2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Analyzing Fuzzy Association Rules with Fingrams in KEEL

David P. Pancho, Jose M. Alonso, Jesús Alcalá-Fdez *Member, IEEE*, Luis Magdalena *Member, IEEE*

***Abstract*— This work presents the full integration of fuzzy inference-grams (Fingrams) in KEEL to visual analysis of fuzzy association rules. Fingrams graphically represent fuzzy rule-based systems (FRBSs) in 2D graphs that illustrate the interaction among fuzzy rules in terms of rule cofiring, i.e., paying attention to rule pairs simultaneous fired by a given input. The new module allows to generate Fingrams for fuzzy association rules created in the suite KEEL, that can be afterwards analyzed to comprehend the system behavior and improve it. We sketch the use and potentials in an illustrative example built in KEEL over a real-world dataset including qualitative assessments of a set of design chairs.**

I. Introduction

FUZZY ASSOCIATION RULES [1] permit the uncovering of dependencies among items in datasets. They have been successfully applied to a wide variety of problems [2] such as, effective fuzzy associative classification [3], mining of medical databases [4], and so on. Unfortunately, systems made up of automatically extracted fuzzy association rules are rarely as interpretable as desired [3]. A crucial problem is the often huge number of rules that can be found from a database, making quite hard to understand and analyze this obtained systems.

Fingrams have arisen as a powerful and very useful tool for analyzing fuzzy systems [5]. We have designed them for dealing with fuzzy rule-based classifiers, regressors and fuzzy associate rules. Fingrams facilitate the analysis of those fuzzy systems at inference level from a comprehensibility viewpoint. With that aim, fuzzy rule bases are graphically represented in the form of social networks that display the interactions among rules. Experts can comfortably analyze Fingrams to understand the structure and behavior of the represented fuzzy system, even for large systems. Moreover, they support an interpretability-driven design of fuzzy systems [6].

Fingrams can be built and analyzed in the software tools GUAJE [7] and KNIME [8]. Also, a stand alone software tool, Fingrams Generator[1], allows to construct them independently of the fuzzy design tool used [9].

This contribution presents the improved Fingram module for the software suite KEEL presented in [9]. The new implementation permits the complete creation of Fingrams in SVG format without additional software. It allows an easy visualization, analysis and interpretation of fuzzy association rules generated with algorithms provided by KEEL. We use it to generate Fingrams in a real world problem and analyze the behavior of the system according to them.

The rest of the manuscript is structured as follows. Section II introduces some preliminaries about fuzzy association rules, Fingrams and KEEL. Section III describes the new Fingram module implemented in KEEL. Its use in an illustrative but real example is given in Section IV. Finally, Section V points out some conclusions and future work.

II. Preliminaries

This section introduces the basic definitions and measures of fuzzy association rules; the basic aspects and methodology of Fingrams; and the data mining suite KEEL emphasizing its main features.

A. Fuzzy Association Rules

Association rules uncover and represent dependencies among items in a dataset [10]. These are represented like $X \rightharpoonup Y$, where X and Y are itemsets and $X \cap Y = \oslash$ [11]. We should understand them as if X appears in a pattern is highly probable that Y appears there as well. For instance, in market basket analysis the association $\{computer, keyboard, screen\} \rightharpoonup \{mouse\}$ points out that when you buy a computer, a keyboard and a screen you also order a mouse.

Although many works in the field of association rules focus on discrete or binary datasets, quantitative datasets turned out as highly interesting due to its appearance in real-world applications. Thus, fuzzy set theory appeared as a profitable solution in order to represent associations. It avoids unnatural boundaries in the partitioning and improves linguistic interpretability of the rules. In recent years, many methods have been proposed to mine fuzzy association rules from quantitative datasets [2], [3], [12], [13].

Let us show an example of fuzzy association rule from a dataset with three attributes (A_1, A_2 and A_3) and three linguistic terms each one (LOW, MEDIUM, and HIGH):

$$\{A_1 \; is \; \text{LOW} \; and \; A_2 \; is \; \text{HIGH} \} \rightharpoonup \{A_3 \; is \; \text{MEDIUM}\}.$$

David P. Pancho, Jose M. Alonso and Luis Magdalena are with the European Centre for Soft Computing, Edificio de Investigación, C/ Gonzalo Gutiérrez Quirós s/n, 33600 Mieres, Asturias, Spain (email: {david.perez, jose.alonso, luis.magdalena}@softcomputing.es).

Jesús Alcalá-Fdez is with the Department of Computer Science and Artificial Intelligence, University of Granada, CITIC-UGR, 18071 Granada, Spain (email: jalcala@decsai.ugr.es).

This work has been funded by the Spanish Ministry of Economy and Competitiveness under Grants TIN2011-29824-C02-01 and TIN2011-29824-C02-02 (ABSYNTHE project); the Spanish Ministry of Education and Science under Grant TIN2011-28488; the Andalusian Government under Grant P10-TIC-6858; and the GENIL program of the CEI BioTIC GRANADA under Grant PYR-2014-2. The dataset used in the case study comes from the project EIBT11-05, supported by PCTI Asturias, in collaboration between Vortica (www.vortica.es) and the European Centre for Soft Computing (www.softcomputing.es).

[1] Available in: http://sourceforge.net/projects/fingrams/

Support and *Confidence* are well known measures used to assess fuzzy association rules. Considering the rule $R : X \rightharpoonup Y$, they are defined as:

$$Support(R) = \frac{\sum_{x_p \in D} \mu_R(x_p)}{\mid D \mid} \quad (1)$$

$$Confidence(R) = \frac{\sum_{x_p \in D} \mu_R(x_p)}{\sum_{x_p \in D} \mu_X(x_p)} \quad (2)$$

being $\mu_X(x_p)$ the matching degree of the pattern x_p with the rule antecedents; $\mu_R(x_p)$ the matching degree of the pattern x_p with the rule antecedents and consequent; and $\mid D \mid$ the cardinality of the dataset used D.

Most of the classic algorithms to generate fuzzy association rules attempt to mine rules whose *Support* and *Confidence* are greater than a *minimum Support* and a *minimum Confidence*. Though this framework is the most employed in the literature it is well known that it has many drawbacks. In particular, if the *Support* of the consequent of a rule is very high the *Confidence* of that rule will be very high despite the combinations of items in the antecedent [14], [15]. For instance, let us consider a rule $A \rightharpoonup B$ with *Confidence* 0.9 and a rule $C \rightharpoonup E$ with *Confidence* 0.7. If the *Support* of B is 0.95 and the *Support* of E is 0.1, the rule $C \rightharpoonup E$ will be better than the rule $A \rightharpoonup B$ since in this case the first rule corresponds to a negative dependence (observing A reduces the probability of B), whilst in the second case the probability of E increases significantly when we observe C. For this reason, several authors have arisen some measures for the selection and ranking of patterns according to their potential interest to the user [16]. One of them is the measure *Lift* [17], which represents the ratio between the *Confidence* and the expected *Confidence* of the rule. This measure is defined as:

$$Lift(R) = \frac{Confidence(R)}{\sum_{x_p \in D} \mu_Y(x_p) / \mid D \mid} \quad (3)$$

being $\mu_Y(x_p)$ the matching degree of the pattern x_p with the rule consequent. This measure obtains values in [0,∞), detecting negative dependence (*Lift* < 1), independence (*Lift* $= 1$) or positive dependence among items (*Lift* > 1).

B. Fingrams

Usually, FRBSs heavily cover the input space, i.e. a given input can fire simultaneously (co-fire) various rules. Taking advantage of this quirk, Fingrams show graphically the interaction among rules at the inference level in terms of co-fired rules.

Fingrams represent fuzzy systems as social networks made out of rules that collaborate/compete to produce a final behavior. Nodes represent fuzzy rules and their relations represent the interaction among rules. These relations are computed using a specific metric, usually a rule co-firing metric, that gives weighted links between nodes.

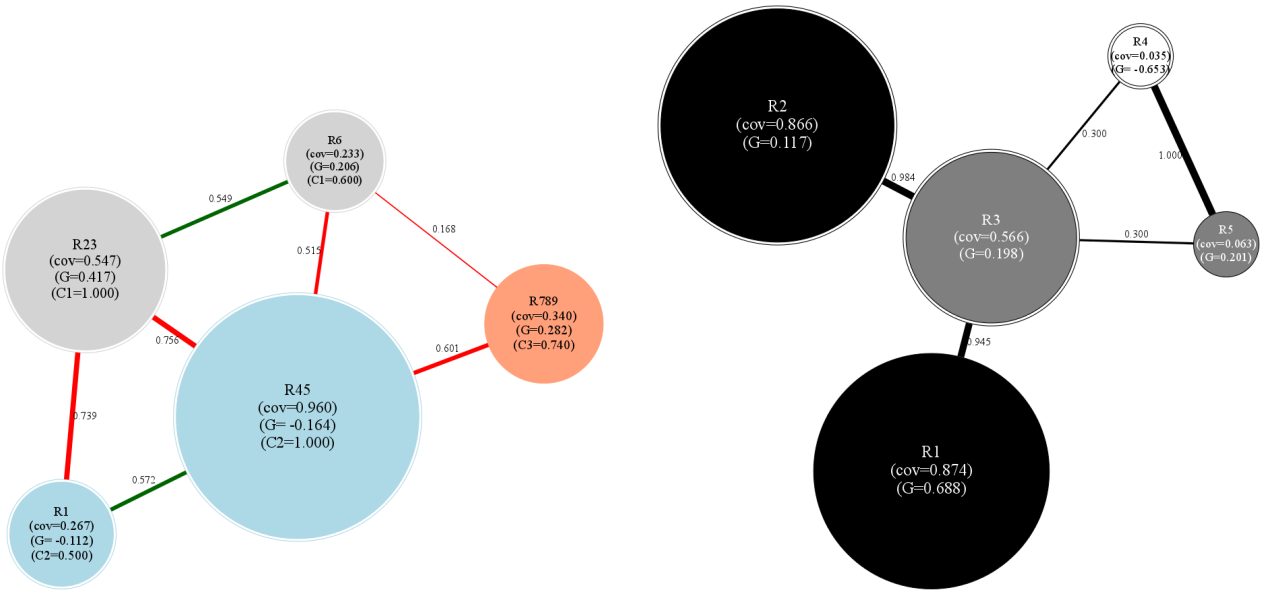
We have proposed a methodology for visual representation and exploratory analysis of the fuzzy inference process in FRBSs based on Fingrams [5]. The procedure to build Fingrams is as follows: (1) create the network; (2) scale it while keeping the most important information; and finally (3) display the network using an appropriate drawing algorithm.

1) **Fingram generation**: The original network is built by means of a dataset, a rule base, a fuzzy reasoning mechanism and a rule co-firing metric. This complete set of relations is formalized in a square matrix and represented by a graph.
   Web social networks usually use the friendship or the mentions as metric to construct a network. Fingrams relate rules according to the instances they cover.
2) **Fingram scaling**: The complete network is usually very dense and complex to analyze, therefore, a scaling process is demanded. As result, we obtain a simplified social network that keeps all the nodes but just the most relevant relations among them.
   Given a network, the scaling algorithms look at proximity information and yield structures uncovering the underlying organization. They consider similarities, correlations or distances in order to prune the initial network regarding the proximity between pairs of nodes.
   Fingrams remove the less important relations among rules by means of Pathfinder algorithm [18]. This algorithm preserves the most important relations, producing no new unconnected nodes and keeping the backbone of the network.
3) **Fingram drawing**: A layout algorithm automatically places the nodes and links of the scaled network guided by aesthetical criteria.
   Force-based or force-directed algorithms are widely used for drawing networks in the area of information science [19], [20]. Their purpose is to locate the elements of a graph in a 2D or 3D space so that all the links are approximately of equal length and there are as few crosses as possible, trying to obtain the most aesthetically pleasing view.

Fingrams deal, so far, with fuzzy rule-based classifiers and regressors [5] and fuzzy association rule systems [9], so-called FAR-Fingrams. Fig. 1 shows two illustrative examples of Fingrams. The picture on the left (Fig. 1(a)) represents a fuzzy rule-based classifier and the one on the right (Fig. 1 (b)) a fuzzy rule-based regressor, both formed by 5 rules[2]. Each node represents a rule enriched with information of the rules (identifier, coverage, and so on). The size of the nodes is proportional to the coverage, i.e. the number of examples covered by the rule. A link shows relation between a rule pair and its thickness is proportional to the level of interaction between them. You can find more details about the representation in [5].

Fingrams were firstly implemented into the fuzzy modeling tool GUAJE [21], [7] to deal with fuzzy rule-based classifiers and regressors. Then Fingrams Generator [9], a stand alone command line software, was developed to create

[2]An example of FAR-Fingram will be explained in detail in Section III-B.



(a) Classification Fingram. (b) Regression Fingram.

Fig. 1. Examples of Fingrams.

and visualize any of the three types of Fingrams from a configuration file. Later, we integrated them into the data mining framework KNIME [8]. Finally, we implemented a module in KEEL [9] to obtain a configuration file with the information of fuzzy association rules that can be afterwards used by Fingrams Generator.

C. KEEL

The suite KEEL[3] is aimed to assess computational intelligence algorithms for Data Mining problems including regression, classification, clustering, association rule learning and so on [22], [23]. The last version of KEEL consists of five parts (Fig. 2):

Fig. 2. Screenshot of the main window of KEEL.

[3]Available in: http://www.keel.es/

1) **Data Management**: It consists of a set of tools which allow us to export/import datasets in KEEL format or other formats, to edit and apply transformations to datasets, to make partitions for different kinds of validation models, etc.
2) **Experiments**: This block has two main objectives. On the one hand, you can use the software as a test and evaluation tool during the development of an algorithm. On the other hand, it is also a good option in order to compare new developments with standard algorithms already implemented and available in KEEL. To do so, this block provides a Graphical User Interface (GUI) based on data flow that allows us to easily design our experiments considering the available datasets, algorithms, visualization and analysis tools into KEEL. Then, this GUI generates a directory structure with all the necessary files needed to run our experiments in the local computer. Thus, users can forget scripts and other parameter files that made arduous the design of an experiment.
3) **Educational**: The main objective of this block is provide to the user a visual feedback of the learning progress of relevant algorithms in different areas (classification, regression, unsupervised learning) in order to make the evaluation and understanding of their behavior easier. This block has a similar structure to the previous, allowing us to easily design our experiments from a reduced set of relevant algorithms and datasets and run them on-line in order to display the learning process of the algorithms. Moreover, these experiments can be halted and resumed.
4) **Modules**: In this block we can access to several modules that extend KEEL, including an imbalan-

ced learning module [24], a non-parametric statistical analysis module [25], and a multiple instance learning module [26].

5) **Help**: It informs about the KEEL possibilities and how to use the graphic environment.

These blocks allow KEEL to be useful for different kinds of users. In what follows, we comment the main characteristics of KEEL:

- This suite contains a large number of evolutionary learning algorithms (supervised and unsupervised) for predicting models, preprocessing and postprocessing. Moreover, it also presents many algorithms for different areas of data mining such as fuzzy rule based systems, association rules, and so on.
- KEEL is aimed to design experiments with different algorithms and datasets using a simple GUI based on data flow in order to analyze the behavior of the algorithms. These experiments can be designed with a double goal, research and educational, being these experiments run off-line or on-line respectively.
- It contains a statistical analysis library to perform parametric and nonparametric analysis of the obtained results by the analyzed algorithms.
- KEEL provides a friendly GUI focused on the analysis of algorithms.

Finally, KEEL presents an environment where the interested developers can contribute with their own methods. The modular development of KEEL allows to easily integrate these new methods. The format of configuration files, data files, an API dataset and more resources are available for the support of developers[4].

III. Fingrams in KEEL

We have designed and developed a new KEEL module that permits the creation of FAR-Fingrams. This module takes as input the fuzzy association rules generated by an algorithm into KEEL and constructs FAR-Fingrams in vectorial SVG format. Notice that the previous version of this module, presented in [9], required the use of additional software to obtain FAR-Fingrams.

A. FAR-Fingrams module requirements

Fingrams module can be used over fuzzy association rules created by the algorithms Alcalaetal-A, FuzzyApriori-A, GeneticFuzzyAprioriDC-A, and GeneticFuzzyApriori-A. Fig. 3 illustrates a possible use of the new module. The blue nodes in background create fuzzy association rules, while the brown ones construct FAR-Fingrams. The dialog presented in the foreground of the figure shows the possibilities Fingram module provides.

Fingrams module requires some software libraries installed in the computer. Graphviz is demanded for Fingrams drawing. Thus, a pop-up warning message turns up when first use of the module.

[4]Available in: http://sci2s.ugr.es/keel/development.php

B. FAR-Fingrams module options

We will overview here the possibilities the Fingram module provides and how the different parameters should be selected (see Fig. 3).

0) **Rule selection**: This option allows selecting those rules with a value for the *Lift* measure higher than a threshold. This reduces the number of rules and allows the user to focus his/her attention in those more relevant.

1) **Fingram generation**: Two parameters can be selected in the dialog for this step. *Fingrams Generation* $\rightarrow$ *Blank threshold* (BT) lets the user to discard instances that fire rules below a threshold. A parameter (*Fingrams Generation* $\rightarrow$ *Metric*) permits constructing FAR-Fingrams through two different co-firing metrics. We can select the desired one in the pop-up dialog of the module.

   - *Symmetric relation:* It reflects how related the rules are according to the number of instances they cover.
   $$m_{i,j} = \frac{|D_{R_i R_j}|}{\sqrt{|D_{R_i}| \cdot |D_{R_j}|}}$$
   with D_{R_j} the sets of instances firing rule R_j, i.e. $D_{R_j} = \{x_p \in D \mid \mu_{R_j} > BT\}$; and $D_{R_i R_j}$ the sets of instances firing both rules R_i and R_j, i.e. $D_{R_i R_j} = \{x_p \in D \mid \mu_{R_i}(x_p) > BT \,\&\, \mu_{R_j}(x_p) > BT\}$. More instances covered in common means higher relation, with 1 if both rules cover exactly the same instances.
   The symmetry of this metric produces undirected networks.
   - *Asymmetric relation:* It characterizes generalization/specialization relations between rule pairs [27].
   $$m_{i,j} = 1 - \frac{\sum_{x_p \in D_{R_i}} (|\mu_{R_i}(x_p) - \mu_{R_j}(x_p)|)}{\sum_{x_p \in D_{R_i}} \mu_{R_i}(x_k)} \quad (4)$$
   Note that rule R_i is highly related with R_j, i.e. $R_i \xrightarrow{\approx 1} R_j$, when R_j is fired at similar degrees by the same set of examples that fires R_i.
   It yields a directed network, i.e., each link has associated two possible arrows (one per direction). In case both links between two nodes have the same weight, they are substituted by an undirected link.

2) **Fingram scaling**: This option allows us to create the original and scaled FAR-Fingrams, allowing the user to study both in detail.
Pathfinder algorithm [18] requires a q parameter that constrains the number of indirect proximities examined when generating the network. It must be an integer value between 2 and $N-1$, where N is the number of nodes to take into account. The configuration window of the module allows changing that value.

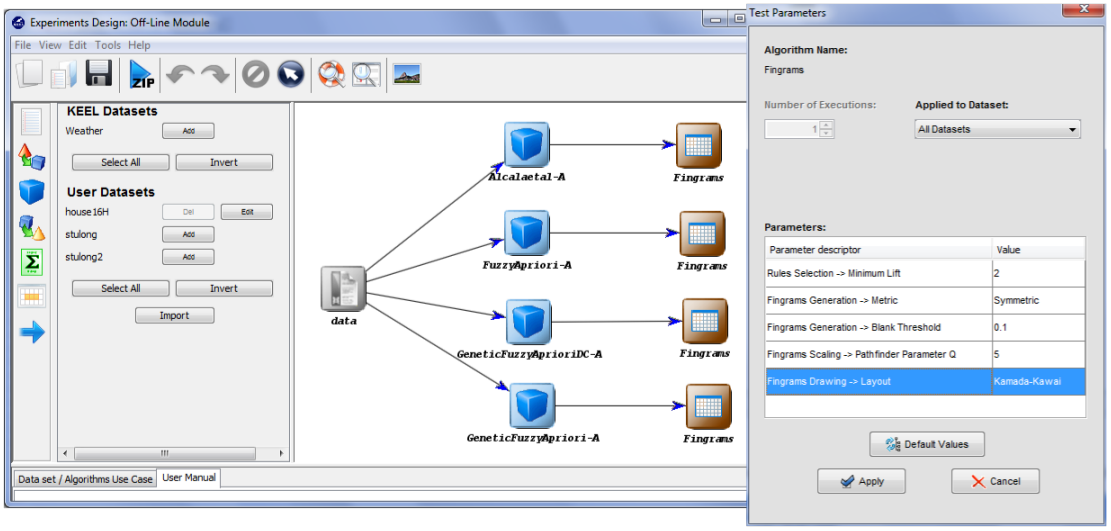


Fig. 3. Experiments window of KEEL.

3) **Fingram drawing**: Fingrams can be displayed using Kamada-Kawai [28] and Fruchterman-Reingold [29], two of the most representative and used methods of force-directed algorithms. As result, we obtain SVG images enriched with additional information. The use of this vectorial format permits a confortable analysis, zooming and moving around the interesting zones.

Fig. 4 shows an example and legend of FAR-Fingram created with KEEL. The legend shows the info FAR-Fingrams provide. Each fuzzy association rule is represented by a node that shows rule identifier, *Support*, *Confidence* and *Lift*. Node sizes and colors are proportional to *Support* and *Lift*, respectively. Moreover, the number of borders indicates the number of attributes compounding the rule. A link indicates level of relation between rules according to the metric used, arrowed if we used an asymmetric relation. The link thickness is proportional to the related weight $m_{i,j}$, whereas its absence means no interaction or a link pruned.

This illustrative example is made by more than one hundred fuzzy association rules and at first hand it seems hard to understand. However, the structure of the representation gives very valuable information about the system. We can detect highly related rules (as the ones marked inside the dotted blue ellipse), or more disperse ones (as the highlighted inside the red solid ellipse). Note that related rules cover parts of the input space in common, and a relation of 1 means that two rules are covering exactly the same set of instances. Moreover, FAR-Fingram reveals rules with lower/higher *Support* and *Lift* thanks to the use of sizes and colors, which can be appreciated by zooming the SVG image. Hence, the rules marked inside the red ellipse have higher *Support* and lower *Lift* than those inside the blue one.

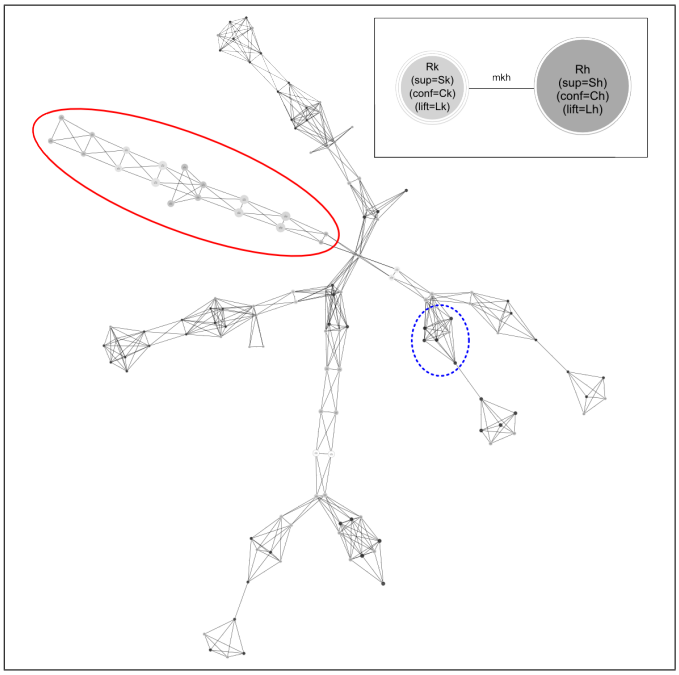


Fig. 4. Illustrative example and legend of FAR-Fingrams built with KEEL.

IV. Case Study

Here we present the use of FAR-Fingrams to analyze a set of fuzzy association rules created from real data. We deal with a dataset including qualitative assessments of a set of design chairs. Namely, we analyze how a set of users evaluated the degree of femininity of the considered set of chairs. We focus on finding out the physical characteristics of the chairs that influence the collected evaluations.

The dataset comes from a research project[5] where 28 users (11 males and 17 females) evaluated the degree of femininity of 23 chairs models. The users gave their appreciations in a fuzzy scale (as presented in Fig. 5). Trapezoidal fuzzy sets represent the degree of femininity from 0% to 100% with high semantic expressivity in this scale. The models are shown sequentially and in different random order for each

[5]More information in: http://bit.ly/18ZsEt7

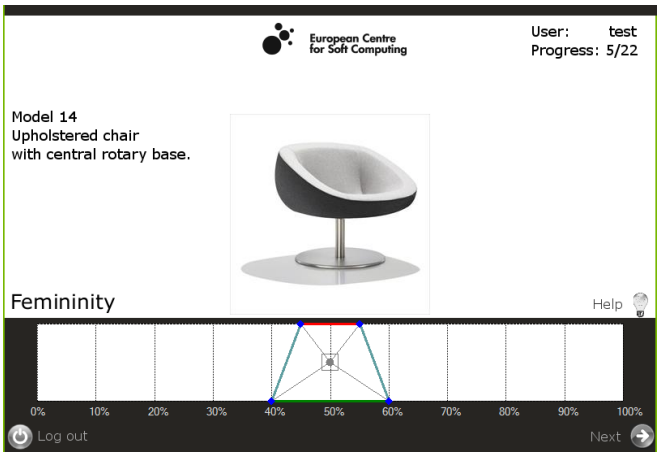


Fig. 5. Example of the chairs in the poll.

user, trying to avoid bias and conditioned responses. We used a reduced version of the dataset with just male responses.

It is possible to relate the degree of femininity associated to each chair with its physical properties. To do so, we first induced fuzzy association rules and then we analyzed subsets of them from an expert analysis viewpoint.

We used a learning algorithm [30] that extracts both membership functions (MFs) and fuzzy association rules for the given dataset. It tackles with quantitative values by means of a genetic learning of the MFs based on the 2-tuples linguistic representation model and the use of a basic method for mining fuzzy association rules. The initial linguistic partitions comprised 3 linguistic terms with uniformly distributed triangular MFs. We used the recommended default parameters proposed by the authors plus a minimum *Support* of 0.25 and a minimum *Confidence* of 0.9. As result, the algorithm produces 53 rules involving a subset of all the variables, namely, *Femininity*, *Distance between legs*, *Distance between armrests*, *Distance from the seat to the ground*, *Type of base*, and *Type of structure*.

Then we constructed the asymmetric FAR-Fingram related to this system, showed in Fig. 6. We can observe at first sight that there were different levels of *Support* and *Lift* in the rules as we saw different sizes and darkness in the nodes respectively. The left branch of the FAR-Fingram shows rules with the highest *Support*, namely, the biggest nodes. We studied the differences in detail and we decided to maintain those rules with higher values of *Lift*, more valuable ones. We used the dialog shown in Fig. 3 to discard rules with *Lift* below 1.3. Hence, a new FAR-Fingram was constructed with the remaining 28 rules (Fig. 7).

Analyzing the FAR-Fingram of Fig. 7 we detected different subsets of nodes mutually related by 1.0 links. This means that the corresponding rules are covering the same set of examples at equal level. They are highlighted with red rectangles. The 4 most left nodes being a subset, the second line of nodes another subset and so on. Moreover, the corresponding rules are quite similar, adding no valuable

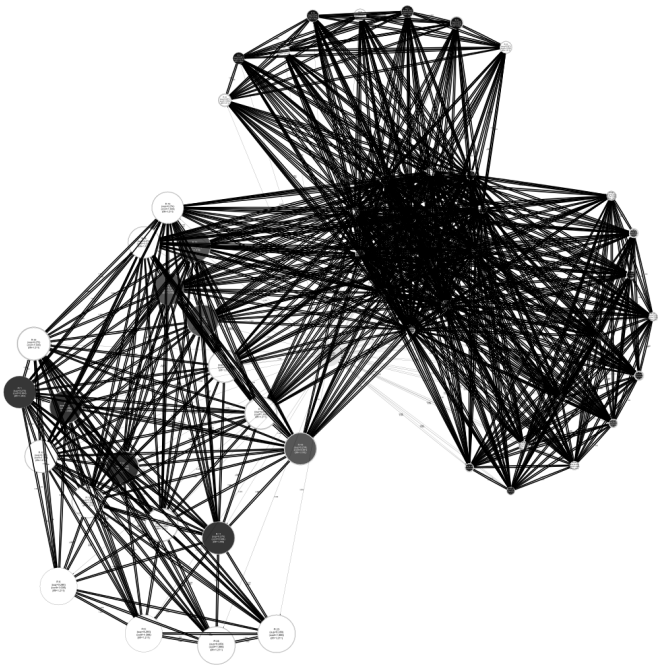


Fig. 6. Original FAR-Fingram of 53 rules.

attributes. For the first set of rules, we have:

R1: {*Femininity is* MEDIUM} ⇀ {*Distance between legs is* LOW}
R9: {*Femininity is* MEDIUM *and Type of base is* TRADITIONAL} ⇀ {*Distance between legs is* LOW}
R11: {*Femininity is* MEDIUM *and Type of structure is* GEOMETRIC LINES} ⇀ {*Distance between legs is* LOW}
R31: {*Femininity is* MEDIUM *and Type of base is* TRADITIONAL *and Type of structure is* GEOMETRIC LINES} ⇀ {*Distance between legs is* LOW}

Therefore, we can get these subsets of rules and just maintain one rule per each, the most representative and general one. We filtered all but the rule with less antecedents (and when tie, the one with higher *Lift*), and so for, the more general. In the previous example we kept R1. Again, and supported by the FAR-Fingram representation, we filtered the entire FAR-Fingram obtaining a final set of 5 rules. We can see the FAR-Fingram associated to this system and the textual description of these rules in Fig. 8. They suggest that the distances between legs and armrests are relevant for the degree of femininity medium. The associated FAR-Fingram shows that rules R8 and R14 are covering the less number of examples (smaller nodes) but with higher *Lift* (darker nodes). In fact, these two rules have two antecedents, so are more specific than the other three rules that just have one antecedent. On the contrary R1 and R3 are covering much more examples (bigger nodes) but with not that high *Lift* (lighter nodes). Notice that rule pairs R1–R3, R4–R14 and R8–R14 are highly related and with the same link weight in both directions ($m_{1,3} = m_{3,1} = 0.985$, $m_{4,14} = m_{14,4} = 0.989$, $m_{8,14} = m_{14,8} = 0.991$).

V. Conclusions and Future Work

We have presented the full integration of FAR-Fingrams in the software suite KEEL to deal with fuzzy association rules. The new module permits the use of different parameters such as metrics, scaling parameters or layout algorithms.

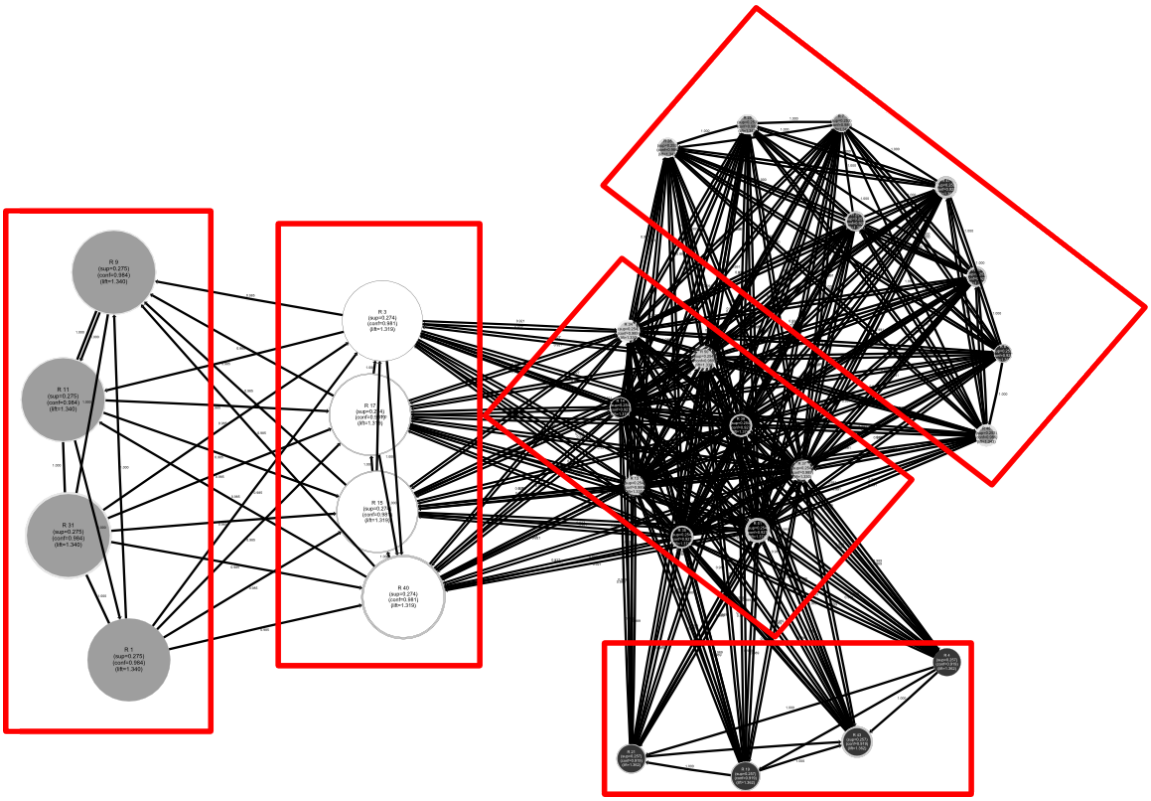
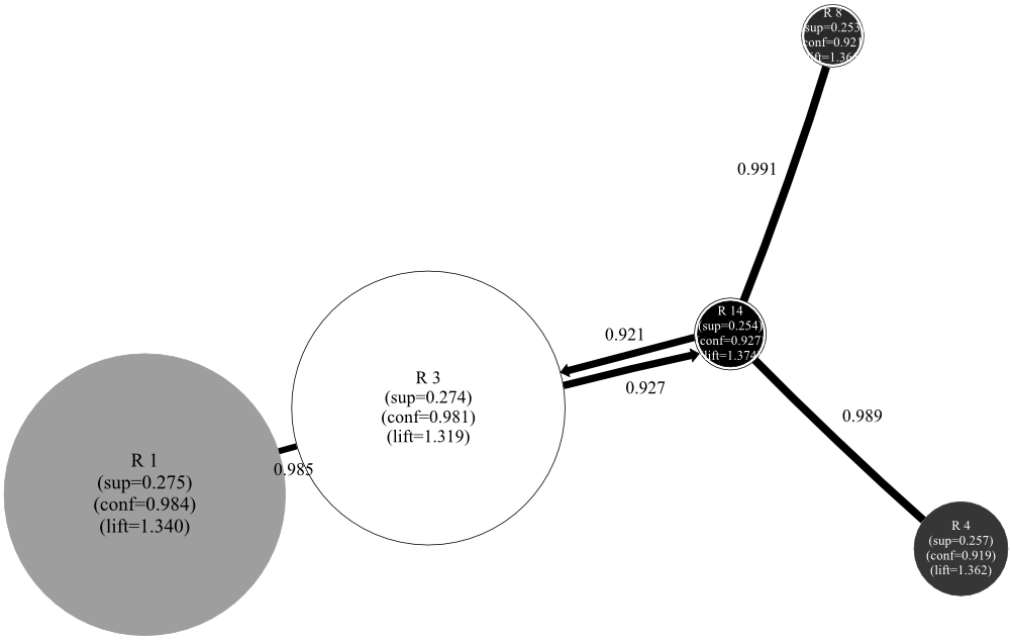


Fig. 7. FAR-Fingram built with the 28 rules of the highest *Lift*.



R1: {*Femininity is* MEDIUM} ⇀ {*Distance between legs is* LOW}
R3: {*Femininity is* MEDIUM} ⇀ {*Distance between legs is* MEDIUM}
R4: {*Femininity is* MEDIUM} ⇀ {*Distance between armrests is* MEDIUM}
R8: {*Distance between legs is* LOW *and Femininity is* MEDIUM} ⇀ {*Distance between armrests is* MEDIUM}
R14: {*Distance between legs is* MEDIUM *and Femininity is* MEDIUM} ⇀ {*Distance between armrests is* MEDIUM}

Fig. 8. FAR-Fingram constructed with the 5 selected rules.

We constructed a set of fuzzy association rules from real data to exhibit some of the potentials of FAR-Fingrams. Their analysis allows us to understand them focusing our attention in those more valuable rules.

The possibilities of Fingrams in the expert analysis of fuzzy rules is undoubtable. We will make an effort to collect common cases and situations, and annotated with descriptions that explain the situation and possible actions. This will allow us to systematically understand and improve fuzzy systems.

The concept of Fingram can be extended to relate not only fuzzy rules, but also attributes/fuzzy terms appearing in the fuzzy rules. So for, new metrics will be proposed to produce complementary information about the system to the designer.

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