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Sliding mode control of fuzzy descriptor systems with time delay

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***Abstract*— This paper investigates the problem of sliding mode control (SMC) for a class of uncertain Takagi-Sugeno (TS) fuzzy descriptor systems with time-varying delay. An integral-type sliding function is proposed and a new delay-dependent stability criterion, in terms of linear matrix inequality(LMI), is derived which guarantees to the sliding mode dynamics to be regular, impulse free and stable for all admissible uncertainties and time-varying delay. Moreover, a SMC law is synthesized to ensure the reaching condition. A numerical example is given to demonstrate the effectiveness of the proposed method SMC scheme.**

Keywords: fuzzy descriptor systems, time-varying delay, Integral sliding surface, SMC control

I. INTRODUCTION

To properly describe the behavior of some practical systems, it is interesting to consider the nature of their dynamic and static states at the same time. Singular systems are also referred to as implicit systems and are those dynamics of which are governed by a mixture of algebraic and differential equations [11], [5]. On the other hand, time delay often occurs in various complex engineering systems such as communications systems, long transmissions lines, chemical processes, etc. Many problems for the class of singular delayed-systems either in continuous-time and discrete-time have been tackled and interesting results have been reported in the literature [15], [17], [8], [3], [10].

Recently, the (TS) fuzzy model has been extended to deal with descriptor nonlinear systems with time delay [16], [1], [7]. The stability problem of this class of system is more complicated than that for regular systems because it requires considering not only stability but also regular and impulse free.

Over the past two decades, the study on sliding mode control (SMC) has increased considerably owing to its various attractive features such as fast response, good transient performance, and insensitiveness to the uncertainties on the sliding surface [13], [6], [4], [9]. Recent research [4], [14] has studied a control scheme called integral sliding mode control in which an integral controller is added to a sliding mode controller. The main advantages of integral SMC are that it offers the robustness of system's stability and can eliminate the steady-state error under step input.

In this paper, a SMC approach is proposed to address the problem of stabilizing for a class of uncertain (TS) descriptor time-delay systems. First, we develop a delay-dependent sufficient condition, in term of linear matrix inequality (LMI), which ensures to the sliding mode dynamics to be robustly admissible. Second, we investigate the synthesis of SMC law to drive the system trajectories onto the predefined Integral sliding surface. This paper is organized as follows: Section 2 formulates the system descriptions and presents some preliminaries. The main results are presented in section 3. An illustrative example is given in section 4. Section 5 provides a conclusion.

Notations. Throughout this paper, $X \in \mathbb{R}^n$ denotes the $n-$dimensional Euclidean space, while $X \in \mathbb{R}^{n\times m}$ refers to the set of all $n \times m$ real matrices. The notation $X > 0$ (respectively, $X \geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). The symbol $(*)$ stands for matrix block induced by symmetry, $\mathrm{sym}(X)$ stands for $X + X^T$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated. If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

The (TS) fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. A continuous fuzzy descriptor model with delay and parameter uncertainties can be described by :

$$\begin{cases} E\dot{x}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(\theta)\Big\{A_i(t)x(t) + A_{hi}(t)x(t-h(t)) \\ \qquad + B(u(t) + f(t, x(t))\Big\} \\ x(t) = \varphi(t),\ t \in [-h_M, 0]. \end{cases} \tag{1}$$

where $\mu_i(\theta) = \frac{\prod_{j=1}^{s} F_j^i(\theta_j)}{\sum_{i=1}^{r}\prod_{j=1}^{s} F_j^i(\theta_j)}$, $i = 1, 2, \cdots, r$, are the normalized weight functions, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $f(t, x(t))$ represents the system nonlinearity and any model uncertainties in the system including external disturbances or unknown input. F_j^i $(j = 1 \ldots s)$ are fuzzy sets, $\theta = [\theta_1, \ldots, \theta_s]$ is the premise variable vector. The delay $h(t)$ is time-varying and satisfies

$$0 \leq h(t) \leq h_M, \quad \dot{h}(t) \leq h_d. \tag{2}$$

where h_M is constants representing the bounds of the delay, h_d is a positive constant. $\varphi(t)$ is a compatible vector-valued initial function in $[-h_M, 0]$ representing the initial condition of the system. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular and

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assume that $\mathrm{rank}(E) = q \leq n$. $A_i(t) = A_i + \Delta A_i(t)$ and $A_{hi}(t) = A_{hi} + \Delta A_{hi}(t)$ are time-varying system matrices. A_i, A_{hi} and B_i are constant matrices with appropriate dimensions. Note that the normalized weights $\mu_i(\theta)$ satisfy

$$\mu_i(\theta) \geq 0, \quad i = 1, 2, \cdots, r \quad \sum_{i=1}^{r} \mu_i(\theta) = 1. \tag{3}$$

Without loss of generality, we introduce the following assumption for technical convenience.

1) $\Delta A_i(t)$ and $\Delta A_{hi}(t)$ are the unmatched uncertainties satisfying

$$\begin{bmatrix}\Delta A_i(t) & \Delta A_{hi}(t)\end{bmatrix} = M_i F(t) \begin{bmatrix} N_i & N_{hi}\end{bmatrix}, \tag{4}$$

where M_i, N_i and N_{hi} are known real constant matrices and $F(t)$ is unknown time-varying matrix function satisfying $F^T(t)F(t) \leq I$.

2) The matched nonlinearities $f(t, x(t))$ satisfies the inequality

$$f(t, x(t)) \leq \eta(t, x(t)) \tag{5}$$

where $\eta(x)$ is a positive known vector-valued function.

First of all, we recall some definitions.
Consider an unforced linear descriptor system with delay described by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_h x(t - h(t)), \quad 0 \leq h(t) \leq h_M \\ x(t) &= \varphi(t), \; t \in [-h_M, 0]. \end{aligned} \tag{6}$$

Definition *1:* [5] System (6) is said to be admissible if it is regular ($det\Big(sE - A\Big) \neq 0$), impulse-free ($\deg\Big(\det\Big(sE - A\Big)\Big) = \mathrm{rank}(E)$) and stable.

Lemma *1:* [12] For given real matrices Q, a and b with appropriate dimensions such that $Q > 0$, the following statements are equivalent

1)

$$\begin{bmatrix} Q & a \\ a^T & 0 \end{bmatrix} + \mathrm{sym}\left\{ \begin{bmatrix} F \\ G \end{bmatrix} \begin{bmatrix} b^T & -I \end{bmatrix} \right\} < 0 \tag{7}$$

is feasible in variable F and G

2) Q, a and b satisfy

$$Q + \mathrm{sym}(ab^T) < 0 \tag{8}$$

III. Main results

SMC design involve two basic steps. The first one is to design an appropriate switching surface such that the sliding mode dynamics restricted to the surface is admissible. The second step consists to synthesize a variable structure controller to guarantee that the system state trajectories converge to the predefined sliding surface and maintain them there for all subsequent time.

A. Integral sliding mode surface

The integral sliding-mode control completely eliminating the matched-type nonlinearities and uncertainties of (1) while keeping $s = 0$.
In this work, the following integral sliding surface is considered:

$$\begin{aligned} s(x,t) &= \mathcal{M}Ex(t) - \mathcal{M}Ex_0 \\ &\quad + \int_0^t \sum_{i=1}^{r} \mu_i \Big\{ \big(A_i + BK_i\big)x(\theta) + A_{hi}x(\theta - h(\theta))d\theta \Big\} \end{aligned} \tag{9}$$

where $K_i \in \mathbb{R}^{m\times n}$ is real matrix to be designed and $\mathcal{M} \in \mathbb{R}^{m\times n}$ is designed to satisfy that $\mathcal{M}B$ is nonsingular. According to SMC theory, when the system trajectories reach onto the sliding surface, it follows that $s(x,t) = 0$ and $\dot{s}(x,t) = 0$. Therefore, from $\dot{s}(x,t) = 0$, the equivalent control law can be established as

$$\begin{aligned} u_s = (\mathcal{M}B)^{-1}\mathcal{M} \sum_{i=1}^{r} \mu_i \Big\{ \big(\Delta A_i(t) + BK_i\big)x(t) \\ + \Delta A_{hi}(t)x(t - h(t))\big)\Big\} - f(t, x(t)) \end{aligned} \tag{10}$$

Substituting (10) into (1), we obtain the following sliding mode dynamics:

$$E\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta) \Big\{ \overline{A}_i(t)x(t) + \overline{A}_{hi}(t)x(t - h(t)) \Big\} \tag{11}$$

where $\overline{\mathcal{M}} = I - B(\mathcal{M}B)^{-1}\mathcal{M}$ and

$$\begin{aligned} \overline{A}_i(t) &= \overline{A}_i + \Delta\overline{A}_i(t), & \overline{A}_i &= A_i + BK_i, \\ \overline{A}_{hi}(t) &= A_{hi} + \Delta\overline{A}_{hi}(t), & \overline{M}_i &= \overline{\mathcal{M}}M_i, \end{aligned} \tag{12}$$

$$\begin{bmatrix}\Delta\overline{A}_i(t) & \Delta\overline{A}_{hi}(t)\end{bmatrix} = \overline{M}_i F(t) \begin{bmatrix} N_i & N_{hi}\end{bmatrix}. \tag{13}$$

B. Sliding Mode Dynamics synthesis

We focus on this section to determine the gain K_i in the switching surface function of (9) such that the sliding mode dynamics (11) is robustly admissible.

Theorem *1:* For given h_M and h_d, sliding mode dynamics (11) is regular, impulse free and robustly stable, if there exist positive-definite matrices P, Q_1, W, and matrices F_l, $l = 1, 2, 3$ and $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$ of appropriate dimensions, and a positive scalar ε such that the following conditions:

$$\mathbb{M}(\overline{A}_i, A_{hi}) = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \sqrt{h_M}V_1^T & 0 & X - F_1 + \overline{A}_i^T F_3^T & N_i^T & \varepsilon F_1 \overline{M}_i \\ * & \Psi_{22} & \sqrt{h_M}V_2^T & 0 & -F_2 + A_{hi}^T F_3^T & N_{hi}^T & \varepsilon F_2 \overline{M}_i \\ * & * & -W & 0 & 0 & 0 & 0 \\ * & * & * & -W & \sqrt{h_M}W & 0 & 0 \\ * & * & * & * & -\mathrm{sym}(F_3) & 0 & \varepsilon F_3 \overline{M}_i \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{14}$$

are feasible.

where

$$\Psi_{11} = \mathrm{sym}(V_1^T E + F_1 \overline{A}_i) + Q_1,\ X = E^T P + S R^T,$$
$$\Psi_{12} = -V_1^T E + E^T V_2 + F_1 A_{hi} + \overline{A}_i^T F_2^T,$$
$$\Psi_{22} = \Phi_{22} + \mathrm{sym}(F_2 A_{hi}) \tag{15}$$

and $R \in \mathbb{R}^{n\times(n-q)}$ is any matrix with full column rank and satisfies $E^T R = 0$.

Proof: For more details see [2]. ■

For the controller synthesis purpose, we introduce some auxiliary variables K_s and K_{ds} in sliding mode dynamics (11). We get

$$\begin{aligned} \mathcal{A}_i &= \overline{A}_{si} + B(K_i - K_s), & \mathcal{A}_{hi} &= \overline{A}_{hsi} - BK_s, \\ \overline{A}_{si} &= A_i + BK_s, & \overline{A}_{hsi} &= A_{hi} + BK_{ds}. \end{aligned} \tag{16}$$

Based on the result in Theorem 1, sliding mode dynamics (11) is regular, impulse free and robustly stable, if there exist matrices $P > 0$, $Q_1 > 0$, $W > 0$, F_l, $l = 1,2,3$ and $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$ of appropriate dimensions, and a positive scalar ε such that the following condition holds:

$$\mathbb{M}(\mathcal{A}_i, \mathcal{A}_{hi}) < 0 \tag{17}$$

Inequality (17) may be also written as

$$\mathbb{M}(\overline{A}_{si}, \overline{A}_{hsi}) + \mathrm{sym}(\mathbb{F}\mathbb{K}) < 0 \tag{18}$$

where

$$\mathbb{K} = \begin{bmatrix} (K_i - K_s) & -K_{ds} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbb{F} = \begin{bmatrix} B^T F_1^T & B^T F_2^T & 0 & 0 & B^T F_3^T & 0 & 0 \end{bmatrix}^T$$

By applying Lemma 1 to (18), we obtain the following result.

Theorem *2:* Let h_M and h_d be given positive scalars, and K_s and K_{ds} given matrices with appropriate dimensions. Sliding mode dynamics (11) is regular, impulse free and robustly stable, if there exist symmetric positive-definite matrices P Q_1, W, matrices F_l, $l = 1,2,3$ and $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$ of appropriate dimensions and a positive scalar ε such that the following conditions hold:

$$\Upsilon_i = \begin{bmatrix} \mathbb{M}(\overline{A}_{si}, \overline{A}_{hsi}) & \mathbb{F} \\ * & 0 \end{bmatrix} + \mathrm{sym}(\mathbb{I}\mathbb{Y}_i) < 0 \tag{19}$$
$$i = 1, 2, \cdots, r$$

where

$$\mathbb{Y}_i = \begin{bmatrix} Y_i - GK_s & -GK_{ds} & 0 & 0 & 0 & 0 & 0 & -G \end{bmatrix}$$
$$\mathbb{I} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}^T$$

The gain matrices K_i is given by :

$$K_i = G^{-1} Y_i \tag{20}$$

C. SMC law synthesis

Now, we are in position to synthesize a SMC law, by which the trajectories of the uncertain fuzzy singular time-delay systems (1) can be driven onto the pre-specified switching surface $s(t) = 0$ and then are maintained there for all subsequent time.

Theorem *3:* Consider the uncertain singular time-delay system (1). Suppose that the switching surface function is given by (9), then the trajectories of system (1) can be driven onto the switching surface $s(t) = 0$ by the following SMC law:

$$u(t) = \sum_{i=1}^{r} \mu_i(\theta)\big(K_i x(t) - \alpha_i \frac{s(t)}{\|s(t)\|}\big) \tag{21}$$

where

$$\begin{aligned} \alpha_i = \lambda + \eta(t, x(t)) + \|\big(\mathcal{M}B\big)^{-1}\mathcal{M}M_i\| \\ \Big(\|N_i x(t)\| + \|N_{hi} x(t - h(t))\|\Big) \end{aligned} \tag{22}$$

Proof: Choose $\mathcal{M}$ under the condition of $\mathcal{M}B$ is nonsingular. Consider the following Lyapunov function:

$$V_s(t) = \frac{1}{2} s^T(t)\big(\mathcal{M}B\big)^{-1} s(t) \tag{23}$$

According to (9), we have

$$\begin{aligned} \dot{s}(t) = \mathcal{M} \sum_{i=1}^{r} \mu_i \Big\{ &\big(\Delta A_i(t) - BK_i\big)x(t) \\ &+ \Delta A_{hi}(t) x(t - h(t))) \\ &+ B\Big(u(t) + f(t, x(t))\Big)\Big\} \end{aligned} \tag{24}$$

Thus, taking the derivative of $V_s(t)$ and considering the above equation, we have

$$\begin{aligned} \dot{V}_s(t) =& s^T(t)\big(\mathcal{M}B\big)^{-1}\dot{s}(t) \\ =& s^T(t)\big(\mathcal{M}B\big)^{-1}\mathcal{M}\sum_{i=1}^{r}\mu_i\Big\{\Delta A_i(t)x(t) \\ &+ \Delta A_{hi}(t)x(t - h(t)))\Big\} \\ &+ s^T(t)\Big(u(t) + \sum_{i=1}^{r}\mu_i\big(f(t, x(t)) - K_i x(t)\big)\Big) \\ \leq& \|s(t)\| \sum_{i=1}^{r}\mu_i\Bigg\{\|\big(\mathcal{M}B\big)^{-1}\mathcal{M}M_i\|\Big\{\|N_i x(t)\| \\ &+ \|N_{hi} x(t - h(t))\|\Big\} + \eta_i(x)\Bigg\} \\ &+ s^T(t)\Big(u(t) - \sum_{i=1}^{r}\mu_i K_i x(t)\Big) \end{aligned} \tag{25}$$

Substituting (21) into (25), we have

$$\dot{V}_s(t) = -\lambda\|s(t)\| < 0, \quad \forall \|s(t)\| \neq 0 \tag{26}$$

Then the system trajectories converges to the predefined sliding surface and is restricted to the surface for all subsequent time. ■

IV. Numerical example

In this section, a numerical example is provided to show the effectiveness of the proposed methods. Consider the nonlinear time delay system borrowed from [16]

$$\begin{aligned}&\Big(1+(a+\delta a)\cos(\theta(t))\Big)\ddot{\theta}(t)=-b\dot{\theta}^3(t)\\&+(c+\delta c)\theta(t)+(c_h+\delta c_h)\theta(t-h(t))\\&+d(u(t)+f(t,x(t)))\end{aligned} \tag{27}$$

where the range of $\dot{\theta}(t)$ is assumed to satisfy $|\dot{\theta}(t)|<\phi$, $\phi=2$, $c_h=0.8$, $h(t)=1.1+0.15\sin(t)$ (thus $h_M=1.25, h_d=0.15$), $u(t)$ being the control input. For simulation purposes, we set $a=b=d=e=1$, $c=1$. As in [16], time-delay system (27) can be expressed exactly by the following fuzzy descriptor form :

$$\begin{aligned}E\dot{x}(t)=\sum_{i=1}^{3}\mu_i\Big\{&A_i(t)x(t)+A_{di}(t)x(t-h(t))\\&+B(u(t)+f(t,x(t)))\Big\}\end{aligned} \tag{28}$$

where

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A_1=\begin{bmatrix}0&1&0\\0&0&1\\c&-b(\phi^2+2)&a-1\end{bmatrix},$$

$$A_2=\begin{bmatrix}0&1&0\\0&0&1\\c&0&-a-1-a\phi^2\end{bmatrix},\ A_3=\begin{bmatrix}0&1&0\\0&0&1\\c&0&a-1\end{bmatrix},$$

$$A_{hi}\begin{bmatrix}0&0&0\\0&0&0\\c_h&0&0\end{bmatrix},\ B=\begin{bmatrix}0\\0\\d\end{bmatrix},$$

$$\mu_1=\frac{x_2^2(t)}{\phi^2+2},\ \mu_2=\frac{1+\cos(x_1(t))}{\phi^2+2},$$

$$\mu_3=\frac{\phi^2-x_2^2(t)+1-\cos(x_1(t))}{\phi^2+2}$$

Assume that $\delta a(t)=\beta\Delta(t)\bar{a}$, $\delta c(t)=\beta\Delta(t)c$ and $\delta c_h(t)=\beta\Delta(t)c_h$, the uncertain matrices can be described as (4) with

$$M_i=\begin{bmatrix}0\\0\\\beta\end{bmatrix}\quad N_{1,3}=\begin{bmatrix}c&0&\bar{a}\end{bmatrix}\quad N_2=\begin{bmatrix}c&0&-\bar{a}(\phi^2+1)\end{bmatrix},$$

$$N_{hi}=\begin{bmatrix}\bar{c}_h&0&0\end{bmatrix}\quad N_{wi}=0,\quad i=1,2,3.$$

In this example we choose $R=\begin{bmatrix}0&0&I\end{bmatrix}^T$, $K_s=\begin{bmatrix}-4.5&-5.5&-1\end{bmatrix}$, $K_{ds}\begin{bmatrix}-0.1&-0.1&-0.03\end{bmatrix}$ and $\mathcal{M}=\begin{bmatrix}0.3&0.1&1\end{bmatrix}$ According to Theorem 3, a feasible solution is obtained and the associate controller gains are

$$\begin{aligned}K_1&=\begin{bmatrix}-4.5754&-5.2778&-1.7405\end{bmatrix},\\K_2&=\begin{bmatrix}-4.5739&-6.3750&-0.6305\end{bmatrix},\\K_3&=\begin{bmatrix}-4.5823&-6.3835&-1.7364\end{bmatrix}.\end{aligned}$$

Set $\lambda=0.65$, $\beta=2$, $\bar{a}=0.2a$, $\Delta(t)=cos(0.1t)$ and $f(t,x(t))=1.5sin^2(x_1(t))x_1^3(t)$ then the sliding surface and the SMC law can be designed according to (9) and (21)-(22), respectively. To prevent the control signals from chattering, we change sign $\frac{s(t)}{\|s(t)\|}$ with $\frac{s(t)}{0.06+\|s(t)\|}$.

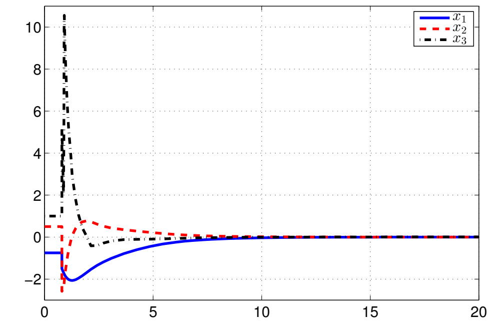

Fig. 1. State trajectories.

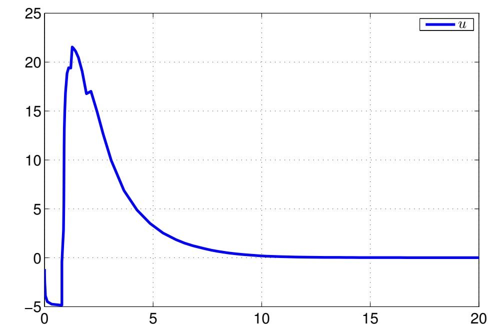

Fig. 2. Control input trajectory

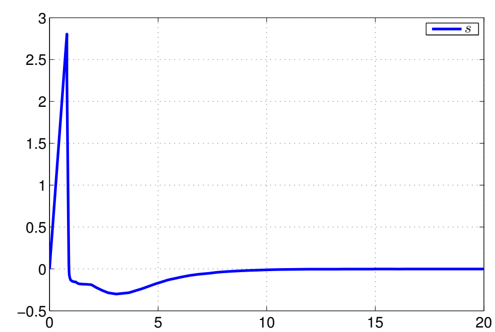

Fig. 3. Switching surface trajectory

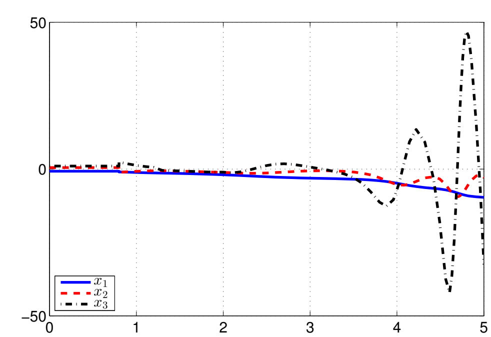

Fig. 4. States trajectories without sliding mode term

Figures (1)-(4) illustrate the simulation results using the initial conditions $x(0) = \begin{bmatrix} -0.75 & 0.5 & 1 \end{bmatrix}^T$. Figure (1) plots the evolution of the system outputs and Figure (2) depicts the control input vector. The response of $s(t)$ is given in Figure (3). From Figure (1), we see that the SMC achieves asymptotic stability of states in spite presence of uncertainty, time delay and mismatched disturbance. Figure (4) shows the state responses of the closed-loop system without sliding mode term. From this figure, we can see the effectiveness of the sliding mode term to compensate the effect of unknown input.

Assume now that $f(t, x(t)) = 0$ and for time $t \geq 15s$ the model parameters a, c and c_h abruptly change. For $\beta = 3$ and $\Delta(t) = sin(10t)$, applying control law (21) to the system, one gets the state trajectory evolutions shown in Figure (5). However, when the control law is applied to the system without sliding mode term, the stability of the uncertain system with poor performance is shown in Figure (6). It is clearly that the proposed SMC scheme effectively eliminates the effects of parameter uncertainties and guarantees the asymptotic stability of the closed-loop systems.

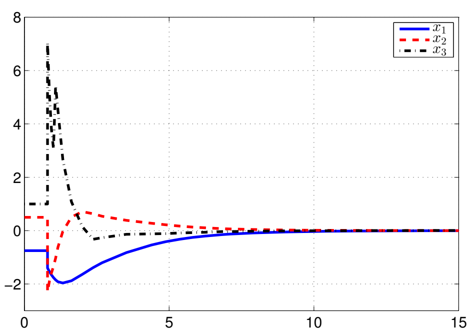

Fig. 5. State trajectories for $\beta = 3$.

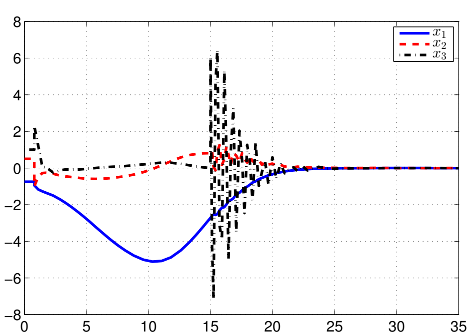

Fig. 6. State trajectories without sliding mode term for $\beta = 3$.

Regarding this results, we conclude that the proposed SMC law yields a good performance and stabilizes the nonlinear system with time varying delay, unknown parameters uncertainties and nonlinear input.

V. Conclusion

In this paper, SMC design for (TS) fuzzy descriptor time-delay models is reported. The major theoretical findings are as follows. First, the delay-dependent sufficient condition is developed to guarantee that the sliding mode dynamics is robustly admissible. Then by synthesizing a SMC law it has been shown that the system trajectories can be driven onto the predefined switching surface. The illustrative example validates the reported method.

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