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Design of Fuzzy Synergetic Controller

Chi-Hua Liu and Ming-Ying Hsiao*

Abstract—**In this paper, a fuzzy synergetic control (FSC) scheme is proposed for a class of nonlinear systems with system uncertainties and external disturbances. The control law is introduced by using methods of synergetic control theory and fuzzy logic control technique. The simulations results of the proposed scheme are compared with those of the conventional sliding mode control and synergetic control. All the simulation results demonstrate the effectiveness and feasibility of the proposed control.**

I. Introduction

BASICALLY, the synergetic control theory (SCT) [1-3] is beginning by defining a macro-variable to construct a manifold, and then an associated self-convergence evolution constraint is given to design the desired controller. From the inherent behavior, the synergetic control can govern the system states of the controlled system to be converged to the manifold smoothly. With the given evolution constraint function, it can guarantee that the system states will be attracted toward the manifold theoretically and retained hereafter. To date, the synergetic control approach has been successfully applied to power converters for pulse current charging [4], nonlinear power system stabilizer [5], DC-DC boost converter [6], synergetic optimal controllers [7], and robot manipulators control [8].

Fuzzy Logic Control (FLC) has been extendedly studied and successfully applied in various fields for decades since Zadeh [9] introduced the fuzzy set theory. Basically, human control expertise is emulated fuzzy control schemes; therefore, complicated mathematical theories are not required in designing Mamdani-type fuzzy control [10]. Sometimes, tuning the parameters of the FLC can achieve better system performance, so, the FLC is credited as one of the methodologies in designing robust controllers when system uncertainties and external disturbances are existed. But, to select proper fuzzy rules and the membership functions is crucial in fuzzy controller design. In order to overcome these drawbacks of choosing associated parameters, it is necessary to integrate fuzzy logic control and other methodologies [11].

SMC techniques [12-13] provide discontinuous control laws to drive the system states to a specified sliding surface and to retain them on the sliding surface. The control command is adequately designed such that the states will move toward the neighborhood of a switching boundary zone and reach on the sliding surface, if the states reach on the sliding surface then the system is said to be in the sliding mode. During the sliding mode, the system possesses some invariance properties, such as robustness, order reduction and disturbance rejection. But, one of the drawbacks of using SMC is the high frequency switching control input. Because a discontinuous switching control is applied to the plant, chattering always appears as a source to excite the un-modeled high-frequency dynamics of the controlled system. One commonly used method to eliminate the chattering is to replace the relay control by a saturating approximation [14]. Another method is to apply fuzzy logic control to the SMC system

such that a smooth and reasonable hitting control can be generated to reduce the chattering effect.

Fuzzy sliding-mode control (FSMC) [15-16], a hybrid of the SMC and FLC, gives a simple way to design the controller systematically and provides the asymptotical stability of the system. In general, the FSMC can also reduce the rule number in the FLC and still possess robustness in the face of model uncertainties and external disturbances.

In this paper, we propose a fuzzy synergetic control scheme for controlling a class of nonlinear systems with system uncertainties and external disturbances. This technique inherits the merit of synergetic control theory and fuzzy control to overcome the major deficiency of the synergetic control which only can be applied to well-defined systems.

This paper is organized as follows. In Section II, synergetic control theory is reviewed briefly. Next, based on the synergetic theory and fuzzy logic control, the design procedure of the fuzzy synergetic controller for a class of nonlinear systems is addressed in Section III. In Section IV, computer simulations are performed to examine the feasibility and effectiveness of the proposed scheme. The conclusions are drawn in Section V.

II. A Brief Review of Synergetic Control Theory

Basic SC theory will be reviewed here before the proposed fuzzy synergetic control scheme is introduced.

Consider a class of nonlinear system as following:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{b}(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x}\in R^{n\times 1}$ is the states vector, $\boldsymbol{f}(\boldsymbol{x})\in R^{n\times 1}$ is some nonlinear function, $\boldsymbol{b}(\boldsymbol{x})\in R^{n\times m}$ is the control matrix, and $\boldsymbol{u}\in R^{m\times 1}$ is the control input.

Basically, the design procedure of synergetic controller for well-defined system is very similar to that of the sliding-mode

Ming-Ying Hsiao is with the Electrical Engineering Department, Fortune Institute of Technology, Kaohsiung 83160, Taiwan (corresponding author to provide phone: 886-7-788-9888; fax: 886-7-788-9777; e-mail: myhsiao@gmail.com, myhsiao@fotech.edu.tw).

Chi-Hua Liu is with the Department of Industrial Engineering and Administration, Fortune Institute of Technology, Kaohsiung 83160, Taiwan (e-mail: chliu0914@gmail.com, chliu@fotech.edu.tw).

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control controller. To design the synergetic controller, a macro-variable $\boldsymbol{\sigma}$ is defined to construct a manifold as:

$$M = \left\{ \boldsymbol{x} : \boldsymbol{\sigma} = \boldsymbol{s}(\boldsymbol{x}), \boldsymbol{s}(\boldsymbol{x}) \in R^{m\times1} \right\} \tag{2}$$

where $\boldsymbol{\sigma} = [\sigma_1 \ \sigma_2 \ \cdots \ \sigma_m]^T$ is function of system states $\boldsymbol{x}$. Sometimes, for design simplicity, the macro-variable $\boldsymbol{\sigma}$ is constructed by a linear combination of system states with weighting factors, and this scheme is very similar to constructing the sliding function in the sliding-mode control. Then, an evolution constraint law is given to create the dynamical system with attractors at $\boldsymbol{\sigma} = 0$ which will automatically govern the system states of the controlled system to converge to the specified manifold M exponentially.

The evolution constraint law can be stated as

$$\boldsymbol{\tau}\dot{\boldsymbol{s}}(\boldsymbol{x}) + \boldsymbol{s}(\boldsymbol{x}) = 0 \tag{3}$$

where $\boldsymbol{\tau} = diag(\tau_1 \ \tau_2 \ \cdots \ \tau_m)$ is a strict positive real value diagonal matrix which will affect the speed of convergence the system states directly.

Next, by solving the nonlinear system (1) with the evolution constraint (3), then, the resulting synergetic control law $\boldsymbol{u}_{SC}$ can be obtained as

$$\begin{aligned} \boldsymbol{u}_{\mathrm{SC}} &= -\left(\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) - \left(\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}(\boldsymbol{x}) \\ &= \boldsymbol{u}_{\mathrm{eq}} + \boldsymbol{u}_{\mathrm{sy}} \end{aligned} \tag{4}$$

where $\boldsymbol{u}_{\mathrm{eq}} = -\left(\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x})$ is the equivalent control term of the SC which is identical to that of the SMC in nominal system, and $\boldsymbol{u}_{\mathrm{sy}} = -\left(\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}(\boldsymbol{x})$ is the synergetic term of the synergetic control.

Unlike the conventional SMC, the synergetic control provides a smooth and continuous term $\boldsymbol{u}_{\mathrm{sy}}$ instead of a switching term. This controller can force the system trajectory to exponential approach the manifold smoothly. As the trajectory reaches the manifold, the synergetic controller will also maintain it there thereafter. The synergetic controller prevails over the sliding-mode controller is that the former is smoothly continuous while the latter is discontinuous with abrupt change resulting the chattering phenomena.

Theorem 1. *Consider the nonlinear system (1). The states and their rates will converge exponentially to zero with the speed of convergence depending on the selected parameter matrix* $\boldsymbol{\tau}$ *, if the control law is exerted as (4).*

Proof. A Lyapunov candidate function is selected as $V = 0.5\boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{\tau}\boldsymbol{s}(\boldsymbol{x})$, then one can have

$$\dot{V} = \frac{d}{dt}\left(0.5\boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{\tau}\boldsymbol{s}(\boldsymbol{x})\right) = \boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\dot{\boldsymbol{x}}$$

$$= \boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\left(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x})\boldsymbol{u}_{\mathrm{SC}}\right)$$

$$\begin{aligned} &= \boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\Big[\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x})\Big(-\left(\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{f}(\boldsymbol{x}) \\ &\quad -\left(\boldsymbol{\tau}\boldsymbol{s}_{\boldsymbol{x}}(\boldsymbol{x})\boldsymbol{b}(\boldsymbol{x})\right)^{-1}\boldsymbol{s}(\boldsymbol{x})\Big)\Big] \\ &= -\boldsymbol{s}(\boldsymbol{x})^T\boldsymbol{s}(\boldsymbol{x}) \le 0 \end{aligned} \tag{5}$$

From the inequality (5), it will guarantee the stability of the controlled system (1) with the control input $\boldsymbol{u}_{\mathrm{SC}}$. ∎

III. Fuzzy Synergetic Controller Design

In this section, the design procedure of fuzzy synergetic controller is described. As the synergetic control can not tackle the system with uncertainties and disturbances, the fuzzy logic control is adopted to handle these unmolded dynamics. Now, we consider a class of the nonlinear system with system uncertainties and external disturbances which can be expressed as

$$\dot{\boldsymbol{x}}(t) = \left(\boldsymbol{f}(\boldsymbol{x}) + \Delta\boldsymbol{f}(\boldsymbol{x})\right) + \left(\boldsymbol{b}(\boldsymbol{x}) + \Delta\boldsymbol{b}(\boldsymbol{x})\right)\boldsymbol{u} + \boldsymbol{w}(x,t) \tag{6}$$

where $\Delta\boldsymbol{f}(\boldsymbol{x})$ and $\Delta\boldsymbol{b}(\boldsymbol{x})$ are system uncertainties, and $\boldsymbol{w}(x,t)$ is the external disturbances. We denote $\boldsymbol{d}(x,t) = \Delta\boldsymbol{f}(\boldsymbol{x}) + \Delta\boldsymbol{b}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{w}(x,t)$ which is bounded, then, eq. (6) becomes

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}(x,t) \tag{7}$$

where $\boldsymbol{d}(x,t) = [d_1 \ \ d_2 \ \ \cdots \ \ d_n]^T$, $\left|d_j\right| \le \kappa_j$, and κ_j , $j = 1 \to n$ are bounded positive constants.

Now, we combine the synergetic control and fuzzy logic control, the resulting controller becomes

$$\boldsymbol{u}_{\mathrm{FSC}} = \boldsymbol{u}_{\mathrm{eq}} + \boldsymbol{u}_{\mathrm{sy}} + \boldsymbol{u}_{\mathrm{fuzz}} \tag{8}$$

where $\boldsymbol{u}_{\mathrm{fuzz}} = [v_{\mathrm{f1}} \ \ v_{\mathrm{f2}} \ \ \cdots \ \ v_{\mathrm{fn}}]^T$. We choose the macro-variable as the input of the fuzzy logic controller, then, the *j*-th fuzzy rule can be expressed as

$$\text{Rule } j\text{: IF } s_{\mathrm{j}} \text{ is } F_{\mathrm{j}}^{\mathrm{i}},\ \text{THEN } v_{\mathrm{j}} \text{ is } \psi_{\mathrm{j}}^{\mathrm{i}}, \tag{9}$$

For simplicity, we choose the weighted average defuzzification, then, the output of the fuzzy inference system can be written as

$$v_{\mathrm{jc}} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{M}}\mu_{\mathrm{i}}(s_{\mathrm{j}})\times\psi_{\mathrm{j}}^{\mathrm{i}}}{\sum_{\mathrm{i=1}}^{\mathrm{M}}\mu_{\mathrm{i}}(s_{\mathrm{j}})} = \boldsymbol{\Psi}_{\mathrm{j}}^T\boldsymbol{Z}_{\mathrm{j}} \tag{10}$$

where M is the number of rules, $\psi_{\mathrm{j}}^{\mathrm{i}}$ is the associated singleton membership function of v_{j} and $\mu_{\mathrm{i}}(s_{\mathrm{j}})$ is the firing strength of the j-th rule. Then, the output of the fuzzy control term is

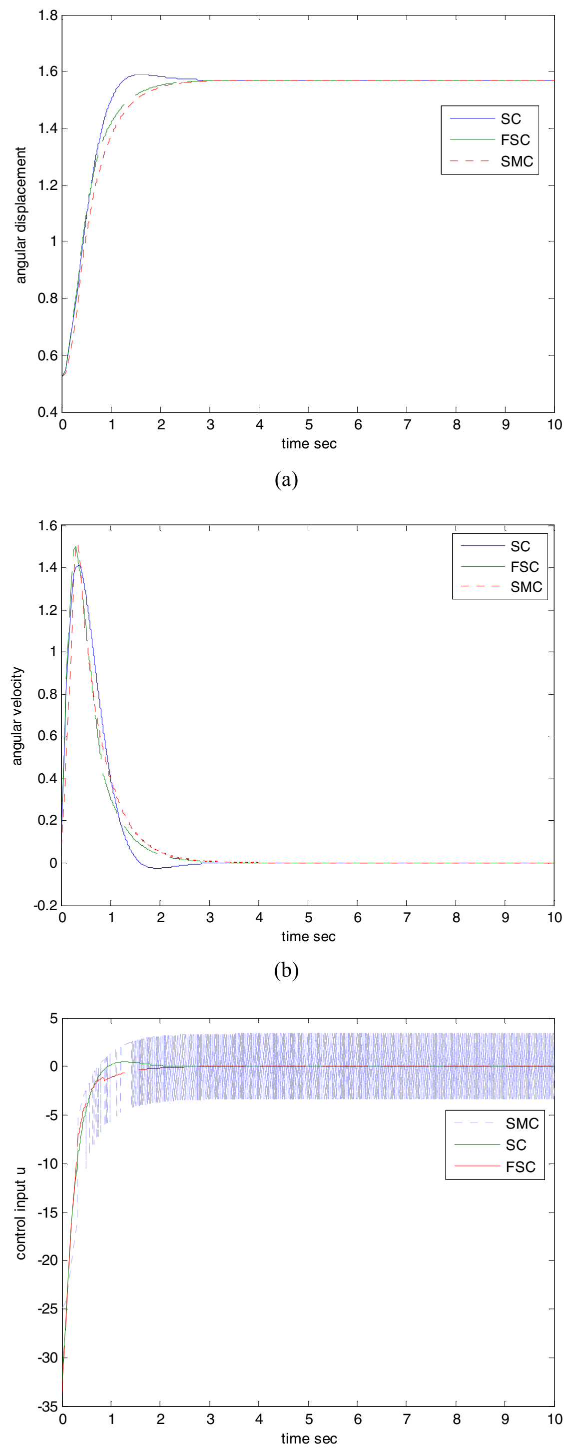
$$v_{\mathrm{fi}} = f_{\mathrm{i}}v_{\mathrm{ic}},\mathrm{i} = 1 \to n \tag{11}$$

where f_{i} is the scaling factor of output variable. Here we complete the fuzzy synergetic controller design.

IV. Numerical Example

Example: Considering an inverted pendulum system

$$\dot{x}_1 = x_2 \qquad (12a)$$



(a)

(b)

(c)

Fig. 1. The inverted pendulum controlled to the upright position by the SMC, SC, and FSC schemes with the initial condition $x_1(0) = \pi/6$ and $x_2 = 0$: (a) angular displacement; (b) angular velocity; (c) control input.

$$\dot{x}_2 = \frac{(m+M)mgL\sin x_1 - m^2L^2x_2^2\sin x_1\cos x_1)}{h(x_1) = (I+mL^2)(m+M) - m^2L^2\cos^2 x_1} \qquad (12b)$$
$$+u + d(x,t)$$

where x_1 and x_2 are the angular displacement and angular velocity of the pendulum, respectively, m is the pendulum mass, L is the length from the center of gravity to the pivot, I is the moment of inertia of the pendulum with respect to the center of gravity, g is the gravity constant, u is the control input, and $d(x,t)$ is unmolded dynamics. The control objective is to maintain the pole in the upright position by means of the force from any initial position of the pole.

In this example, the pendulum mass is 0.05 kg, cart mass is 1 kg, and the pendulum length is 1.5 m. Firstly, the control objective is to maintain the pole in the upright position by means of the control input force.

For the proposed FSC scheme, the membership functions selected for the IF-part and THEN-part are triangular and singleton types with three equal partitions, respectively. The fuzzy rules are listed below:

R^1 : IF s_i is *PO* THEN u_i is *NE*;

R^2 : IF s_i is *ZE* THEN u_i is *ZE*;

R^3 : IF s_i is *NE* THEN u_i is *PO*.

where *NE*, *ZE*, and *PO* denote negative, zero, and positive, respectively and $i = 1 \to 3$.

Simulation is done first to keep the inverted pendulum at the upright position without unmolded dynamics. Fig. 1 shows the angular displacements, angular velocities, and control inputs of the inverted pendulum controlled by the SC, SMC, and FSC schemes with the initial conditions $x_1(0) = \pi/6$ and $x_2(0) = 0$. All these three schemes can always perform well to upright the pole; however, one can read from this figure that the FSC is the best controller among these three controllers. One can also see clearly that chattering phenomena is occurred when SMC is applied to the inverted pendulum system, whereas, the continuous and

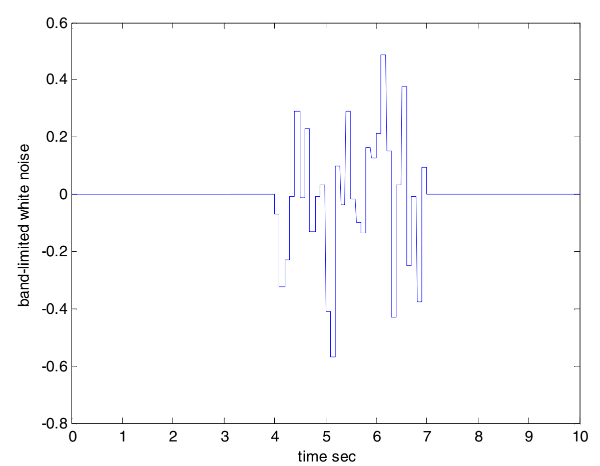
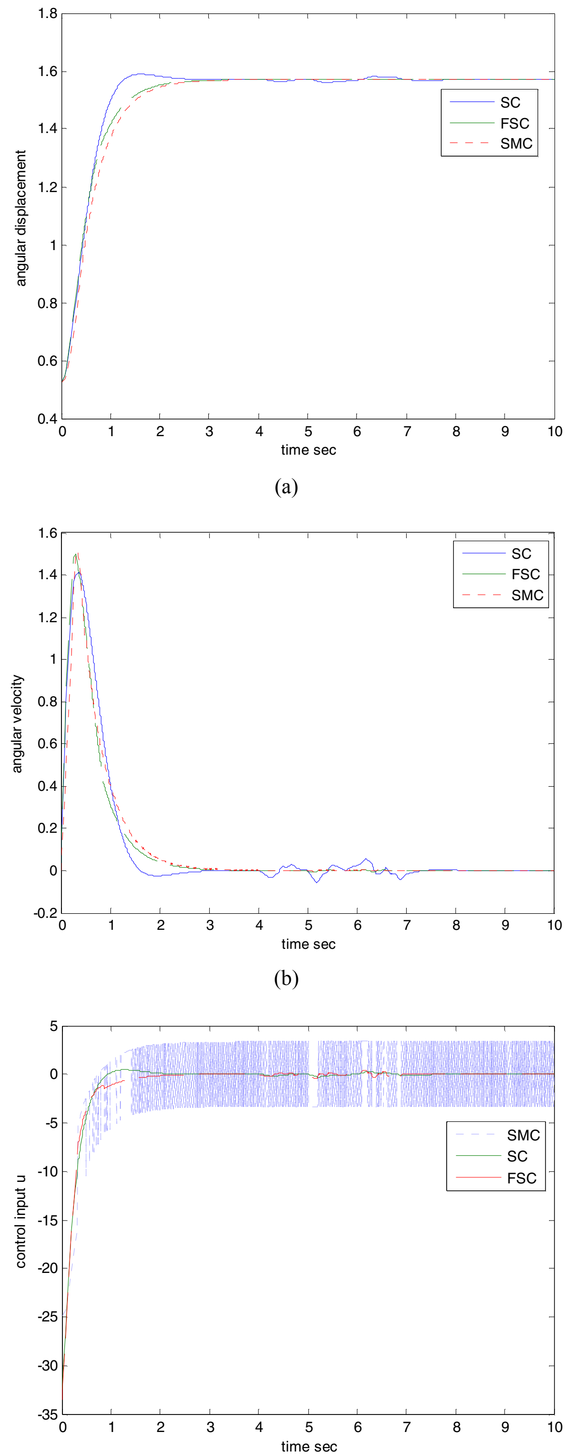


Fig. 2. The band-limited white noise applied to the inverted pendulum beginning at 4 seconds and ended at 7 seconds.

smooth control is occurred when SC and FSC schemes are applied to the system.

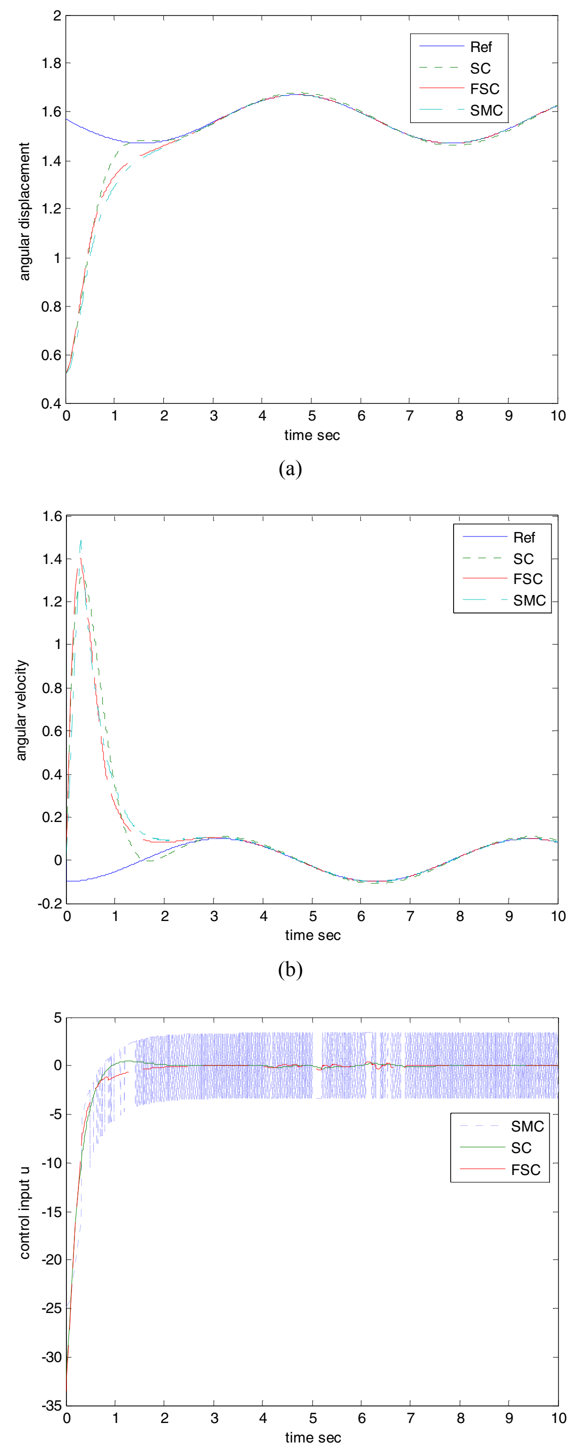
Next, unmolded dynamics is considered for simulation. We take the band-limited white noise as the sum-up effect of existed system uncertainties and disturbances as shown in Fig. 2. Now, we apply the noise to the inverted pendulum system and the simulation result is shown in Fig. 3 which reveals the capability of SC to tackle the noise is poor.

Suppose that the desired trajectory of angular displacement is $x_d = \pi/2 - 0.1\sin(t)$, and now we let the



(c)

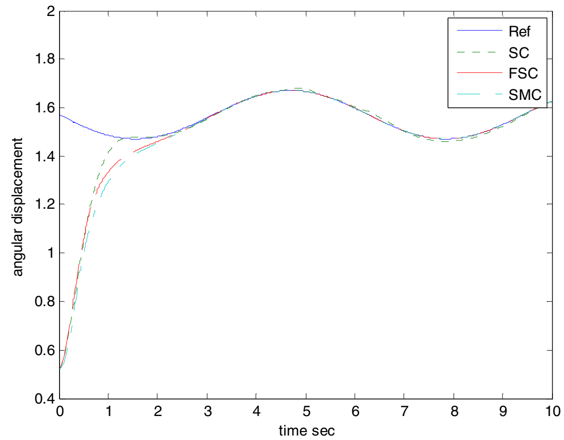
Fig. 3. The noise applied to the inverted pendulum controlled to the upright position by the SMC, SC, and FSC schemes with the initial condition $x_1(0) = \pi/6$ and $x_2 = 0$: (a) angular displacement; (b) angular velocity; (c) control input.



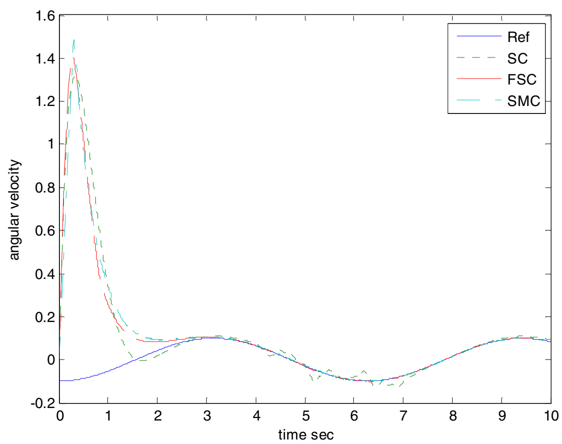
(c)

Fig. 4. The inverted pendulum controlled to track the desired trajectory $x_d = \pi/2 - 0.1\sin(t)$ by the SMC, SC, and FSC schemes with the initial condition $x_1(0) = \pi/6$ and $x_2 = 0$: (a) angular displacement; (b) angular velocity; (c) control input.

inverted pendulum track this reference trajectory. Fig. 4 and Fig. 5 show the simulation results of these three controllers with and without noise, respectively. In these two figures, one can easily find that the tracking performance of the FSC is the best among these three control schemes.



(a)



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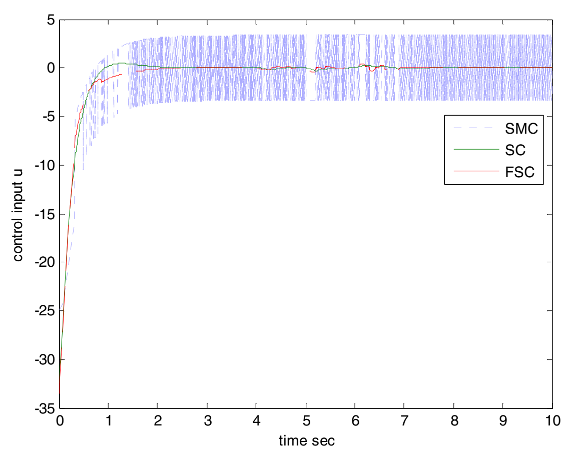

(c)

Fig. 5. The noise applied to the inverted pendulum controlled to track the desired trajectory $x_d = \pi/2 - 0.1\sin(t)$ by the SMC, SC, and FSC schemes with the initial condition $x_1(0) = \pi/6$ and $x_2 = 0$: (a) angular displacement; (b) angular velocity; (c) control input.

V. Conclusions

In this paper, we have developed a fuzzy synergetic controller for regulating and tracking control of a class of nonlinear systems. The control law has been introduced by using methods of synergetic theory and fuzzy logic control which can handle the nonlinear systems with system uncertainties and external disturbances. The Lyapunov stability method has been adopted to verify the stability of the controlled system. All the simulation results demonstrate the effectiveness and feasibility of the proposed control method.

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