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The Simplest Interval Type-2 Fuzzy PID Controller: Structural Analysis

Ahmet Sakallı, Tufan Kumbasar, M. Furkan Dodurka, Engin Yesil

Abstract— **In this paper, we will present analytical derivations of the simplest the Interval Type-2 Fuzzy PID (IT2-FPID) controller output which is composed of only 4 rules. Thus, we will first propose a new visualizing method called Surface of the Switching Points (S-MAP) in order to better analyze the derivation of the Switching Points (SPs) of the Karnik-Mendel algorithms. We presented mathematical explanation of the S-MAP and showed that the SPs are determined by only two Boundary Functions (BFs) for the simplest IT2-FPID controller. We will then give the simplified analytical derivation of the simplest IT2-FPID controller around the steady state via the employed BFs and S-MAP. We have illustrated that the simplest IT2-FPID controller is in fact analogous to a conventional PID controller around the steady state. We presented the simplest IT2-FPID controller output in terms of the parameters of the antecedent IT2-FSs. We examined the effect of the design parameter over IT2-FPID control system performance. In the light of the observations, we presented a simple self-tuning mechanism to enhance the transient state and disturbance rejection performance.**

Keywords—Simplest Interval type-2 fuzzy PID controllers; Karnik- Mendel Method, Switicing Points.

I. INTRODUCTION

Recently, the research focus is on Interval Type-2 Fuzzy Logic Controllers (IT2-FLCs) since the they achieve satisfactory and robust control performance because of the additional degree of freedom provided by the Footprint of Uncertainty (FOU) in their Membership Functions (MFs) [1-2]. However, since the IT2-FLCs employ Interval Type-2 Fuzzy Sets (IT2-FSs), a Type-Reduction (TR) procedure is required to compute the crisp output. The Karnik-Mendel (KM) method is widely used as the TR method [3]-[4]. IT2-FLCs have been much interest especially in control applications and the superiority of the IT2-FLCs over the type-1 counterparts is shown in [1], [5]-[12]. In [1], it is shown that the IT2-FLCs have better handle with uncertainties and unknown dynamics. The control surface of the IT2-FLC around the origin is smoother than Type-1 Fuzzy Logic Controllers (T1-FLC), thus the risk of oscillations is reduced and the disturbances cause smaller changes in control signals [5], [13]. Although the advantages of the IT2-FLCs have been shown, the systemic design and stability analysis of the IT2-FLCs is still open problem since the analytic derivation of the IT2-FLCs is relatively more complex in comparison to its type-1 counterpart [13], [14]. So far, the stability analysis of the interval type-2 fuzzy model based control systems is examined in [15] and the stability of the IT2-FLCs is presented in [16] by using an alternative type-reduction method [17] (which enables to write the output of the IT2-FLC in a closed form). Moreover, analytical derivations of the Interval Type-2 Fuzzy PI and PD (IT2-FPI and IT2-FPD) controllers are obtained in [18] and [19] by dividing the input space to several number of sub-regions. Kumbasar [11] derive the closed-from formulation of the single input Interval Type-2 Fuzzy PID (IT2-FPID) controller. The research interest on this topic still continues to understand and analyze the internal structure of the IT2-FLCs better and design more systematic controllers.

In this study, we will present analytical structural analysis of the simplest IT2-FPID controller which is composed of four rules. We will present a novel graphical method called Surface of Switching Points Map (S-MAP) to visualize and understand better the variation of the Switching Points (SPs) selection of the KM algorithm. The S-MAP is a novel plot that illustrates the SP selection of the KM algorithm which is based on the Boundary Function (BF) based KM (BF-KM) [21]. The BF-KM TF method is an enhancement to the original KM TR algorithms and eliminates the iterative nature of the KM algorithm. With the aids of the S-MAP and BF-KM, we will derive the closed-from formulation of the simplest IT2-FPID controller around the steady state and show that the IT2-FPID controller is analogous to a PID controller. We will present the output of the simplest IT2-FPID controller in terms of the parameters of the antecedent IT2-FSs of the rules. We will present a detailed investigation of the design parameter to show and examine the effect of this parameter over IT2-FPID control system performance. In the light of the observations, we will present a simple self-tuning mechanism and the effectiveness is shown on simulation studies.

The paper is organized as follows. Section II gives brief information about the general structure of the simplest IT2-FPID controllers. Section III presents the proposed the S-MAP. Section IV presents the analytical structure of the simplest IT2-FPID controllers and a simple tuning mechanism. Section VI includes the conclusions.

II. THE STRUCTURE OF THE SIMPLEST IT2-FPID CONTROLLERS

In this section, we will first present the general structure of the IT2-FPID controllers and then give the properties of the simplest IT2-FPID controller studied in this paper. The IT2-FPID controllers are constructed by choosing the inputs

A. Sakalli, T. Kumbasar, M. F. Dodurka and E. Yesil are with the Control and Automation Engineering Department, Istanbul Technical University (e-mail: {sakallia, kumbasart, dodurkam, yesileng} @itu.edu.tr)

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as error (e) and derivative of error ($\dot{e}$) and the output as the control signal (u) as illustrated in Fig. 1. Here, the input scaling factors (SFs) K_e and K_d normalize the inputs to the universe of discourse where the MFs of the two inputs are defined. Thus, the inputs e and $\dot{e}$ are transformed to E and $\dot{E}$ while the output (U) of the fuzzy controller is scaled by the output SFs K_a and K_b to the control signal (u) as follows:

$$u = K_a U + K_b \int U\,dt \tag{1}$$

The fuzzy rules of the simplest IT2-FLC are

$$R_q:\ \text{IF}\ \ E \text{ is } \tilde{A}_i \ \text{ and } \ \dot{E} \text{ is } \tilde{B}_j \ \ \text{THEN} \ \ U \text{ is } C_q,\quad q = 1, \ldots, Q = 4 \tag{2}$$

where $\tilde{A}_i$ and $\tilde{B}_j$ $(i, j \in \{1,2\})$ are the Interval Type-2 Fuzzy Sets (IT2-FSs) which are the antecedent MFs of the inputs E and $\dot{E}$ respectively, C_q is the consequent crisp set and Q is the total number of the rules. The crisp output of the IT2-FLC can be described with the type reduced sets as shown in [20], then, the output of the IT2-FLC can be formulized as:

$$U = \frac{U_l + U_r}{2} \tag{3}$$

where U_l and U_r are the end points of the type reduced sets and determined as follows:

$$U_l = U_l^{L^*} = \min(U_l^L) \tag{4}$$

$$U_l = U_l^{R^*} = \max(U_r^R) \tag{5}$$

where $U_l^{L^*}$ and $U_l^{R^*}$ are the type reduced set calculated by the KM algorithm with respect to the optimal SFs (L^* and R^*) that minimize and maximize respectively following equations:

$$U_l^L = \frac{\sum_{q=1}^{L} \overline{f_q}\, C_q + \sum_{q=L+1}^{Q=4} \underline{f_q} C_q}{\sum_{q=1}^{L} \overline{f_q} + \sum_{q=L+1}^{Q=4} \underline{f_q}} \tag{6}$$

$$U_r^R = \frac{\sum_{q=1}^{R} \underline{f_q} C_q + \sum_{q=R+1}^{Q=4} \overline{f_q} C_q}{\sum_{q=1}^{R} \underline{f_q} + \sum_{q=R+1}^{Q=4} \overline{f_q}} \tag{7}$$

where L and R are the candidate SPs of the KM algorithm and $\underline{f_q}$ and $\overline{f_q}$ represent the lower and upper bounds of a Firing Interval (FI) (F_q) which are calculated as

$$\underline{f_q} = \underline{\mu}_{\tilde{A}_i} \cap \underline{\mu}_{\tilde{B}_j} \tag{8}$$

$$\overline{f_q} = \overline{\mu}_{\tilde{A}_i} \cap \overline{\mu}_{\tilde{B}_j} \tag{9}$$

$$F_q = \left[\underline{f_q}, \overline{f_q}\right] \tag{10}$$

where $\underline{\mu}_{\tilde{A}_i}$ and $\overline{\mu}_{\tilde{A}_i}$ are the Lower MFs (LMFs) and Upper MFs (UMFs) of the input E, $\underline{\mu}_{\tilde{B}_j}$ and $\overline{\mu}_{\tilde{B}_j}$ are the LMFs and UMFs of the input $\dot{E}$ respectively, $\cap$ denotes the t-norm operator which is the algebraic product in this study.

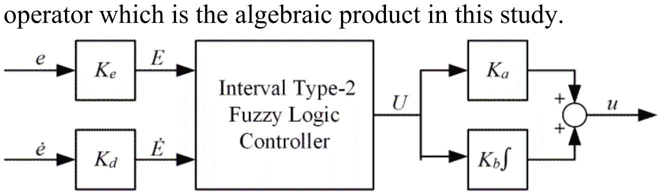

Fig. 1. IT2-FPID controller structure

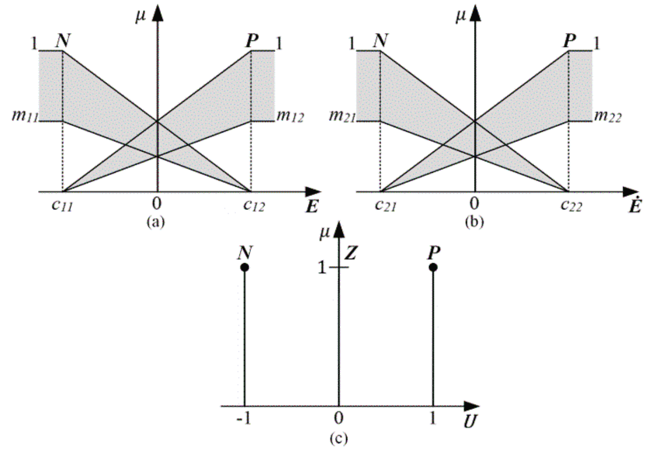

Fig. 2. The antecedent and consequent MFs of IT2-FLC

In the simplest IT2-FPID controller structure, we will define the antecedent MFs of IT2-FLC with two uniformly distributed symmetrical triangular IT2-FSs as shown in Fig. 2a and Fig. 2b and the employed IT2-FSs can be described with two parameters, the core of the UMF and LMF (c_{ij}) and the height of the LMF (m_{ij}). Here, for the employed IT2-FLC structure, $\tilde{A}_1$, $\tilde{A}_2$, c_{11}, c_{12}, m_{11} and m_{12} are used for the error input (E) while $\tilde{B}_1$, $\tilde{B}_2$, c_{21}, c_{22}, m_{12} and m_{22} are used for the derivative of the error input ($\dot{E}$). In the employed IT2-FLC, the cores of the IT2-FSs are assigned as $c_{11}= c_{21}=-1$ and $c_{12}= c_{22}=1$ in order to have universe of discourse and the heights of the LMFs are assumed to equal, i.e. $m_{11}=m_{12}=m_{21}=m_{22}=M$ to generate symmetrical FOU in each IT2-FS. The linguistic terms *Negative (N)* and *Positive (P)* are represented by $\tilde{A}_1$ and $\tilde{A}_2$ IT2-FSs for the first input (E), besides for the second input ($\dot{E}$) they are characterized by $\tilde{B}_1$ and $\tilde{B}_2$. The IT2-FS antecedent MFs of the employed IT2-FLC can be expressed as follows:

$$\underline{\mu}_{\tilde{A}_1} = \left(\frac{1-E}{2}\right)M, \qquad \overline{\mu}_{\tilde{A}_1} = \left(\frac{1-E}{2}\right) \tag{11}$$

$$\underline{\mu}_{\tilde{A}_2} = \left(\frac{1+E}{2}\right)M, \qquad \overline{\mu}_{\tilde{A}_2} = \left(\frac{1+E}{2}\right) \tag{12}$$

$$\underline{\mu}_{\tilde{B}_1} = \left(\frac{1-\dot{E}}{2}\right)M, \qquad \overline{\mu}_{\tilde{B}_1} = \left(\frac{1-\dot{E}}{2}\right) \tag{13}$$

$$\underline{\mu}_{\tilde{B}_2} = \left(\frac{1+\dot{E}}{2}\right)M, \qquad \overline{\mu}_{\tilde{B}_2} = \left(\frac{1+\dot{E}}{2}\right) \tag{14}$$

where M is the height of the LMFs. The MFs of the consequent part are defined with three singleton consequents which are *Negative* (N)=-1, *Zero* (Z)=0 and *Positive* (P)=1 as shown in Fig.2c. In this context, the simplest IT2-FPID controller has totally four rules as shown in Table 1.

TABLE 1 - THE TYPE-2 FUZZY RULE BASE

If	E is N	and	$\dot{E}$ is N	Then	U is N		
If	E is N	and	$\dot{E}$ is P	Then	U is Z		
If	E is P	and	$\dot{E}$ is N	Then	U is Z		
If	E is P	and	$\dot{E}$ is P	Then	U is P		

Hence, the firing intervals of each rule can be expressed as:

$$F_1=\left[\underline{f_1},\ \overline{f_1}\right]=\left[\underline{\mu}_{\tilde{A}_1}\cap\underline{\mu}_{\tilde{B}_1},\ \ \overline{\mu}_{\tilde{A}_1}\cap\overline{\mu}_{\tilde{B}_1}\right]$$
$$=\left[\frac{1-E-\dot{E}+E\dot{E}}{4}M^2,\ \ \frac{1-E-\dot{E}+E\dot{E}}{4}\right] \tag{15}$$

$$F_2=\left[\underline{f_2},\ \overline{f_2}\right]=\left[\underline{\mu}_{\tilde{A}_1}\cap\underline{\mu}_{\tilde{B}_2},\ \ \overline{\mu}_{\tilde{A}_1}\cap\overline{\mu}_{\tilde{B}_2}\right]$$
$$=\left[\frac{1-E+\dot{E}-E\dot{E}}{4}M^2,\ \ \frac{1-E+\dot{E}-E\dot{E}}{4}\right] \tag{16}$$

$$F_3=\left[\underline{f_3},\ \overline{f_3}\right]=\left[\underline{\mu}_{\tilde{A}_2}\cap\underline{\mu}_{\tilde{B}_1},\ \ \overline{\mu}_{\tilde{A}_2}\cap\overline{\mu}_{\tilde{B}_1}\right]$$
$$=\left[\frac{1+E-\dot{E}-E\dot{E}}{4}M^2,\ \ \frac{1+E-\dot{E}-E\dot{E}}{4}\right] \tag{17}$$

$$F_4=\left[\underline{f_4},\ \overline{f_4}\right]=\left[\underline{\mu}_{\tilde{A}_2}\cap\underline{\mu}_{\tilde{B}_2},\ \ \overline{\mu}_{\tilde{A}_2}\cap\overline{\mu}_{\tilde{B}_2}\right]$$
$$=\left[\frac{1+E+\dot{E}+E\dot{E}}{4}M^2,\ \ \frac{1+E+\dot{E}+E\dot{E}}{4}\right] \tag{18}$$

Then, the derived FIs are used in the type-reduced set formulation of the IT2-FLC given in Equations (6) and (7). Clearly, the FIs are the functions of inputs (E and $\dot{E}$) and the height of the LMFs (M) then the crisp output of the IT2-FLC (U) is determined with respect to E, $\dot{E}$ and M. Therefore the height of the LMFs, M, is the only design parameter of the simplest IT2-FPID controller. Hence, the performance of the simplest IT2-FPID directly depends on the choice of M parameter.

III. Surface of Switching Points Map

In this section, we will discuss the switching point selection of the KM algorithm and then using two BFs we will employ a method called Boundary Functions Based KM (BF-KM) [21] for determining the switching points of the simplest IT2-FPID controller.

The output of the IT2-FLC is determined with respect to the optimal switching points (L^* and R^*) in Equations (4) and (5). In this context, the KM algorithm is used to determine the switching points for computing U_l and U_r [20] and the KM algorithm calculates the switching points iteratively with respect to Equations (6) and (7) by following conditions:

$$C_L\le U_l^L\le C_{L+1},\qquad L=1,\dots,Q-1=3 \tag{19}$$

$$C_R\le U_r^R\le C_{R+1},\qquad R=1,\dots,Q-1=3 \tag{20}$$

As mentioned in the previous section, in the employed simplest IT2-FPID controller structure there are only four consequents (C_1=N=-1, C_2=Z=0, C_3=Z=0, C_4=P=1) and four fuzzy IF-THEN rules. Initially, these consequents have been sorted in ascending order. In the simplest IT2-FLC structure, there are only three possible SPs ($\forall\, L,R\in[1,2,3]$) are obtained as given in Equations (19) and (20). Also, it is shown in [21] that the SPs never equal to 2 for the simplest IT2-FLC. Therefore, only three feasible SPs which KM algorithm will compute, are determined as {L=1, R=1}, {L=1, R=3} and {L=3, R=3}. Reminding that L^* is the SP that minimizes U_l and R^* is the SP that maximizes U_r with respect to Equations (6) and (7). Then, the optimal SPs must satisfy the following conditions (KM conditions):

$$L^*=\begin{Bmatrix}1 & \text{if} & U_l^1\le U_l^2 & \text{and} & U_l^1\le U_l^3\\ 2 & \text{if} & U_l^2\le U_l^1 & \text{and} & U_l^2\le U_l^3\\ 3 & \text{if} & U_l^3\le U_l^1 & \text{and} & U_l^3\le U_l^2\end{Bmatrix} \tag{21}$$

$$R^*=\begin{Bmatrix}1 & \text{if} & U_r^1\ge U_r^2 & \text{and} & U_r^1\ge U_r^3\\ 2 & \text{if} & U_r^2\ge U_r^1 & \text{and} & U_r^2\ge U_r^3\\ 3 & \text{if} & U_r^3\ge U_r^1 & \text{and} & U_r^3\ge U_r^2\end{Bmatrix} \tag{22}$$

where U_l^w and U_r^w are the candidate solutions of type reduced sets (in Equations (6) and (7)) calculated with respect to w, and w is the candidate value of the corresponding SP. By solving Equations (4) and (5) in terms of Equations (19) and (20) after substituting Equations (11) to (18), the BFs for the selection of the optimal SPs (L^* and R^*) can be obtained as follows:

$$B_L(\dot{E},M)=\frac{1-\dot{E}-M^2-\dot{E}M^2}{1-\dot{E}+M^2+\dot{E}M^2} \tag{23}$$

$$B_R(\dot{E},M)=\frac{1+\dot{E}-M^2+\dot{E}M^2}{-1-\dot{E}-M^2+\dot{E}M^2} \tag{24}$$

Here, the BFs are determined as a function of only $\dot{E}$ and M. The optimal SPs (L^* and R^*) are determined with respect the following conditions:

$$L^*=\begin{Bmatrix}1 & \text{if} & E\le B_L(\dot{E},M)\\ 3 & \text{if} & E>B_L(\dot{E},M)\end{Bmatrix} \tag{25}$$

$$R^*=\begin{Bmatrix}1 & \text{if} & E\le B_R(\dot{E},M)\\ 3 & \text{if} & E>B_R(\dot{E},M)\end{Bmatrix} \tag{26}$$

Thus, we can determine the SPs of the KM algorithm with no need to perform the iterative procedure of the KM algorithm. Using the two BFs given in Equations (23) and (24) we can easily visualize the SPs. With the help of this analysis, we will present a novel visualizing method called the Surface of Switching Points Map (S-MAP) which helps us to understand and analyze the selection of SPs of the KM algorithm without using its iterative procedure. In this visual method, the values of the SPs are project to a three dimensional space with respect to the input variables. In the three-dimensional space, a point is expressed with three terms; the values of E and $\dot{E}$ (for x-axis and y-axis) and the SPs values (for z-axis).

We will consider four cases in order to show how the S-MAP can be used for analyzing the selection of SPs of the KM for the simplest IT2-FPID controller. Thus, the BFs defined in Equations (23) and (24) for obtaining S-MAPs are illustrated in Fig. 3 when the height of the LMFs is selected as M=0.2, 0.5, 0.8, 1. Then, the S-MAPs is obtained using the BFs and the optimal SPs given in Equations (25) and (26). As shown in Fig. 4a, Fig. 4b and Fig. 4c, there are only three different colored regions with respect to the three SPs, which are mentioned above for the simplest IT2-FPID controller; the blue region for {L=1, R=1}, the green region for {L=1, R=3} and the red region for {L=3, R=3}.

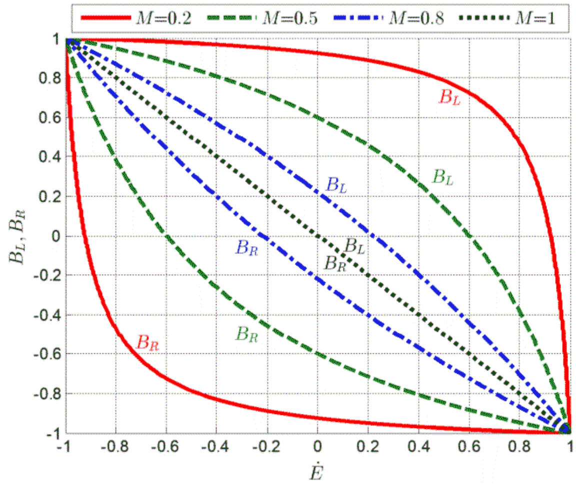

Fig. 3. The boundary functions for M=0.2, 0.5, 0.8, 1

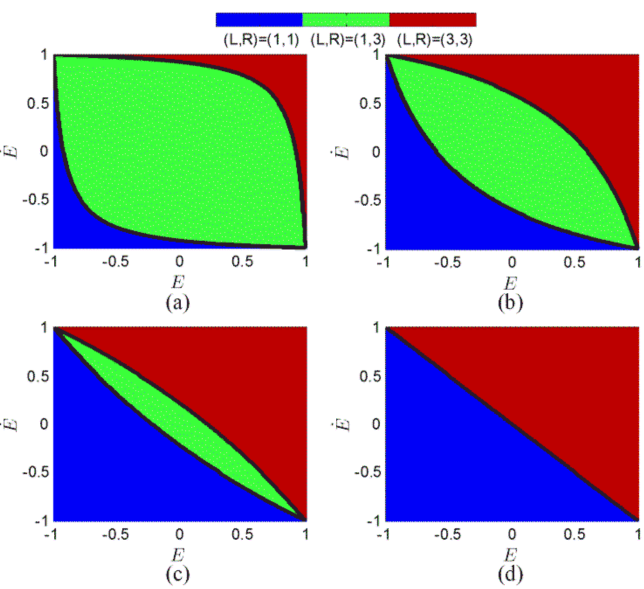

Fig. 4. The S-MAPs for (a) M=0.2, (b) M=0.5, (c) M=0.8, (d) M=1

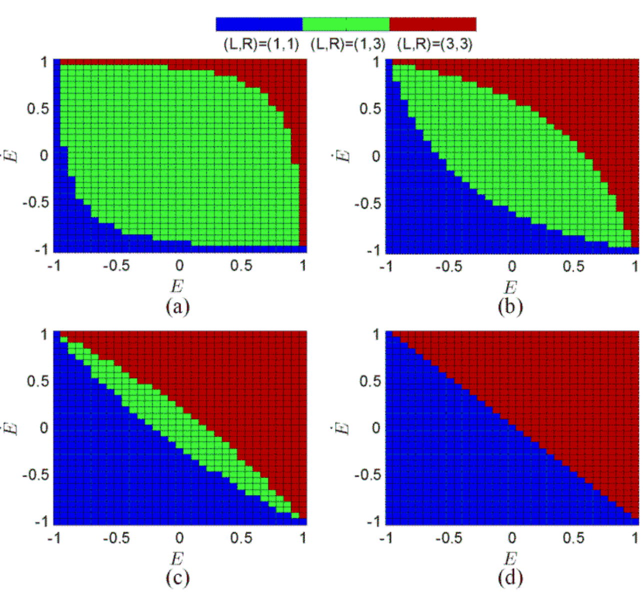

Fig. 5. The SPs found using the KM algorithm by quantizing the input space for (a) M=0.2, (b) M=0.5, (c) M=0.8, (d) M=1

The same results can be obtained when the KM algorithm would be used in the input space ($E \in$[-1, 1] and $\dot{E} \in$[-1, 1]) and iteratively. The SPs are solved by various iterations after quantizing the input space. Fig. 5 shows the results obtained for M={0.2, 0.5, 0.8, 1} and here the black lines represent the quantized input values. As it can easily be observed, the proposed S-MAPs illustrated in Fig. 4a, Fig. 4b, Fig. 4c and Fig. 4d exactly matches with the SPs obtained from the KM algorithm illustrated in Fig.5, Fig. 5b, Fig. 5c and Fig. 5d respectively. In addition, it should be stated that when M is chosen to be 1, as expected the simplest IT2-FLC reduces to the T1-FLC since the LMFs and the UMFs fully overlap and only two regions on the S-Map appear since the FOU vanishes. The detailed information that gives more generalized remarks on this subject can be found in [21].

Consequently, the outcomes of the S-MAP analysis can be interpreted for simplest IT2-FPID controller as follows:

- The left and the right SPs of the KM algorithms can be clearly displayed corresponding to the values of the inputs, E and $\dot{E}$, in the S-MAP
- Each region on the S-MAP points out different SPs of the KM algorithm (for instance {L=1, R=3})
- The size of the FOU (in terms of M) changes the area around the origin of S- MAP
- The three regions on the S- MAP are determined by two conditions given in Equations (25) and (26) with two BFs given in Equations (23) and (24).

IV. The Analytical Structure of The Simplest IT2FPID Controller

The main goal of the analytical analysis of the S-MAP presented in the previous section is to determine the switching points of the KM algorithm using a conditional expression involving two quite simple BFs with respect to E, $\dot{E}$ and M. This observation will lead us to derive the output of the IT2-FLC (U) in a closed-form formulation since the iterative nature of the KM algorithm is eliminated for the particular circumstances.

As it is known, when the value of M decreases, the size of the FOU increases, and as observed from analysis of the S-MAP of the simplest IT2-FPID controller, the region at where L^*=1 and R^*=3 and which is located around the origin (E=$\dot{E}$=0), expands. From the point of the control theory view, the origin of the S-MAP characterizes the steady-state point of the closed-loop system since the error (e) and derivative of error ($\dot{e}$) of the system are defined as

$$E = K_e e \quad (27)$$

$$\dot{E} = K_d \dot{e} \quad (28)$$

approach to zero when the inputs of the IT2-FLC approach to zero. Then, the motivating result of this remark will lead us further analysis performed only on the region of the origin as the robustness of IT2-FPI controllers are examined in [14].

The output of the simplest IT2-FLC can be defined with three output terms with respect to the SPs that can be derived easily by the BFs given in Equations (23) and (24). Hence, the output of the IT2-FLC can be expressed as follows:

$$U = \begin{cases} U_{11} & if \quad E \le B_L \quad and \quad E \le B_R \\ U_{13} & if \quad E \le B_L \quad and \quad E > B_R \\ U_{33} & if \quad E > B_L \quad and \quad E > B_R \end{cases} \tag{29}$$

where

$$U_{11} = \frac{\left(-\overline{f_1} + \underline{f_4}\right)}{2\left(\overline{f_1} + \underline{f_2} + \underline{f_3} + \underline{f_4}\right)} + \frac{\left(-\underline{f_1} + \overline{f_4}\right)}{2\left(\underline{f_1} + \overline{f_2} + \overline{f_3} + \overline{f_4}\right)} \tag{30}$$

$$U_{13} = \frac{\left(-\overline{f_1} + \underline{f_4}\right)}{2\left(\overline{f_1} + \underline{f_2} + \underline{f_3} + \underline{f_4}\right)} + \frac{\left(-\underline{f_1} + \overline{f_4}\right)}{2\left(\underline{f_1} + \underline{f_2} + \underline{f_3} + \overline{f_4}\right)} \tag{31}$$

$$U_{33} = \frac{\left(-\overline{f_1} + \underline{f_4}\right)}{2\left(\overline{f_1} + \overline{f_2} + \overline{f_3} + \underline{f_4}\right)} + \frac{\left(-\overline{f_1} + \underline{f_4}\right)}{2\left(\underline{f_1} + \underline{f_2} + \underline{f_3} + \overline{f_4}\right)} \tag{32}$$

In this study, we will present and examine the analytic derivations of the simplest IT2-FLC around the steady state; therefore U_{13} is more essential for this study because it represents the region around the origin (the green region in S-MAP). By using FIs and (31), U_{13} can be expressed as

$$U_{13} = \frac{A\dot{E} + AE + C(E\dot{E}^2 + \dot{E}\,E^2)}{B + 4CE\dot{E} + D\dot{E}^2 + DE^2 + DE^2\dot{E}^2} \tag{33}$$

where

$$\begin{aligned} A &= (6M^2 + 2M^4), \qquad & B &= (1 + 3M^2)^2, \\ C &= (2M^2 - 2M^4), \qquad & D &= (-1 + 2M^2 - M^4) \end{aligned} \tag{34}$$

Here, U_{13} constructs a nonlinear control law, which depends on a nonlinear function of the height of the LMFs, M. The simplified output of the simplest IT2-FLC around the origin can be approximated for

$$\frac{1 + \dot{E} - M^2 + \dot{E}M^2}{-1 - \dot{E} - M^2 + \dot{E}M^2} < E \le \frac{1 - \dot{E} - M^2 - \dot{E}M^2}{1 - \dot{E} + M^2 + \dot{E}M^2} \tag{35}$$

when the nonlinear terms E^2, $\dot{E}^2$ and $E\dot{E}$ are taken as zero. Then, U_{13} can be expressed simply as

$$U_{13} = V(M)E + V(M)\dot{E} \tag{36}$$

where $V(M)$ is the nonlinear gain and defined as

$$V(M) = \frac{A}{B} = \frac{6M^2 + 2M^4}{1 + 6M^2 + 9M^4} \tag{37}$$

Then by substituting Equations (1) and (36), the closed-form formulation of the simplest IT2-FPID controller for the steady state can be constructed as follows:

$$u = K_a V(M)\left(E + \dot{E}\right) + K_b V(M) \int \left(E + \dot{E}\right) dt \tag{38}$$

By substituting Equations (27) and (28) into Equation (38), we obtain the following control law

$$\begin{aligned} u = {} & K_a V(M)(K_e e + K_d \dot{e}) \\ & + K_b V(M) \int (K_e e + K_d \dot{e})\, dt \end{aligned} \tag{39}$$

After rearranging the terms in Equation (39), we obtain the control law around the origin as follows:

$$\begin{aligned} u = {} & V(M)(K_a K_e + K_b K_d)e \; + V(M) K_b K_e \int e\, dt \\ & + V(M) K_a K_d \dot{e} \end{aligned} \tag{40}$$

Furthermore, for the sake of the simplicity, the ratios of input SFs (α) and output SFs (β) and the additional term (γ) which is a function of ratios of SFs (α and β), is defined as:

$$K_d = \alpha K_e \tag{41}$$

$$K_b = \beta K_a \tag{42}$$

$$\gamma = 1 + \alpha\beta \tag{43}$$

After substituting Equations (41)-(43) into (40), the simpler expression of the control law is obtained as follows:

$$\begin{aligned} u = {} & V(M) K_a K_e (1 + \alpha\beta) e \; + V(M) K_a K_e \beta \int edt \\ & + V(M) K_a K_e \alpha \dot{e} \\ = {} & V(M) K_a K_e \gamma \left(e + \frac{\beta}{\gamma} \int edt + \frac{\alpha}{\gamma} \dot{e}\right) \end{aligned} \tag{44}$$

Clearly, the control law of the simplest IT2-FPID is equivalent to the conventional PID controller structure given in Equation (45). However the main difference between the simplest IT2-FPID controller and the conventional PID controller is that the gain of the IT2-FPID controller is variable while the gain of the conventional PID controller is static. The PID controller formulation is given below.

$$u = K_c \left(e + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right) \tag{45}$$

Here, K_c represents the static controller gain for the conventional PID controller and the variable controller gain for the simplest IT2-FPID controller. By matching Equations (44) and (45) term by term, we obtained the variable controller gain (K_c), integral time constant (T_i) and derivative time constant (T_d) of the simplest IT2-FPID controller for the steady state as follows:

$$K_c = V(M) K_a K_e \gamma \tag{46}$$

$$T_i = \gamma/\beta \tag{47}$$

$$T_d = \alpha/\gamma \tag{48}$$

Consequently, we obtained the closed-form formulation of the simplest IT2-FPID controller around the steady state in terms of the nonlinear function $V(M)$ and the SFs. As it can be seen from the above derivations, $V(M)$ changes directly the proportional gain of the simplest IT2-FPID controller. Thus, we can adjust the proportional gain of the simplest IT2-FPID controller by simply tuning the value, M. Hence, the change of $V(M)$ respect to M is analyzed and illustrated in Fig. 6. From the prior knowledge of the IT2-FLCs, we can obtain a T1-FLC when M is chosen as 1, then the value of nonlinear gain turns $V(M)$ to be 0.5 as shown in Fig. 6. An interesting observation from Fig. 6 is that we also obtain a T1-FLC when M is assigned as 0.448. The gain of the simplest IT2-FPID controller increases which means that the closed–loop system has aggressive behavior when the nonlinear function $V(M)$ is high, similarly, the closed-loop system has sluggish behavior if $V(M)$has relatively a low value. Consequently, we will obtain a smoother controller than T1-FPID when M is chosen less than 0.448. Also, when M is chosen between 0.448 and 1, we will be designing an IT2-FPID controller more aggressive than type-1 counterpart.

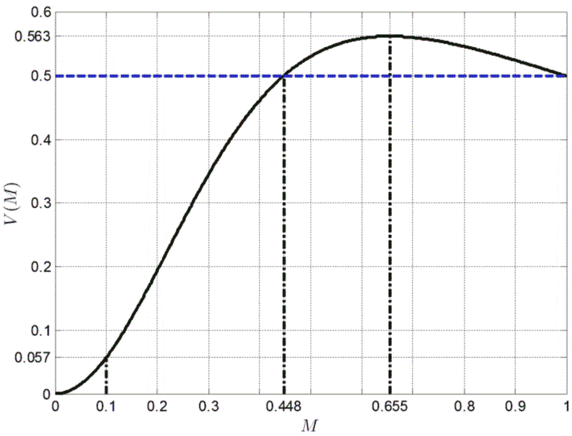

Fig. 6. The change of $V(M)$ with respect to the height of the LMFs, M

As it is known from the control theory, there is a tradeoff between the set point tracking and the disturbance rejection performances [22]. If a closed loop system has an aggressive behavior, then the disturbances can be rejected quickly but high overshoots and oscillations will occur. On the other hand, if a closed loop system has a sluggish behavior, then the overshoots and oscillations reduce while the disturbances are rejected in longer time. Using these observations and above mentioned assumptions, the design parameter M can be assumed as the online tuning parameter which directly changes the gain of the simplest IT2-FPID controller. In this context, we consider the obtained closed-form formulation of IT2-FPID controller for the steady state (in Equations (45) to (48)) in order to improve both set point tracking and disturbance rejection performances for determining and tuning the value of M in an online manner.

In order to improve the closed-loop control performance the following meta-rules are obtained:

- When the system output is around the set point, the gain should be larger in order to reject disturbances as fast as possible;
- When the system is far away from the set point, the gain should be smaller in order to reduce overshoots.

So, we claim that after a set point change the controller gain should increase by time, so overshoots and oscillations will be reduced with negligible comprise of the settling time. In addition, the disturbances it will be quickly rejected. Then, we consider only possible IT2-FPID gains corresponding to the interval of $M \in [0.1, 0.448]$ since we desire to more smooth controller action. The lower value of M is set as 0.1, since any value less than 0.1 results a very low controller gain, which extremely slows the closed loop response.

The following simple heuristic function, which has only error as the input, is proposed for tuning M in an online manner:

$$M(E) = (1 - |E|)h_2 + h_1 \tag{49}$$

Here, h_1 determines the feasible lower limit of $V(M)$, which is set to 0.1, and h_1+h_2 determines the upper limit of the $V(M)$, which is set to 0.448. Note that, h_1 and h_2 assure the minimum and maximum values of the $V(M)$ since $V(M)$ is monotonic for $M \in [0.1, 0.448]$ interval as seen in Fig. 6.

V. Simulation Study

In order to show the benefit of the proposed simple tuning method, the unit step and disturbance rejection performances of the Self Tuning IT2-FPID (STIT2-FPID), IT2-FPID and T1-FPID controllers are examined via simulations on the first order plus dead time processes defined with the following transfer function:

$$G(s) = \frac{K}{\tau s + 1} e^{-\theta s} \tag{50}$$

So as to make a fair assessment the overshoot (%OS), the settling time (Ts) and the Integral Absolute Error (IAE) performance measures are considered. The robustness against parameter uncertainty of the proposed method is also investigated. In this context, four different cases are considered; the nominal system $G_0(s)$, (K=1, τ=2, θ=0.6) and the three perturbed systems; $G_1(s)$, (K=1.3, τ=1, θ=1.2), $G_2(s)$, (K=0.7, τ=2.3, θ=0.9), $G_3(s)$, (K=1.5, τ=1.5, θ=0.3).

The consequent and antecedent MFs of T1-FLC and IT2-FLCs are selected as mentioned in Section II. The SFs are set to K_e=1, K_d=0.2, K_a=2, K_b=2, note that the T1-FPID and the IT2-FPID controllers have identical SFs. The height of the LMF of the employed IT2-FPID controller is constant and it is set to 0.22 to show that the response of the simplest IT2-FPID controller will be slow compare to T1-FPID.

The responses of the controllers are illustrated in Fig. 6a for the nominal system for a step set-point change and an input disturbance. For all the simulations, an input disturbance is applied at 20th second with magnitude of 0.3. The responses are illustrated for the perturbed systems in Fig. 6b, Fig. 6c and Fig. 6d. The overall results are tabulated in Table 2. The STIT2-FPID controller improves the overall performances in every sense compared to the simplest IT2-FPID and T1-FPID controllers. For example, as show in Fig. 6a, for the nominal system response, the T1-FPID has the Ts with 7.11 s but the highest OS with 13.68% because of the aggressive nature of the T1-FLC, while the simplest IT2-FPID has less OS with 11.12% but the longest Ts with 10.35 s because of the smooth nature of the IT2-FPID (M=0.22).However, the STIT2-FPID structure has the best OS% and approximately same Ts with T1-FPID with 10.61% and 7.58 s respectively.

Table 2. Simulation Results

	$G_0(s)$			$G_1(s)$			$G_2(s)$			$G_3(s)$		
	OS%	**Ts**	**IAE**	**OS%**	**Ts**	**IAE**	**OS%**	**Ts**	**IAE**	**OS%**	**Ts**	**IAE**
T1-FPID	13.68	7.11	2.33	52.76	11.15	3.20	20.29	13.49	3.50	4.04	4.85	1.37
IT2-FPID	11.12	10.35	2.91	49.19	7.13	3.12	15.83	13.48	4.04	2.89	6.56	1.90
STIT2-FPID	10.61	7.58	2.40	45.02	7.92	2.83	14.91	13.59	3.48	3.41	5.01	1.48

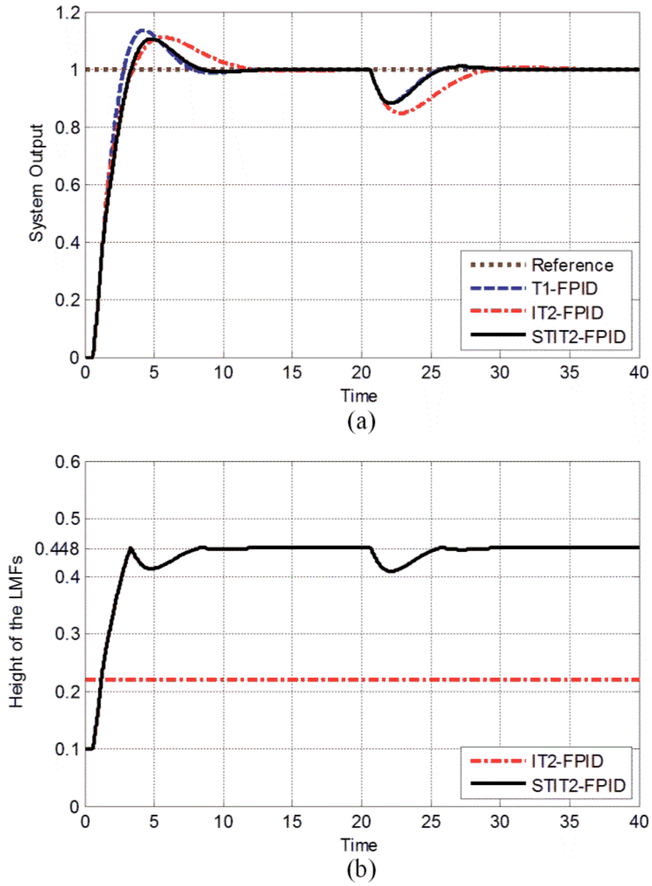

Fig. 7. (a) The closed loop control performances for the nominal system $G_0(s)$ and (b) the variation of the height of the LMFs, M

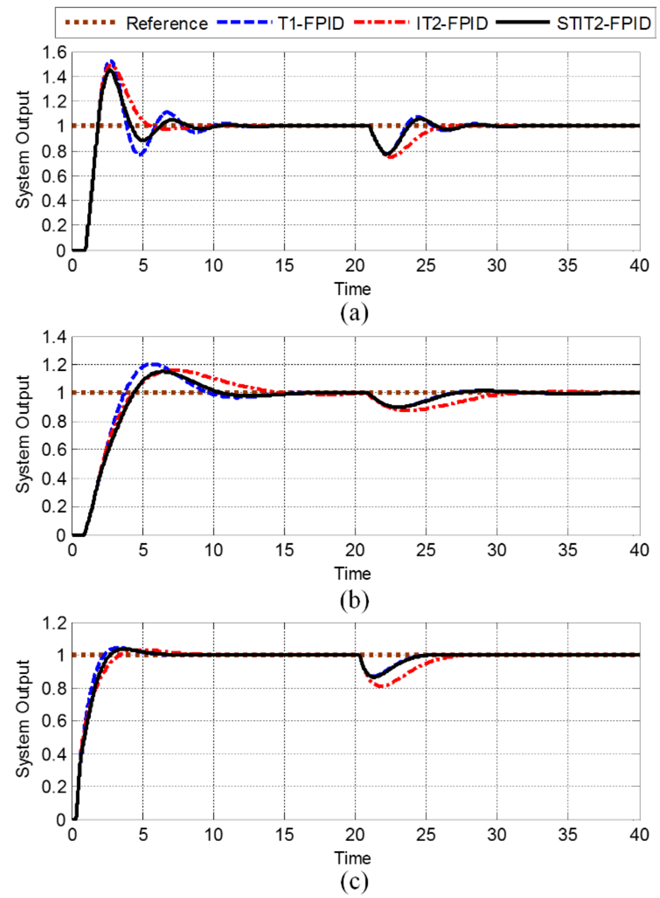

Fig. 8. The closed loop performances for (a) the perturbed system 1, $G_1(s)$ (b) the perturbed system 2, $G_2(s)$ and (c) the perturbed system 3, $G_3(s)$

The reason behind this is that the proposed STIT2-FPID act like the IT2-FPID when the system far away the reference which provides smooth control action for set point tracking and then it act like the T1-FPID when the system output about the set point which provides aggressive control action against disturbances. Hence, it can be interpreted that the proposed STIT2-FPID controller benefits the superiorities of the T1-FPID and IT2-FPID controllers. Similar comments can be made for the perturbed systems.

In contrast, the STIT2-FPID structure may slightly increase the possibility of oscillation. For example, for the perturbed system 1 response illustrated in Fig. 6c, IT2-FPID has less oscillation around the set point, but STIT2-FPID controller still improves the overall performance since STIT2-FPID has nearly 10% better Ts and IAE values than IT2-FPID. Hence, the STIT2-FPID structure is robust against disturbances and parameter uncertainties because of its IT2-FPID base.

VI. Conclusions and Future Work

In this study, we presented the analytical structural analysis of the simplest IT2-FPID controller which is composed of four rules. We will present a novel graphical method called S-MAP to visualize and understand better the variation of the SPs selection of the KM algorithm. With the aids of the S-MAP and BF-KM, we derived the closed-from formulation of the simplest IT2-FPID controller around the steady state and showed that the IT2-FPID controller is in fact analogous to a nonlinear PID controller. We have investigated how the design parameter, i.e. the FOU size, affects the controller performance by examining the controller gain. In the light of the observations, we proposed a simple self-tuning structure based on a heuristic function to enhance control performance of the IT2-FPID controllers. We have presented comparative simulation studies where the STIT2-FPID structure is compared to the IT2-FPID and T1-FPID controllers. The outcomes of the studies showed that the superiority of the proposed STIT2-FPID structure is related to hybrid nature of the self-tuning structure because it benefits the advantages of the T1-FPID and IT2-FPID controllers by changing the size of the FOU in an online manner.

For our future work, we aim to focus on more sophisticated tuning mechanism which might improve the control performance better.

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