2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Boundary Function based Karnik- Mendel Type Reduction Method for Interval Type-2 Fuzzy PID Controllers

Mehmet Furkan Dodurka, Tufan Kumbasar, Ahmet Sakalli, Engin Yesil

Abstract—**In this paper, we will present a Boundary Function (BF) based type reduction/ defuzzification method for Interval Type-2 Fuzzy PID (IT2-PID) controllers. Thus, we have presented a novel representation of the optimal Switching Points (SPs) of the Karnik Mendel (KM) method by first decomposing the IT2-FPID controller into SubControllers (SCs) and then derived Boundary Functions (BFs) to determine the optimal SPs of each SCs. Since the optimal SPs are calculated without an iterative algorithm, the explicit expressions of how the SPs are determined is represented in analytical structure via the proposed BFs. We have presented comparative studies where the computational time performance of the proposed BF-KM method is compared to the KM and the decomposition based KM methods. The presented results show that proposed method is superior in comparison to the other compared methods and feasible for especially real time control applications where there is a need of small sampling times.**

Keywords—Interval type-2 fuzzy PID controllers; Karnik-Mendel Method, Decomposition, Switicing Points.

I. INTRODUCTION

Recently, it has been shown in various works that Interval Type-2 Fuzzy Logic Controllers (IT2-FLCs) may achieve better control performances because of the additional degree of freedom provided by the Footprint of Uncertainty (FOU) in their Interval Type-2 Fuzzy Sets (IT2-FSs) in comparison to its type-1 counterpart [1]. Nevertheless computational cost of calculating the crisp output of an IT2-FLC is a problematic issue [2]. Since to obtain a crisp value from an IT2-FLC, a Type-Reduction (TR) mechanism is required to reduce the IT2-FSs into Type-1 Fuzzy Sets (T1-FSs) and then defuzzification process is applied to obtain the crisp output [3]. Here, the problematic issue is arisen from type-reduction mechanism mainly [3]. The Karnik-Mendel (KM) method is the most commonly used algorithm for TR mechanism [4]. In the KM algorithm the type reduced set is obtained by determining defined Switching Points (SPs). However, due to fact that these SPs cannot be represented as functions of the inputs of the fuzzy system, the KM algorithm determines the SPs iteratively [3]-[4]. Thus the TR process tends to be a bottleneck for interval IT2-FLCs in the sense of computational time [5]. In [6], several TR methods are compared and categorized as alternative TR methods and enhancements to the KM TR algorithms. The alternative TR algorithms are closed-form approximations to the original KM TR algorithm without any iterative nature [11]-[13]. Yet, alternative TR algorithms cannot represent the significant features of KM such as novelty and adaptiveness [14]. While in the enhancements to the KM TR algorithms, the aim is to reduce the computational complexity in comparison to the original KM TR [7]-[10]. Moreover, in order to reduce the computational complexity, it has been shown that a certain class of IT2-FLCs can be decomposed into several SubControllers (SCs) and then the KM TR can be employed only for the activated SC [15].

In this study, we will propose an explicit solution to determine optimal switching points of the KM method for IT2-PID controllers which are constructed from two inputs and one output. We will present a novel representation of the SPs by first decomposing the IT2-FPID controller into SubControllers (SCs) and then derive Boundary Functions (BFs) to determine the optimal SPs of each SC. We will demonstrate that by employing the proposed BFs based KM (BF-KM) method, the iterative nature of the KM method can be simply eliminated and the computational complexity of the KM method is significantly reduced. At first, according to decomposition property, the input space is divided into subspaces, and IT2-PID is decomposed into SCs. We will then derive inequality equations which determine optimal the SPs of the KM for each SC. Then, the analytical formulation of determining optimal switching points is derived from by solving determined inequality type equations in order to obtain the crisp output of the IT2-FPID controller.

This paper is organized in five sections. In Section II, background information about IT2-FSs used in the IT2-PIDs and components of IT2-PIDs are briefly presented. In Section III the proposed BF-KM method is presented. In Section IV a comparative study is presented to show the effectiveness of the proposed BF-KM method. In Section V conclusions and future works are presented.

II. THE GENERAL STRUCTURE OF THE INTERVAL TYPE-2 FUZZY PID CONTROLLERS

In this section, we will present the general structure of the two input IT2-FPID controller. The IT2-FPID controller is constructed by choosing the inputs to be error (e) and change of error ($\dot{e}$) and the output as the control signal (u) as shown in Fig.1. Here, the input scaling factors K_e (for e) and K_d (for $\dot{e}$) normalize the inputs to the universe of discourse where the inputs (E and $\dot{E}$) of the IT2-FLC are defined. The output (U) of the IT2-FLC is then converted into the control signal (u) by the output scaling factors K_a and K_b [18].

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This research is supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the project (113E206). All of these supports are appreciated.

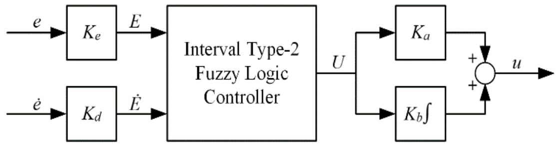


Fig. 1. IT2-FPID controller structure

In fuzzy control design strategies, symmetrical and monotonic rule bases are preferred and widely employed [18-24]. The rule structure of the IT2-FLC is as follows:

$$R_n\text{: IF } E \text{ is } \tilde{A}_i \text{ and } \dot{E} \text{ is } \tilde{B}_j \text{ THEN } U \text{ is } C_h \quad n = 1 \dots N \tag{1}$$

where $\tilde{A}_i, \tilde{B}_j$ $(i, j \in \{-g, \dots, -2, -1, 0, 1, 2, \dots, g\})$ are the antecedent IT2-FSs. The total number of the antecedent IT2-FSs is $T = 2g + 1$ and the total number of rules is equal to $N = TxT$. The rule index (n) of the IT2-FLC is defined with respect to indexes of the antecedent MFs (i, j) as follows:

$$n = T * (i + g) + j + g + 1 \tag{2}$$

Here * denotes the product operator. The consequent parts of the rules are defined with crisp sets C_h $(h = \mathrm{i} + \mathrm{j})$ and satisfy the following constraint to ensure the monotonicity in the rule base as shown in Table I [25].

$$C_{h-1} \leq C_h \leq C_{h+1} \tag{3}$$

Moreover, since the rule base is symmetric (as shown in Table I), the total number of the consequent MFs will reduce to $2T - 1$. The corresponding indexes (h) of the consequent MFs (C_h) are defined as:

$$h = H(n) = ceil\left(\frac{n}{T}\right) * (1 - T) + n - 1 \tag{4}$$

where $H(n)$ is a simple mapping between the rule index (n) and consequent MF index (h) so that the rule base is monotonic and symmetrical around the $E = \dot{E}$ line as illustrated with the dashed line in Table I.

The employed antecedent IT2-FSs are defined with the parameters $a_{i-1}, a_i, a_{i+1}, m_i$ and $b_{j-1}, b_j, b_{j+1}, m_j$ for $\tilde{A}_i$ and $\tilde{B}_j$ as shown in Fig.2a and Fig.2b, respectively. The parameters m_i, m_j are the heights of LMF which have the values from the interval [0, 1] and form the FOUs of the IT2-FSs. The IT2-FSs $(\tilde{A}_i, \tilde{B}_j)$ can be described by Upper MFs (UMFs) $(\overline{\mu}_{\tilde{A}_i}, \overline{\mu}_{\tilde{B}_j})$ and Lower MFs (LMFs) $(\underline{\mu}_{\tilde{A}_i}, \underline{\mu}_{\tilde{B}_j})$ and have the following properties:

$$\underline{\mu}_{\tilde{A}_i} = \overline{\mu}_{\tilde{A}} * m_i \qquad \underline{\mu}_{\tilde{B}_j} = \overline{\mu}_{\tilde{B}_j} * m_j \tag{5}$$

As illustrated in Fig.2, the antecedent IT2-FSs of each input variable will partition the universe of discourse fully in the sense of UMFs and LMFs. In this study, we will define the antecedent IT2-FSs in the same universe of discourse (D) which is defined as:

$$D = [-a_g, a_g] = [-b_g, b_g] \tag{6}$$

Since we have employed 50% overlapping UMFs, there will be an equal spread (S) between two consecutive IT2-FSs in each universe of discourse, i.e. $S = a_{i+1} - a_i = b_{j+1} - b_j$. This property can be easily deduced by investigating Fig. 2.

The output of the IT2-FLC is obtained by using the center-of-sets TR which is defined as [17]:

$$U_{cos} = \bigcup_{f^n \in F^n} \frac{\sum_{n=1}^{N} f^n * C_{H(n)}}{\sum_{n=1}^{N} f^n} \qquad U_{cos} = [U_l, U_r] \tag{7}$$

where U_l, U_r are the end points of the type reduced set and F^n is the total firing strength of n^{th} rule is defined as:

$$F^n = \left[\underline{f}^n, \overline{f}^n\right] = \left[\overline{\mu}_{\tilde{A}_i}(E) m_i * \overline{\mu}_{\tilde{B}_j}(\dot{E}) m_j, \overline{\mu}_{\tilde{A}_i}(E) * \overline{\mu}_{\tilde{B}_j}(\dot{E})\right] \tag{8}$$

The type reduced set $[U_l, U_r]$ can be found via the KM method [17] by defining U_l^L and U_r^R as follows:

$$U_l^L = \frac{\sum_{n=1}^{L} \overline{f}^n * C_{H(n)} + \sum_{n=L+1}^{T} \underline{f}^n * C_{H(n)}}{\sum_{n=1}^{L} \overline{f}^n + \sum_{n=L+1}^{N} \underline{f}^n} \tag{9}$$

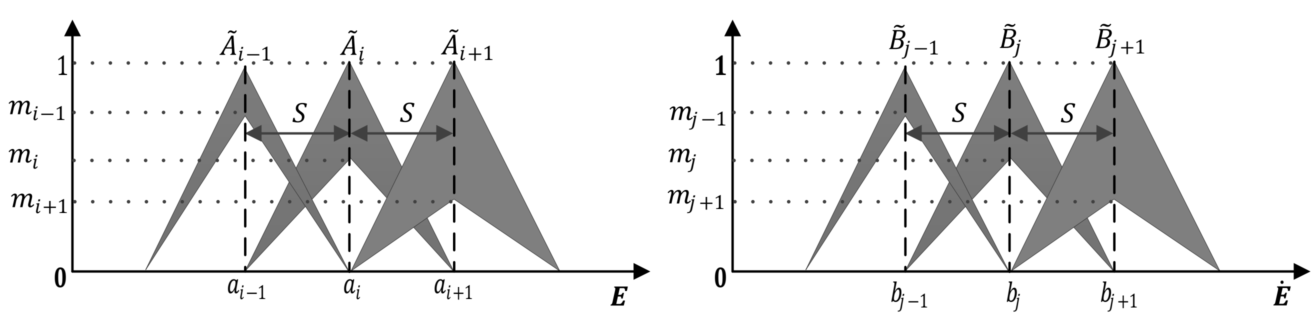
$$U_r^R = \frac{\sum_{n=1}^{R} \underline{f}^n * C_{H(n)} + \sum_{n=R+1}^{N} \overline{f}^n * C_{H(n)}}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \overline{f}^n} \tag{10}$$

Then, U_l can be found as

$$U_l = \min_{L \in [1, N-1]} (U_l^L) = U_l^{L^*} \tag{11}$$

and consequently U_r can be found as

$$U_r = \max_{R \in [1, N-1]} (U_r^R) = U_r^{R^*} \tag{12}$$



(a) (b)

Fig.2. Illustration of the antecedent IT2-FSs for the input (a) E and (b) $\dot{E}$

Here, L^*, R^* represents the optimum SPs of Equation (11) and (12), respectively and can be found via the KM method [17]. Then, the crisp output of the IT2-FLC can be found as:

$$U = (U_l + U_r)/2 \tag{13}$$

TABLE I
THE GENERAL STRUCTURE OF IT2-FLC RULE BASE

$\dot{E}$ \ E	$\tilde{A}_{-g}$	...	$\tilde{A}_{-1}$	A_0	A_{+1}	...	A_{+g}
$\tilde{B}_{-g}$	C_{-g-g}	...	C_{-g-1}	C_{-g}	C_{-g+1}	...	C_0
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\tilde{B}_{-1}$	C_{-1-g}	...	C_{-2}	C_{-1}	C_0	...	C_{-1+g}
$\tilde{B}_0$	C_{-g}	...	C_{-1}	C_0	C_{+1}	...	C_g
B_1	C_{1-g}	...	C_0	C_{+1}	C_{+2}	...	C_{1-g}
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
B_g	C_0	...	C_{+g-1}	C_{+g}	C_{+g+1}	...	C_{+g+g}

III. ANALYTICAL DERIVATIONS TO REACH THE OUTPUT OF THE IT2-FLC

In this section, we will propose a method based on analytical derivations to determine the optimal SPs of the KM for IT2-FPID controllers with the aids of the decomposition property. The employed decomposition property is a simple constructive procedure which has been used for T1-FLCs [16] and generalized for IT2-FLCs [15]. The decomposition property will facilitate the determination of the optimum SPS methodology significantly which will be explained in detail in the following subsection.

A. *Boundary Function based KM TR/Defuzzification method*

In this subsection, we will present an analytical method to determine the SPs of the KM method and consequently the output of the IT2-FPID. The proposed methodology will eliminate the iterative nature of the KM algorithm. The proposed methodology consists of two main steps, decomposing the IT2-FPID model into type-2 fuzzy SubControllers (SCs) and then determining the optimum SPs of the SCs to obtain the output of the IT2-FPID using the presented derived set of inequalities.

In [15], it has been shown that the fuzzy system can be decomposed into SCs so that the rule base can be portioned into square blocks in which all the fuzzy inference operations can be performed. According to the decomposition property, the rule base is partitioned in the 2^{2*g} SCs as shown in Fig.3 [15]. Moreover the partitioned SCs can be indexed as $SC_{q,w}$ as follows:

$$SC_{q,w}\ (q, w \in \{-g, \dots, -2, -1, 0, 1, 2, \dots, g-1\}) \tag{14}$$

where index q depends on the smallest index of the fired MFSs of $\tilde{A}_i$ in a SC and similarly w depends on the smallest index of the fired MFs of $\tilde{B}_j$ in a SC.

In this study, we will examine the generic $SC_{q,w}$ for an easier analysis (the grey area in Fig.3). For the generic $SC_{q,w}$, the corresponding antecedent IT2-FS are $\tilde{A}_i$, $\tilde{A}_{i+1}$ (for E) and $\tilde{B}_j$, $\tilde{B}_{j+1}$ (for $\dot{E}$) and consequents are C_h, C_{h+1}, C_{h+2} as illustrated in Fig.3 (the grey area). Thus, in total the number of possible activated rules is four. Moreover by using the rule indexing function $H(n)$ (given in Equation (4)), the Activated rules Index set (AI) for the $SC_{q,w}$, is as follows:

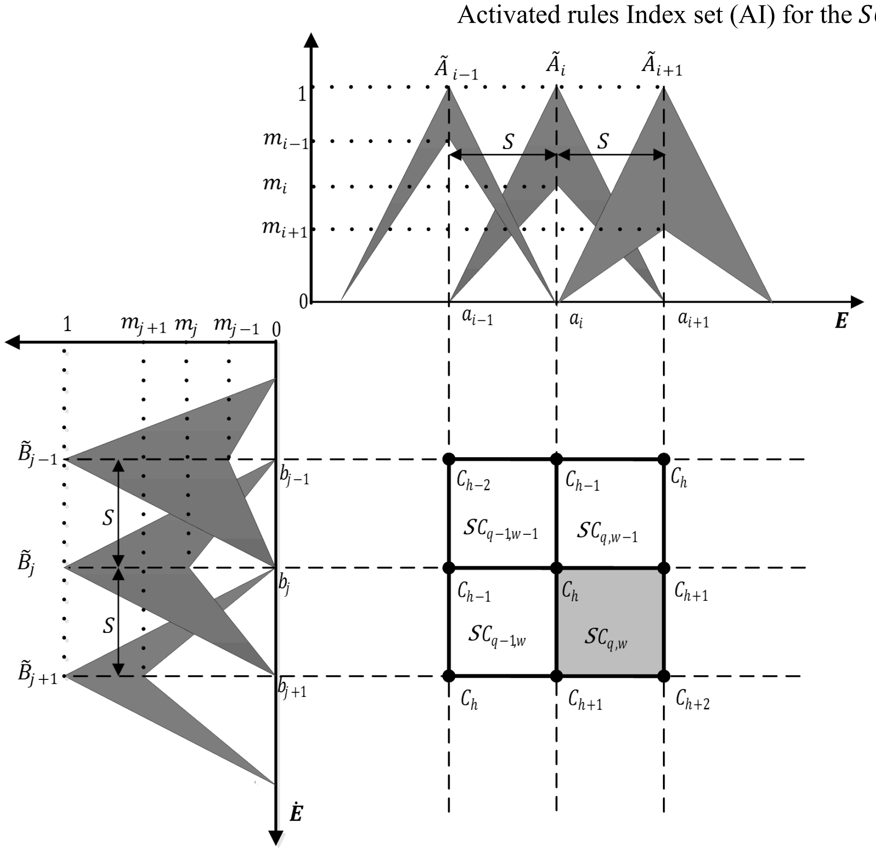


Fig. 3. Illustration of the decomposition property for the IT2-PID controller

$$AI_{IC(q,w)} = [n, n+1, n+T, n+T+1] \quad (15)$$

Thus, Equations (11) and (12) can be redefined as follows:

$$U_l = \min_{L\in[n,n+1,n+T]} (U_l^L) = U_l^{L^*} \quad (16)$$

$$U_r = \max_{R\in[n,n+1,n+T]} (U_r^R) = U_r^{R^*} \quad (17)$$

It can be clearly seen that there only three possibilities for assigning L^* and R^* since $L, R \in [n, n+1, n+T]$. Taking in account of that, the possible values of L^* and R^* can be defined under the following conditions (KM conditions) [17].

$$L^* = \begin{cases} n, & \text{if } U_l^n \le U_l^{n+1} \text{ and } U_l^n \le U_l^{n+T} \\ n+1, & \text{if } U_l^{n+1} \le U_l^n \text{ and } U_l^{n+1} \le U_l^{n+T} \\ n+T, & \text{if } U_l^{n+T} \le U_l^n \text{ and } U_l^{n+T} \le U_l^{n+1} \end{cases} \quad (18)$$

$$R^* = \begin{cases} n, & \text{if } U_r^n \ge U_r^{n+1} \text{ and } U_r^n \ge U_r^{n+T} \\ n+1, & \text{if } U_r^{n+1} \ge U_r^n \text{ and } U_r^{n+1} \ge U_r^{n+T} \\ n+T, & \text{if } U_r^{n+T} \ge U_r^n \text{ and } U_r^{n+T} \ge U_r^{n+1} \end{cases} \quad (19)$$

The presented KM conditions can be stated explicitly if and only if the inequalities on the right hand side in Equation (18) and (19) can be determined. To solve these inequalities, firstly the functions in inequalities, U_l^L and U_r^R ($\forall\, L, R \in [n, n+1, n+T]$) must be determined. Thus, the firing interval sets F^n are calculated for $\forall\, n \in AI_{IC(q,w)}$, i.e. for each activated SC. The corresponding firing interval sets can be calculated with respect to Equation (8) as follows:

$$F^n = [m_i * \overline{\mu}_{\tilde{A}_i}(E) * m_j * \overline{\mu}_{\tilde{B}_j}(\dot{E}), \; \overline{\mu}_{\tilde{A}_i}(E) * \overline{\mu}_{\tilde{B}_j}(\dot{E})] \quad (20)$$

$$F^{n+1} = [m_i * \overline{\mu}_{\tilde{A}_i}(E) * m_{j+1} * \overline{\mu}_{\tilde{B}_{j+1}}(\dot{E}), \; \overline{\mu}_{\tilde{A}_i}(E) * \overline{\mu}_{\tilde{B}_{j+1}}(\dot{E})] \quad (21)$$

$$F^{n+T} = [m_{i+1} * \overline{\mu}_{\tilde{A}_{i+1}}(E) * m_j * \overline{\mu}_{\tilde{B}_j}(\dot{E}), \; \overline{\mu}_{\tilde{A}_{i+1}}(E) * \overline{\mu}_{\tilde{B}_j}(\dot{E})] \quad (22)$$

$$F^{n+T+1} = [m_{i+1} * \overline{\mu}_{\tilde{A}_{i+1}}(E) * m_{j+1} * \overline{\mu}_{\tilde{B}_{j+1}}(\dot{E}), \; \overline{\mu}_{\tilde{A}_{i+1}}(E) * \overline{\mu}_{\tilde{B}_{j+1}}(\dot{E})] \quad (23)$$

where the membership grades of the $\overline{\mu}_{\tilde{A}_i}(E)$, $\overline{\mu}_{\tilde{A}_{i+1}}(E)$ for $E \in [a_i, a_{i+1,}]$ can be calculated as:

$$\overline{\mu}_{\tilde{A}_i}(E) = \frac{a_{i+1} - E}{a_{i+1} - a_i} \quad (24)$$

$$\overline{\mu}_{\tilde{A}_{i+1}}(E) = \frac{E - a_i}{a_{i+1} - a_i} \quad (25)$$

Similarly, the values of $\overline{\mu}_{\tilde{B}_j}(\dot{E}), \overline{\mu}_{\tilde{B}_{j+1}}(\dot{E})$ for $\dot{E} \in [b_j, b_{j+1}]$:

$$\overline{\mu}_{\tilde{B}_j}(\dot{E}) = \frac{b_{j+1} - \dot{E}}{b_{j+1} - b_j} \quad (26)$$

$$\overline{\mu}_{\tilde{B}_{j+1}}(\dot{E}) = \frac{\dot{E} - b_j}{b_{j+1} - b_j} \quad (27)$$

Replacing the membership grades given in Equations (24-27) into the corresponding firing interval sets given in Equations (20-23), the firing interval sets F^n for $\forall\, n \in AI_{IC(q,w)}$ are:

$$F^n = [m_i * \frac{a_{i+1} - E}{a_{i+1} - a_i} * m_j * \frac{b_{j+1} - \dot{E}}{b_{j+1} - b_j}, \; \frac{a_{i+1} - E}{a_{i+1} - a_i} * \frac{b_{j+1} - \dot{E}}{b_{j+1} - b_j}] \quad (28)$$

$$F^{n+1} = [m_i * \frac{a_{i+1} - E}{a_{i+1} - a_i} * m_{j+1} * \frac{\dot{E} - b_j}{b_{j+1} - b_j}, \; \frac{a_{i+1} - E}{a_{i+1} - a_i} * \frac{\dot{E} - b_j}{b_{j+1} - b_j}] \quad (29)$$

$$F^{n+T} = [m_{i+1} * \frac{E - a_i}{a_{i+1} - a_i} * m_j * \frac{b_{j+1} - \dot{E}}{b_{j+1} - b_j}, \; \frac{E - a_i}{a_{i+1} - a_i} * \frac{b_{j+1} - \dot{E}}{b_{j+1} - b_j}] \quad (30)$$

$$F^{n+N+1} = [m_{i+1} \frac{E - a_i}{a_{i+1} - a_i} m_{j+1} \frac{\dot{E} - b_j}{b_{j+1} - b_j}, \; \frac{E - a_i}{a_{i+1} - a_i} \frac{\dot{E} - b_j}{b_{j+1} - b_j}] \quad (31)$$

Replacing equations (28-31) into the KM conditions for L^* and R^* given in Equation (18) and (19) respectively, the obtained inequality equations can be solved with the aids of cylindrical algebraic decomposition method. The obtained explicit solution switching points L^* and R^* are found as follows:

$$L^* = \begin{cases} n, & E \le BF_2(\dot{E}) \\ n+T, & E > BF_2(\dot{E}) \end{cases} \quad (32)$$

$$R^* = \begin{cases} n, & E \le BF_1(\dot{E}) \\ n+T, & E > BF_1(\dot{E}) \end{cases} \quad (33)$$

where $BF_1(\dot{E})$ and $BF_2(\dot{E})$ are the Boundary Functions (BFs) for determining optimal switching points and are given in Equations (34-35). It is worth to mention that BFs in Equation (34) and (35) are just functions of $\dot{E}$ while the other parameters are the known structural parameters of the IT2-FPID controllers.

$$BF_1(\dot{E}) = \frac{(C_{h+1} - C_{h+2}) * (b_j - \dot{E}) * (b_j + a_i - a_{i+1}) + m_i * m_j * (C_h - C_{h+1}) * a_i * (\dot{E} - b_{j+1})}{(C_{h+1} - C_{h+2}) * (b_{j+1} - \dot{E}) + m_i * m_j * (C_h - C_{h+1}) * a_i * (\dot{E} - b_{j+1})} \quad (34)$$

$$BF_2(\dot{E}) = \frac{(C_{h+1} - C_{h+2}) * (b_j - \dot{E}) * (b_j + a_i - a_{i+1}) * (m_{i+1} * m_{j+1}) + (C_h - C_{h+1}) * a_{i+1} * (\dot{E} - b_{j+1})}{m_{i+1} m_{j+1} * b_j * (C_{h+1} - C_{h+2}) + (C_{h+1} - C_h) * b_{j+1} + \dot{E} * (C_h + m_{i+1} * m_{j+1} * C_{h+1} - C_{h+1} * (1 + m_{i+1} * m_{j+1}))} \quad (35)$$

TABLE II
PSEUDO CODE OF THE BF-KM TR /DEFUZZIFICATION METHOD

	Decomposition of the IT2-FPID structure
1.	Determine the a_i, a_{i+1} and i being index of $\tilde{A}_i$ from $a_i \le E < a_{i+1}$ where , $i \in \{-g, \dots, -2, -1, 0, 1, 2, \dots, g\}$ Determine b_j, b_{j+1} and j being index of B_j from $b_j \le \dot{E} < b_{j+1}$ where , $j \in \{-g, \dots, -2, -1, 0, 1, 2, \dots, g\}$
2.	Find the consequents C_h, C_{h+1} and C_{h+2}, where $h = i + j$
3.	Find rule index n from Equation (4)
	Calculation of the optimal SPs
4.	Calculate Boundary Functions $BF_1(\dot{E})$ and $BF_2(\dot{E})$ from Equation (34) and (35)
5.	If $E \le BF_2(\dot{E})$ Then assign $L^* = n$ Else assign $L^* = n + T$
6.	If $E \le BF_1(\dot{E})$ Then assign $R^* = n$ Else assign $R^* = n + T$
	Calculation of the crisp output of the IT2-FPID structure
7.	Calculate the firing intervals $F^n, F^{n+1}, F^{T+n}, F^{T+n+1}$ from Equation (8)
8.	Calculate the $U_l^{L^*}$ from Equation (9) Calculate the $U_r^{R^*}$ from Equation(10)
9.	Calculate the defuzzified output U via Equation (13)

Now, at any certain time (when an input signal activates the IT2-FPID) the inputs $(E, \dot{E})$ are known and their corresponding membership grades can be calculated (since the antecedent and the consequent MFs are known). Consequently the optimal SPs can be determined via Equations (32) and (33). In fact, by using Equations (32) and (33), the L^* and R^* switching points can be determined without using iteration algorithm of KM. Once the switching points are determined, then Equations (9) and (10) can be evaluated to obtain the output of an IT2-FLC can be found. The pseudo code of the proposed BF based KM (BF-KM) algorithm is given in Table II.

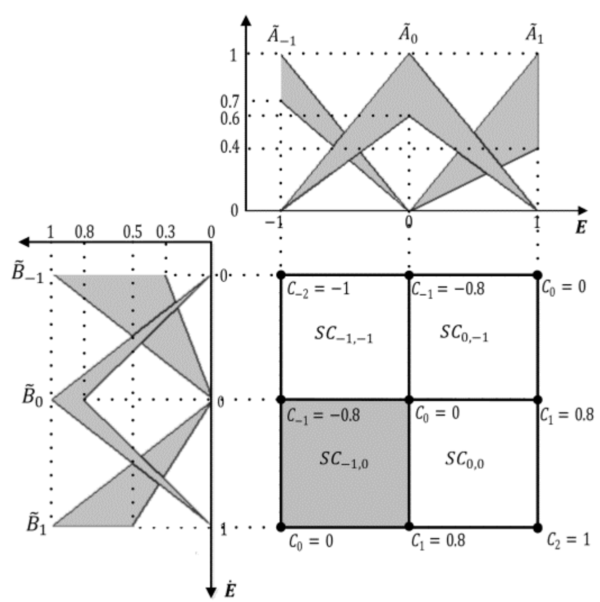


Fig.4. Illustration of the decomposition of the IT2-FPID composed of 3x3 rules

B. Illustrative Example

In this subsection, we will present an illustrative example to demonstrate the proposed BF-KM TR/defuzzification method to have a clear and easier explanation. We will handle the widely used IT2-PID controller composed of $3x3$ rules, consequently $T = 3$ and $g = 1$. The rule structure is as:

$$R_n: \text{ IF } E \text{ is } \tilde{A}_i \text{ and } \dot{E} \text{ is } \tilde{B}_j \text{ THEN } U \text{ is } C_h, n = 1 \dots 9.$$

The antecedent and consequent MFs are shown in Fig. 4. According to the decomposition property, the rule base is partitioned into the four SCs as shown in Fig.4. So, the BFs for each SC can be found via the Equations (34) and (35). The BFs for all SCs are derived and presented in Table 3.

Now, for an input vector $(E, \dot{E}) = (-0.6, 0.7)$, the activated rules are spanned by $SC_{-1,0}$ (the shaded area in Fig.4) which is constructed from the antecedent MFs $\tilde{A}_{-1}, \tilde{A}_0$ (for E) and $\tilde{B}_0, \tilde{B}_1$ (for $\dot{E}$). The corresponding consequent MFs can be determined via the mapping given in Equation (4), where $h = i + j = -1$ .Thus, the consequent MFs are $C_{-1} = -0.8, C_0 = 0, C_1 = 0.8$. Thus the necessary structural parameters are for calculating BFs are as:

$$a_i = -1, a_{i+1} = 0, m_i = 0.7, m_{i+1} = 0.6$$
$$b_i = 0, b_{i+1} = 1, m_j = 0.8, m_{j+1} = 0.5$$
$$C_h = -0.8, C_{h+1} = 0 , C_{h+2} = 0.8$$

The corresponding the rule index n (given in Equation (2)) can be found as:

$$\begin{aligned} n &= T(i + g) + j + g + 1 \\ n &= 3(-1 + 1) + 0 + 1 + 1 \\ n &= 2 \end{aligned} \quad (36)$$

From Table 3 for $SC_{-1,0}$ and for $\dot{E} = 0.7$, we obtain $BF_1(0.7) = -0.806$ and $BF_2(0.7) = -0.412$. Then, the SPs are determined from Equation (32) and (33) for $E = -0.6$ as:

$$\begin{aligned} L^* = n = 2, &\quad -0.6 \le -0.412 \\ R^* = n + T = 5, &\quad -0.6 > -0.806 \end{aligned} \quad (37)$$

Consequently, the type reduced interval can be found via Equations (9) and (10) as $U_l = -0.1639$ and $U_r = 0.2449$ and the crisp output of the IT2-FLC as $U = 0.0405$.

TABLE III
BFS FOR CORRESPONDING SUBCONTROLLERS

$SC_{q,w}$	$BF_1(\dot{E})$	$BF_2(\dot{E})$
$SC_{-1,-1}$	$\frac{0.8 + 0.8\dot{E}}{-0.8 - 0.758\dot{E}}$	$\frac{0.384 + 0.38\dot{E}}{-0.384 - 0.184\dot{E}}$
$SC_{0,-1}$	$\frac{0.144\dot{E}}{-0.8 - 0.656\dot{E}}$	$\frac{0.8\dot{E}}{-0.256 + 0.544\dot{E}}$
$SC_{-1,0}$	$\frac{0.8\dot{E}}{-0.448 - 0.352\dot{E}}$	$\frac{0.24\dot{E}}{-0.8 + 0.56\dot{E}}$
$SC_{0,0}$	$\frac{-0.384 + 0.384\dot{E}}{-0.384 + 0.184\dot{E}}$	$\frac{-0.8 + 0.8\dot{E}}{-0.8 + 0.76\dot{E}}$

IV. COMPARISON STUDY

In this section, the computational times of proposed BF-KM TR will be compared with the original KM TR [17], and Decomposition based KM (D-KM) TR [15] strategies to show the efficiency of the proposed method. The comparison study has been performed on a personal computer with CPU i7-3960X 3.3 GHz and 10GB.

In order to compare the algorithms computational time performances, we will measure the total computation time needed to calculate the defuzzified output of IT2-FLC. During the experiments, we will discretize the universe of discourses ($D = [-1,1]$) of the normalized inputs ($E, \dot{E}$) into 21 equal points separately with a fixed step of 0.1. Thus, for each experiment the algorithms are evaluated for 21x21=421 times. We will compare the execution times of the TR methods on three different TxT rule bases where $T = 3, 5,$ and 7. To make a fair comparison and to provide statistical information about the measured computation times, we have repeated each experiment 250 times for each handled TxT rule base. Consequently, a total of 250x3 randomly IT2-FLC has been generated. In the generation of an IT2-FLC with a TxT rule base, the antecedent MF parameters of m_i, m_j and consequent parameters C_h are randomly generated with a uniform distribution. In each experiment the number of different antecedent parameters m_i , m_j are generated randomly according to their rule base sizes (TxT) where $i, j \in \{-(T-1)/2, \dots, -2, -1, 0, 1, 2, \dots, (T-1)/2\}$ and where antecedent MF parameters are between $0 < m_i, m_j < 1$, and to construct symmetrical rule bases $2 * T - 1$ different consequents are generated such that $C_h < C_{h+1} < C_{h+2}$ where $C_h \in [-1,1]$. For instance, in the generation of an IT2-FLC with $5x5$ rule base, 5 antecedent parameters (m_i and m_j) and 9 consequent parameters will be random generated for each experiment.

For comparing the computation time performance of the TR algorithms, the minimum, maximum, mean and standard deviation times are measured and calculated for 250 experiments. The measured times for the 3x3, 5x5 and 7x7 rule base are given in Table IV, Table V and Table VI, respectively. Moreover, the total computation times of the 250 experiments of each TR method is illustrated in Fig. 5.

TABLE IV
COMPUTATION TIMES (ms) FOR 3X3 RULE BASE IT2-FLC

T. R. Algorithm	Min time	Max time	Mean and Standard deviation
BF-KM	134	152	137±0.184
D-KM	374	464	400±1.011
KM	373	434	405±1.091

TABLE V
COMPUTATION TIMES (ms) FOR 5X5 RULE BASE IT2-FLC

T. R. Algorithm	Min Time	Max Time	Mean and Standard deviation
BF-KM	139	158	142±0.236
D-KM	379	473	398±0.920
KM	470	539	510±1.002

TABLE VI
COMPUTATION TIMES (ms) FOR 7X7 RULE BASE IT2-FLC

T. R. Algorithm	Min Time	Max Time	Mean and Standard deviation
BF-KM	146	170	152 ±0.408
D-KM	377	458	392±1.097
KM	620	751	660±1.501

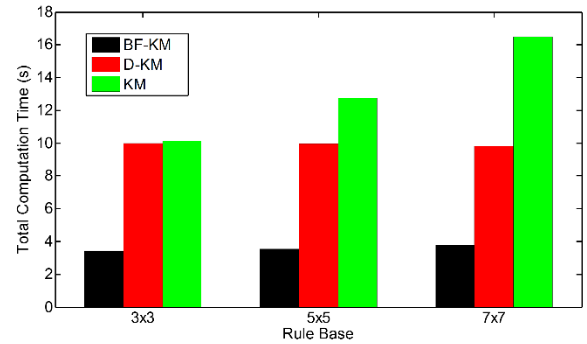


Fig. 5. Total Computation Time of 250 experiments respect to different rule bases

It can be clearly seen from the measured times that, the proposed BF-KM method always outperforms the other methods with respect to the minimum, maximum, mean and standard deviation times. For 3x3 rule base IT2-FLC performance of D-KM and KM are nearly same while the BF-KM method has the lowest time values. Moreover, it can be clearly seen that, as the size of rule base increases, the computation times of BF-KM and D-KM methods do not vary too much whereas the computation times of KM are increasing exponentially. This is mainly caused because of the decomposition of the rule base since the number of possible activated rules in a decomposed rule base is always fixed, and consequently the computation times do not increase for both the BF-KM and D-KM methods. However, the superiority of the BF-KM method can be seen clearly especially for the $7x7$ rule base structure. As tabulated in Table VI it can be seen that BF-KM is approximately 3 times faster than KM and 1.5 times faster than the D-KM method. Since the SPs are obtained explicitly without the need of an iterative algorithm, the computation times dramatically decreases. It can be also seen that the presented results shown in Fig.5 coincide with assertions. It can be concluded that superiority of BF-KM, it's computational time does not increases as the size of the rule base increases since it does not have an iterative structure (as the original KM) and due to the employed decomposition theory.

V. CONCLUSIONS AND FUTURE WORK

In this study, an explicit solution for determining the switching points (L^* and R^*) of the KM method is presented. We have presented a novel representation of the SPs by first decomposing the IT2-FPID controller into SCs and then derive BFs to determine the optimal SPs of each SC. Thus, since the optimal SPs are determined through the BFs, the proposed method simply eliminates the bottleneck part of KM algorithm. In other words, the proposed BF-KM method determines the SPs directly without using an iterative algorithm. The computational time performance of the proposed BF-KM method is compared to the original KM and

D-KM. The presented results show that proposed method is a practical and superior method for especially controller applications where there is a need of low computation times in small sampling times. Moreover, we would like to underline that, by employing the proposed BF-KM method, the explicit expression of how the switching points L^* and $R^*$are determined are represented in analytical structure.

Future work will focus on generalizing and employing the BF-KM method to general type-2 fuzzy controller structures in real-time control applications.

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