$

Theorem 3 For any $(h_i,c_j),(h_k,c_l),(h_s,c_t)\in\mathcal{L}_5\times\mathcal{L}_2$, if $(h_i,c_j)\le(h_k,c_l)$, then

$$(h_i,c_j)\otimes(h_s,c_t)\le(h_k,c_l)\otimes(h_s,c_t)\ ,$$
$$(h_i,c_j)\oplus(h_s,c_t)\le(h_k,c_l)\oplus(h_s,c_t)\ .$$

It can be proved from the Definition 3 easily.

According to the properties of the operations $\otimes$ and $\oplus$ in lattice implication algebra, we know the operation $\otimes$ and $\oplus$ satisfy the Commutative law, i.e., $(h_i,c_j)\otimes(h_k,c_l)$ $=(h_k,c_l)\otimes(h_i,c_j)$, $(h_i,c_j)\oplus(h_k,c_l)$ $=(h_k,c_l)\oplus(h_i,c_j)$. Then

$$(h_s,c_t)\otimes(h_i,c_j)\le(h_s,c_t)\otimes(h_k,c_l)\ ,$$
$$(h_s,c_t)\oplus(h_i,c_j)\le(h_s,c_t)\oplus(h_k,c_l)\ .$$

Corollary 2. Support that x_i , x_j are two evaluations of a certain criteria and ω_i , ω_j are two linguistic weights. If $x_i\le x_j$ and $\omega_i\le\omega_j$, Then

$$\omega_i\otimes x_i\le\omega_j\otimes x_i$$
$$\omega_i\otimes x_i\le\omega_i\otimes x_j$$

That is to say, the criterion is more important, the weighted evaluation value is larger; the evaluation value is larger, the weighted evaluation value is larger.

Theorem 4 For any $(h_i,c_j),(h_k,c_l),(h_s,c_t)\in\mathcal{L}_5\times\mathcal{L}_2$,

$$(h_i,c_j)\otimes(h_5,c_2)=(h_i,c_j)\ ,$$
$$(h_i,c_j)\oplus(h_1,c_1)=(h_i,c_j)\ .$$

We obtain the results of the operation $\otimes$ for 10LV_LIA in table II.

TABLE II

THE OPERATION $\otimes$ IN $\mathcal{L}_5\times\mathcal{L}_2$

$\otimes$	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)
(h_5,c_2)	(h_5,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)
(h_4,c_2)	(h_4,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_1,c_2)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)
(h_3,c_2)	(h_3,c_2)	(h_2,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)
(h_2,c_2)	(h_2,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)
(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_2)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)
(h_5,c_1)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_5,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)
(h_4,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_4,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)
(h_3,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_3,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)
(h_2,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_2,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)
(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)	(h_1,c_1)

B. The decision making approach based on 10LV_LIA

In the linguistic decision making problem, there is always incomparable information. For example, when someone evaluates a shirt, he may say "It looks very good" or "It looks slightly bad". In this case, we use some linguistic values to describe the information, rather than the numerical values. The LIA can express both comparable and incomparable information. For example, (h_4,c_2) and (h_5,c_1) is incomparable, and $g^{-1}(h_4,c_2)=(Ve,\ Y)$, $g^{-1}(h_5,c_1)=(Sl,\ N)$. So we can process the decision making problem based on LIA.

A linguistic multi-criteria decision making problem can be described as follows.

Assume there be a non-empty alternative set

$A=\{A_1,A_2,\cdots,A_N\}$ and $C=\{C_1,C_2,\cdots,C_m\}$ is the criteria set. $W=\{\omega_1,\omega_2,\cdots,\omega_m\}$ is the weight set. ω_i $i=\{1,2,\cdots,m\}$ is the importance degree respect to C_i The decision making approach based on 10LV_LIA phases are summarized as follows.

Step 1. Collect the evaluation data. The evaluation is made with the value in $\mathcal{L}_{V(5\times2)}$. Because the values in $\mathcal{L}_{V(5\times2)}$ are all linguistic value, people can make their evaluations more easily.

Step 2. Transform the values in $\mathcal{L}_{V(5\times2)}$ into ten-element LIA $\mathcal{L}_5\times\mathcal{L}_2$ by the mapping g. For example, the evaluation "The flowers smell very sweet", and if the smell is the flower's j^{th} criteria, then we say the evaluation of the j^{th} criteria of the flower is g(*Ve*, *Y*)=(h_4,c_2) .

Step 3. When the importance of the criteria is different, we take the weight into account. From corollary 2 and corollary 3, the operation $\otimes$ can be used to mark the weighted evaluations. For example, suppose the evaluation of the j^{th} criteria C_j of the i^{th} alternative A_i is $x_{ij}=(h_s,c_t)\in\mathcal{L}_5\times\mathcal{L}_2$, and the weight respect to C_j is $\omega_j=(h_k,c_l)\in\mathcal{L}_5\times\mathcal{L}_2$, then the weighted evaluation is

$$y_{ij}=(h_k,c_l)\otimes(h_s,c_t)\,. \qquad (3)$$

The rule such as "If the flower smells very good, then I will buy it." is represented as follows.

If C_j is (h_4,c_2) , then Q .

Where the Q is the conclusion, and (h_4,c_2) = g(*Ve*, *Y*) ,.

If take the weight into account, and C_j is exactly not important, the rule above is represented like that

If C_j is $(h_4,c_2)\otimes(h_3,c_1)$, then Q.

Step 4. Aggregate the weighted evaluations results obtained in step 3，according to the relation of the alternative. Rank the final results with respect to each alternative.

Because there exists incomparable information, there may be more than one choice.

IV. Illustration Example

There is a flower shop to purchase a number of flowers. Now there are three varieties to choose from: $A=\{A_1,A_2,A_3\}$. The linguistic evaluation is made from four criteria: looks (C_1), smell (C_2), resistance (C_3), price (C_4). The evaluation value is constructed by five linguistic hedges in 10LV_LIA and two opposite evaluating values for each criterion. The evaluating values with respect to "looks" are beautiful (*BE*) and ugly (*UG*), respectively; the evaluating values with respect to "smell" are sweet (*SW*) and bad (*BA*), respectively; the evaluating values with respect to "resistance" are strong (*ST*) and weak (*WE*), respectively; The evaluating values with respect to "price" are cheap (*CH*) and expensive (*EX*), respectively.

TABLE III
The Evaluations to the flowers

	C_1	C_2	C_3	C_4
A_1	(*Ve*, *BE*)	(*So*, *BA*)	(*Ab*, *WE*)	(*Ve*, *EX*)
A_2	(*Ab*, *BE*)	(*Ve*, *SW*)	(*Ve*, *WE*)	(*Ve*, *EX*)
A_3	(*Ve*, *BE*)	(*Ex*, *SW*)	(*Ab*, *ST*)	(*Ve*, *CH*)

From table 3, A_1 looks very beautiful, smells somewhat not bad, with absolutely weak resistance, and very expensive flower variety. The others express in the similar way.

For the shop selling decorative flowers, the looks and smell is more important than resistance. But for the one selling planting flowers, the resistance is more important obviously. Assuming this shop sells planting flowers, the weight set with respect to criteria is $W=\{\omega_1,\omega_2,\omega_3,\omega_4\}$. The evaluating values with respect to weight are important (*I*) and not important (*NI*), respectively

$\omega_1=(Ex,\ NI)$ means that the flower looks is exactly not important;

$\omega_2=(Ab,\ NI)$ means that the smell absolutely not important;

$\omega_3=(Ve,\ I)$ means that the resistance is very important;

$\omega_4=(Ve,\ I)$ means that the resistance is very important;

Table 4 shows the results transforming the 10LV-LIA evaluation to ten-element LIA evaluation by the mapping g from $\mathcal{L}_{V(5\times2)}$ to $\mathcal{L}_5\times\mathcal{L}_2$.

TABLE IV
The Evaluations to the Flowers

	C_1	C_2	C_3	C_4
A_1	(h_4,c_2)	(h_4,c_1)	(h_5,c_1)	(h_4,c_1)
A_2	(h_5,c_2)	(h_4,c_2)	(h_4,c_1)	(h_4,c_1)
A_3	(h_4,c_2)	(h_3,c_2)	(h_5,c_2)	(h_4,c_2)

In the same way, we represent the weight with ten-element LIA.

$\omega_1=(h_3,c_1)$, $\omega_2=(h_5,c_1)$, $\omega_3=(h_4,c_2)$, $\omega_4=(h_4,c_2)$.

We obtain the weighted evaluations using the operation $\otimes$ in ten-element LIA, from (3). For example, the weighted evaluation of the first criteria of $A_1$is (h_3,c_1) $\otimes(h_4,c_2)$ = (h_2,c_1) . Table V is the weighted evaluations.

TABLE V
The Weighted Evaluations to the Flowers

	C_1	C_2	C_3	C_4
A_1	(h_2,c_1)	(h_4,c_1)	(h_5,c_1)	(h_3,c_1)
A_2	(h_3,c_1)	(h_4,c_1)	(h_3,c_1)	(h_3,c_1)
A_3	(h_2,c_1)	(h_3,c_1)	(h_4,c_2)	(h_3,c_2)

Aggregate the evaluations according to the operator

$$(C_1\wedge C_3)\vee(C_2\wedge C_3)\vee(C_4\wedge C_3)=(C_1\vee C_2\vee C_4)\wedge C_3$$

That is to say, the man wants to buy beautiful and strong flowers, or sweet and strong flowers, or cheap and strong flowers. Denote the final result to A_i as q_i.

q_1=(h_4,c_1) , q_2=(h_3,c_1) , q_3=(h_3,c_2) .

Hence, we know that the man does not want to buy the first variety flower very much, and he does not want to buy the second one exactly, and he want to buy the third one exactly.

Rank the q_i.

$q_3> q_1> q_2$.

Then he will buy A_3, which agrees with his intention value more strong resistance.

V. Conclusion

People use natural language to express their preferences in a qualitative decision making problems. This paper proposed a linguistic decision making approach based on ten-element LIA. In this approach, we process linguistic value directly and the both comparable and incomparable information is considered.

The importance of the criteria is always different. For the linguistic weight, we use the operation $\otimes$ to calculate the weighted criteria evaluation. The example illustrates that the proposed approach is effective when we solve the linguistic decision making problems. We will study more about the multi-experts decision making problems with linguistic information.

References

[1] L. A. Zadeh, "Fuzzy logic = computing with words", IEEE Transactions on Fuzzy Systems, vol. 4, no. 2, 1996, pp. 103–111.

[2] L. A. Zadeh, "The concept of linguistic variable and application to approximate reasoning, Part I , Part II, Part III," *Information Science*, vol.8, 1975, pp.199-249, vol.8, 1975, pp.301-357, vol.9, 1975, pp.43-80,.

[3] D. Meng, Z. Pei, "On weighted unbalanced linguistic aggregation operators in group decision making", *Information Sciences*, vol. 223, 2013, pp. 31-41.

[4] J. M. Mendel, H. Wu, "Type-2 fuzzistics for nonsymmetric interval type-2 fuzzy sets: forward problems," *Transactions On Fuzzy Systems*, 2007, vol. 15, no. 5, pp. 916-930.

[5] Y. Xu, S. W. Chen, and J. Ma, "Linguistic truth-valued lattice implication algebra and its properties," *In Proc. CESA'06*, USA, Piscataway, NJ, 2006, pp. 1413–1418.

[6] Y. Xu, X. Li, J. Liu, and D. Ruan, "Determination of α−resolution for lattice-valued first-order logic based on lattice implication algebra," *International Conference on Intelligent Systems and Knowledge Engineering*, China, Chengdu, 2007, pp. 1567–1574.

[7] Y. Xu, J. Liu, D. Ruan, and X.B. Li, "Determination of a-resolution in lattice-valued first-order logic LF(X)," *Information Sciences*, vol. 181, 2011, pp. 1836–1862.

[8] J. J. Lai, and Y. Xu, "Linguistic truth-valued lattice-valued propositional logic system ℓP(X) based on linguistic truth-valued lattice implication algebra," *Information Sciences*. Vol. 180, no.10, 2010, pp.1990–2002.

[9] B. D. Liu, "Membership functions and operational law of uncertain sets," *Fuzzy Optimization and Decision Making*, vol. 11, no. 4, 2012, pp. 387–410.

[10] B. D. Liu, *Uncertain Theory: A Branch of Mathematics for Modeling Human Uncertainty*. Berlin, CA: Springer, 2010.

[11] Z. Li, D. Ruan, and Z. Pei, etc., "A linguistic-valued lattice implication algebra approach for risk analysis," *Journal of Multiple-Valued Logic and Soft Computing*, vol. 17, no. 4, 2011, pp. 293–303.

[12] B. Li, Z. Pei, C.J. Li etc., "Multi-criteria decision-making and extracting fuzzy linguistic summaries based on intuitionistic fuzzy sets," *IEEE International Conference on Fuzzy Systems*, 2009, pp. 1845–1850.

[13] Z. Pei, D. Ruan, J. Liu, and Y. Xu, *Linguistic Values Based Intelligent Information Processing: Theory, Methods and Application*, Singapore, CA: Atlantispress & World Scientific, 2009.

[14] L. Zou, D. Ruan, Z. Pei, and Y. Xu, "A Linguistic-Valued Lattice Implication Algebra Approach for Risk Analysis." *Journal of Multi-Valued Logic and Soft Computing*, 2011, vol.17, no. 4, pp. 293–303.

[15] L. Zou, D. Ruan, Z. Pei and Y. Xu. "A linguistic truth-valued reasoning approach in decision making with incomparable information," *Journal of Intelligent and Fuzzy Systems*, 2008, vol.19, no. (4-5), pp. 335–343.

[16] S. W. Chen, J. Liu, H. Wang, J. C. Augusto, "A linguistic multi-criteria decision making approach based on logical reasoning," I*nformation Sciences*, vol. 258, 2014, pp. 266－276.