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A New Fuzzy Graph and Signature Based Approach to Describe Fuzzy Situational Maps

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***Abstract*— Computational tasks involving intelligent agents often need to process complex structured information. The way of describing this information greatly influences the performance of the agent. Therefore, a big issue is how the complex data describing that valuable information is not lose while it can also be processed in tractable time. Fuzzy signatures and their multidimensional geometric extension, fuzzy situational maps, are used to describe such complex structured data. These problems are examined in the context of a cooperative mobile robot task and a new method is developed for the simplified describing and processing of the complex inner relations in fuzzy situational maps. This paper mainly deals with the fundamentals of this method.**

I. INTRODUCTION

Computational tasks involving intelligent agents often need to process complex structured information. The way of describing this information greatly influences the performance of the agent. In certain cases, e.g. intelligent robot tasks, logistic processes, etc., the decision making would be made in real-time whit limited computational capacity. Therefore, a big issue is how the complex data describing that valuable information is not lose while it can also be processed in tractable time.

The second problem in an information system is the lack or distortion of data. The goal is that the system remain in the state of decision-making capability even if some data are missing.

These problems are examined in a cooperative mobile robot task. The basic idea of this system inherited from a partly unpublished research project at LIFE [1]. There is a cooperation system where a group of autonomous intelligent mobile robots is supposed to solve logistic problems according to the exact instruction given to the Master Robot (R_0). The other assistant robots have no direct communication links with R_0 and all others, but can solve the task by intention guessing from the actual movements and positions of other robots, even though they might not be unambiguous.

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Fuzzy signatures and their multidimensional geometric extension, fuzzy situational maps, are used to describe such complex structured data. We have developed a method for the simplified describing and processing of the complex inner relations in fuzzy situational maps. This paper mainly deals with the fundamentals of this method.

II. FUZZY SIGNATURES

The original definition of fuzzy sets [2] was $A: X \to [0,1]$, and was soon extended to *L-fuzzy sets* by Goguen [3].

This definition is $A_L : X \to L$, L being an arbitrary algebraic lattice. A practical special case, *Vector Valued Fuzzy Sets* was introduced in [4], where $A_{V,k} : X \to [0,1]^k$, and the range of membership values was the lattice of k-dimensional vectors with components in the unit interval. A further generalization of this concept is the introduction of fuzzy signatures and signature sets, where each vector component is possibly another nested vector (Fig. 1).

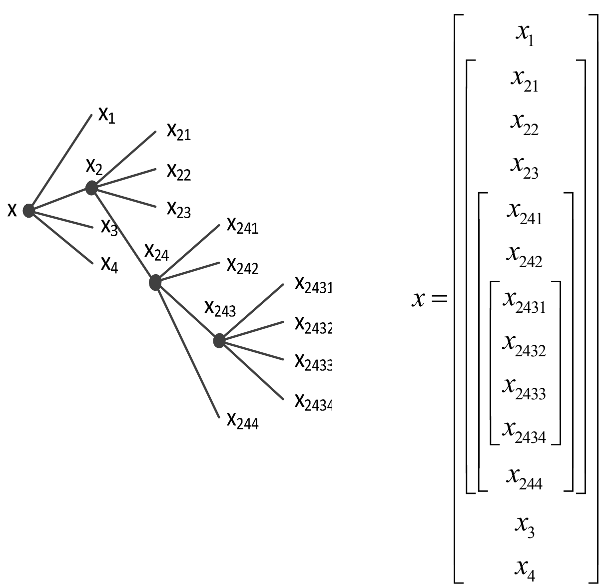

Fig. 1. Example for Fuzzy Signature Structure

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III. FUZZY SITUATIONAL MAPS

We propose a novel approach to support difficult decision-making and to depict situation or context dependent structured data. A special form of fuzzy signatures [5-8] with a spatial structure is used, namely the Fuzzy Situational Map (FSM).

Fuzzy situational maps as multidimensional extended fuzzy signatures (FS) are suitable to describe complex multidimensional system conditions in cases where the information is fragmented, distorted or noisy [9-12].

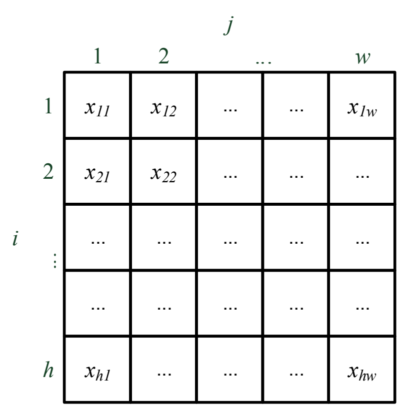
The FSM can be two-, three- or even n-dimensional. Let us see the simplest case, two-dimensional fuzzy situational maps (Fig. 2).

A. Two-dimensional Fuzzy Situational Maps

Two-dimensional FSM may be considered as a geometric lattice, where each node has a fuzzy value or a whole fuzzy set, in extended case.

FSM can be represented as lattices or in matrix form as shown in Fig. 2 and in (1).

$$FSM = \begin{bmatrix} x_{011} & x_{012} & x_{013} & \begin{bmatrix} x_{111} & x_{112} \\ x_{121} & \begin{bmatrix} x_{211} & x_{212} & x_{213} \\ x_{221} & x_{222} & x_{223} \\ x_{231} & x_{232} & x_{233} \end{bmatrix} \end{bmatrix} \\ x_{021} & x_{022} & x_{023} & x_{024} \\ x_{031} & x_{032} & x_{033} & x_{034} \\ x_{041} & x_{042} & x_{043} & x_{044} \end{bmatrix} \quad (1)$$



$$x_{ij} = [0,1]$$
$$i = 1,\ldots,h$$
$$j = 1,\ldots,w$$

Fig. 2. Fuzzy Situational Map

The values in the individual nodes can be interpreted as elements of a fuzzy signature, so fuzzy situational maps can be described as multidimensional spatially structured fuzzy signatures, see Fig. 3.

In the example a very simple case of "refining" the situational map by 2x2 grids is presented.

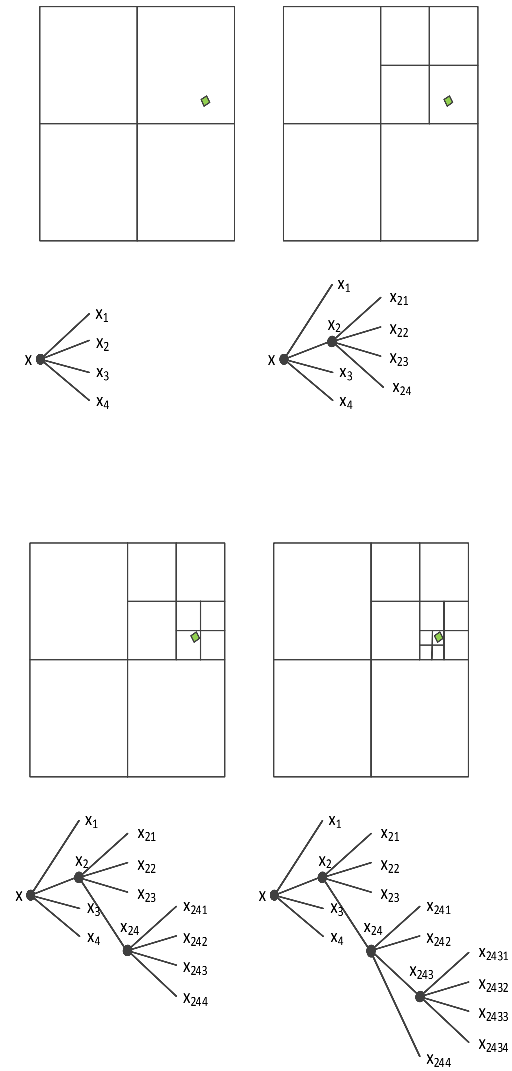


Fig. 3. FSM as multidimensional Fuzzy Signatures

Following this interpretation, it can be said that each node in a FSM can be a further nested FSM and continued iteratively, this extension may go to depth z (applying increased resolution). The approach can lead to a fine structured FSM in each node as Fig. 4 shows. The resolutions of nodes are independent of each other. This depth extension operation of FSM is called "zoom-in".

The individual nodes and the corresponding sub-lattices (high-resolution lattices) are related in the sense that the subgroups of sub-lattices jointly determine the features of the higher (parent) level. This structure, each node can store significant amount of additional information which processed only in the necessary resolution depth, can greatly reduce the computational requirements. FSM can describe hierarchically structured multidimensional data in a more concise way than simple fuzzy signatures.

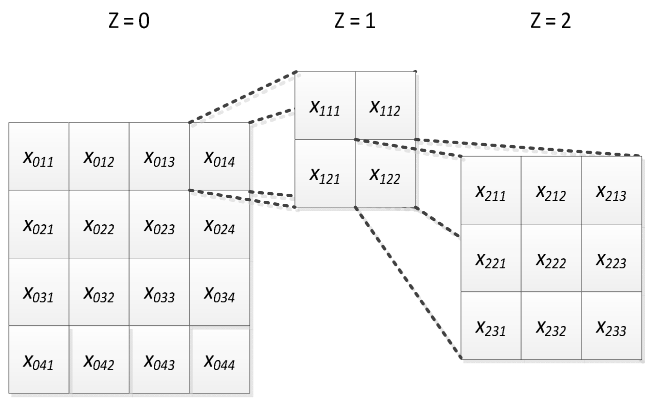


Fig. 4. Sub-lattices of a Fuzzy Situational Map

IV. The internal relations of nodes in fuzzy situational maps

So far fuzzy situational maps were considered as extended multi-dimensional fuzzy signatures. The further advantage of fuzzy situational maps is not only to be able to sort out the external information in a hierarchical structured form, but to be able to take into account the effects of the individual nodes on each other, as well. This means each node is in relation with the adjacent nodes and its actual value, change this value, may have impact on their neighbor nodes. These effects can be directed or mutual (bidirectional) and may affect the whole situational map or only some sub-lattice of the map.

The description of the interrelation in fuzzy situational maps is built up on the theoretical foundations of fuzzy graphs [13-22], taking into account these dependencies, mainly due to their mutuality, may be very complex and require high processing capacity. To solve this problem, we worked out a method that creates appropriate, easy to compute fuzzy vectors and fuzzy signatures from the actual sub-lattice of the fuzzy situational map.

In the next sections the essence of this method will be introduced.

A. Fuzzy graphs

The foundations of fuzzy graphs were laid down by Rosenfeld in 1975 [13]. Some structures and concepts which are known in traditional graph theory have been incorporated in the field of fuzzy graphs later [15-22]. We showed a set of theoretical frameworks and application possibilities of *fuzzy vertex graphs* in [14]. The internal relations of fuzzy situational maps are described by such graph structure or, with certain restrictions, some combination of fuzzy edge and vertex graphs.

Essentially there are two main groups of fuzzy graphs: the fuzzy edge graphs and fuzzy vertex graphs.

The *fuzzy edge graph* is defined as a fuzzy graph F is taken in a complete graph G, so that each edge has a fuzzy membership value in [0, 1] interval.

The μ_e gives the membership value (presence) of the edge e in fuzzy graph F. So if $\mu = 0$ then the edge is not in F, if $\mu = 1$, then the edge is crisp in graph F, the other values of the interval mean real fuzzy edges. Fig. 5 shows examples for fuzzy edge graphs (F_1).

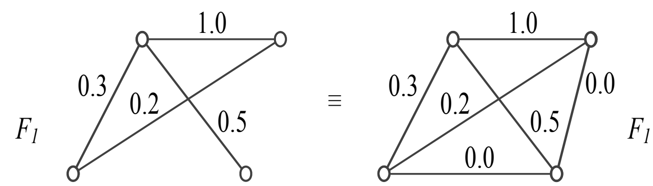


Fig. 5. Fuzzy edge graph

In case of *fuzzy vertex graphs* an arbitrary (crisp) reference graph G is given. In the fuzzy graph F a fuzzy membership value is assigned to each vertex of G and indicate the strength of belonging to the graph. Each edge that connects two points (V_1, V_2) gets a calculated membership value. This membership value is calculated by the formula $\mu(V_1)@\mu(V_2)$, where @ is a connecting operator, in majority it is some of t-norm or averaging aggregator. A fuzzy vertex graph is shown in Fig. 6.

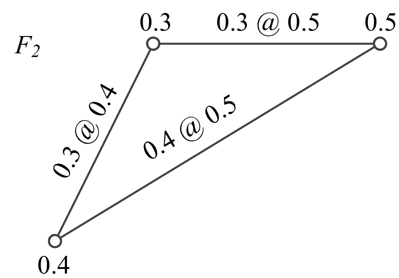


Fig. 6. Fuzzy vertex graph

The combination of fuzzy vertex and edge graphs is also conceivable, where both the edges and the vertex are assigned with fuzzy membership values. These kind of models are quite complicated, and thus rarely used in practice.

B. The description of the relations between the fuzzy situational map nodes

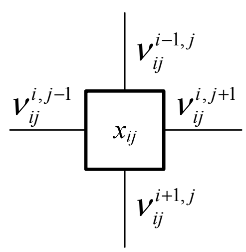
The inner relations of the fuzzy situational map nodes are described by modified fuzzy edge and vertex graphs.

Consider the simple two-dimensional fuzzy situational map A, which could be a reduced map as well. Each node in A has fuzzy membership value, these nodes are leaf-nodes. The nodes are denoted by x_{ij} simplified membership value notation in the lattice (Fig. 2), where $i = 1,\ldots,h;\ j = 1,\ldots,w$ are lattice point indexes.

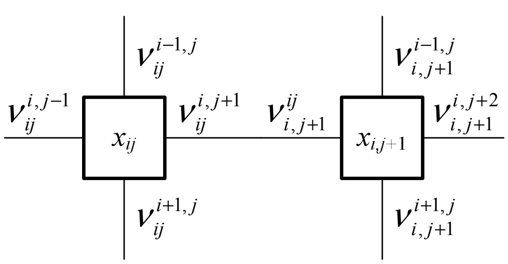
Consider the connection possibilities of node x_{ij} to the adjacent nodes. Fig. 7.a shows that the four possible edges of the actual node may have dissimilar membership values. The $v_{ij}^{i-1,j}$ membership value means the *directed edge membership value* between nodes x_{ij} and $x_{i-1,j}$, where $i = 2,\dots,h;\ j = 1,\dots,w$. The membership values of the other three edges can be given in the same way.

The edge membership value between two adjacent nodes can be interpreted as follows (Fig. 7.b):

- if $\nu_{ij}^{i,j+1} = 0$ and $\nu_{i,j+1}^{ij} = 0$ then between nodes x_{ij} and $x_{i,j+1}$ there is no any connection, there is not edge,
- if $\nu_{ij}^{i,j+1} \neq 0$ and $\nu_{i,j+1}^{ij} = 0$ then there is a *directed edge* (path) between the nodes x_{ij} and $x_{i,j+1}$, which is passable only the $x_{ij} \rightarrow x_{i,j+1}$ direction (x_{ij} has impact to $x_{i,j+1}$, but $x_{i,j+1}$ has not any impact to x_{ij}),
- if $\nu_{ij}^{i,j+1} \neq 0$, $\nu_{i,j+1}^{ij} \neq 0$ and $\nu_{ij}^{i,j+1} \neq \nu_{i,j+1}^{ij}$ then there are two *directed edges* or paths between nodes x_{ij} and $x_{i,j+1}$, which is passable in both directions $\left(x_{ij} \rightarrow x_{i,j+1} \text{ or } x_{i,j+1} \rightarrow x_{ij}\right)$, the usage of edge membership values depends on the traversal direction,
- if $\nu_{ij}^{i,j+1} \neq 0$, $\nu_{i,j+1}^{ij} \neq 0$ and $\nu_{ij}^{i,j+1} = \nu_{i,j+1}^{ij}$ then there are mutual connections between nodes x_{ij} and $x_{i,j+1}$, which can be described with single one edge membership value.



a.



b.

Fig. 7. a) FSM edge membership value on a node, b) edge membership values between two nodes

The inner connection graph of an *h* x *w* sized fuzzy situational map is shown in Fig. 8. Rather complicated relationship graphs may be written down in the case of a not so large fuzzy situational map too, which can be processed with difficulties, especially in case of mutual relationship between the nodes.

Because an effective description of complex data relationship is the goal of our algorithm, so for simplifying the processing tasks the following restrictions and structure reformatting are proposed:

- simple graphs are used, there are not loops and parallel edges,
- in each test step a root-node is selected, then the whole graph is re-described about this reference node,
- the direction of edge membership values are preserved, but the paths starting from the reference node are taken into account,
- the edges with $\nu_{ij} = 0$ membership value are erased from the graph.

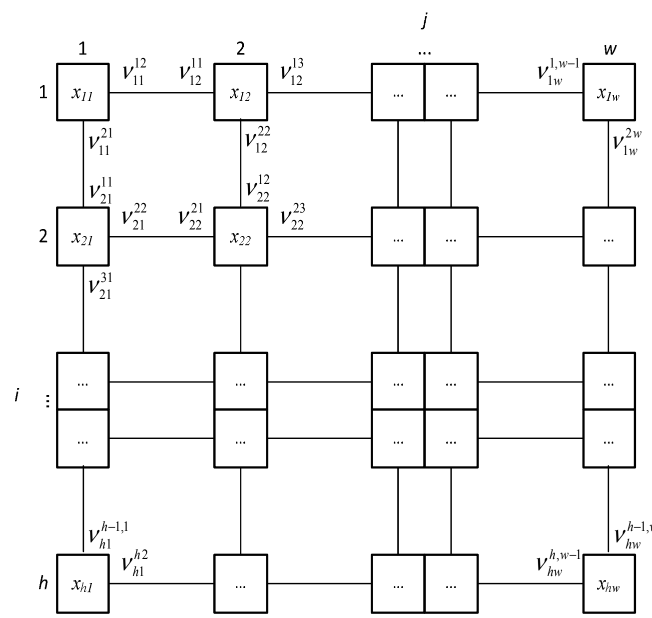


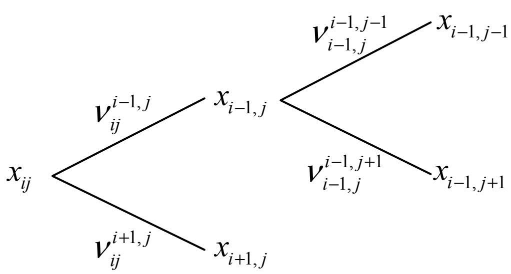
Fig. 8. Inner connection graph of the fuzzy situational map

On the base of above described, consider node x_{ij} as the root-node of the actual investigation. Suppose it is known that certain edges have 0 membership values, then the x_{ij} centered graph is obtained as Fig. 9 shows. For each node a similar graph can be got, which are sub-maps of the original fuzzy situational map. Thus, each node can be represented by its vertex membership value x_{ij} and its edge membership vector $\mathbf{e}_{ij}$ which is formed from the four edge membership values (2).

$$V_{ij} = \left(x_{ij}, \mathbf{e}_{ij}\right)$$

$$\mathbf{e}_{ij} = \begin{bmatrix} v_{ij}^{i-1,j} \\ v_{ij}^{i,j+1} \\ v_{ij}^{i+1,j} \\ v_{ij}^{i,j-1} \end{bmatrix},\ i = 1,\ldots,h;\ j = 1,\ldots w \tag{2}$$

From the connection graph, which has been created in this way, a descriptive structure, more precisely, an extended fuzzy signature can be clearly defined. Considering the connection graph in Fig. 9 the x_{ij} root-node can be described by the next fuzzy signature:



This signature uniquely describes the effect of the surrounding nodes to the actual root-node x_{ij} in a particular structure. The edge membership values v_{ij} play crucial role in formatting inner structure of the fuzzy situational map and in the mediation of the nodal effects.

V. Conclusion

In this paper we described the inner relations of fuzzy situational map nodes, for which fuzzy vertex and edge graphs were used. The paths may be very complex on the graph. To avoid complicated calculations, we proposed a transformation method by which the situational map could be examined node by node. In a properly constructed graph, nodes are selected as the root step by step, and they respective dependency can be written by fuzzy signature which is processed by the already well-known signature operators.

Fuzzy signatures expressed in such a way are well usable in practice. This method has been successfully tried out in a mobile robot cooperative task for describing the complex environment.

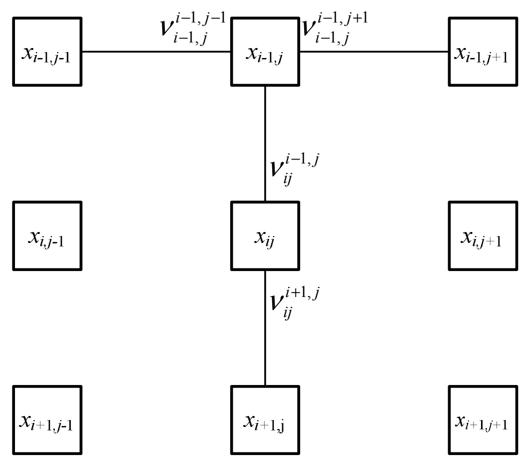


Fig. 9. Connection graph on x_{ij} root-node

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