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Fuzzy Multi Entity Bayesian Networks: A Model for Imprecise Knowledge Representation and Reasoning in High-Level Information Fusion

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Abstract— **This paper presents a novel comprehensive Fuzzy extension to Multi-Entity Bayesian Networks (MEBN) that is deemed a well-studied and theoretically rich language that expressively handles semantics analysis, and effectively model uncertainty management. However, MEBN lack the capability of modeling the inherent conceptual and structural ambiguity that is delivered with the knowledge gained through human language. In this paper, Fuzzy MEBN that is a new version of MEBN which is based on First-order Fuzzy Logic, and Fuzzy Bayesian Networks is introduced. Furthermore, its applicability is evaluated by implementing an application related to Vehicular Ad-hoc Networks area. The results demonstrate that Fuzzy MEBN is capable of dealing with ambiguous semantical and uncertain causal relationships between the knowledge entities very efficiently.**

I. Introduction

SITUATION awareness is undoubtedly the main result of data and information fusion that is followed by knowledge insight extraction. This is made more achievable nowadays as connectivity and mobility have been improved, and further, resulted in the availability of data and information surrounding us. However, although it is now easier to access and make use of them, vast amounts of data and information that come from various sources with different levels of abstraction have become challenging to store, process, and handle. Subsequently, this increases the need for attaining reasonable automated knowledge discovery methodologies, and makes their role more critical than ever. Nevertheless, studies for acquiring knowledge rather than dealing with unstructured information has been considered and argued by researchers as early as the 1980s (see [17]).

First of all, it is necessary to note that in this paper, *data* is deemed any low-level fact that specifies the features of a certain entity, whereas *information* encloses the facts about an already recognized entity and/or its relationships with other entities. Moreover, Low-level Data Fusion (LLDF) is defined as the fusion of low-level data produced by physical sensors, and recognition of context-related entities in a specific environment to form a unified picture [12]. High-level Information Fusion (HLIF) is the stepping stone that combines theories, algorithms, and tools to explore the knowledge that lies within the information generated from multiple sources and exists among the *relationships* of various entities to draw a generic awareness of the situation for the purpose of improving the accuracy and robustness of the final decision and action. [7]

LLDF is deemed a well-studied discipline, as it has been under thorough investigation for many years [12]. In contrast, the HLIF research has only started to attract much attention in the information fusion community as reflected by the review articles published in that community within the last few years [3], [4], [20]. Nonetheless, HLIF faces major issues making its usage impractical . As a matter of fact, Blasch et al. [4] outline the top ten trends of research in HLIF among which the issues of introducing reference models, managing uncertainty and analyzing semantics/ontologies are deemed the most important areas of study that have not received enough attention in the past.

Multi-Entity Bayesian Networks, introduced by Laskey in [16] is a well-defined and theoretically rich language for HLIF that tackles uncertainty management and semantics analysis simultaneously. MEBN is a combination of First-order Logic (FOL) and Bayesian Networks (BN), and is considered as a powerful tool for modeling knowledge for situation assessment. However, despite being a strong bridge connecting structured knowledge (that is often expressed by domain experts) to computational models, MEBN lacks the capability of modeling some imperfect aspects of data such as ambiguity[1] that is an inherent characteristic of human language, and the observations gained from the environment. For instance, when referring to an entity in an environment, various sources may use different identifiers that although all can be semantically positioned in one category, they may not be completely the same as identifiers used for defining the semantic relationships. This is basically referred to as semantic similarity in the literature [1].

In this paper, we propose a novel Fuzzy extension to Multi Entity Bayesian Networks that completes our previously introduced Fuzzy extension in [8] by adding fuzzy capability in representing semantics relations. Accordingly, we first redefine the semantics specifications of conventional MEBN by incorporating notions of First-order Fuzzy Logic that is mainly inspired by works of [19]. As a result, contextual constraints of MEBN are generalized in a way to represent the ambiguity that is usually delivered with the imperfect semantic information. Furthermore, a new way of representing

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[1]The terms “ambiguity” and ”vagueness” are used interchangeably throughout this paper.

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Fuzzy Bayesian Networks (FBN) is also presented in this paper, and the well-known Belief Propagation inference algorithm in regular BN is updated to include fuzzy states with a certain likelihood. Finally, to demonstrate the applicability of Fuzzy MEBN, a road safety task in the area of Vehicular Ad-hoc Networks (VANET) is realized, and the results are compared to our previous ones.

The rest of the paper is organized as follows: Section 2 reviews some recent related works and positions our contribution. In section 3 Fuzzy MEBN is introduced, and section 4 demonstrates one of its applications in VANET area. Section 5 highlights the experimental results, and conclusions are presented in section 6.

II. Related Works

Major topics of current research in HLIF are presented by Blasch *et al.* in [4]. In their survey paper, the authors extract the top ten trends of HLIF from the conference papers and panel discussions published within years 2000 to 2011, and outline uncertainty management, and semantics and ontology representation as the most important ones.

There are a wide range of discussions in the literature about general and abstract models of HLIF. For instance, D.A. Lambert in [15] introduces a comprehensive schema with seven building blocks for designing an HLIF system. In his proposed model, the blocks model a wide range of fundamental concepts in HLIF starting from lower-level data fusion methodologies to situation and impact assessment, with added machine-readability capability, and modeled human mental status. However, despite being a well-designed model, no implementation is provided for it.

A comprehensive study on information integration using ontologies is conducted by Wache *et al.* in [22]. Accordingly, the authors introduce structural and semantic heterogeneity groups for heterogeneous sources of information. Nevertheless, some relative problems, such as confounding scaling and naming conflicts rise when semantic heterogeneity is taken into account. Besides, lack of common vocabulary causes some difficulties in comparing ontologies that is resulted to the introduction of hybrid methods that incorporate a common vocabulary on top of the other ontologies.

Integration of semantic information and annotating sensors based on their semantic structure is done by Heintz and Dragisic in [9]. The authors use semantic web technologies and Web Ontology Language (OWL) [21] to model an ontology, define its vocabulary, and to determine the semantic relationships between sources and destinations (*i.e.*, sensors and services) of data.

The fundamentals of HLIF methods for handling uncertainty is outlined by Karlsson in [10]. In his technical report, Karlsson categorizes the methods dealing with uncertainty, so-called Uncertainty Management Methods (UMMs), into three groups of Bayesian, Dempster-Shafer, and Imprecise probability approaches. Moreover, Costa *et al.* in [5] propose the Uncertainty Representation and Reasoning Evaluation Framework (URREF) to improve the system-level metrics such as timeliness, accuracy, and confidence. In other words, their main goal is to study the effect of uncertainty on IF systems. Therefore, they present an abstract model in which different uncertainty handling tools such as probabilistic methods, Dempster-Shafer theory, and Fuzzy Sets, can be used in a plug-and-play fashion.

Another tool that models knowledge extraction in situation assessment is Multi-entity Bayesian Networks (MEBN) proposed by Laskey in [16]. MEBN are a combination of First-order Logic (FOL) and Bayesian Networks (BN). In other words, MEBN are defined as a language that encompasses the expressiveness of FOL, and uncertainty management power of BN. In general, MEBN are BN whose embodied knowledge are structurally arranged using first-order logic concepts, and thus handles both semantics representation, and uncertainty management. A novel Fuzzy extension to MEBN is proposed in this paper that is comprehensively introduced in next section.

III. Fuzzy Multi-entity Bayesian Networks

Multi-entity Bayesian Networks (MEBN) is a First-order Logic (FOL) based language that models probabilistic knowledge by utilizing Bayesian Networks as its uncertainty analysis core. The main notion in MEBN are MEBN Fragments (MFrags) that semantically and causally organize the conceptually related pieces of knowledge. In fact, contextually consistent MFrags (MFrags whose consistency constrained are satisfied) reflect a local distribution on the possible values of their residing random variables. Accordingly, a set of MFrags whose consistency constraints are satisfied constitute an MTheory with a unique joint probability distribution that is computed using the local probability distributions of MFrags.

Modeling semantic and causal relationships is the main power of MEBN, but still, it lacks the capability to deal with the ambiguity that is inherent in human language, and since the knowledge base fed to MEBN is constructed based on the information provided by a domain expert, inevitable delivered ambiguity cannot be modeled properly. To deal with this problem, a fuzzy extension to MEBN is previously proposed in [8] that is deemed the first steps towards a fully Fuzzy MEBN. In that model, fuzzy sets are defined on the possible values of random variables, and besides, fuzzy rule-sets are added to MFrags (see [16]) to enhance regular Bayesian Network inference engine with fuzzy inference system. Nonetheless, the proposed fuzzy extension in [8] is still incapable of modeling the implicit ambiguity in the entities that constitute the knowledge base, and is too naive to model ambiguous relationships that exist between the entities.

Here, we improve our previously introduced fuzzy extension to MEBN by replacing the FOL representation with First-order Fuzzy Logic (FOFL) [19] when defining contextual and semantic constraints. Therefore, the original MEBN language is updated, and accordingly, some of its definitions are refined. The interested reader is referred to [16] for more thorough discussion about original MEBN and its different aspects.

A. Fuzzy Entities and Random Variables

A specific domain in MEBN language is modeled using a predefined set of attributed entities, and determining semantic and causal relationships between them. These entities are identified by constants that are included in an infinite collection of domain-specific constants (with meanings fixed by the language), and are referred to by variables that are included in an infinite collection of variable symbols. Besides, features of entities and the relationships between them are modeled using random variables that are drawn from an infinite collection of both logical and domain specific random variables. Following shows how the entities and random variables are refined in Fuzzy MEBN language.

- *Ordinary variable symbols:* The ordinary[2] variables are deemed containers that refer to non-specific entities. Ordinary variables names are alphanumeric strings that begin with a lower case letter, *e.g.*, $veh13$, env.
- *Phenomenal constant symbols:* Constants are represented by fuzzy sets with just a single member, *i.e.*, fuzzy singletons. Constant names may contain both letters and number, but must start with an uppercase letter, and should be followed by a real-valued membership degree subscript within range $[0, 1]$ *e.g.*, $Vehicle_{0.85}$, $Environment_{1.0}$.
- *Unique identifier symbols:* The entities are assigned a unique identifier symbol that are annotated with a fuzzy membership degree, and are arranged in one of the groups below:
  - *Truth value symbols and undefined symbol:* Truth values can either be a real number within range $[0, 1]$, or a member of a finite chain of truth values $L =< l_1, l_2, ..., l_n >$ predefined by the language.
  - *Entity identifier symbols:* Shown by $\mathcal{E}$, the set of entity identifier symbols are used by an interpretation of the theory to label the specific entities. Entity identifier symbols can be either numbers of alphanumeric symbols starting with an exclamation point and are subscripted with a real-valued membership degree ranging from 0 to 1, *e.g.*, $!V428_{0.75}$.
- *Logical connectives (random variables):* All the logical connective symbols, $\neg$,$\vee$,$\wedge$,$\Rightarrow$, $\Leftrightarrow$, and $=$ are deemed reserved logical random variables whose fuzzy interpretations are predefined by the language. Therefore, expressions such as $(\psi \vee \phi)$ will be interpreted by the fuzzy interpretation $\mathcal{D}$ as: $\mathcal{D}(\psi \vee \phi) =_I \mathcal{D}(\psi) \vee \mathcal{D}(\phi)$, in which $=_I$ is read *"is interpreted as"*, and operators such as $\vee$ or $\wedge$ can be substituted with the corresponding fuzzy logic s-norm or t-norm operators, respectively. Finally, logical connectives look more like random variables with truth-valued outputs, if written in prefix notation. For example, $\Rightarrow (\psi, \phi)$ is an implication random variable with two random variables ψ and ϕ.
- *Quantifiers:* Universal $\forall$ and existential $\exists$ quantifiers are interpreted by the fuzzy interpretation $\mathcal{D}$ predefined by the language as $\mathcal{D}(\forall_x \varphi) = \inf_\Delta \mathcal{D}(\varphi_x(\Delta))$ and $\mathcal{D}(\exists_x \varphi) = \sup_\Delta \mathcal{D}(\varphi_x(\Delta))$, respectively, wherein $\Delta = \langle \varepsilon_1^{(\alpha_1)}, \varepsilon_2^{(\alpha_2)}, \cdots, \varepsilon_n^{(\alpha_n)} \rangle$ is a vector of unique entity identifier symbols (*s.t.* $\varepsilon_i^{(\alpha_i)} \in \mathcal{E}$) with a length equal to the number of arguments that the logical (or domain specific) random variable (see below) φ takes, and x is an exemplar symbol.
- *Findings:* observed evidence is called finding in MEBN, and are stored in the set Ω. Logical findings are assigned a truth value within the range $[0, 1]$ or from the finite chain of truth values $L = \langle l_1, l_2, ..., l_n \rangle$.
- *Domain-specific random variable symbols:* random variable names in Fuzzy MEBN are alphanumeric strings beginning with a capital letter. Each random variable is assigned a positive integer that corresponds to the number of argument it takes. Moreover, random variables can have a set of finite or infinite possible values. Accordingly, possible values of *logical* random variables can be either within the continuous range of $[0, 1]$, or the finite chain of truth values $L = \langle l_1, l_2, ..., l_n \rangle$ predefined by the language. Furthermore, possible values of *phenomenal* random variables are defined as a subset of $\mathcal{E} \cup \{\perp\}$. In addition, the degree of membership of phenomenal random variables are predefined by their fuzzy interpretation, so that a phenomenal random variable R maps a vector of unique entity identifier symbols $\Delta = \langle \varepsilon_1^{(\alpha_1)}, \varepsilon_2^{(\alpha_2)}, \cdots, \varepsilon_n^{(\alpha_n)} \rangle$, called *input arguments*, to another vector of unique identifier symbols $\Gamma = \langle \gamma_1^{(\beta 1)}, \gamma_2^{(\beta 2)}, \cdots, \gamma_m^{(\beta_m)} \rangle$, called *fuzzy state* or *fuzzy value assignment*, with a certain degree. [3] In other words, $R : \Delta \rightarrow_\mu \Gamma$, in which the value of μ for various arrangements of arguments and possible values are predefined in the language by the fuzzy interpretation of R. This can also be represented using fuzzy relations [11] in which the truth values of a relation of set of inputs are resulted. Therefore, using fuzzy relations: $R : \langle \Delta, \Gamma \rangle \rightarrow \mu \in \{l_1, l_2, ..., l_n\}$, in which $\langle \Delta, \Gamma \rangle$ is the concatenation of two vectors Δ and Γ. Finally, it is notable to mention that logical and phenomenal random variables in Fuzzy MEBN are analogous to fuzzy predicate and functions, respectively.

Random variable terms in Fuzzy MEBN are created exactly the same way as those of regular MEBN. In general, ordinary random variables (say u and v) are deemed atomic random variable terms that may be used as input arguments for both logical and phenomenal random variables (*e.g.*, $AreInCommunication(u, v)$ and $Driver(v)$ respectively) to make more complex random variable terms. Furthermore, random variable terms can be logically related to each other using logical connectives random variables. The resulting in-

[2]As mentioned by Laskey in [16], the adjective "ordinary" is used to differentiate between ordinary variables and random variables.

[3]For simplicity, vector representations such as $\Delta = \langle \varepsilon_1^{(\alpha_1)}, \varepsilon_2^{(\alpha_2)}, \cdots, \varepsilon_n^{(\alpha_n)} \rangle$ will be denoted by $\Delta = \langle \varepsilon_i^{(\alpha_i)} | \ i = 1 \cdots n \rangle$ from now on, and throughout the paper.

terpretation of a complex random variable term is determined by applying individual interpretations, which are predefined by the language on all symbols of the language, and merging them using the logical connectives interpretations.

B. Fuzzy MEBN Fragments (MFrags)

The building blocks of a MEBN Theory (MTheory) are MEBN Fragments (MFrags) that semantically and causally represent a specific notion of the knowledge.

Definition 1: A Fuzzy MFrag (FMFrag) is defined as $\mathcal{F} = (\mathcal{C}, \mathcal{I}, \mathcal{R}, \mathcal{G}, \mathcal{D}, \mathcal{S})$ which hosts three different types of nodes, namely context nodes $\mathcal{C}$, input nodes $\mathcal{I}$ and resident nodes $\mathcal{R}$. Context nodes represent the semantic structures of knowledge by using First-order Fuzzy Logic sentences. Moreover, input nodes act as bridges to resident nodes in other FMFrags, and faciliate feeding any relevant information to the current FMFrag. Finally, resident nodes are random variables that are conditioned on the values of the context and input nodes. Additionally, in an FMFrag $\mathcal{F}$, $\mathcal{G}$ represents a FMFrag graph, set, $\mathcal{D}$ contains local distributions per each resident node, and $\mathcal{S}$ encompasses a set of fuzzy *if-then* rules to be used by the Fuzzy Inference System (FIS). It should be noted that the sets $\mathcal{C}$, $\mathcal{R}$, and $\mathcal{I}$ are pairwise disjoint, and $\mathcal{G}$ is a Directed Acyclic Graph (DAG) whose nodes belong to $\mathcal{I} \cup \mathcal{R}$, and the root nodes are members of $\mathcal{I}$ only. Finally, context value assignment terms in $\mathcal{C}$ are used for enforcing constraints under which the local distributions apply.

In FMFrags, contextual constraints will be assigned a truth value that implies how much a constraint is satisfied. The *consistency constraint degree* of FMFrags are then determined by referring to the fuzzy interpretations of the terms defined in the FMFrag and built-in FMFrags, and calculating the degree of satisfiability of the constraints as a whole. In addition to the consistency constraints, the local probability distribution in FMFrags are defined as a conditional probability distribution of the resident nodes given input/parent, and context nodes. Calculating this conditional probability is easy when the ordinary random variables used in the parents of the resident random variable, and the resident random variable itself are exactly the same. The problem arises when there exist ordinary variables in the parents that do not exist in the child. Such problems are usually tackled by applying aggregation functions and combining rules [18]. Laskey in [16] uses the notion of influence counts to combine the influence of multiple parents. Here, the same approach is adopted, with some refinements for the new Fuzzy MEBN.

Definition 2: Let us assume that $\mathcal{F}$ is an FMFrag within which there exists a resident random variable $\psi(\Theta)$ parametrized by a vector of ordinary variables $\Theta = \langle \theta_i | \ i = 1 \ldots n \rangle$:

1) $\mathcal{B} = \{(\theta_1, {\varepsilon_1}^{(\alpha_1)}), (\theta_2, {\varepsilon_2}^{(\alpha_1)}), \ldots, (\theta_1, {\varepsilon_n}^{(\alpha_n)})\}$ is a binding set of ordered pairs wherein θ_is are ordinary variables, and ${\varepsilon_i}^{(\alpha_i)}$s demonstrate unique entity identifier symbols that are represented by fuzzy singletons whose membership degrees are shown by α_is. Additionally, $\Delta = \langle \varepsilon_i^{(\alpha_i)} | \ i = 1 \cdots n \rangle$ is a vector of size n with elements arranged in the same order of θ_is in $\psi(\Theta)$.
2) With $\mathcal{B}$ being a binding set, and $\psi(\Delta)$ as the instance of ψ after substituting respective ${\varepsilon_i}^{(\alpha_i)}$ for each θ_i, the value assignment $\{(\Gamma = \phi(\Delta)\}$ is achieved that is called a *potential influencing fuzzy configuration* for $\psi(\Delta)$ in which $\phi(\Delta)$ is either an instance of one of its parents, or a context random variable residing in its FMFrag. Accordingly, Γ is a truth value (membership degree) for context random variables, and denotes a possible fuzzy state of $\phi(\Delta)$ for parent random variables. The fuzzy states are gained using the local distribution $\mathcal{D}$ and the fuzzy rule-sets $\mathcal{S}$ defined in the FMFrag $\mathcal{F}$.
3) With $\mathcal{B}$ as the binding set, and upon substituting each unique entity identifier ${\varepsilon_i}^{(\alpha_i)}$ with ordinary random variables θ_i, context constraints, which are reflected by context random variables, are satisfied to some degree based on their predefined fuzzy interpretation. Thus, the truth value of context random variable ϕ_j is calculated using Eq. 1:
$$L_{\mathcal{B}}^{*} = \sup_{\Delta_{\mathcal{B}}}(\phi_j(\Delta_{\mathcal{B}})) \tag{1}$$
where the size of $\Delta_{\mathcal{B}}$ is equal to the number of inputs that ϕ_j takes, and its elements are borrowed from the binding set $\mathcal{B}$. $L_{\mathcal{B}}^{*}$ is also considered to be a set that contains all the equal supremum values. [4]. Accordingly, an *influencing fuzzy configuration* is a potential fuzzy configuration whose unique identifier assignments are found using Eq. 2.
$$\Delta_{\mathcal{B}}^{*} = \arg\sup_{\Delta_{\mathcal{B}}}(\phi_j(\Delta_{\mathcal{B}})) \tag{2}$$
in which $\Delta_{\mathcal{B}}^{*}$ is a set of all the potential fuzzy configurations that yield the supremum value in Eq. 1. Using Eq. 2, *equivalent influential fuzzy configurations* are those in which $\phi(\Delta_{\mathcal{B}}^{i}) = \phi(\Delta_{\mathcal{B}}^{j})$ for $\Delta_{\mathcal{B}}^{i}, \Delta_{\mathcal{B}}^{j} \in \Delta_{\mathcal{B}}^{*}$ and $i \neq j$, and *equivalence classes* are distinct fuzzy configurations of parents of $\psi(\theta)$.
4) Assuming that $\mathcal{E} = \{{\varepsilon_1}^{(\alpha_1)}, {\varepsilon_2}^{(\alpha_1)}, ..., {\varepsilon_n}^{(\alpha_n)}\}$ is a set of unique identifier symbols, a partial fuzzy world $\mathcal{W}_f$ for $\psi(\Theta)$ is constructed by instantiating its parents as well as context random variables with each member of $\mathcal{E}$. Moreover, a partial fuzzy world state $\mathcal{S}_{\mathcal{W}_f}$ will be the fuzzy value assignments of the generated partial world.
5) Finally, the influence counts $|\mathcal{S}_{\mathcal{W}_\psi}|$ for $\psi(\Delta)$ is defined as the number of influencing fuzzy configurations that $\mathcal{S}_{\mathcal{W}_f}$ has for each equivalence class.

It is obvious that finding influence counts in Fuzzy MEBN is exactly the same as in regular MEBN when all the context random variables are assigned the same truth value in different potential configurations. Otherwise, the number

[4] Supremum is used here for the sake of generality. In real experiments with finite sets, it can be substituted with maximum.

of cases in which the consistency constrained are satisfied are reduced by applying Eq. 1 and Eq. 2.

Upon having their consistency constrained analyzed, and after determining the configuration parent nodes, FMFrags will have the probability distribution of their resident nodes calculated as a conditional probability on the possible values of the resident node given the values of its parents (input nodes or findings), and context nodes. Next definitions show how regular Bayesian Networks are replaced with Fuzzy Bayesian Networks.

Definition 3: Let us assume that $\mathcal{E}$ is the set of unique entity identifiers, and in an FMFrag $\mathcal{F}$, N_ψ is the set of all possible values of an instance of the resident node $\psi(\Theta)$ residing in $\mathcal{F}$ (showed by $\psi(\Delta)$). Then:

1) The *fuzzy state* of $\psi(\Delta)$ is defined as the vector $\Gamma_\psi = \langle {\gamma_j}^{(\beta_j)} |\ j = 1 \ldots |N_\psi| \rangle$ in which ${\gamma_j}^{(\beta_j)} \in N_\psi$ for $\gamma_j \in \mathcal{E}$ and β_j as the degree of being in the individual state γ_j.
2) The local probability distribution $\pi_\psi(\Gamma_\psi | \mathcal{S}_\mathcal{W})$ is a conditional probability density function that shows the likelihood of resident random variable ψ being in fuzzy state Γ_ψ given partial fuzzy world state $\mathcal{S}_{\mathcal{W}_f}$ that contains the fuzzy states of both parent and context nodes.
3) Since $\pi_\psi(\Gamma_\psi | \mathcal{S}_\mathcal{W})$ is a probability density function, then $\eta \sum_k \pi_\psi(\Gamma_{k,\psi} | \mathcal{S}_\mathcal{W}) = 1$ wherein $\Gamma_{k,\psi}$s are various fuzzy states that can be generated by exchanging the membership degree β_j of each individual state γ_j, and η is the normalization factor.

Regular Bayesian Networks (BN), are Directed Acyclic Graphs (DAG) in which nodes represent random variables with finite states, and edges specify conditional dependencies between random variables. For each random variable R, a Conditional Probability Table (CPT) is defined that determines the likelihood of R being in one of its states conditioned on the configuration of its parents.

Inference on BN is performed by first adding the observations to the network and then finding the likelihood of the desired random variable by running one of the common inference algorithms in BN such as Variable Elimination, or Belief Propagation [14].

The Fuzzy Bayesian Networks (FBN) presented in this paper is mainly based on the regular BN. However, major differences include defining fuzzy states for random variable nodes, and enhancing them with a set of fuzzy *if-then* rules (coupled with a Fuzzy Inference System) that reflect the knowledge of domain experts. Moreover, observations are assigned a fuzzy state before being added to the FBN, through passing a fuzzification process. Fuzzification of continuous variables is straight forward and follows exactly the same conventional common method presented by L.A. Zadeh in [23] that maps a continuous value to fuzzy set labels with their relative membership degrees. For instance, measured speed of vehicle v (shown by $Speed(v)$ in MEBN language) will be fuzzified and lied in the fuzzy state $\Gamma_{Speed} = \langle Slow^{\beta_1}, Normal^{\beta_2}, Fast^{\beta_3} \rangle$, wherein unique entity identifier symbols *Slow*, *Normal*, and *Fast* are fuzzy sets defined on the universe of discourse of vehicle speed, and the membership degrees are normalized. Furthermore, and for discrete variables, linguistic modifiers that precede them are analyzed to dynamically increase/decrease the fuzziness of the sets defined on a particular discrete universe of discourse. Subsequently, the fuzziness is altered using an ambiguity factor imposed by function $f(\mu_A, \alpha) = (\mu_A)^\alpha$, for μ_A as the membership function defined over set A, and α as the ambiguity factor. Function f is deemed the general form of *dilation* and *contradiction* in fuzzy set theory.

As one of the major inference algorithms in BN, Belief Propagation is presented next for three different cases. The first case is a single node with no parents which may have one, or many fuzzy rule sets (FRS) attached to it. In the second case, a single node (with one or many FRS) is demonstrated that just has a single parent with either a single fuzzy state with a certain probability, or many fuzzy states with probabilities that sum to one. Finally, the third case represents a single node (again with one or many FRS) that has multiple parents which all can be in their only fuzzy states, or if having more than one fuzzy state, may produce various configurations of fuzzy states. Following explains each of these three cases more in detail.

Let us assume that $\psi(\Delta)$ is an instance of resident random variable ψ, with possible values ${\gamma_j}^{(\beta_j)}$, $j = 1 \ldots |N_\psi|$, that is preceded by an arrangement of m parents $\Phi = \{\phi_1, \phi_2, \ldots, \phi_m\}$ that in total create C configurations. Therefore, $P(\psi = {\gamma_j}^{(\beta_j)} | \Phi) = p_{ij}$ will be the probability of ψ being at state ${\gamma_j}^{(\beta_j)}$ given the ith configuration of parents, and is stored in the CPT assigned to resident random variable ψ. In FBN, random variables are in a fuzzy state with a probability that represent the certainty about being in that state. This is represented by vector $\Gamma_\psi^{p_\psi} = \langle {\gamma_j}^{(\beta_j)} |\ j = 1 \ldots |N_\psi| \rangle$ wherein $|N_\psi|$ is the number of possible values of ψ, and p_ψ is the likelihood of being in fuzzy state Γ_ψ.

1) No Parents: The fuzzy state of a random variable node with no parents is obtained using the fuzzification process described above, and the likelihood of being in that state is obtained using the *a priori* probabilities defined for each of its possible values. Accordingly, let us assume that ψ_0 is a random variable with possible values ${\gamma_j}^{(\beta_j)}$, which has no parents ($\Phi = \emptyset$), and is in the fuzzy state $\Gamma_{\psi_0} = \langle {\gamma_j}^{(\beta_j)} |\ j = 1 \ldots |N_{\psi_0}| \rangle$. Therefore, the likelihood p_{ψ_0} of being in fuzzy state Γ_{ψ_0} is found by calculating the following inner product:

$$p_{\psi_0} = \vec{\beta}.\pi_{\psi_0} \tag{3}$$

in which $\vec{\beta} = \langle \beta_1, \beta_2, \ldots, \beta_{|N_{\psi_0}|} \rangle$ is a vector of membership degrees of possible values of ψ_0 that are normalized to 1, and vector $\pi_{\psi_0} = \langle p_1, p_2, \ldots, p_{|N_{\psi_0}|} \rangle^T$ contains *a priori* probabilities of being at each individual state γ_j. If more than one FRS is assigned to ψ_0, then

$$\vec{p}_{\psi_0} = B.\pi_{\psi_0} \tag{4}$$

in which vector $\vec{p}_{\psi_0}$ contains the calculated probability using Eq. 3 for each FRS, and $B = \langle \vec{\beta}_j^T |\ j = 1 \ldots |\mathcal{S}_{\psi_o}| \rangle^T$ is a matrix with row j as the membership degrees $\vec{\beta}_j$ that jth FRS produces.

2) One Parent: Let us assume that random variable ψ_1 has only one parent node $\Phi = \{\phi_1\}$ in fuzzy state $\Gamma_{\phi_1}^{p_{\phi_1}} = \langle {\varphi_r}^{(\rho_r)} |\ r = 1 \ldots |N_{\phi_1}| \rangle$, and its own fuzzy state is also inferred to be $\Gamma_{\psi_1}^{p_{\psi_1}} = \langle {\gamma_j}^{(\beta_j)} |\ r = 1 \ldots |N_{\psi_1}| \rangle$ using the *if-then* rules defined in the FRS assigned to ψ_1. Therefore, with $\vec{\rho}$ as the vector of membership degrees of possible values of parent ϕ_1, p_{ψ_1} is calculated as:

$$p_{\psi_1} = \eta(\vec{\rho}.\pi_{\psi_1}(\psi_1|\phi_1).\vec{\beta}^T)p_{\phi_1} \tag{5}$$

wherein η is the normalization factor, $\vec{\rho}$ is a $1 \times |N_{\phi_1}|$ vector, $\pi_{\psi_1}(\psi_1|\phi_1)$ is a $|N_{\phi_1}| \times |N_{\psi_1}|$ CPT matrix, and $\vec{\beta} = \langle \beta_j |\ j = 1 \ldots |N_{\psi_1}| \rangle^T$ is the vector of membership degrees of possible values of ψ_1. Assuming that more than one FRS is assigned to ψ_1, and by constructing B same as the No Parents case, Eq. 5 will be rewritten as:

$$\vec{p}_{\psi_1} = \eta(\vec{\rho}.\pi_{\psi_1}(\psi_1|\phi_1).B)^T p_{\phi_1} \tag{6}$$

There is also a case in which a parent node might be in different uncertain fuzzy states. For instance, let us assume that parent ϕ_1 is in K different fuzzy states $\Gamma_{k,\phi_1}^{p_{k,\phi_1}}$ (with probabilities $p_{k,\phi_1} > 0$ and $\sum_k p_{k,\phi_1} = 1$). Therefore, Eq. 5 is updated to:

$$p_{\psi_1} = \eta(R.\pi_{\psi_1}(\psi_1|\phi_1).\vec{\beta}^T)^T.\vec{p}_{\phi_1} \tag{7}$$

where matrix $R = \langle \vec{\rho}_j^T |\ j = 1 \ldots K \rangle^T$, contains the membership degrees of each fuzzy state of parents at each row, and $\vec{p}_{\phi_1} = \langle p_{j,\phi_1}\ j = 1 \ldots K \rangle^T$ is the arrangement of their likelihoods.

3) Multiple Parents: If random variable ψ_2 has more than one parent $\Phi = \{\phi_1, \phi_2, \ldots, \phi_m\}$ each in their corresponding fuzzy states $\Gamma_{\phi_i}^{p_{\phi_i}}$, $i = 1 \ldots m$, then vector $\vec{\rho}_T$ is created as follows:

$$\vec{\rho}_T = T(\Gamma_{\phi_1} \times \Gamma_{\phi_2} \times \ldots \times \Gamma_{\phi_m}) \tag{8}$$

in which T is the *t-norm* operator [23], and $\phi_i \times \phi_j$ is the Cartesian product of possible values of ϕ_i and ϕ_j. Accordingly, p_{ψ_2} is calculated as:

$$p_{\psi_2} = \eta(\vec{\rho}_T.\pi_{\psi_2}(\psi_2|\Phi).\vec{\beta}) \prod_{i=1}^{m} p_{\phi_i} \tag{9}$$

in which η is the normalization factor, p_{ϕ_i} is the probability of parent ϕ_i being at fuzzy state $\Gamma_{\phi_i}^{p_{\phi_i}}$, and $\pi_{\psi_2}(\psi_2|\Phi)$ is a CPT matrix whose first and second dimensions are equal to the number of different configurations that parents in Φ can make, *i.e.*, $\prod_{i=1}^{m} |N_{\phi_i}|$, and the number of the possible values of ψ_1, *i.e.*, $|N_{\psi_1}|$, respectively. Furthermore, for the case of parents having more than one fuzzy state, $K^* = \prod_{i=1}^{m} K_i$ will be all the number of configurations that m parents make, and Eq. 8 will be updated to:

$$R_T = \langle T(\Gamma_{\phi_1} \times \ldots \times \Gamma_{\phi_m})_j^T |\ j = 1 \ldots K^* \rangle^T \tag{10}$$

that is a $K^* \times \prod_{i=1}^{m} |N_{\phi_i}|$ matrix within which row i represents the *t-norm* of ith configuration of fuzzy states of parents. Moreover, the probabilities of each configuration are represented by vector $\vec{p}_\Phi$ that is:

$$\vec{p}_\Phi = \langle \prod_{j=1}^{m} p_{i,\phi_j} |\ i = 1 \ldots K^* \rangle^T \tag{11}$$

in which p_{i,ϕ_j} is the probability of parent ϕ_j at the ith configuration of parents. Finally, p_{ψ_2} is found using equation below.

$$p_{\psi_2} = \eta(R_T.\pi_{\psi_2}(\psi_2|\Phi).\vec{\beta})^T.\vec{p}_\Phi \tag{12}$$

At the end, if ψ_2 has more than one FRS collected in the set $\mathcal{S}_{\psi_2}$, then Eq. 12 will be upgraded to:

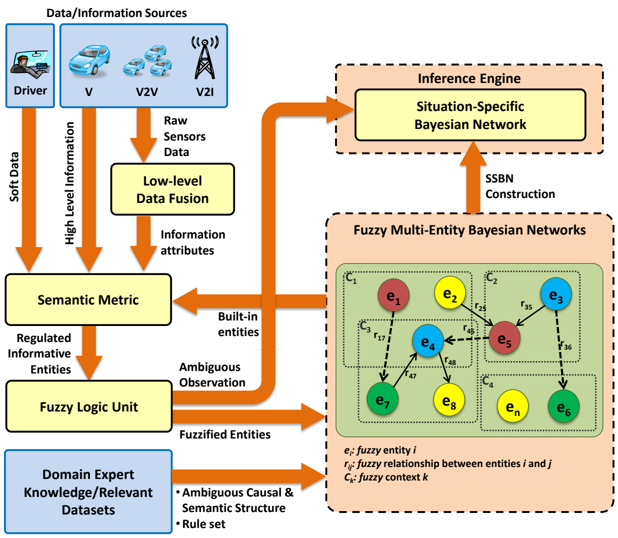
$$\vec{p}_{\psi_2} = \eta(R_T.\pi_{\psi_2}(\psi_2|\Phi).B)^T.\vec{p}_\Phi \tag{13}$$

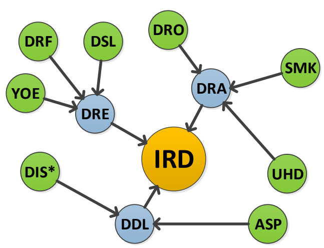
which is a $\mathcal{S}_{\psi_2} \times 1$ matrix, and B is created the same way as Eq. 6.

The rest of this paper demonstrates a practical application in Vehicular Ad-hoc Networks (VANET) area for the theory presented so far.

IV. Case Study

As a case study, Collision Warning System (CWS) in VANET is chosen and implemented (see Fig. 1). The proposed framework is mainly based on our previous work presented in [8]. In this model, the data/information originates from different sources (*i.e.*, the vehicle, surrounding vehicles, infrastructure, or the driver) with various levels of abstraction. Upon having their information attributes set, interpretation of input entities are compared with the built-in entities (predefined by domain experts within Fuzzy MEBN structures) using semantic similarity methods such as those introduced in [1]. Thereafter, the input entities are annotated with a value between 0 and 1 (where 1 means that both entities have exactly the same semantic meaning), and are represented by fuzzy singletons. The inner structure of Fuzzy MEBN is also determined by domain experts within which the semantic side is modeled using First-order Fuzzy Logic, and causal side is represented using our novel Fuzzy Bayesian Network. Finally, the proposed belief propagation algorithm in previous section is performed on a Situation-Specific Fuzzy Bayesian Network (SSFBN) that is constructed based on the imprecise contextual information provided by domain experts and also input entities. According to [13], four main entities that are involved in road safety issues are Vehicle, Environment, Driver, and Demographic which are in both semantic and causal relationships with each other and their sub-entities.Some of the entities related to Irregular Driving behavior, which is one of the major entities involved in a near collision situation (see [8]) along with their causal relationships are demonstrated in Fig. 2.





ASP	Average Speed
DDL	Distance Danger Level
DIS	Distance
DRA	Driver Attentiveness
DRO	Drowsiness
DSL	Driving Skill Level
DRF	Driver Faults
IRD	Irregular Driving
DRE	Driving Experience
SMK	Smoking
UHD	Using Hand-held Devices
YOE	Years of Experience

Fig. 2. Involved entities in an Irregular Driving Behavior, and their causal relations

V. Simulation Results

Two distinct scenarios are designed to evaluate the performance of the proposed framework, and to emphasize the applicability of the proposed model in a real world problem. The first scenario models a single vehicle that obtains the necessary data/information regarding itself, the environment, and the driver locally, and using its own sensors. Furthermore, the second scenario adds vehicle-to-vehicle communication, with fixed number of neighbors (8 in our simulations), and vehicle-to-infrastructure communication features to the first scenario and therefore, enables the vehicle to obtain the necessary data/information both locally and globally. Table I briefly displays the specifications of these two scenarios. It should be noted that the relative fuzzy states of each fuzzy random variable associated with each of the input entities are specified using the fuzzy rule-sets predefined by domain experts. The generated SSFBN for the last situation (after observing the driver smoking) of scenario 1 is depicted in Fig. 3. As it is clearly shown in the figure, upon receiving the observations, respective fuzzy states are calculated using the fuzzy rule-sets attached to each fuzzy random variable. Moreover, the probability of being in each of those fuzzy states for each node is computed based on the number of parents they have (see Section III).

TABLE I

Knowledge Base (KB) of Scenario 1 and 2

Entity	State	
	Scenario 1	Scenario 2
Speed	120 km/h	60 km/h
Road Type	Highway	Highway
Daytime	4:30pm	10:00pm
Weather	Sunny	Partly Cloudy
Skill Level	Very Professional	So Amateur
Driving Faults	6	3
Years of Experience	10	2
Drowsy	Yes	Somehow
Using Device	Somehow	Yes
Smoking	Yes	No
Distance(VEH515)	N/A	Very Close
Distance(VEH516)	N/A	Very Close
Distance(VEH517)	N/A	Very Close

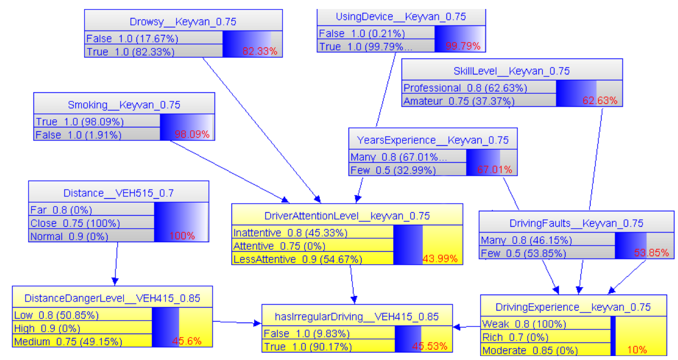


Fig. 3. Generated SSFBN for Scenario 1

The correct performance of the proposed framework is demonstrated by adding the observations in a way to intentionally make the situation close to an irregular driving behavior, and then measuring the probability of being in the fuzzy state assigned to Irregular Driving Behavior fuzzy random variable. As it is shown in Fig. 4, along with adding the observations the probability of being in an irregular driving behavior fuzzy state (9.83% not having, and 90.17% having irregular driving in our case) goes up to 48.37%. In other words, the increase in the graph indicates that the framework is correctly assessing an irregular driving behavior situation. This is also observed in the second scenario as the communication with three more vehicles are initiated,

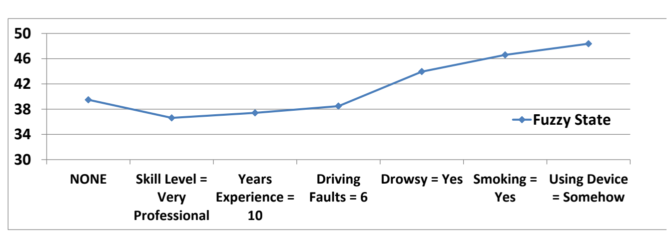


Fig. 4. Evolution of the probability of being in fuzzy state upon adding observations in scenario 1

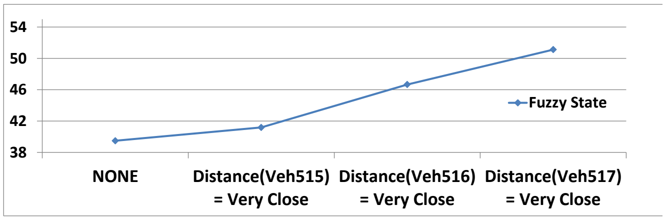


Fig. 5. Evolution of the probability of being in fuzzy state upon adding observations in scenario 2

The HLIF-related evaluation criteria introduced by Costa *et al.* in [5] can be interpreted using the results presented here. The proposed framework is specialized to be deployed in VANET-related tasks, which by nature, benefit from a broad range of data and information sources. The semantically relevant data and information sources are grouped into Fuzzy MFrags to construct a particular entity, and furthermore, a specific task make use of a subset of them to assess a situation. Such capability of AAS makes sure that only the relevant inputs enter the system and other (irrelevant) ones are kept away from the reasoning system. Furthermore, as is was also shown in Figures 4 and 5, different evidences have their own impact on the final outcome. Knowledge and evidence handling is also perfectly managed by the capabilities of the proposed Fuzzy MEBN in such a way that it takes advantage of the expressiveness of FOFL and uncertainty management power of BN. Moreover, the results show that the proposed framework performs correctly in assessing the Irregular Driving Behavior when it is run in different scenarios. Finally, the proposed model is robust since it is able to perform in different scenarios with varying number of available evidence and still assess the desired situation.

VI. Conclusion

In this paper, the theoretical foundation of Fuzzy Multi-Entity Bayesian Networks (MEBN) is comprehensively discussed. Fuzzy MEBN handles the semantics analysis by making use of First-order Fuzzy Logic, and manages uncertainty by employing Fuzzy Bayesian Networks as its causal reasoning core. In other words, Fuzzy MEBN adds the imprecise knowledge representation and reasoning capability to the conventional MEBN by incorporating Fuzzy logic into both its semantics and causal sides.

Results show that proper implementation of Fuzzy MEBN enables imprecise knowledge representation and reasoning, which can be used to tackle many real world applications such as collision warning in Vehicular Ad-hoc Networks area. Our future work will involve the incorporation of imprecise contextual information represented by Fuzzy MFrags when generating the Situation-Specific Fuzzy Bayesian Network and performing inference on it.

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