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# Attitude Tracking Control for Hypersonic Vehicles Based on Type-2 Fuzzy Dynamic Characteristic Modeling Method

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Abstract-It is obvious that the highly nonlinear nature of dynamic behavior in hypersonic vehicle system impose very challenging obstacles to the controller design. In this paper, we discuss the design of a novel type-2 fuzzy dynamic characteristic modeling method to attitude tracking control problem for hypersonic vehicles in gilding phase. The type-2 (T2) fuzzy logic is introduced into the characteristic modeling (CM) method. Unlike the traditional fuzzy dynamic modeling method normally with a fixed local linear model in every fuzzy subspace, our approach performs CM in subspace, which actually can handle the nonlinearity well while reducing the number of fuzzy rules. After dividing the whole restriction region into several subspaces, the whole nonlinear system can be regarded as a T2 fuzzy "blending" of each individual characteristic model. Then this novel T2 fuzzy logic system modelled by decomposition of a complex nonlinear system into a collection of local CMs, can overcome the deficiencies of traditional fuzzy dynamic modeling and CM approaches. Therefore, it can achieve a better trade-off between tracking accuracy and convergence efficiency in the controller design for hypersonic vehicles. Simulation results under the conditions of certainty and uncertainty are given to show the effectiveness of the novel method for the attitude tracking control of hypersonic vehicles in gilding phase.

# I. INTRODUCTION

Hypersonic vehicle generally refers to the winged or wingless aircraft whose speed is more than five times of the speed of sound. The study related to hypersonic vehicles has witnessed a growing interest in recent years due to its ability of high speed transportation and affordable space access. As a key pillar of making access to space routine and affordable, the hypersonic vehicle can provide a wide range of application in civil, military and aerospace field [1]. And it also has a huge difference with the traditional aircrafts. The characteristic of high speed makes it be influenced by many factors such as the dynamic pressure effect, viscous effects, low-density, and so on [2]. Moreover, its structure using advanced airframe/propulsion integration technology makes itself highly coupled [3]. Therefore the flight controller design has been a long-term challenging problem.

Over the past decades, some control methods of hypersonic vehicles have been proposed. Among the available approaches, i.e., implicit model following control [4] and linear output feed-back control [5], they are linearized implementation under a specific trim condition, which can simplify the controller design for nonlinear model. However, it is still a difficult task with the linearization of the dynamics. Others try to use the feedback linearization techniques to design a nonlinear controller. For instance, in [6], a methodology was presented to design robust nonlinear controller for the rigidbody longitudinal hypersonic vehicle, which employed only the elevator as aerodynamic control surface. In [7], a control structure was developed by combining the inputs from the pilot model, baseline controller, and adaptive controller. The sequential loop closure controller design method was proposed with the decomposition of the dynamic equations into functional subsystems [8]. This method followed the approach that combined robust adaptive dynamic inversion with back-stepping arguments to obtain control architecture. In [9], fuzzy logic system was added into the dynamic surface control method to compensate the unknown nonlinearities and modeling errors caused by changes of flight conditions.

With the help of the existing studies, it has also demonstrated some other applications in dealing with complex nonlinear systems. But there is still a need for higher accuracy and less convergence time while guaranteeing the robustness of the system under practical applications [10][11]. As an adaptive control algorithm, characteristic modeling (CM)-based all-coefficient control method was proposed with the simplicity for design, convenience of adjustment, and strong robustness [12]. The idea behind CM is to model according to dynamic characteristics of plant and control performance requirements, rather than merely according to accurate dynamic analysis for plant. Then the characteristic model is expressed with the timevarying difference equation. In a certain sense, this method has solved three principal problems in practical applications: it can guarantee the stability of a closed-loop system in the start period of a transient process or when parameter estimates have not converged to their "true values", the number of parameters to be estimated is small, and the number of parameters to be regulated online is also small. The research on this method has been going on for more than 20 years. It has already been applied successfully to more than 400 systems belonging to nine kinds of engineering plants in the field of astronautics and industry. For its high adaptability, we try to use it to handle the nonlinearity of hypersonic vehicle.

CM based control method actually has been applied to the attitude control of a hypersonic vehicle X-34 [13]. The experiment showed that the controller worked well during the whole climbing phase of hypersonic vehicle. But we also find that those several parameters used in CM seems unlikely to enable the controller to perform well enough in every condition in the gliding phase. Essentially, for some complex rapid tracking nonlinear systems, since traditional CM needs to compress all the information of the high-order system model into several given characteristic parameters, it is difficult to meet the control performance requirements by merely using a characteristic model. Thus by combining CM with some other intelligent methods, i.e., fuzzy logic, CM can be improved to describing some complex systems in some ways [14].

Generally speaking, traditional fuzzy logic controller uses the type-1 (T1) fuzzy set to deal with uncertainty. But when the parameter uncertainty becomes bigger and there exist rule uncertainties, it will be less effective. Then type-2 (T2) fuzzy set provides additional design degrees of freedom in Mamdani and Takagi-Sugeno (T-S) fuzzy logic systems (FLSs), which may be useful when such systems are used in situations where lots of uncertainties are present [15]. The most fundamental difference is that membership functions of T2 fuzzy set are fuzzy whereas the ones of T1 fuzzy set are crisp. In this way, T2 FLSs have the potential to provide better performance than T1 FLSs. Meanwhile, a type reduce process is necessary to convert the T2 fuzzy output set to T1 fuzzy set. It is always computationally intractable when the number of input dimension is large. More recently, there has been a growing interest in the study of interval T2 (IT2) fuzzy sets in which computational efforts are very manageable in practice. Based on this, there exists controller design methods using interval T2 FLS [16][17]. Motivated by it, this paper adds the IT2 FLS into CM method to achieve a better trade-off between accuracy and convergence in the controller design.

Specifically, a controller using CM and IT2 FLS is designed to implementing the attitude control of hypersonic vehicles. In our proposed design approach, on the one hand, while dividing the whole state space into many subspaces via fuzzy logic, it reduces the difficulty of modeling for the whole complex hypersonic vehicle system by using single characteristic model. On the other hand, since in each subspace the CM is performed dynamically according to the different features, it also reduces the dependence on input data to some extent. It means that the number of fuzzy rules in the hypersonic vehicle system is properly controlled with performance guarantees to deal with uncertainties within IT2 FLS framework.

This paper is organized as follows. The nonlinear dynamics model of hypersonic vehicle is described in Section 2. The controller design based on CM and T2 FLS is proposed in Section 3. Simulation results and discussion are presented in Section 4. Finally, conclusions are given in Section 5.

## II. MODEL DESCRIPTION

The longitudinal mathematical model of a generic hypersonic vehicle was developed at NASA Langley Research Center [18]. In gilding phase, it can be described by a set of five-order differential equations in terms of forward velocity V, flight-path angle  $\gamma$ , altitude h, angle of attack  $\alpha$ , and pitch rate q [17]. Specifically, in gilding phase the engine is close and thrust forces is zero. Then the only controllable parameter is elevator deflection  $\delta_e$ . So the model of hypersonic vehicle in gilding phase can be described

$$\dot{V} = -\frac{D}{m} - \frac{\mu \sin \gamma}{r^2} \tag{1}$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{Vr^2} \tag{2}$$

$$h = V \sin \gamma \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = \frac{M_y}{I_y} \tag{5}$$

where m,  $I_y$  and  $\mu$  represent the vehicle mass, moment of inertia, and gravity constant, respectively. And L, D,  $M_y$ , and r represent lift, drag, pitching moment, and radial distance from Earth's center, which can be defined as

$$L = \frac{1}{2}\rho V^2 s C_L \tag{6}$$

$$D = \frac{1}{2}\rho V^2 s C_D \tag{7}$$

$$M_y = \frac{1}{2}\rho V^2 s\bar{c}C_M \tag{8}$$

$$\rho = \rho_s \times e^{(-h/h_s)} \tag{9}$$

$$r = h + R_e \tag{10}$$

where  $C_L$ ,  $C_D$ , and  $C_M$  represent the lift, drag, and moment coefficients due to elevator deflection  $\delta_e$ . The parameters  $\rho$ , s,  $\bar{c}$ , and  $R_e$  represent the air density, the reference area, the mean aerodynamic chord, and the radius of the Earth, respectively. And  $\rho_s$  is a standard value at altitude  $h_s$ , which is used to find  $\rho$  at the altitude h.

In addition, because the flight has a long span of velocity and altitude, the aerodynamic parameters mentioned above cannot be described with accurate mathematical formula. In simulation we get their current values by looking up the tables.

In this paper the physical coefficients are simplified around the nominal cruising flight condition in which V = 6000 m/s, h = 60000 m,  $\gamma = 0$  rad, and q = 0 rad/s, respectively. The parametric uncertainty is defined as an additive perturbation  $\triangle$  to its nominal values.

$$m = m_0(1 + \Delta m) \tag{11}$$

$$s = s_0(1 + \Delta s) \tag{12}$$

$$I_y = I_0(1 + \Delta I_y) \tag{13}$$
$$a_z = a_0(1 + \Delta a) \tag{14}$$

$$\bar{c} = \bar{c}_0 (1 + \Delta \bar{c}) \tag{15}$$

where the nominal values of the above parameters and  $\mu,$   $R_{e},\,h_{s}$  are listed as follows

$$\begin{split} m_0 &= 4353 \text{ kg} \\ s_0 &= 3.45 \text{ m}^2 \\ I_0 &= 34979.599 \text{ kg} \cdot \text{m}^2 \\ \rho_0 &= 1.225 \text{ kg/m}^3 \\ \bar{c}_0 &= 12.7 \text{ m} \\ \mu &= 3.989 \times 10^{14} \text{ N} \cdot \text{m}^2/\text{kg} \\ R_e &= 6371300 \text{ m} \\ h_s &= 7200 \text{ m} \end{split}$$

Here, the maximum values of the additive perturbation terms  $\triangle$  are up to 25% [19]. The aim of hypersonic vehicle control is to make the velocity and altitude track the command target with the help of the control of  $\delta_e$ . However, the V and h change slowly through manipulating the  $\delta_e$ while others changes fast, which may lead to the control fail. Thus the control procedure is divide into two steps: guiding and attitude control. Firstly, through the control of  $\delta_e$ , we get an expected attitude angle (for longitudinal control task, it is  $\alpha$ ). Then  $\alpha$  is used to control V or h. This paper will focus on the attitude controller design. So, in the model, the control input is the elevator deflection  $\delta_e$ , while the controlled output is the angle of attack  $\alpha$ .

# III. CONTROLLER DESIGN BASED ON IT2 FUZZY DYNAMIC CHARACTERISTIC MODELING METHOD

# A. Characteristic modeling (CM)

Generally, it is necessary to get the mathematical model to describe the system's movement law and its environment. However, in particular, it is not easy for some controlled plants because their characteristics and environment may change unpredictably. Moreover, even if some plants may be presented by an accurate mathematical model, the order of the model is very high and the structure of the model is very complicated [14]. Therefore all of these impose very challenging obstacles to the satisfactory controller design problem. In view of this, CM was proposed to solve the problem. We use this method to model the relationship between angle of attack and elevator deflection.

The so-called CM is a modeling method based on plant dynamic characteristics and control performance requirements, rather than merely based on accurate plant dynamic analysis [12]. According to the CM theory, the single input single output system can be expressed with the following second-order time-varying difference equation

$$y(k+1) = f_1(k)y(k) + f_2(k)y(k-1) + g(k)u(k)$$
 (16)

where y(k) is the output, u(k) is the control variable,  $f_1(k)$ ,  $f_2(k)$ , and g(k) are the coefficients at moment k.

Sample time satisfies certain condition to assure the coefficients of (16), i.e.,  $f_1(k)$ ,  $f_2(k)$ , and g(k), are slowly time-varying. Then it can obtain a stable model to generate the right control value. In contrast, if the coefficients are varying quickly, it means that the system changes greatly in the sampling period. So when it generates a control value according to the quickly time-varying coefficients, the

control task is failed because the control signal is lagging. Moreover, in the CM method, as the sample time converges to zero,  $f_1(\infty) + f_2(\infty) + g(\infty) \rightarrow 1$ .

In the hypersonic vehicle system, it is hard to get the above coefficients through deformation, order reduction, or approximate treatment of mathematical model. So in the CM, all the coefficients are updated by online training of realtime data. In this paper, gradient algorithm is used to update the coefficients.

$$\begin{cases} \theta(k+1) = \theta(k) + \frac{\lambda_1 \phi(k)}{\lambda_2 + \phi^T(k)\phi(k)} [y(k+1) - \phi^T(k)\theta(k)] \\ \theta(k) = [f_1(k), f_2(k), g(k)] \\ \phi(k) = [y(k), y(k-1), u(k)] \end{cases}$$
(17)

where  $\lambda_1$  and  $\lambda_2$  are known positive constants.

Then, with the system difference equation (16), the controller can be designed directly as follows

$$u(k) = \frac{y_r(k+1) - f_1(k)y(k) - f_2(k)y(k-1)}{g(k)}$$
(18)

where  $y_r(k)$  is the expected output at moment k.

It uses the minimum variance self-tuning control strategy to change the variable into the expected value in next sampling period. Because the system status may change greatly, which cannot be described by (18) while using control output in this model. So the system cannot achieve the desired control effect. Then the golden section adaptive (GSA) method proposed by Wu [12], is used to solve the problem. It is a strategy which is increasingly close to the desired target and very valuable for practical applications. The GSA control strategy can be described as follows

$$u_g(k) = \frac{-l_1 f_1(k) e(k) - l_2 f_2(k) e(k-1)}{g(k)}$$
(19)

where e(k) is the error between the actual output and the expected one at moment k. The parameters  $l_1 = 0.382$  and  $l_2 = 0.618$  (namely, the golden section coefficients).

Although it cannot guarantee the output error is the minimum, the dynamic characteristics are satisfactory, and whether the parameter estimates are accurate or not, the closed-loop system is stable [20]. In addition, many other auxiliary control strategies have been used in CM method to make the result more accurate. They are defined as follows [13]

$$u_0(k) = \frac{y_r(k) - f_1(k)y_r(k-1) - f_2(k)y_r(k-2)}{g(k)}$$
(20)

$$u_t(k) = u_t(k-1) - K_t(k)e(k)$$
(21)

$$u_d(k) = -K_d(k)e(k) \tag{22}$$

$$K_t(k) = \begin{cases} K_{t1}, & e(k)[e(k) - e(k-1)] > 0; \\ K_{t2}, & e(k)[e(k) - e(k-1)] \le 0; \end{cases}$$
(23)

$$K_d(k) = C_d \sqrt{\sum_{j=0}^{l_d} |e(k-j)|}$$
(24)

where  $u_0$  is the keep controller,  $u_t$  is the logic integration controller generated by the history errors aiming to reduce the vibration,  $u_d$  is the logic differential controller used to accelerate the convergence rate. The parameters  $K_t$ ,  $C_d$  and  $l_d$  represent integral coefficient, differential coefficient, and the number of sampling points.

So the final control output is

$$u(k) = u_g(k) + u_0(k) + u_t(k) + u_d(k)$$
(25)

For a specific control case, the above control strategies will be chosen to combine a complete controller according to the real effect. In this paper, GSA controller is the main controller. It controls the system by the model equation directly and can embody the adaptive characteristic well. The other control strategies are used as auxiliary controller.

# B. IT2 fuzzy dynamic characteristic modeling (IT2-FDCM)

The idea behind CM method is to compress all the information of the high-order model into several given characteristic coefficients. So it is very important to restrict the result coefficients in a reasonable range in order to make the identifier precise enough. The CM is a process of searching the coefficients in a reasonable range [21]. If the range is too small to contain the model, the control will fail. On the contrary, if the solution space is too large, the time of searching the right coefficients will be long which will make the control fail because of lagging.

Generally, the proper range is set by deducing an approximate analytical model. However, the system of hypersonic vehicle changes drastically, the searching space will be very large. To solve this problem, we introduce the IT2 fuzzy logic to partition the large space into several smaller subspaces. T2 fuzzy sets was firstly introduced as an extension of an ordinary T1 fuzzy sets, in which membership grades are also T1 fuzzy sets and are called as secondary memberships [22]. It is believed to be able to enhance modeling uncertainty about noisy data and different information meanings. However, since the computation in rule inference and type-reduction process is complexity and it is difficult to determine those secondary memberships, most of real-world applications were implemented based on the simplified version, i.e., IT2 fuzzy set, in which secondary memberships are either zero or one [23].

The simplification makes all computations in twodimensional plane. So the IT2 FLS can be described as

$$\tilde{A} = \int_{x \in X} \int_{\tau \in J_x} 1/(x,\tau), J_x \subseteq [0,1]$$
(26)

where

x: primary variable of one T2 fuzzy set.  $x \in X$ ,  $\tau$ : secondary variable of one T2 fuzzy set, in which $\tau \in J_x$ 

 $J_x$ : primary membership of x, in which  $J_x \subseteq [0,1]$ 

For IT2 FLS, all of the possible values form an area. The area between the upper MF (UMF) and the lower MF (LMF) is called footprint of uncertainty (FOU) [24]. Since all of the secondary grades of an IT2 FLS equal 1, the IT2 FLS can be and only be represented by LMF and UMF [25].

$$A = 1/\text{FOU}(A) \tag{27}$$

$$\operatorname{FOU}(\tilde{A}) = \begin{cases} \left\{ \underline{\tau}_{\tilde{A}}(x), ..., \bar{\tau}_{\tilde{A}}(x) \right\}, \forall x \in X_d \\ \left[ \underline{\tau}_{\tilde{A}}(x), \bar{\tau}_{\tilde{A}}(x) \right], \forall x \in X \end{cases}$$
(28)

Here the top line is for a discrete universe of discourse  $X_d$ , and the bottom line is for a continuous universe of discourse X. The parameters  $\underline{\tau}$  and  $\overline{\tau}$  represent the LMF and UMF.

The structure of IT2 FLS is shown in Fig. 1. In this figure, the type-reducer is a module used to convert the IT2 fuzzy results into T1 fuzzy set.



Fig. 1. The structure of IT2-FLS

In our method, the searching range is divided into some subspace. If the IT2 fuzzy system has p inputs and the total number of fuzzy rules is N. The *i*-th fuzzy rule is in the following form

Rule *i* : If 
$$x_1(k)$$
 is  $F_1^i$  and ... and  $x_p(k)$  is  $F_p^i$ , then  
 $y^i(k+1) = f_1^i(k)y(k) + f_2^i(k)y(k-1) + g^i(k)u(k)$ 

where  $\tilde{F}$  is the IT2 fuzzy sets,  $y^i$  is the output, and u is the control value. Meanwhile,  $f_1^i$ ,  $f_2^i$ , and  $g^i$  are the coefficients used in CM. And the p inputs are  $x_1, \dots, x_p$ .

According to (19), the *i*-th fuzzy rule of the GSA controller design can be described as

Rule 
$$i$$
: If  $x_1(k)$  is  $F_1^i$  and ... and  $x_p(k)$  is  $F_p^i$ , then  
 $u_g^i(k) = \frac{-l_1 f_1^i(k) e(k) - l_2 f_2^i(k) e(k-1)}{g^i(k)}$ 

where  $e(k) = |y_r(k) - y(k)|$  and  $u_g^i$  is the GSA control output.

Then the computation is to merge all the GSA controller of subspace into a final GSA controller. It can be calculated as the following steps [26]:

(1) Computing the membership degree of  $\tilde{F}_l^i$  on each  $x_l$ ,  $[\underline{\tau}_{\tilde{F}_l^i}(x_l), \overline{\tau}_{\tilde{F}_l^i}(x_l)], l = 1, 2, \cdots, p, i = 1, 2, \cdots, N$ 

(2) Computing the firing interval of the *i*-th rule  $w^{i}(x)$ 

$$w^{i}(x) = \left[\underline{\tau}_{\tilde{F}_{1}^{i}}(x_{1}) \times \cdots \times \underline{\tau}_{\tilde{F}_{p}^{i}}(x_{p}), \bar{\tau}_{\tilde{F}_{1}^{i}}(x_{1}) \times \cdots \times \bar{\tau}_{\tilde{F}_{p}^{i}}(x_{p})\right]$$
$$= \left[\underline{w}^{i}, \bar{w}^{i}\right], i = 1, 2, \cdots, N$$
(29)

(3) By using the center-of-sets (COS) type reducer [27], the type-reduced set is given by

$$u_{gCOS} = \int_{w^1} \cdots \int_{w^N} 1 / \frac{\sum_{i=1}^N w^i u_g^i}{\sum_{i=1}^N w^i} = [u_l, u_r] \quad (30)$$

where

$$u_{l} = \min_{w^{i}} \frac{\sum_{i=1}^{N} w^{i} u_{g}^{i}}{\sum_{i=1}^{N} w^{i}}$$
(31)

$$u_r = \max_{w^i} \frac{\sum_{i=1}^{N} w^i u_g^i}{\sum_{i=1}^{N} w^i}$$
(32)

Here  $u_l$  and  $u_r$  can be computed efficiently using the Karnik-Mendel (KM) algorithms listed as follows [28].

## Algorithm 1 The Karnik-Mendel algorithms

1: Sort  $u_g^i$  in increasing order, get  $u_g^1 \le u_g^2 \le \cdots \le u_g^N$ 2: Initialize  $w_0^i$  by setting  $w_0^i = (\underline{w}^i + \overline{w}^i)/2, \ i = 1, 2, \cdots, N$ 3: Compute  $u'_g = \frac{\sum_{i=1}^N u_g^i w_0^i}{\sum_{i=1}^N w_0^i}$ 4: repeat  $u_g = u'_g$ Find the switch point  $h(1 \le h \le N - 1)$  such that 5: 6:  $\begin{array}{l} u_g^h \leq u_g \leq u_g^{h+1} \\ \text{for } 1 \leq i \leq h \quad \text{do} \end{array}$ 7:  $w_0^i = \bar{w}^{\overline{i}};$ 8: end for 9: for  $h+1 \leq i \leq N$  do 10:  $w_0^i = \underline{w}^i;$ 11: 12: end for 13:  $u'_g = \frac{\sum_{i=1}^{N} u_g^i w_0^i}{\sum_{i=1}^{N} w_0^i}$ 14: until  $u'_g == u_g$ 15:  $u_l = u'_a$ 

The computation of  $u_r$  is similar to that of  $u_l$ . The difference is that set  $w_0^i = \underline{w}^i$  when  $i \leq h$  and set  $w_0^i = \overline{w}^i$  when i > h. The  $u_r$  is the result of  $u'_a$ .

(4) Computing the defuzzied output as  $u_q = (u_l + u_r)/2$ .

The design of keeping controller is similar with the GSA controller.

The overall control scheme is shown in Fig. 2. According to the error between expected value and actual value, the system is divided into a series of subsystems. The coefficients will be updated after getting a new sampling point. So the final GSA control value can be computed after merging the coefficients of all subsystems through IT2 FLS. According to the actual effect, the auxiliary controllers are chosen from the keep controller, the logic integration controller and the logic differential controller. After optimizing the auxiliary controller, we can get the final control output.



Fig. 2. The overall control scheme of characteristic model method based on IT2 FLS  $% \left( {{\rm{TL}}} \right)$ 

It can be described as algorithm 2 listed as follows. We call it IT2 fuzzy dynamic characteristic modeling (IT2-FDCM) method.

# Algorithm 2 The IT2-FDCM algorithm

1: Initialize the simulation time T, number of rules N, and the target  $y_r$ ;

2: k = 1;

3: while  $k \neq T$  do

4: Update 
$$e(k) = |y_r(k) - y(k)|;$$

5: for the *i*-th rule  $(1 \le i \le N)$  do

6: compute the coefficients 
$$f_1^i, f_2^i, g^i$$

7: 
$$u_q^i(k) = \frac{-l_1 f_1^i(k) e(k) - l_2 f_2^i(k) e(k-1)}{q^i(k)};$$

8: 
$$u_0^i(k) = \frac{y_r(k) - f_1^i(k)y_r(k-1) - f_2^i(k)y_r(k-2)}{g^i(k)};$$

- 9: end for
- 10: Merge all the  $u_g^i(k)$  and  $u_0^i(k)$  to generate the final control  $u_g$  and  $u_0$  according to IT2 fuzzy logic;
- 11: Compute the auxiliary control  $u_t, u_d$ ;
- 12:  $u(k) = u_g + u_0 + u_t + u_d;$
- 13: Apply the u(k) to the model of hypersonic vehicle to get the output y(k);
- 14: k = k + 1;
- 15: end while
- 16: Output y;

#### IV. SIMULATION RESULTS

In this section, to evaluate the effectiveness of the proposed IT2-FDCM, the simulation studies are carried out with and without parameters uncertainties.

# A. Simulation condition and initialization

Several experiments were conducted to evaluate the effectiveness of our proposed method. The initial condition  $(V_0, h_0, \gamma_0, \alpha_0, q_0)$  of the cruise flight of hypersonic vehicle is listed in table I. We use the absolute value of error of angle of attack as the inputs of the controller. The elevator deflection is used as the control signal which is chosen in the range of [-30, +30] degree. The experiment lasted 120 s. To be more realistic, we have added noise to the system of hypersonic vehicle. Noise is chosen as both uniform noise and gaussian noise.

TABLE I. INITIAL CONDITION OF HYPERSONIC VEHICLE

States	Value
$V_0$	6000 m/s
$h_0$	60000 m
$\gamma_0$	0 rad
$\alpha_0$	4 rad
$q_0$	0 rad/s

Input is divided into five IT2 fuzzy sets, which are Gaussian membership functions (MFs) with the same center but different widths. The lower width is chosen as 0.1 and the upper width is 0.2. It is shown in Fig. 3. Therefore there are 5 fuzzy rules which are formed according to the prior knowledge.

## B. Simulation results without uncertainty

In this simulation, fixed parameter uncertainty and noise do not exist and all parameters are used in their nominal values. The result of angle of attack is shown in Fig. 4. The response curves using IT2-FDCM track the command timely and steadily. The mean squared error (MSE) is 0.0433. Fig.5



Fig. 3. Membership functions of IT2-FLS

shows the result using type-1 fuzzy dynamic characteristic modeling(T1-FDCM) and the MSE is 0.0460. The result using CM is shown in Fig. 6 and the MSE is 0.0633. With the application of IT2 fuzzy logic, the MSE and convergence time decrease.



Fig. 4. Observed value of angel of attack without noise using IT2-FDCM



In Fig. 7, it shows tracking error in 120 s using the CM, T1-FDCM and IT2-FDCM. At the beginning, the curve of IT2-FDCM tracks the target more quickly than single CM method. In addition, the amplitude of the curve of IT2-



Fig. 6. Observed value of angel of attack without noise using CM

FDCM is smaller than single CM method. At the end of the curve, the signal CM cannot keep steady well. And the IT2-FDCM can maintain tracking of target in the whole process. Because the number of inputs, it not obviously to show the accuracy from Fig.4 and Fig.5. But from the MSE and Fig.7, IT2-FDCM increases the accuracy in a degree compared with T1-FDCM. It shows the effectiveness and stability of our proposed method.



Fig. 7. Error results using IT2-FDCM, T1-FDCM, and CM

#### C. Simulation results with uncertainty

In this section, we add both uniform and Gaussian noise to the system. Fig. 8 shows the result with uniform 25% noise and its MSE is 0.0445. The result with the Gaussian noise which variance and mean are 0.1 and 0 is shown in Fig. 9 and MSE with Gaussian noise is 0.0474. Although noise is added into the system of hypersonic vehicle, MSE of IT2 fuzzy dynamic characteristic modeling changes a little. The curve can can quickly track the target in small error. Simulation results show the robustness of IT2 fuzzy dynamic characteristic modeling.

In Fig. 10, it shows the result with uniform noise under the single characteristic modeling. When the noise is gaussian noise, performance of single characteristic modeling control is shown in Fig. 11. Compared with the result in Fig. 8 and Fig. 9, the single characteristic modeling cannot



Fig. 8. Observed value of angel of attack with uniform noise



Fig. 9. Observed value of angel of attack with gaussian noise

keep tracking the target. The curve vibrates heavily at the beginning and then slowly converges to the target. But at the end of the simulation, when the target changes heavily, it still vibrates heavily. The traditional characteristic modeling cannot track the angle of attack well. When our proposed method is applied into the control, in the whole process, the curve can keep tracking the target steadily.



Fig. 10. Control performance with uniform noise using CM



Fig. 11. Control performance with gaussian noise using CM

# V. CONCLUSIONS

In this paper, a controller using IT2-FDCM is designed to solve the hypersonic vehicle attitude tracking control problem. CM method is mainly used to generate the control value and IT2 FLS with five rules is added into the CM to improve accuracy and the searching speed. In this paper, we just consider the attitude control in gilding phase. Simulation results show the robustness and practicality of our proposed method.

In the future, the method will be applied to some more complex models of hypersonic vehicle to verify its feasibility and effectiveness. Building a complete model to control the velocity and altitude of hypersonic vehicle will be considered in our next research. Meanwhile, other type reduction methods will be discussed to reduce the computation cost of our method.

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