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Handling Preferences Under Uncertainty in Recommender Systems

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***Abstract*—While uncertainty can't be ignored in real-world problems, there is almost no research work addressing this issue in the recommender systems framework, especially all that relates to user ratings preferences. Indeed, the subjectivity of user's rating and his/her changing preferences over time, make them subject to uncertainty. Usually, user's imprecise rating for an item (product or service) is time-dependent information and generally provided much later. Meantime the item may change either by degrading or improving its inherent quality. The rating therefore may deviate, since it doesn't describe faithfully the actual current state of the item. This deviation leads to a form of uncertainty on user preferences that we handle in this paper. We show that uncertainty is an ubiquitous aspect in building recommender systems and its taking into account can help predicting the most accurate items by improving their certainty degrees.**

I. Introduction and Motivating Example

Multi-criteria Recommender systems are widespread used in e-commerce as a tool in on-line retail to customize content according to customers' preferences, in order to promote new products and thereby increase sales. They are usually classified into three categories [12]: content-based that recommends items similar to the ones the user preferred in the past, collaborative filtering, which recommends items that users with similar preferences have liked in the past and hybrid approaches that combine content-based and collaborative methods in order to get more accuracy of the recommender. Recommender systems were traditionally based on single-rating systems that have been successfully used in many applications. In contrast, multi-criteria rating systems have started receiving attention in recommender systems research and are regarded as one of the important issues for the next generation of recommender systems. They are being more and more commonly employed in many industries [1]. Multi-criteria ratings may help to better understand each user's preferences, as a result enabling to provide users with more powerful, accurate and focused recommendations, i.e, by recommending for instance a restaurant that will score best on the food criterion, if this is the most important one for some users [2].

To improve accuracy of the recommender, recent research has introduced modelling of user preferences by means of fuzzy logic theory [5] [17]. Such a framework offers more flexibility in expressing user's preferences, which impacts favourably on recommender accuracy. Meanwhile, other research works suggest operating on preference relations instead of absolute preferences, whether they are fuzzy or not. Some encouraging results have proven the validity of these kind of relations and the benefits of their use to improve the accuracy of system recommendations [4] [8] [7]. However and to the best of our knowledge, almost no research studied the uncertainty phenomena in users' fuzzy ratings in the recommender systems framework, although there exists some kind of uncertainty behind using imprecise preferences. Note that some emerging research field dealing with group recommender systems, has already addressed the issue of reaching a joint decision under uncertainty [16].

Yet, uncertainty of users' ratings is so pervasive, that can't be ignored. For instance, a recommender system based on collaborative filtering may recommend to user u_1 an item such as $Tassili$ which is a restaurant that is renowned for its good food according to similar users' past reviews. After consuming the service, user u_1 may be disappointed because the food wasn't that good as recommended by the system. The $Tassili$ restaurant doesn't really meet the user's expectations therefore (s)he may dismiss the system because it becomes less trustworthy. The question is to which extent the system's recommendation was certain? The system relies on users' ratings which normally translate theirs faithful evaluations about the item. However, it turns out that the past users' evaluations are time-dependent that do not reflect the current item's state. Actually, between the moment they consume the service of $Tassili$ restaurant and the moment they evaluate the quality of its food, there's an elapsed time during which the state of the item may change. This leads to uncertainty about whether, the quality of the $Tassili$s food is still good or not. In general, uncertainty occurs whenever information pertaining to a situation is incomplete, contradictory or fluctuating [19]. users' preferences are fuzzy and time-aware ratings, they are automatically subject to uncertainty. In this work, we are mainly concerned by uncertainty of these preferences, more than their fuzzy side. In this paper, we particularly discuss the following contributions: (i) We emphasize the presence of uncertainty's hidden facet in recommender systems, through fuzzy preferences' analysis. (ii) We describe a method to aggregate and to predict items under uncertainty. (iii) We show how to select the highly certain among the top-K most interesting items to recommend to users. The rest of this paper is organized as follows. In Section 2, we present the context of our work and outline the necessity of managing uncertainty in the recommender systems domain. We describe our aggregation process that handle uncertainty dimension in Section 3. In Section 4, we formulate our prediction process of new items to a target user. In Section 5, we propose a new recommendation technique under uncertainty. Finally, we conclude the paper in Section 6.

II. Problem description

Taking into account the temporal dynamics of user preferences is a challenging issue. Recent works highlighted its impact in improving the recommender systems accuracy [15]. In other cases, time-aware ratings influence prediction quality more than various algorithmic enhancements [13] and user tastes are more correlated with recent rated items than old ones [18]. Users' ratings are then time-dependent according to theirs tastes changing or the current state of the items they rate. The recommender system must continuously tune this changing in order to perform more accuracy in recommending interesting items for users. Unfortunately, temporal dynamics of the ratings is rarely taken into account in the recommender systems framework. This lack of information leads to uncertainty about the veracity of the recommendation. In the setting of this work, we first bring out this uncertainty dimension of preference relations in a collaborative recommender system framework, afterwards we look at how to deal with this form of uncertainty in order to improve not only the accuracy of recommendation, but also the users' confidence.

A. Preference relations

Among relatively recent recommender approaches, it has already been proven the benefits of using preference relations over absolute ratings of preferences, to perform more accurate predictions for users [8] [7] [4]. Our work is a part of a collaborative recommendation system where users preferences are expressed in multi-criteria fuzzy ratings through linguistic terms. Preference relations are then deduced from those multi-criteria fuzzy preferences and quantified with a preference intensity degree that expresses to which extent a characterisation is more preferable than another.

1) Fuzzy multi-criteria preferences: Multi-criteria ratings help improving the system accuracy because it can faithfully represent more sophisticated user's preferences.

In this work, we are dealing with user multi-criteria preferences that are expressed qualitatively, using linguistic terms. It's an appropriate way to describe imprecise preferences. The criteria were inspired from those used in *TripAdvisor*[1] the well-known restaurant recommender system, which are *Food*, *Service*, *Value* and *Atmosphere* and the five linguistic terms to characterize a criterion or an item preference are: *Terrible*, *Poor*, *Average*, *Very good* or *Excellent*. Each linguistic term is represented through a triangular fuzzy number $\tilde{T}=(t_1, t_2, t_3)$, which defines a certain precision around a value [3]. The linguistic terms' distribution on the real line, is depicted through figure 1 below.

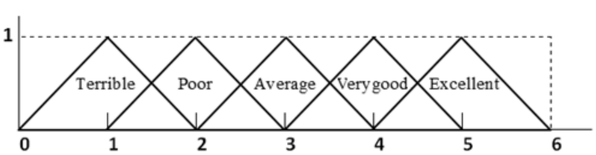

Fig. 1: Fuzzy triangular numbers scale

[1]More information is available in www.tripadvisor.com/Restaurants.

It is worth noting that, when criteria ratings are provided in crisp numerical scale, it is quite easy to translate each crisp rating in its corresponding linguistic term, provided that the number of values for both scales are the same. However, when there are much more, crisp ratings than fuzzy qualitative quantifiers to translate in, we split the crisp ratings into intervals that corresponds to our rating scale. We assign each interval's rating to one of the fuzzy qualitative quantifiers corresponding to inherent order of the values.

We assign a weight w_i to each fuzzy number that represents a criterion preference, according to its predominance among the others. The reason is that, some criteria are more prominent to the user than the others, such as the $Food$ quality may be more important than the $Value$ for him and therefore, the two criteria shouldn't be treated equally on the same rating scale. By doing so, we emphasize more the highest predominant criteria preference intensity among the others.
If predominant criteria respect the fuzzy values' rankings, one can assign *1*, for instance to the most predominant criterion preference that corresponds to the highest fuzzy number and deduce the remaining values respectively as summarized in the table I below [3]:

Fuzzy number	(0,1,2)	(1,2,3)	(2,3,4)	(3,4,5)	(4,5,6)
Weight w_i	0.2	0,4	0.6	0.8	1.0

TABLE I: Weights distribution

Example 1: Criteria preferences for *"Tassili"*

As an example, assume that a user u_1's criteria preferences for a restaurant item, such as $Tassili$, are described in the following table II:

Criterion(c_i)	Preference level(i)	Fuzzy number ($\tilde{c_i}$)	Coefficient (w_i)
Food	Excellent	(4,5,6)	1.0
Service	Very good	(3,4,5)	0.8
Value	Average	(2,3,4)	0.6
Atmosphere	Terrible	(0,1,2)	0.2

TABLE II: Criteria preferences for *"Tassili restaurant"*

Which can be represented on the real-line of the following figure 2 below:

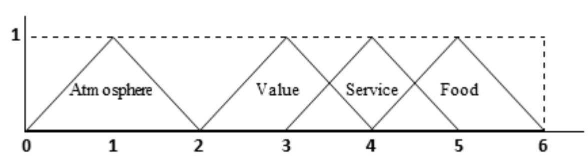

Fig. 2: Fuzzy criteria preferences of *Tassili* restaurant

Example 2: Item preferences

The rating of $Tassili$ is deduced from it's criteria ratings by calculating their fuzzy weighted arithmetic average (See [3] for more details):

$$\tilde{T_1} = \frac{\sum_{i=1}^{n} w_i \times \tilde{c_i}}{\sum_{i=1}^{n} w_i} \tag{1}$$

Where $\tilde{T_1} = (\tilde{t_1}, \tilde{t_2}, \tilde{t_3})$ is the fuzzy rating of the item, $\tilde{c_i}$ the rating of the criterion i among n available criteria and w_i the weight of the fuzzy number $\tilde{c_i}$. Therefore, when the item is *”Tassili”* and the criteria are *’Food’*, *”Service”*, *”Value”* and *”Atmosphere”*, translated into their respective fuzzy ratings, namely $\tilde{c_1} = (0, 1, 2)$, $\tilde{c_2} = (2, 3, 4)$, $\tilde{c_3} = (3, 4, 5)$ and $\tilde{c_4} = (4, 5, 6)$, the overall rating $\tilde{T_1}$ of the item is then estimated as follows:

$$\begin{aligned} \tilde{t_1} &= \tfrac{(1.0\times4)+(0.8\times3)+(0.6\times2)+(0.2\times0)}{1.0+0.8+0.6+0.2} \approx 3 \\ \tilde{t_2} &= \tfrac{(1.0\times5)+(0.8\times4)+(0.6\times3)+(0.2\times1)}{1.0+0.8+0.6+0.2} \approx 4 \\ \tilde{t_3} &= \tfrac{(1.0\times6)+(0.8\times5)+(0.6\times4)+(0.2\times2)}{1.0+0.8+0.6+0.2} \approx 5 \end{aligned} \quad (2)$$

Therefore the rating of *”Tassili”* is the fuzzy number $\tilde{T_1} = (3, 4, 5)$ which corresponds to the linguistic term *”Very good”*.

Similarly, a user u_1, can have other item preference ratings, for instance assume that there are *”Terrible”* for *”Atlas”* and *”Average”* for *”Cirta”* whose ratings were deduced as above, namely $\tilde{T_3} = (0, 1, 2)$ and $\tilde{T_2} = (2, 3, 4)$. The user u_1's item preferences are then summarized in the figure 3 as follows:

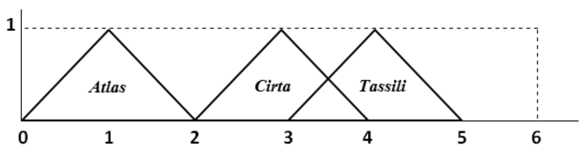

Fig. 3: Fuzzy item preferences

2) Extracting preference relations intensity: A preference relation P on a set of fuzzy preference alternatives $\tilde{T} = (\tilde{T_1}, \tilde{T_2}, ..., \tilde{T_n})$ is a scalar number α on the product $\tilde{T} \times \tilde{T}$, which represents the separating surface area between two fuzzy alternatives $(\tilde{T_i}, \tilde{T_j})$. The preference relations can be represented by $(n \times n)$ matrix $P = (p_{ij}) = \alpha_{(\tilde{T_i}, \tilde{T_j})}$ $\forall i, j \in \{1, 2, ..., n\}$, which characterizes to which extent a preference alternative $\tilde{T_i}$ is more preferred than another $\tilde{T_j}$, i.e degree or intensity of the alternative $\tilde{T_i}$ over $\tilde{T_j}$. Therefore $\tilde{T_i}$ is α more preferable than $\tilde{T_j}$. For successive *n* linguistic terms, we record *n* possible surface values of α.
Generally, the surface can be deduced from any $(\tilde{T_i}, \tilde{T_j})$ fuzzy preferences, as follows:

$$\alpha_{(\tilde{T_i}, \tilde{T_j})} = \begin{cases} 0 & \text{if } \tilde{T_j} = \tilde{T_i} \\ 0.25 & \text{if } \tilde{T_j} = \tilde{T_i} + \tilde{1} \\ k-1 & \text{if } \tilde{T_j} = \tilde{T_i} + \tilde{k} \quad \forall\, k > 1 \end{cases} \quad (3)$$

Where $\tilde{1} = (1, 1, 1)$ and $\tilde{k} = (k, k, k)$. In the context of this work, possible values of α are $\{0, 0.25, 1, 2, 3\}$, since we are dealing with five successive linguistic terms.

Let consider the previous figure 3, for which we want to estimate the surfaces area between pairs of its three fuzzy numbers, namely $\tilde{T_1} = (3, 4, 5)$, $\tilde{T_2} = (2, 3, 4)$ and $\tilde{T_3} = (0, 1, 2)$ representing respectively *”Tassili”*, *”Cirta”* and *”Altas”*, as illustrated in figure 4 below:

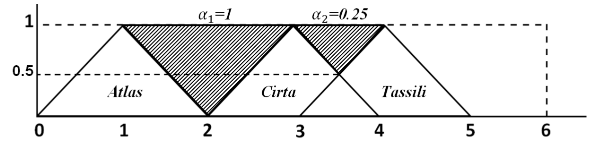

Fig. 4: Pairwise surface evaluation

Through the equation 3, we can express each fuzzy number according to another fuzzy number and by doing so, the surface between them is easily determined.

$\tilde{T_2} = (\tilde{T_1} + \tilde{2}) \Rightarrow k = 2 \Rightarrow \alpha_{1(\tilde{T_1}, \tilde{T_2})} = (k-1) = 1$
$\tilde{T_3} = (\tilde{T_1} + \tilde{3}) \Rightarrow k = 3 \Rightarrow \alpha_{3(\tilde{T_1}, \tilde{T_3})} = (k-1) = 2$
$\tilde{T_3} = (\tilde{T_2} + \tilde{1}) \Rightarrow \alpha_{2(\tilde{T_2}, \tilde{T_3})} = 0.25$, which estimates the triangle area between the two fuzzy numbers $\tilde{T_2}$ and $\tilde{T_3}$.

Example 3: Preference relations matrix of u_1
1. The table III shows the preference relations matrix $P_{(u_1, Tassili)}$ corresponding to the user u_1's criteria preferences for, *Tassili* restaurant:

	Atmosphere	Value	Service	Food
Atmosphere	0			
Value	1	0		
Service	2	0.25	0	
Food	3	1	0.25	0

TABLE III: Criteria preference relations matrix $P_{(u_1, Tassili)}$

For instance, *Food* is *3* more preferable than *Atmosphere*, *1* more preferable than *Value* and *0.25* more preferable than *Service*.

2. The preference relations matrix P_{u_1} corresponding to the above user u_1's item preferences, is then summarized in the table IV as follows:

	Atlas	Cirta	Tassili
Atlas	0		
Cirta	1	0	
Tassili	2	0.25	0

TABLE IV: Item preference relations matrix P_{u_1}

For instance, *Cirta* restaurant is *1* more preferable than *Atlas* but *-0.25* less preferable than *Tassili*.

B. Uncertainty modeling

User preferences are levied from two different but complementary points of view. Firstly, preference relations offer users a means to express their preferences more naturally than using fuzzy preferences individually. That is for instance, *”I like Tassili restaurant more than Cirta”* is much more faithful than *”I like Tassili restaurant very much”* and *”Cirta about average”*. In this context, preference relations have the benefit to overcome not only the users’ subjectivity but also the indistinguishable preferences owing to a reduced scale of values. Secondly, given a fuzzy preference of an item such as the restaurant

$Tassili$ for instance, to which extent this preference is certain?
It's worth noting that given the changes over time of preferences and the user's intrinsic subjectivity, preferences become subject to uncertainty. Thenceforth, the $Tassili$ preference may deviate since it doesn't describe faithfully the actual state of the restaurant.
In this regard, whether the preference is certain or not, is an important issue which should be reviewed more closely. The reason why is, the certainty of the preferences helps setting more reliable recommender systems that are able to predict interesting items with high degree of certainty and thereby improve user trust. Thereby, our preference relations are characterized with two kind of metrics which are the preference relation intensity, between two alternatives and the certainty degree of their relationship.

1) Extracting item preferences uncertainty: Users evaluate well after, an item that they have already consumed since a while. From the time when they consumed the item till they rated it, there is a lack of information indicating that the sustainability of the item's quality, is somewhat tainted with uncertainty. Thus, the uncertainty is so pervasive in such dynamic environment, that can't be ignored. One way to fill such information missing is to assess uncertainty through an analysis of users' reviews.

As mentioned in the introduction, there are at least two causes of uncertainty: the variability of phenomena and the incompleteness of the available information [10]. Users may review differently the same item over time, according to their tastes' changing, the current service quality that is offered and mainly for their subjectivity. However, apart from users' subjectivity, the ratings must be consistent and close enough since they are related roughly to the same temporal state of the item. It turns out that, contrary assessments are source of uncertainty since there is no agreed opinion about an item's rating. This form of uncertainty caused by the more or less heterogeneous / conflictual users' ratings, can be estimated via the Shannon entropy for instance.

$$H(P) = -\sum_{j=1}^{n} p_j \times log_b\ (p_j) \tag{4}$$

Where p_j is the probability of the outcome j. Because we are dealing with five possible state of preferences corresponding to our fuzzy numbers, we select the base b equals to 5 in order to normalize the uncertainty degrees.

Example 4: Computing uncertainty for *"Tassili restaurant"*
Assume the following ratings case of *"Tassili"* restaurant, as shown in table V, that has been rated at time $t = 0$ by users u_1, u_2, u_3, u_4, u_5 and u_6, as denoted below:

r_{u_1}	r_{u_2}	r_{u_3}	r_{u_4}	r_{u_5}	r_{u_6}
Very good	Average	Average	Poor	Very good	Average
(3,4,5)	(2,3,4)	(2,3,4)	(1,2,3)	(3,4,5)	(2,3,4)

TABLE V: Different item ratings for *"Tassili restaurant"*

The Shannon entropy is then estimated in table VI as follows:

j	$P(p_j = j)$	$log_5\ (p_j)$	$p_j \times log_5\ (p_j)$
(1,2,3)	0.17	-1.11	-0.19
(2,3,4)	0.50	-0.43	-0.22
(3,4,5)	0.33	-0.68	-0.23
$H(P) = -\sum_{j=1}^{n} p_j \times log_5(p_j)$			0.63

TABLE VI: Shannon entropy for *"Tassili restaurant"*

Shannon entropy measures the uncertainty, that is the error between different users' rating from which we extrapolate the certainty measure. This latter expresses to which extent the user ratings were certain about *"Tassili"* restaurant. Notice that it has already been mentioned by [9], that the entropy and imprecision capture the same facet of uncertainty, albeit in different guises. We take then into account this uncertainty to characterize an item overall rating, that is $R^t_{item} = (r_{user}, \eta)$, where r is the rating of the item that has been generated from the user criteria ratings and η is the certainty degree (one can take this degree equals to 1 minus the measure of Shannon entropy) deduced by the system. For *"Tassili"* restaurant for instance, it will be $R^0_{Tassili} = (\textit{Very good}, 0.37)$.

2) Expressing preference relations uncertainty: Given two item preference alternatives $\tilde{T}_i$ and $\tilde{T}_j$ whose respective certainty degrees are η_i and η_j, the question is, what is the common certainty degree η of their preference relation $p_{(\tilde{T}_i,\tilde{T}_j)}$? Intuitively, the two alternatives $\tilde{T}_i$ and $\tilde{T}_j$ are at least as certain as the minimum of their inherent certainty degrees η_i and η_j, which represents a kind of a minimum consensus of certainty between both. Therefore $\eta(p_{(\tilde{T}_i,\tilde{T}_j)}) = min(\eta_i, \eta_j)$.

Example 5: Preference relations uncertainty for u_1
Let's suppose that the certainty values for the user u_1 item preferences are: $\eta_1(Tassili) = 0.37$, $\eta_2(Atlas) = 0.58$ and $\eta_3(Cirta) = 0.25$, represented through the figure 5 below:

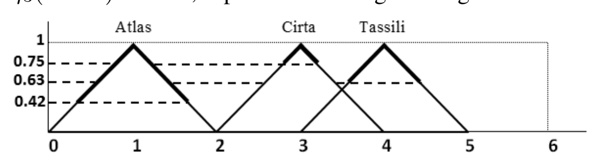

Fig. 5: Uncertain fuzzy item preferences

The table VII illustrates the overall certainty measure between the item relations is denoted as follows:

	Atlas	Cirta	Tassili
Atlas	0.58		
Cirta	0.25	0.25	
Tassili	0.37	0.25	0.37

TABLE VII: Certainty degrees of the user u_1's preference relations

For example, $Tassili$ and $Atlas$ mutual certainty degree is equals to 0.37.

Handling both preference relations intensity and certainty degree as rating for the relation between two alternatives is then expressed by $r_u(\alpha_{(\tilde{T}_i,\tilde{T}_j)}, \eta_{(\tilde{T}_i,\tilde{T}_j)})$, where $\alpha_{(\tilde{T}_i,\tilde{T}_j)}$ is the intensity degree of preference and $\eta_{(\tilde{T}_i,\tilde{T}_j)}$ is the certainty degree, between two fuzzy preference alternatives $\tilde{T}_i$ and $\tilde{T}_j$ for user a u.

Example 6: Preference relations ratings for user u_1

The preference relations ratings M_{u_1} for both preferences intensities and certainty degrees, for a user u_1, will be expressed through the table VIII below:

	Atlas	Cirta	Tassili
Atlas	(0 , 0.58)		
Cirta	(1 , 0.25)	(0 , 0.25)	
Tassili	(2 , 0.37)	(0.25 , 0.25)	(0 , 0.37)

TABLE VIII: Preference relations ratings matrix M_{u_1} of a user u_1

For instance on one hand, *Tassili* restaurant is *2* more preferable than *Atlas*, which both have their consensual certainty degree equals to *0.37*. On the other hand, *Cirta* is *-0.25* less preferable than *Tassili*, with their mutual certainty degree equals to *0.25*.

III. Aggregation process

A. Users' similarity dual metric

One of the most approaches commonly used in the recommender systems framework, to measure similarity between pairs of users, is the famous Pearson correlation coefficient. It discards the fact that there is a possible difference between users' habits ratings, to make them comparable. Roughly, even if some users' ratings are slightly different from the others, there's still a linear correlation of ratings and hence the similarity can be sensed [12].

$$Sim_{(u,v)} = \frac{\sum_{i \in S_{(u,v)}} (r_{u_i} - \overline{r_u})(r_{v_i} - \overline{r_v})}{\sqrt{\sum_{i \in S_{(u,v)}} (r_{u_i} - \overline{r_u})^2} \sqrt{\sum_{i \in S_{(u,v)}} (r_{v_i} - \overline{r_v})^2}} \quad (5)$$

Where u and v is a pair of users, r_{u_i}, r_{u_i} are the users u's and v's rating of items i and j, $S_{(u,v)}$ is the items set that users u and v have both rated in common and $\overline{r_u}$ and $\overline{r_v}$ are the average of non-zero ratings of the two users. Given both criteria and items matrix of all users; the aim is to choose a target user for whom we want to predict some un-rated items with high degree of certainty. We want to strengthen the user confidence toward the system so that it becomes a more trustworthy recommender, on which user can rely. For this purpose, we select the most similar user to him/her. Two users are considered as similar, when first of all, they order resources the same way, even if they don't rate them identically, thereafter they agree on a minimum certainty degree. In our context, we apply the Pearson correlation on preference relations instead of absolute ratings baseline, taking into account their temporal dimension and the fact that they are also subject to uncertainty.

As for the temporal dimension, we claim that similarity between two users is better detected for any item that was rated by both, referring the same period of time of the item's state since it wouldn't have changed too much. The ratings that were assigned into the same period will be then, more valuable than those belonging to different periods and therefore for discerning the two categories, we over-weighted with *0.75* the former and *0.25* the latter (See formulas 6 and 7). The reason why, we want to emphasize more the ratings among those with less variability of time than those which are temporarily far from each other.

Concerning the preference relations uncertainty, combined together, they produce an overall uncertainty equals to the minimum of their inherent values. It reflects the fact that the preference relations are at least fairly certain, to each other about the items' assessment. The similarity is gradually built, taking into account firstly the time-aware dimension of preference relations, secondly the uncertainty dimension, finally the overall composed similarity between pairs of users, as illustrated bellow:

1) Time-aware preference relations similarity: The similarity is based on the degree α of each preference relation, for which we consider its time-stamp information $t = |t_i - t_j|$ of the related preferences ($\tilde{T}_i$, $\tilde{T}_j$).

$$Sim^{(t)}_{(u,v)} = \frac{(0.75) \times \left(\sum_{i \in S_{(u,v)}} (\alpha_{u_i} - \overline{\alpha_u})(\alpha_{v_i} - \overline{\alpha_v}) \right)^t}{\left(\sqrt{\sum_{i \in S_{(u,v)}} (\alpha_{u_i} - \overline{\alpha_u})^2} \right)^t \left(\sqrt{\sum_{i \in S_{(u,v)}} (\alpha_{v_i} - \overline{\alpha_v})^2} \right)^t} \quad (6)$$

$$Sim^{(t_j,t_k)}_{(u,v)} = \frac{(0.25) \times \sum_{i \in S_{(u,v)}} (\alpha_{u_i} - \overline{\alpha_u})^{t_j} \ (\alpha_{v_i} - \overline{\alpha_v})^{t_k}}{\left(\sqrt{\sum_{i \in S_{(u,v)}} (\alpha_{u_i} - \overline{\alpha_u})^2} \right)^{t_j} \left(\sqrt{\sum_{i \in S_{(u,v)}} (\alpha_{v_i} - \overline{\alpha_v})^2} \right)^{t_k}} \quad (7)$$

After calculating the arithmetic mean of all similarities of the same period's evaluations $\overline{Sim^{(t)}_{(u,v)}}$ and the arithmetic mean of all similarities of different period's evaluations $\overline{Sim^{(t_j,t_k)}_{(u,v)}}$, the overall temporal similarity will be:

$$Sim^{Temporal}_{(u,v)} = \overline{Sim^{(t)}_{(u,v)}} + \overline{Sim^{(t_j,t_k)}_{(u,v)}}.$$

2) Uncertainty similarity: The certainty degree, of an item which has been rated by a particular user, is estimated through the variation of previous reviews of the other users for the same item. The users' similarity regarding the uncertainty dimension is composed of the minimum of their certainty degrees, which is considered as a form of certainty consensus between them about common items' ratings. For instance, if a user u_1's certainty degree, for instance is more specific than users u_2 and u_3, then all what is certain for u_2 and u_3 is also at least as certain as u_1. Therefore,

u_1, u_2 and u_3 are at least similar at the minimum of their certainty degrees. The uncertainty similarity of two users u an v users, is then estimated as follows:

$$Sim^{Uncertainty}_{(u,v)} = Min(\eta_u, \eta_v) \tag{8}$$

where η_u and η_v are the certainty degrees of users u and v respectively, for the common rated items. The overall composed similarity between user u and user v is therefore:

$$Sim^{Overall}_{(u,v)} = (Sim^{Temporal}_{(u,v)} \ , \ Sim^{Uncertainty}_{(u,v)}) \tag{9}$$

We select then the highly certain among the top-K most similar users to aggregate their test items ratings for the target user u_1.

B. Preference relations aggregation

Several operators were proposed to aggregate preference relations such as OWA, IOWA in group decision-making and LOWA to solve group decision making problems from individual linguistic preference relations and many others techniques [6][11]. In our context, given a target user u_1 to whom we want to predict some un-rated items and a set of his/her similar users $U_k = \{u_2, u_3, \dots, u_k\}$, the preference of u_1 for an unrated item i can be predicted by aggregating, first the ratings of the criteria preference relations pairs of users U_k, after their certainty degrees of the same item i.

1. For the criteria preference relations pairs $\alpha_{(\tilde{T_a},\tilde{T_b})}$ of the test item i, the aggregation consists in compiling the weighted sum of similar users u_k's criteria preference relations matrix into a unique outcome matrix, as follows:

$$\alpha_{(\tilde{T_a},\tilde{T_b})} = \sum_{j=1}^{k} sim(u_1, u_j) \cdot [\alpha_{(\tilde{T_a},\tilde{T_b})}]^{u_j} \tag{10}$$

Where $[\alpha_{(\tilde{T_a},\tilde{T_b})}]^{u_j}$ are the criterion preference relation to be aggregated for each user $u_j \in U_k$ and $sim(u_1, u_j)$ is the similarity between the target user u_1 and the user u_j which acts as a weight. The reason why is, the more a user u_j is similar to the target user u_1, the more his/her preference relations are involved in the aggregating process.

2. Aggregating certainty η_{u_1} of the test item i of users U_k consists in applying the minimum operator on their inherent certainty degrees $\{\eta_{u_2}, \eta_{u_3}, \dots, \eta_{u_k}\}$. Therefore $\eta_{u_1} = min(\eta_{u_2}, \eta_{u_3}, \dots, \eta_{u_k})$.

Example 7: Aggregating criteria preference relations of (u_2, u_3)

Let's take a situation summarized in table IX and X where criteria matrices of two identical preference restaurants test items are ***'Essofra'*** and ***'Jenina'*** so that $P_{(u_2,Essofra)}$ equals to $P_{(u_2,Jenina)}$ and $P_{(u_3,Essofra)}$ equals to $P_{(u_3,Jenina)}$, of two users u_2 and u_3, similar to the target user u_1, such as:

$$Sim^{Essofra}_{(u_1,u_2)} = (0.95\,,\,0.37) \qquad Sim^{Essofra}_{(u_1,u_3)} = (0.89\,,\,0.56)$$

$$Sim^{Jenina}_{(u_1,u_2)} = (0.95\,,\,0.80) \qquad Sim^{Jenina}_{(u_1,u_3)} = (0.89\,,\,0.92)$$

	Atmosphere	Value	Service	Food
Atmosphere	0			
Value	-2	0		
Service	-3	-0.25	0	
Food	0	2	3	0

TABLE IX: Criteria relations matrix $P_{(u_2,Essofra)}$ and $P_{(u_2,Jenina)}$

	Atmosphere	Value	Service	Food
Atmosphere	0			
Value	-0.25	0		
Service	0.25	1	0	
Food	2	3	1	0

TABLE X: Criteria relations matrix $P_{(u_3,Essofra)}$ and $P_{(u_3,Jenina)}$

The aggregation of the two users matrix is processed in two steps.

1. First, we aggregate the two matrices $P_{(u_2,Essofra)}$ and $P_{(u_3,Essofra)}$ and similarly $P_{(u_2,Jenina)}$ and $P_{(u_3,Jenina)}$ by adding preference relations degrees of $P_{(u_2,Essofra)}$ and $P_{(u_3,Essofra)}$ and thus, $P_{(u_2,Jenina)}$ and $P_{(u_3,Jenina)}$ that we multiply by their corresponding users similarities, for each pair of criteria alternatives. The similar aggregate outcome matrix $A_{(Essofra)}$ (similarly $A_{(Jenina)}$), presented in table XI, will be as follows:

	Atmosphere	Value	Service	Food
Atmosphere	0			
Value	-2.12	0		
Service	-2.63	0.65	0	
Food	1.78	4.57	3.74	0

TABLE XI: Aggregated relations matrix $A_{(Essofra)}$ and $A_{(Jenina)}$

2. The second step of the process is to specify the item's overall certainty by aggregating the certainty degrees of both users u_2 and u_3. Therefore the certainty degree for the two items $Essofra$ and $Jenina$ will be:

$$\eta^{Essofra}_{u_1} = min^{Essofra}(\eta_{u_2}, \eta_{u_3}) = min(0.37, 0.56) = 0.37$$

$$\eta^{Jenina}_{u_1} = min^{Jenina}(\eta_{u_2}, \eta_{u_3}) = min(0.92, 0.80) = 0.80$$

The steps of the aggregation process are summarized into the algorithm "*CriMatrixAggr*" below:

Algorithm 1 : *CriMatrixAggr*

Input: u : Target user , I_u: Test items list of u
P=$\{P_{(u_1,i)}, \cdots , P_{(u_k,i)}\}$: k Criteria matrices of test item i
$[\alpha]^u_{(\tilde{T}_a,\tilde{T}_b)}$: Relation in $A_{(u,i)}$ between $(\tilde{T}_a, \tilde{T}_b)$ of user u
$[\alpha]^{u_j}_{(\tilde{T}_a,\tilde{T}_b)}$: Relation in $P_{(u_j,i)}$ between $(\tilde{T}_a, \tilde{T}_b)$ of user u_j
S_u=$(sim_{(u,u_1)}$, $sim_{(u,u_2)}$, ..., $sim_{(u,u_k)})$: k similarity degrees list, between (u,u_j) / j= 1, 2, 3, ..., k
η^i_u: Uncertainty value for the test item i
Output: $A_{(u,i)}$: Aggregated matrix for test item i of user u

/* Aggregating criteria preference relations */
for all test item $i \in I_u$ **do**
  Initialize $A_{(u,i)}$ with $P_{(u_1,i)}$
  for all Preference relation $(\tilde{T}_a, \tilde{T}_b) \in A_{(u,i)}$ **do**
    $[\alpha]^u_{(\tilde{T}_a,\tilde{T}_b)} = ([\alpha]^u_{(\tilde{T}_a,\tilde{T}_b)} \times sim_{(u,u_1)}$
  end for
  $\eta^i_u = \eta_u$
  for all $(P_{(u_j,i)} \in P)$ s.t. $(j = 2$ **to** $k)$ **do**
    for all Preference relation $(\tilde{T}_a, \tilde{T}_b) \in A_{(u,i)}$ **do**
    $[\alpha]^u_{(\tilde{T}_a,\tilde{T}_b)}$= $[\alpha]^u_{(\tilde{T}_a,\tilde{T}_b)} + ([\alpha]^{u_j}_{(\tilde{T}_a,\tilde{T}_b)} \times sim_{(u,u_j)})$
    end for
    /* Aggregating certainty of the test item i */
    $\eta^i_u = min(\eta^i_u, \eta_{u_j})$
  end for
end for

IV. Prediction process

The prediction of the criteria preference relations is achieved by rounded dividing each preference relation intensity with the sum of the involved similarities. Therefore, the prediction for criteria preference relations of both test items $Essofra$ and $Jenina$ for the target user P_{u_1} is given by the same following matrix (Table XII):

	Atmosphere	Value	Service	Food
Atmosphere	0			
Value	-1	0		
Service	-1	0	0	
Food	1	2	2	0

TABLE XII: Predicted criteria preference relations for both $Essofra$ and $Jenina$

Some preference relations intensities, such as $\alpha_{(Value,Atmosphere)}$ and $\alpha_{(Service,Atmosphere)}$ may be revised, in order to preserve not only the transitivity of the relation but also the coherence of total order of the preference relations. For instance, $\alpha_{(Value,Atmosphere)}$ and $\alpha_{(Service,Atmosphere)}$ must be equal to *-0.25* (see [3] for more detail). Hence, possible criterion fuzzy ratings that correspond to this preference relations matrix are presented in the table XIII below:

Criterion	Atmosphere	Value	Service	Food
Fuzzy numbers alternatives	(2,3,4) (1,2,3)	(1,2,3) (0,1,2)	(1,2,3) (0,1,2)	(4,5,6) (3,4,5)

TABLE XIII: Possible criteria ratings

By computing the fuzzy weighted average of those criteria, the item fuzzy rating outcome will be either *(2,3,4)* or *(3,4,5)* which correspond to the linguistic terms *Average* and *Very good*. We choose from those two alternatives of item fuzzy ratings, the one that is consistent and coherent with the initial item preference relations matrix of the target user. It turns out that in this example the two alternatives are eligible and one can be chosen indifferently, for instance the *(2,3,4)* rating. Note that, the certainty degree for the items *Essofra* is *0.37* and *Jenina* is *0.80*, and therefore the initial item preference relations matrix of user u_1 will be updated as illustrated in the following table XIV:

	Atlas	Cirta	Tassili	Essofra	Jenina
Atlas	(0 , 0.58)				
Cirta	(1 , 0.25)	(0 , 0.25)			
Tassili	(2 , 0.37)	(0.25 , 0.25)	(0 , 0.37)		
Essofra	(1 , 0.37)	(0 , 0.25)	(-0.25 , 0.37)	(0 , 0.37)	
Jenina	(1 , 0.58)	(0 , 0.25)	(-0.25 , 0.37)	(0 , 0.37)	(0 , 0.80)

TABLE XIV: Item preference relations prediction matrix M_{u_1}

V. Recommendation process

The recommendation step is a decision aiding problem. It relates to the issue of choosing one or more potential action(s) from a set of alternatives, ranking them in a descending order, or sorting them into predefined ordered categories. In this context, there are various ways to present recommendations to the user; either by offering the best item (choosing), or by presenting the top-K items as a recommendation list (ranking), or by classifying the items into categories, i.e. *'highly recommended'*, *'fairly recommended'*, *'not recommended'* (sorting) [14]. Note that a recommender system that provides uncertain predictions for items is less reliable and easily dismissed. Besides, a trustworthy system is the one that delivers credible predictions to increase user confidence. Accordingly, what matters is the ability to recommend items with high level of certainty, in such a way to increase the confidence of the system users. And because we target the certainty of recommendations for increasing the trust and confidence of the user towards the system, it's more advisable to recommend the highly certain among the top-K most interesting items. To do so, the top-K most interesting items are suggested to users, sorted by certainty degree and classified into categories such as *'highly recommended'* and *'fairly recommended'*. The *'highly recommended'* are the most certain predicted items and the *'fairly recommended'* are items which certainty level is about average. For our example, *Jenina* will be *'highly recommended'* and *Essofra 'fairly recommended' to the target user* u_1.

VI. Conclusion and future works

In this work, we formalize the collaborative filtering recommendation as a time-dependent prediction under uncertainty problem. The uncertainty occurs whenever an item is affected through time by changing users' assessments. That's the case of preferences when users' ratings are time-dependent and so tainted with uncertainty. Although, the topic of uncertainty addressed hasn't yet fully matured in the literature, we have attempted to emphasize its impact, for enhancing user confidence towards the system and providing more accurate recommendations. For this purpose, we have developed a model that supports uncertainty pertaining to user preferences, that we have extrapolated from the Shannon entropy. Once done, we discuss an aggregation method based on the minimum operator which guarantees a minimum consensus among similar users about the ratings of common items. Then, we recommend to each target user, the highly and the fairly certain among the top-K most interesting items, in order to increase his/her confidence. As future works, we intend to dig deeper in this hidden facet of uncertainty of user's preferences, implement the developed model and conduct some experimentations, to prove and show the validity of our approach.

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