2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)
July 6-11, 2014, Beijing, China

Delay-dependent local stabilization of nonlinear discrete-time system using T-S models through convex optimization

Luís F. P. Silva, Valter J. S. Leite, Eugênio B. Castelan, and Gang Feng

***Abstract*— In this paper we develop convex delay-dependent conditions in terms of linear matrix inequalities (LMIs) for the synthesis of fuzzy stabilizing feedback controllers. The condition is developed from a novel Lyapunov-Krasovskii fuzzy function. We consider that the T-S fuzzy model represents the nonlinear system only inside a region of validity. Because of this, we determine a domain of stability inside the region of validity, such that the trajectories of the nonlinear system in closed-loop starting from this domain converge asymptotically to origin. The domain of stability is characterized through a Cartesian product of two sets, where the first one is used to treat the initial state vector at the sample $k = 0$, and the second set is used to treat the delayed state vectors and the difference between two sampling of the delayed state vectors. We also develop a convex optimization problem to compute the gains of the fuzzy controllers to maximize the domain of stability. Finally, we show an example to demonstrate the developed conditions.**

I. Introduction

Dynamic systems with time-delay are often found in industrial processes especially when there is transfer of mass, energy, and/or information. In feedback control systems, time-delay usually causes performance deterioration and even loss of stability [9], [10]. This kind of control problem has been receiving attention in recent researches, as can be seen, for example, in [4], [8], and [16].

Fuzzy logic techniques have been widely and successfully used in nonlinear system modeling and control. In a large number of model-based fuzzy control studies and applications, the Takagi-Sugeno (T-S) fuzzy model approach has been shown quite popular and a convenient tool to handle complex nonlinear systems as can be seen in [6], [7], [11], and [17].

Recently, T-S fuzzy control of time-delay systems has been studied in the context of systems with delayed states, as can see in [3], [15], [19], [20], and [21]. The stability analysis and the control synthesis problems for time-varying delay discrete-time systems represented by the T-S fuzzy model are treated in these papers, where the proposed conditions are delay-dependent and based on Lyapunov-Krasovskii (L-K) functionals.

An important issue present in many real systems is the need to work inside a subregion of the state-space due to safe operational conditions, physical constraints or some desired level of energy consumptions. Moreover, an important characteristic of the T-S fuzzy model is that it can represent exactly a class of nonlinear system in a state space region of interest [13], known as region of validity. Inside this region the convex characteristics of T-S fuzzy models are preserved. For convenience, the region of validity of T-S fuzzy models can be chosen accordingly to the cited operational, physical or energy consumption constraints. Because of this, we need to deal with the corresponding region of attraction. This region is the set of all initial conditions such that the corresponding closed-loop trajectories converge to the origin. The exact characterization of the region of attraction is not an easy task [14]. Hence, it is necessary to characterize subsets of safe initial conditions that are inside of the region of attraction. In this paper, we call these subsets the domain of stability.

In this work, we propose convex delay-dependent conditions in terms of linear matrix inequalities (LMIs) for the synthesis of fuzzy stabilizing feedback controllers that stabilize a class of nonlinear systems. The resulting proposals are based on a time-delay fuzzy L-K function that guarantees the convergence of the trajectories for any sequence of initial conditions that belong to some level sets defined from the L-K function and contained in the region of validity of the T-S model. We determine a domain of stability as the Cartesian product of two sets, where one is used to treat the initial state vector at $k = 0$, and the other is used to treat the delayed state vectors and the difference between two sampling of these delayed states. Based on the developed conditions a LMI based optimization problem is proposed to synthesize a fuzzy state feedback controller that maximizes the domain of stability. Finally, we demonstrate the efficiency of the developed conditions through an example.

Notations: The r-th row of the matrix L is denoted as $L_{(r)}$. The symbol $\star$ represents the symmetric blocks in relation the diagonal. The matrices $\mathbf{I}$ and $\mathbf{0}$ de-

L. F. P. Silva and E. B. Castelan are with the Department of Automation and Systems, Universidade Federal de Santa Catarina, Santa Catarina, PPGEAS — DAS/CTC/UFSC, 88040-900, Florianópolis, SC, Brazil (e-mails: luis.filipe.silva@posgrad.ufsc.br and eugenio@das.ufsc.br); V. J. S. Leite is with the PPGEL and *Campus* Divinópolis — CEFET–MG, R. Álvares Azevedo, 400, 35503-822, Divinópolis, MG, Brazil (e-mail: valter@ieee.org); and Gang Feng is with Department of Mechanical and Biomedical Engineering, City University of Hong Kong 83 Tat Chee Avenue, Hong Kong, China (e-mail: megfeng@cityu.edu.hk).

This work was supported by Brazilian Agencies CAPES, CNPq and FAPEMIG.

note, respectively, identity and null matrices of appropriate dimensions. For $d \in \mathbb{N}^*$ and $k \in \mathbb{N}$, $\phi_{d,k} = \{x_{k-d}, x_{k-(d-1)}, \ldots, x_{k-1}\}$ denotes a sequence of d vectors $x_j \in \mathbb{R}^n$, $j \in [-d, -1]$, where $[a, b]$ is the interval of the integers starting from "a" and ending with "b". Consider that $\varphi_{d,k}$ defines a sequence of $d+1$ vectors $x_j \in \mathbb{R}^n$, $j \in [-d, 0]$, such that $\varphi_{d,k} = \{\phi_{d,k}, x_k\}$. The space of the vector sequence $\varphi_{d,k} = \{\phi_{d,k}, x_k\}$, which maps $[-d, 0]$ in $\mathbb{R}^n$, is $\mathcal{D}_d = \mathcal{D}([-d, 0], \mathbb{R}^n)$, with the norm $\|\phi_{d,k}\|_d = \sup_{-d\leq j\leq -1} \|x(k+j)\|$, where $\|\cdot\|$ is the Euclidian norm. The sequence of difference of delayed initial conditions is defined as $\Delta\phi_{d,k} = \{\phi_{d,k-d+1} - \phi_{d,k-d}, \ldots, \phi_{d,k} - \phi_{d,k-1}\}$. The function $Y = \texttt{round}(X)$ rounds the elements of X to the nearest integers.

II. Problem statements

Consider a class of discrete-time nonlinear system with a time-varying delay in the state represented by:

$$x_{k+1} = f(x_k) + f_d(x_{k-d_k}) + g(x_k)u_k, \tag{1}$$

where $f(x_k)$, $f_d(x_{k-d_k})$, and $g(x_k)$ represent nonlinear functions of the state vector $x_k \in \mathbb{R}^n$, d_k is the time-varying delay which $1 \leq \underline{d} \leq d_k \leq \bar{d}$, and $u_k \in \mathbb{R}^m$ is the control input vector with m input signals. The initial condition of the nonlinear system (1) is given by a sequence $\varphi_{\bar{d},0}$, with $\varphi_{\bar{d},0} = \{\phi_{\bar{d},0}, x_0\}$ and $\varphi_{\bar{d},0} \in \mathcal{D}_{\bar{d}}$, $\phi_{\bar{d},0}(j) = x_j$, $j \in [-\bar{d}, -1]$.

A Takagi-Sugeno (T-S) fuzzy model can be used to represent the nonlinear system (1), with the rules IF-THEN given by:

$$\begin{array}{ll} \text{IF} & z_1(k) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_p(k) \text{ is } M_{ip}, \\ \text{THEN} & x_{k+1} = A_i x_k + A_{di} x_{k-d_k} + B_i u_k, \end{array} \tag{2}$$

where $z_j(k)$, $j = 1, \ldots, p$, are the scalar premise variables supposed to be dependent only on the states, M_{ij} are the fuzzy sets, and p is the number of premise variables. The matrices of the subsystems, $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$, are known.

From a center-average defuzzifier, product fuzzy inference, and singleton fuzzifier, the dynamic fuzzy model (2) can be expressed by the following model [1], [2]:

$$x_{k+1} = A(\alpha_k)x_k + A_d(\alpha_k)x_{k-d_k} + B(\alpha_k)u_k, \tag{3}$$

where $\alpha_{k(i)} = w_i(z(k))/\sum_{j=1}^{N} w_j(z(k))$ with $w_i = \prod_{j=1}^{p} M_{ij}(z_j(k))$ and $z(k) = \begin{bmatrix} z_1(k) & z_2(k) & \cdots & z_p(k)\end{bmatrix}^T$. The membership function vector α_k is a state-dependent and time-varying parameter that is measurable or possible to be estimated in real time, and verifies the unitary simplex:

$$\Xi = \left\{ \alpha_k \in \mathbb{R}^N;\ \sum_{i=1}^{N} \alpha_{k(i)} = 1,\ \alpha_{k(i)} \geq 0, i = 1, \ldots, N \right\}. \tag{4}$$

Therefore, matrices in (3) can be rewritten as:

$$\begin{bmatrix} A(\alpha_k) & A_d(\alpha_k) & B(\alpha_k)\end{bmatrix} = \sum_{i=1}^{N} \alpha_{k(i)} \begin{bmatrix} A_i & A_{di} & B_i \end{bmatrix}, \quad \alpha_k \in \Xi. \tag{5}$$

The initial condition that assures existence and unicity of solutions for (3) is given by a sequence $\varphi_{\bar{d},0}$, where $\varphi_{\bar{d},0} = \{\phi_{\bar{d},0}, x_0\}$ with $\varphi_{\bar{d},0} \in \mathcal{D}_{\bar{d}}$, $\phi_{\bar{d},0}(j) = x_j$, $j \in [-\bar{d}, -1]$ and $\bar{d}$ is sufficiently large.

According to [13], an exactly global representation of the nonlinear system (1) by (3) is difficult to find. Furthermore, due to the possible presence of safe operational conditions, physical constraints or some desired level of energy consumptions, we consider that this representation is valid only in a given compact domain $\mathcal{V}_0$, known as the region of validity, and defined as the following:

$$\mathcal{V}_0 = \{x_k \in \mathbb{R}^n;\ |L_{(i)}x_k| \leq \epsilon_{(i)}\}, \tag{6}$$

where $\epsilon_{(i)} > 0$ and $L_{(i)} \in \mathbb{R}^{1\times n}$, for $i = 1, \ldots, \kappa$, with κ representing the number of constraints that characterizes the allowed region in the state-space, where the closed-loop system trajectories can evolve. Thus, it is necessary that $\varphi_{\bar{d},0} \in \mathcal{V}_0$ such that this initial condition sequence leads the states of nonlinear system to be controlled (1) to satisfy $x_k \in \mathcal{V}_0$.

Now consider the following control law:

$$u_k = K(\alpha_k)x_k + K_d(\alpha_k)x_{k-d_k}, \tag{7}$$

when it is possible to access the value of d_k in real time. If the value of d_k is unknown, it is possible to assume that $K_d(\alpha_k) = \mathbf{0}\ \forall k$ in (7). Note that the matrices of the controller are dependent on the membership function and, likewise the matrices of the fuzzy system (3), they are defined as follows:

$$\begin{bmatrix} K(\alpha_k) & K_d(\alpha_k)\end{bmatrix} = \sum_{i=1}^{N} \alpha_{k(i)} \begin{bmatrix} K_i & K_{di}\end{bmatrix},\ \alpha_k \in \Xi, \tag{8}$$

where $K_i \in \mathbb{R}^{m\times n}$ and $K_{d_i} \in \mathbb{R}^{m\times n}$.

Using the fuzzy formulation (3) and (7)–(8), we have the fuzzy closed-loop system with a time-varying delay in the state:

$$x_{k+1} = \hat{A}(\alpha_k)x_k + \hat{A}_d(\alpha_k)x_{k-d_k}, \tag{9}$$

where, by construction,

$$\hat{A}(\alpha_k) = A(\alpha_k) + B(\alpha_k)K(\alpha_k) = \sum_{i=1}^{N}\sum_{j=i}^{N} \mu_{ij}\alpha_{k(i)}\alpha_{k(j)} \frac{A_i + B_iK_j + A_j + B_jK_i}{2}, \tag{10}$$

$$\hat{A}_d(\alpha_k) = A_d(\alpha_k) + B(\alpha_k)K_d(\alpha_k) = \sum_{i=1}^{N}\sum_{j=i}^{N} \mu_{ij}\alpha_{k(i)}\alpha_{k(j)} \frac{A_{di} + B_iK_{dj} + A_{dj} + B_jK_{di}}{2}, \tag{11}$$

with

$$\mu_{ij} = \begin{cases} 2, & i \neq j, \\ 1, & \text{otherwise.} \end{cases} \tag{12}$$

Furthermore, we need to consider a domain of stability denoted Υ_φ, and following what we said, satisfying $\Upsilon_\varphi \subseteq \mathcal{V}_0$. Therefore, the main problem that we try to solve in this paper is exposed as follows:

Problem 1: Determine the T-S fuzzy gains K_i and K_{di} of the controller (7)–(8), and characterize the domain of stability Υ_φ, such that trajectories of the closed-loop system formed by the feedback interconnection of the nonlinear system (1) and the T-S controller (7)–(8) remain in $\mathcal{V}_0$ and converge asymptotically to the origin.

III. Preliminaries

We define the domain of stability through a Cartesian product, that is, $\Upsilon_\varphi = \mathcal{B}(r_1, r_2) \times \mathcal{C}_x = \{((\phi_{\bar{d},0}, \Delta\phi_{\bar{d},0}), x_0) \mid (\phi_{\bar{d},0}, \Delta\phi_{\bar{d},0}) \in \mathcal{B}(r_1, r_2), \text{ and } x_0 \in \mathcal{C}_x\}$ with:

$$\mathcal{C}_x = \left\{x_0 \in \mathcal{D}_{\bar{d}};\ V_1(x_0, \alpha_0) \leq c(\phi_{\bar{d},0}, \Delta\phi_{\bar{d},0})\right\}, \tag{13}$$

and

$$\mathcal{B}(r_1, r_2) = \left\{\phi_{\bar{d},0} \in \mathcal{D}_{\bar{d}};\ \|\phi_{\bar{d},0}\|_{\bar{d}} \leq r_1 \text{ and } \|\Delta\phi_{\bar{d},0}\|_{\bar{d}} \leq r_2\right\}, \tag{14}$$

where $V_1(x_0, \alpha_0)$ is a parameter dependent quadratic form, which we will define later, $c(\phi_{\bar{d},0}, \Delta\phi_{\bar{d},0})$ is a function on $\mathbb{R}^+$ with the sequences $\phi_{\bar{d},0}$ and $\Delta\phi_{\bar{d},0}$ as arguments and $0 \leq r_i \in \mathbb{R}^+$, $i = 1, 2$. Note that the set $\mathcal{C}_x$ is used to treat the initial state vector at $k = 0$, and the set $\mathcal{B}(r_1, r_2)$ is used to treat the delayed state vectors and the difference between two sampling of the delayed state vectors.

Now consider the following fuzzy L-K candidate function, $V(x_k, \alpha_k) : \mathbb{R}^n \times \Xi \to \mathbb{R}^+$:

$$V(x_k, \alpha_k) = \sum_{v=1}^{6} V_v(x_k, \alpha_k) > 0 \tag{15}$$

with $V_1(x_k, \alpha_k) = x_k^T Q^{-1}(\alpha_k) x_k$, $V_2 = \sum_{j=k-\bar{d}}^{k-1} x_j^T R_1 x_j$, $V_3 = \sum_{j=k-\underline{d}}^{k-1} x_j^T R_2 x_j$, $V_4 = \delta \sum_{\ell=\bar{d}}^{-1-\underline{d}} \sum_{m=k+\ell}^{k-1} y_m^T Z_1 y_m$, $V_5 = \bar{d} \sum_{\ell=-\bar{d}}^{-1} \sum_{m=k+\ell}^{k-1} y_m^T Z_2 y_m$ and $V_6 = \underline{d} \sum_{\ell=-\underline{d}}^{-1} \sum_{m=k+\ell}^{k-1} y_m^T Z_3 y_m$, where $y_j = x_{j+1} - x_j$; $\delta = \underline{d} - \bar{d}$, and $Q(\alpha_k) = \sum_{i=1}^{N} \alpha_{k(i)} Q_i$.

We can associate level sets, from the fuzzy L-K function defined above, as follows:

Definition 1: [5, Lemma 4] The intersection of ellipsoidal sets associated with the matrices $Q_i > \mathbf{0}$, $i = 1, \ldots, N$, can be defined as:

$$\mathcal{L}_{V_1}(c) = \left\{\mathcal{E}(Q_i^{-1}, c),\ \forall \alpha_k \in \Xi\right\} = \bigcap_{\alpha_k \in \Xi} \mathcal{E}(Q^{-1}(\alpha_k), c) = \bigcap_{i \in \{1,\ldots,N\}} \mathcal{E}(Q_i^{-1}, c), \tag{16}$$

where c is a scalar positive, and $\mathcal{E}(Q_i^{-1}, c)$, for $i = 1, \ldots, N$, denotes the ellipsoidal sets defined as follows:

$$\mathcal{E}(Q_i^{-1}, c) = \left\{x_k \in \mathbb{R}^n;\ x_k^T Q_i^{-1} x_k \leq c\right\}. \tag{17}$$

The Definition 1 is used for characterizing sets where corresponding trajectories remain confined in. In this definition, if $c = 1$, we use the simplified notation $\mathcal{L}_{V_1} = \mathcal{L}_{V_1}(1)$ and $\mathcal{E}(Q_i^{-1}) = \mathcal{E}(Q_i^{-1}, 1)$. It is used to determine sets of initial conditions $\mathcal{C}_x$.

Definition 2: The set $\mathcal{C}_x$ in (13) can be defined as:

$$\mathcal{C}_x = \left\{x_0 \in \mathcal{D}_{\bar{d}};\ V_1(x_0, \alpha_0) \leq 1 - (\rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2)\right\}, \tag{18}$$

where ρ_1 and ρ_2 are given, respectively, by

$$\rho_1 = \lambda_{\max}(R_1)\bar{d} + \lambda_{\max}(R_2)\underline{d}, \tag{19}$$

and

$$\rho_2 = \lambda_{\max}(Z_1)\frac{\delta^2(\delta+1)}{2} + \lambda_{\max}(Z_2)\frac{\bar{d}^2(\bar{d}+1)}{2} + \lambda_{\max}(Z_3)\frac{\underline{d}^2(\underline{d}+1)}{2}. \tag{20}$$

For the set $\mathcal{B}(r_1, r_2)$ in (14), we have r_1 and r_2 satisfying together $0 \leq r_1 \leq \rho_1^{-\frac{1}{2}}$, $0 \leq r_2 \leq \rho_2^{-\frac{1}{2}}$ and $\rho_1 r_1^2 + \rho_2 r_2^2 \leq 1$.

Proof: Consider the Definition 1 and that $c = 1 - (\rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2)$, we have:

$$\mathcal{C}_x = \mathcal{L}_{V_1}(1 - (\rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2)) = \{x_0 \in \mathbb{R}^n;\ x_0^T Q^{-1}(\alpha_0) x_0 \leq 1 - (\rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2)\}. \tag{21}$$

From V_i, $i = 2, \ldots, 6$, we can define the variables ρ_1 and ρ_2 as follows:

$$\begin{aligned}\sum_{i=2}^{6} V_i \leq{}& \sum_{j=-\bar{d}}^{-1} \phi_{\bar{d},0}^T(j) R_1 \phi_{\bar{d},0}(j) + \sum_{j=-\bar{d}}^{-1} \phi_{\bar{d},0}^T(j) R_2 \phi_{\bar{d},0}(j) \\ &+ \delta \sum_{\ell=\bar{d}}^{-1-\underline{d}} \sum_{m=\ell}^{-1} \Delta\phi_{\bar{d},0}^T(m) Z_1 \Delta\phi_{\bar{d},0}(m) + \bar{d} \sum_{\ell=-\bar{d}}^{-1} \sum_{m=\ell}^{-1} \Delta\phi_{\bar{d},0}(m) \\ &\times Z_2 \Delta\phi_{\bar{d},0}(m) + \underline{d} \sum_{\ell=-\underline{d}}^{-1} \sum_{m=\ell}^{-1} \Delta\phi_{\bar{d},0}(m) Z_3 \Delta\phi_{\bar{d},0}(m) \\ \leq{}& \rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2 \end{aligned} \tag{22}$$

■

Through the following lemma, the sets $\mathcal{C}_x$ and $\mathcal{B}(r_1, r_2)$ can be connected in terms of the confinement of trajectories in $\mathcal{L}_{V_1}$ and local asymptotic stability.

Lemma 1: Let $V(x_k, \alpha_k) > 0$ be given by (15). If $\Delta V(x_k, \alpha_k) = V(x_{k+1}, \alpha_{k+1}) - V(x_k, \alpha_k) < 0$, then

$$V(x_k, \alpha_k) < V(x_0, \alpha_0) \leq x_0^T Q^{-1}(\alpha_0) x_0 + \rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2. \tag{23}$$

Therefore, $\forall x_0 \in \mathcal{C}_x = \mathcal{L}_{V_1}(1 - (\rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2))$ and for all $(\phi_{\bar{d},0}, \Delta\phi_{\bar{d},0}) \in \mathcal{B}(r_1, r_2)$ it ensures that $x_k \in \mathcal{L}_{V_1}$, $\forall k \geq 0$ and $\lim_{k\to\infty} x_k = 0$.

Proof: From (15), if $\Delta V(x_k, \alpha_k) < 0$, $\forall k$ and $\forall \alpha_k \in \Xi$, then it verifies that:

$$x_k^T Q^{-1}(\alpha_k) x_k \leq V(x_k, \alpha_k) < V(x_0, \alpha_0). \tag{24}$$

By using (22), (19) and (20), we have:

$$V(x_0, \alpha_0) \leq x_0^T Q^{-1}(\alpha_0) x_0 + \rho_1 \|\phi_{\bar{d},0}\|_{\bar{d}}^2 + \rho_2 \|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2. \tag{25}$$

From (24) and (25), it can be verified that if $x_0^T Q^{-1}(\alpha_0)x_0 \leq 1-(\rho_1\|\phi_{\bar{d},0}\|_{\bar{d}}^2+\rho_2\|\Delta\phi_{\bar{d},0}\|_{\bar{d}}^2)$, then $x_k^T Q^{-1}(\alpha_k)x_k \leq 1$. Therefore, the local asymptotic stability of the state delayed closed-loop system is ensured. ■

IV. Main results

In the next theorem, it is shown that the T-S fuzzy gains synthesized by new delay-dependent conditions can locally stabilize nonlinear discrete-time systems with time-varying delay in the states.

Theorem 1: Suppose there are symmetric definite positive matrices $Q_i \in \mathbb{R}^{n\times n}$, $i=1,\dots,N$, $\tilde{R}_1 \in \mathbb{R}^{n\times n}$, $\tilde{R}_2 \in \mathbb{R}^{n\times n}$, $\tilde{Z}_1 \in \mathbb{R}^{n\times n}$, $\tilde{Z}_2 \in \mathbb{R}^{n\times n}$ and $\tilde{Z}_3 \in \mathbb{R}^{n\times n}$, and matrices $U \in \mathbb{R}^{n\times n}$, $Y_i \in \mathbb{R}^{m\times n}$ and $Y_{di} \in \mathbb{R}^{m\times n}$ verifying the LMIs (26) and

$$\begin{bmatrix} -Q_i & Q_i L_{(\ell)}^T \\ \star & -\epsilon_{(\ell)}^2 \end{bmatrix} \leq \mathbf{0},\ \forall i=1,\dots,N,\ \forall \ell=1,\dots,\kappa. \quad (27)$$

Then, the controller matrices (7)–(8) obtained with

$$K_i = Y_i U^{-1} \quad \text{and} \quad K_{di} = Y_{di}U^{-1} \quad (28)$$

are such that the origin of the nonlinear system (1) in closed-loop by control law (7)–(8) is asymptotically stable for any set of initial conditions $\Upsilon_\varphi = \mathcal{B}(r_1,r_2)\times\mathcal{C}_x$, ensuring that the respective trajectories remain in $\mathcal{L}_{V_1} \subseteq \mathcal{V}_0$, with $R_1 = U^{-T}\tilde{R}_1U^{-1}$, $R_2 = U^{-T}\tilde{R}_2U^{-1}$, and $Z_r = U^{-T}\tilde{Z}_rU^{-1}$, for $r=1,2,3$.

Proof: See Appendix. ■

A. Convex optimization problem

The objective here is to solve the Problem 1 by using the Theorem 1 through computing the set Υ_φ as large as possible. In this sense, a fundamental issue is to maximize the size of $\mathcal{L}_{V_1} \subseteq \mathcal{V}_0$. Such an optimization can be achieved by considering the maximization of an ellipsoidal set included in the level set $\mathcal{L}_{V_1}$ as follows

$$\mathcal{E}(W) = \left\{x \in \mathbb{R}^n; x^T W x \leq 1\right\} \subseteq \mathcal{L}_{V_1}. \quad (29)$$

This inclusion is equivalent to:

$$\begin{bmatrix} W & \mathbf{I} \\ \mathbf{I} & Q_i \end{bmatrix} \geq \mathbf{0},\ i=1,\dots,N. \quad (30)$$

Thus, a convex optimization problem can be proposed as follows:

$$\begin{array}{ll} \min & \operatorname{trace}(W) \\ \text{subject to} & (26), (27), \text{ and } (30). \end{array} \quad (31)$$

V. An Example

A control problem of a truck-trailer is studied in [12]. In this paper, furthermore, a time-varying delay in the states is also considered in this truck-trailer system as follows:

$$\begin{aligned} x_{1,k+1} =& \quad cx_{1,k}+(1-c)x_{1,k-d_k}-\tfrac{v\bar{t}}{L}\sin(cx_{1,k} \\ & +(1-c)x_{1,k-d_k})+\tfrac{v\bar{t}}{l}u_k \\ x_{2,k+1} =& \quad x_{2,k}-\tfrac{v\bar{t}}{L}\sin(cx_{1,k}+(1-c)x_{1,k-d_k}) \\ x_{3,k+1} =& \quad x_{3,k}+v\bar{t}\cos(cx_{1,k}+(1-c)x_{1,k-d_k}) \\ & \times\sin\Big(x_{2,k}+\tfrac{v\bar{t}}{2L}\sin(cx_{1,k} \\ & +(1-c)x_{1,k-d_k})\Big) \end{aligned} \quad (32)$$

where $x_{1,k}$ represents the angle between the truck and the trailer, $x_{2,k}$ denotes the angle of the trailer and $x_{3,k}$ is the vertical position of the rear, l and L represent the length of the vehicle and of the trailer, respectively. $\bar{t}$ is the sampling time and v is the constant reverse speed. The delay coefficient c is a constant, that satisfies $c \in [0,1]$, and it imposes a time-varying delay, d_k, in the state $x_{1,k}$, because of the image processing time. In this case, we have $l=2.8m$, $L=5.5m$, $v=-1.0m/s$, $\bar{t}=2.0s$, $c=0.7$, and $d_k \in [1,5]$. Note that when $x_1 = \pm\pi/2$, the "jack-knife" occurs. In order to simplify the model (32), we consider $cx_{1,k}+(1-c)x_{1,k-d_k} = \sin(cx_{1,k}+(1-c)x_{1,k-d_k})$, for $|x_{1,k}| \leq 0.675$, which implies a maximum error of 5% and allows we obtain a T-S fuzzy model with less rules. Therewith, we have

$$\begin{aligned} x_{1,k+1} =& \quad cx_{1,k}+(1-c)x_{1,k-d_k}-\tfrac{v\bar{t}}{L}(cx_{1,k} \\ & +(1-c)x_{1,k-d_k})+\tfrac{v\bar{t}}{l}u_k \\ x_{2,k+1} =& \quad x_{2,k}-\tfrac{v\bar{t}}{L}(cx_{1,k}+(1-c)x_{1,k-d_k}) \\ x_{3,k+1} =& x_{3,k}+v\bar{t}\cos(cx_{1,k}+(1-c)x_{1,k-d_k}) \\ & \sin(x_{2,k}+\tfrac{v\bar{t}}{2L}(cx_{1,k}+(1-c)x_{1,k-d_k})). \end{aligned} \quad (33)$$

The model (33) can be described as a T-S model through nonlinear local rules [13]:

$$x_{k+1} = \sum_{i=1}^{2}\sum_{j=1}^{2} S_i(Z_1(k))W_j(Z_2(k)) \times\{\bar{A}_{ij}x_k+\bar{A}d_{ij}x_{k-d_k}+\bar{B}u_k\}, \quad (34)$$

with $\bar{B} = \begin{bmatrix} \frac{v\bar{t}}{l} \\ 0 \\ 0 \end{bmatrix}$, $\bar{A}_{ij} = \begin{bmatrix} c\left(1-\frac{v\bar{t}}{L}\right) & 0 & 0 \\ c\frac{v\bar{t}}{L} & 1 & 0 \\ c\frac{v^2\bar{t}^2}{2L}d_ig_j & v\bar{t}d_ig_j & 1 \end{bmatrix}$ and $\bar{A}_{dij} = \begin{bmatrix} (1-c)\left(1-\frac{v\bar{t}}{L}\right) & 0 & 0 \\ (1-c)\frac{v\bar{t}}{L} & 0 & 0 \\ (1-c)\frac{v^2\bar{t}^2}{2L}d_ig_j & 0 & 0 \end{bmatrix}$. Where we define: $S_1(Z_1(k)) = \frac{\cos(\theta(x_1(k)))-d_2}{d_1-d_2}$, $S_2(Z_1(k)) = \frac{d_1-\cos(\theta(x_1(k)))}{d_1-d_2}$, $W_1(Z_2(k)) = \frac{\sin(p(k))-g_2p(k)}{p(k)(g_1-g_2)}$, $W_2(Z_2(k)) = \frac{g_1p(k)-\sin(p(k))}{p(k)(g_1-g_2)}$, for $p(k)\neq 0$, $W_1(Z_2(k))=1$, and $W_2(Z_2(k))=0$, for $p(k)=0$, $Z_1(k)=\cos(\theta(x_1(k)))$, for $\theta(x_1(k)) = cx_1(k)+(1-c)x_1(k-d_k)$, $Z_2(k)=\sin(p(k))$, for $p(k)=x_2(k)+\frac{v\bar{t}}{2L}\theta(x_1(k))$, $d_1=\max(Z_1(k))=1$, $d_2=\min(Z_1(k))=0.7807$, $g_1=1$, and $g_2=10^{-2}/\pi$.

Note that to obtain the T-S fuzzy model (34), we need to consider $|x_{1,k}| \leq 0.675$, and $|x_{2,k}| \leq \pi$. The state $x_{3,k}$ is "free", but we consider $|x_{3,k}| \leq 500$. Therefore, the region of validity (6) is defined by $L=\mathbf{I}$, and $\epsilon = \begin{bmatrix}0.675 & pi & 500\end{bmatrix}^T$. By utilizing the optimization problem (31) to compute the gains of controller which stabilize the nonlinear system (32) through the control law (7), we obtain: $K_1 = \begin{bmatrix}0.6768 & -0.0424 & 1e^{-4}\end{bmatrix}$, $K_{d1} = \begin{bmatrix}0.5898 & 0 & 0\end{bmatrix}$, $K_2 = \begin{bmatrix}0.6263 & -0.0348 & 1e^{-4}\end{bmatrix}$, $K_{d2} = \begin{bmatrix}0.5827 & 0 & 0\end{bmatrix}$, $K_3 = \begin{bmatrix}0.6749 & -0.038 & 1e^{-4}\end{bmatrix}$, $K_{d3} = \begin{bmatrix}0.5884 & 0 & 0\end{bmatrix}$, $K_4 = \begin{bmatrix}0.6775 & -0.0391 & 1e^{-4}\end{bmatrix}$, and $K_{d4} = \begin{bmatrix}0.5895 & 0 & 0\end{bmatrix}$. For this example we consider that $r_1=r_2=0$ in $\mathcal{B}(r_1,r_2)$. Thus, we have $\|\phi_{5,0}\|_{10}^2=0$, and $\|\Delta\phi_{5,0}\|_{10}^2=0$ and in consequence $\mathcal{C}_x=\mathcal{L}_{V_1}$. In Figure 1, we show the set $\mathcal{C}_x$ obtained.

$$
\begin{bmatrix}
\delta^2\tilde{Z}_1 + \bar{d}^2\tilde{Z}_2 + \underline{d}^2\tilde{Z}_3 - U^T - U & 0.5(A_iU + B_iY_j + A_jU + B_jY_i) - \delta^2\tilde{Z}_1 - \bar{d}^2\tilde{Z}_2 - \underline{d}^2\tilde{Z}_3 & \mathbf{0} & 0.5(A_{di}U + B_iY_{dj} + A_{dj}U + B_jY_{di}) & \mathbf{0} & U^T \\
\star & 0.5(Q_i + Q_j) - U^T - U + \delta^2\tilde{Z}_1 + (\bar{d}^2 - 1)\tilde{Z}_2 + (\underline{d}^2 - 1)\tilde{Z}_3 + \tilde{R}_1 + \tilde{R}_2 & \tilde{Z}_3 & \mathbf{0} & \tilde{Z}_2 & \mathbf{0} \\
\star & \star & -\tilde{Z}_1 - \tilde{Z}_3 - \tilde{R}_2 & \tilde{Z}_1 & \mathbf{0} & \mathbf{0} \\
\star & \star & \star & -2\tilde{Z}_1 & \tilde{Z}_1 & \mathbf{0} \\
\star & \star & \star & \star & -\tilde{Z}_1 - \tilde{Z}_2 - \tilde{R}_1 & \mathbf{0} \\
\star & \star & \star & \star & \star & -Q_q
\end{bmatrix} < \mathbf{0},
$$
$$
\forall i, q = 1, \ldots, N, \;\; j = i, \ldots, N \quad (26)
$$

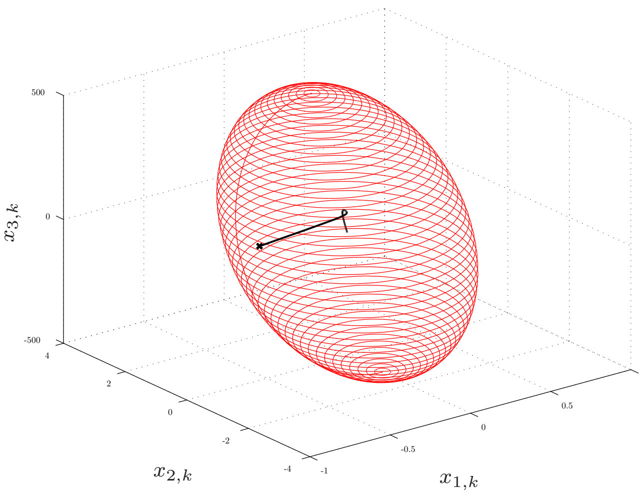

Fig. 1. The set $\mathcal{C}_x = \mathcal{L}_{V_1}$, a point marked by $\times$, and a stable trajectory.

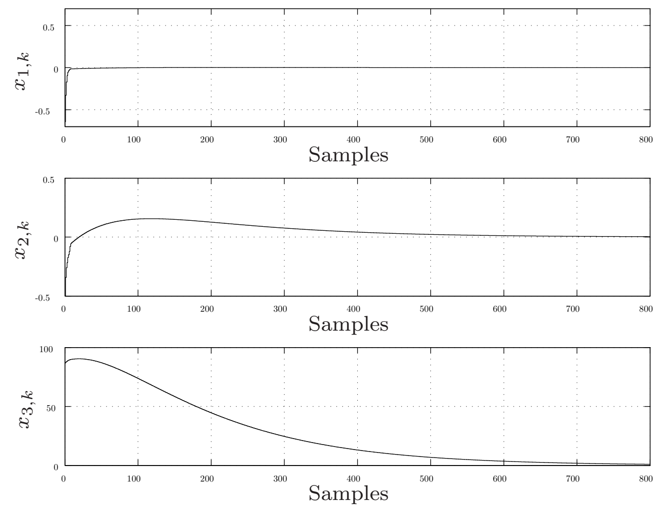

Fig. 2. Trajectories of stable states.

By applying the control law (7) with the gains shown in the previous paragraph in the nonlinear system (32), we obtain the stable trajectory shown in Figure (1). This same trajectory is shown in Figure 2, but now in function of the samples. To obtain this trajectory, we always consider $|x_{1,k}| \leq \pi$, the following expression for the time-varying delay $d_k = \texttt{round}(3 + 2\cos(k))$, and the initial condition $\varphi_{5,0} = \{\phi_{5,0}, x_0\} \in \mathcal{D}_5$ with $\|\phi_{5,0}\|_5^2 = 0$ and $x_0 = \begin{bmatrix} -0.6743 & -0.5009 & 86.73 \end{bmatrix}^T$, marked by $\times$ in Figure 1. Note that the initial condition belongs to the set $\mathcal{C}_x$ and as we hope $\lim_{k \to \infty} x_k = 0$.

Now from the [3, Theorem 2], we compute the gains used in the control law (7) to give feedback to the nonlinear system (32), and obtain: $K_1 = \begin{bmatrix} 1.6387 & -1.6094 & 0.0561 \end{bmatrix}$, $K_{d1} = \begin{bmatrix} 0.7732 & -7e^{-4} & 0 \end{bmatrix}$, $K_2 = \begin{bmatrix} 1.6141 & -1.5266 & 0.0566 \end{bmatrix}$, $K_{d2} = \begin{bmatrix} 0.7651 & 0.0021 & -2e^{-4} \end{bmatrix}$, $K_3 = \begin{bmatrix} 1.5926 & -1.3916 & 0.0571 \end{bmatrix}$, $K_{d3} = \begin{bmatrix} 0.7569 & -7e^{-4} & 0 \end{bmatrix}$, $K_4 = \begin{bmatrix} 1.5926 & -1.3915 & 0.0571 \end{bmatrix}$, and $K_{d4} = \begin{bmatrix} 0.7568 & -7e^{-4} & 0 \end{bmatrix}$. By utilizing the same initial condition and the same expression for the time-varying delay considered in the previous paragraph, we obtain Figure (3). Through this figure, we can observe that the state $x_{1,k}$ tends to the origin, and the $x_{2,k}$ also tends to the origin, because $x_{2,k}$ represents an angle and its final value is 2π. However, the state $x_{3,k}$ does not tend to the origin. Actually, it tends to a value around $150m$ far from the origin. From the results obtained in this example, it shows that it is important to take into account the region of validity in the synthesis of gains for fuzzy controllers.

VI. Conclusions

In this paper convex local delay-dependent conditions for the synthesis of fuzzy stabilizing feedback controllers are developed. These conditions are obtained through fuzzy Lyapunov-Krasovskii function, and they are represented as LMIs. The T-S fuzzy model exactly represents the nonlinear system only inside a region of validity. Because of this, we determine a domain of stability

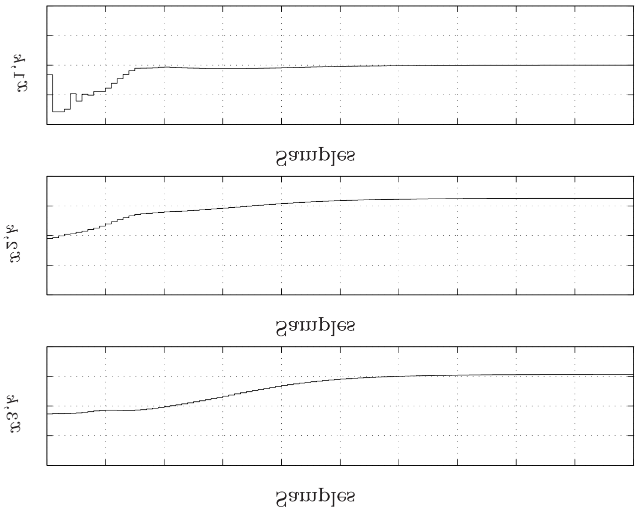

Fig. 3. Trajectories of unstable states.

inside of the region of validity, such that all trajectories of the closed-loop nonlinear system starting from this domain converge asymptotically to the origin and do not leave the region of validity. The domain of stability is characterized by a Cartesian product of two sets, where the first one is used to treat the initial state vector at $k = 0$, and the second set is used to treat the delayed state vectors and the difference between two sampling of the delayed state vectors. A convex optimization problem is developed and through this optimization problem the fuzzy controllers' gains are computed to maximize the domain of stability. By the example we show that the fuzzy controllers' gains designed from our results can be applied in the nonlinear system. However, in the same example we show that the fuzzy controllers' gains designed from results found in the literature cannot be applied in the nonlinear system.

Appendix: Proof Theorem 1

From the fuzzy L-K function (15), we have:

$$\begin{aligned}\Delta V(x_k,\alpha_k) &\leq x_{k+1}^T[Q^{-1}(\alpha_{k+1}) + \delta^2 Z_1 + \bar{d}^2 Z_2 + \underline{d}^2 Z_3] \\ &\times x_{k+1} + 2x_{k+1}^T[-\delta^2 Z_1 - \bar{d}^2 Z_2 - \underline{d}^2 Z_3]x_k + x_k^T \\ &\times[-Q^{-1}(\alpha_k)+R_1+R_2+\delta^2 Z_1+(\bar{d}^2-1)Z_2+(\underline{d}^2-1)Z_3]x_k \\ &+ 2x_k^T Z_3 x_{k-\underline{d}} + 2x_k^T Z_2 x_{k-\bar{d}} + x_{k-\underline{d}}^T[-R_2 - Z_1 - Z_3]x_{k-\underline{d}} \\ &+ 2x_{k-\underline{d}}^T Z_1 x_{k-d_k} + x_{k-d_k}^T[-2Z_1]x_{k-d_k} + 2x_{k-d_k}^T Z_1 x_{k-\bar{d}} \\ &+ x_{k-\bar{d}}^T[-R_1 - Z_1 - Z_2]x_{k-\bar{d}}, \end{aligned} \quad (35)$$

considering that the discrete Jensen inequality was applied in ΔV_i, for $i = 4, 5,$ and 6, like as shown in [18]. From Finsler's Lemma, we have a sufficient condition $\theta(\alpha_k) = \mathcal{Q}(\alpha_k) + \mathcal{X}\mathcal{B}(\alpha_k) + \mathcal{B}(\alpha_k)^T\mathcal{X}^T < \mathbf{0}$, where $\xi_k^T\mathcal{Q}(\alpha_k)\xi_k < \mathbf{0}$ subject to $\mathcal{B}(\alpha_k)\xi_k = \mathbf{0}$, with

$$\xi_k = \begin{bmatrix} x_{k+1}^T & x_k^T & x_{k-\underline{d}}^T & x_{k-d_k}^T & x_{k-\bar{d}}^T \end{bmatrix}^T, \quad (37)$$

$$\mathcal{B}(\alpha_k) = \begin{bmatrix} -\mathbf{I} & \hat{A}(\alpha_k) & \mathbf{0} & \hat{A}_d(\alpha_k) & \mathbf{0} \end{bmatrix}, \quad (38)$$

and

$$\mathcal{X} = \begin{bmatrix} F^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T. \quad (39)$$

Consider the congruence transformation like as $\tilde{\theta}(\alpha_k) = \mathcal{T}^T\theta(\alpha_k)\mathcal{T}$, where $\mathcal{T} = \text{diag}\{U^T, U^T, U^T, U^T, U^T\}$, and $U^T = F^{-1}$. In $\tilde{\theta}(\alpha_k)$ we apply the Schur's complement in block (1,1), and consider that $-U^TQ^{-1}(\alpha_k)U \leq Q(\alpha_k) - U^T - U$. Thus, we obtain (40).

In (40) we consider $\hat{A}(\alpha_k)$ and $\hat{A}_d(\alpha_k)$ are given in (10) and (11), respectively,

$$\begin{aligned} Q(\alpha_k) = \left(\sum_{j=1}^{N}\alpha_j\right) Q(\alpha_k) = \sum_{i=1}^{N}\sum_{j=i}^{N}\mu_{ij}\alpha_{k(i)}\alpha_{k(j)} \\ \times 0.5(Q_i + Q_j), \end{aligned} \quad (41)$$

with μ_{ij} defined in (12), $Q(\alpha_{k+1}) = \sum_{q=1}^{N}\alpha_q Q_q$, $Y_i = K_iU$, and $Y_{di} = K_{di}U$. Therefore, we obtain the condition (26).

Now we assume that (26) is verified and additionally (27) is satisfied. Then, we multiply (27) by $\alpha_{k(i)}$ and sum up on $i = 1, \ldots, N$, getting:

$$\Lambda = \begin{bmatrix} -Q(\alpha_k) & Q(\alpha_k)L_{(\ell)}^T \\ \star & -\epsilon_{(\ell)}^2 \end{bmatrix} \leq \mathbf{0}. \quad (42)$$

By using the congruence transformation $\mathcal{F}^T\Lambda\mathcal{F} = \tilde{\Lambda}$ with $\mathcal{F} = \text{diag}\{Q^{-1}(\alpha_k), 1\}$, we get

$$\tilde{\Lambda} = \begin{bmatrix} -Q^{-1}(\alpha_k) & L_{(\ell)}^T \\ \star & -\epsilon_{(\ell)}^2 \end{bmatrix} \leq \mathbf{0}. \quad (43)$$

Applying Schur's complement in $\tilde{\Lambda}$, we have:

$$L_{(\ell)}^T\epsilon_{(\ell)}^{-2}L_{(\ell)} - Q^{-1}(\alpha_k) \leq \mathbf{0}. \quad (44)$$

In this last inequality, we can pre- and post-multiply by x_k^T and x_k, respectively, and by using the S-procedure, we have that:

$$x_k^T L_{(\ell)}^T\epsilon_{(\ell)}^{-2}L_{(\ell)}x_k \leq 1,\ \forall x_k \in \mathcal{L}_{V_1} = x_k^TQ^{-1}(\alpha_k)x_k \leq 1. \quad (45)$$

Then, by considering (45), we prove that $\mathcal{L}_{V_1} \subseteq \mathcal{V}_0$ and any trajectory that starts in $\Upsilon_\varphi = \mathcal{B}(r_1, r_2) \times \mathcal{C}_x$ remains $\mathcal{L}_{V_1}$ and thus the local stability of nonlinear system (1) in closed-loop by control law (7)–(8) is assured.

References

[1] M. Chen, G. Feng, H. Ma, and G. Chen. Delay-dependent $\mathcal{H}_\infty$ filter design for discrete-time fuzzy systems with time-varying delays. *IEEE Trans. on Fuzzy Systems*, 17(3):604–616, June 2009.

[2] G. Feng. *Analysis and Synthesis of Fuzzy Control Systems: A Model-Based Approach.* CRC Press, Hong Kong, 2009.

[3] H. Gao, X. Liu, and J. Lam. Stability analysis and stabilization for discrete-time fuzzy systems with time-varying delay. *IEEE Trans. on Systems, Man, and Cybernetics – Part B*, 39(2):306–317, April 2009.

[4] H. Gassara, A. E. Hajjaji, and M. Chaabane. Robust control of T-S fuzzy systems with time-varying delay using new approach. *Int. Journal of Robust and Nonlinear Control*, 20(14):1566–1578, August 2010.

$$
\mathcal{Q}(\alpha_k) = \begin{bmatrix} Q^{-1}(\alpha_{k+1})\delta^2 Z_1 + \bar{d}^2 Z_2 + \underline{d}^2 Z_3 & -\delta^2 Z_1 - \bar{d}^2 Z_2 - \underline{d}^2 Z_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & -Q^{-1}(\alpha_k) + \delta^2 Z_1 + (\bar{d}^2 - 1) Z_2 + (\underline{d}^2 - 1) Z_3 + R_1 + R_2 & Z_3 & \mathbf{0} & Z_2 \\ \star & \star & -Z_1 - Z_3 - R_2 & Z_1 & \mathbf{0} \\ \star & \star & \star & -2Z_1 & Z_1 \\ \star & \star & \star & \star & -Z_1 - Z_2 - R_1 \end{bmatrix}, \quad (36)
$$

$$
\tilde{\theta}(\alpha_k) \leq \begin{bmatrix} \delta^2 \tilde{Z}_1 + \bar{d}^2 \tilde{Z}_2 + \underline{d}^2 \tilde{Z}_3 - U^T - U & \hat{A}(\alpha_k)U - \delta^2 \tilde{Z}_1 - \bar{d}^2 \tilde{Z}_2 - \underline{d}^2 \tilde{Z}_3 & \mathbf{0} & \hat{A}_d(\alpha_k)U & \mathbf{0} & U^T \\ \star & Q(\alpha_k) - U^T - U + \delta^2 \tilde{Z}_1 + (\bar{d}^2 - 1)\tilde{Z}_2 + (\underline{d}^2 - 1)\tilde{Z}_3 + \tilde{R}_1 + \tilde{R}_2 & \tilde{Z}_3 & \mathbf{0} & \tilde{Z}_2 & \mathbf{0} \\ \star & \star & -\tilde{Z}_1 - \tilde{Z}_3 - \tilde{R}_2 & \tilde{Z}_1 & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -2\tilde{Z}_1 & \tilde{Z}_1 & \mathbf{0} \\ \star & \star & \star & \star & -\tilde{Z}_1 - \tilde{Z}_2 - \tilde{R}_1 & \mathbf{0} \\ \star & \star & \star & \star & \star & -Q(\alpha_{k+1}) \end{bmatrix} < \mathbf{0} \quad (40)
$$

[5] M. Jungers and E. B. Castelan. Gain-scheduled output control design for a class of discrete-time nonlinear systems with saturating actuators. *Systems & Control Letters*, 60(3):315–325, March 2011.

[6] M. Khairy, A. L. Elshafei, and H. M. Emara. LMI based design of constrained fuzzy predictive control. *Fuzzy Sets and Systems*, 161(6):893–918, March 2010.

[7] D. H. Lee, J. B. Park, and Y. H. Joo. Further improvement of periodic control approach for relaxed stabilization condition of discrete-time Takagi-Sugeno. *Fuzzy Sets and Systems*, 174(1):50–65, July 2011.

[8] Z. Liu, S. Lü, S. Zhoung, and M. Ye. Stabilization analysis for discrete-time systems with time delay. *Applied Mathematical and Computation*, 216(7):2024–2035, June 2010.

[9] M. F. Miranda and V. J. S. Leite. Robust stabilization of polytopic discrete-time systems with time-varying state delay: A convex approach. *Journal of the Franklin Institute*, 348:568–588, 2011.

[10] D. S. Niculescu. *Delay effects on stability: a robust control approach*. Springer, Germany, 2001.

[11] H. C. Sung, J. B. Park, Y. H. Joo, and K. C. Lin. Robust digital implementation of fuzzy control for uncertain systems and its application to active magnetic bearing system. *Int. Journal of Control, Aut., and Systems*, 10(3):603–612, June 2012.

[12] K. Tanaka and M. Sano. A robust stabilization problem of fuzzy control systems and its application to backing up control of a Truck-Trailer. *IEEE Trans. on Fuzzy Systems*, 2(2):119–134, May 1994.

[13] K. Tanaka and H. O. Wang. *Fuzzy control systems design and analysis: A linear matrix inequality approach*. John Wiley & Sons, New York, 2001.

[14] S. Tarbouriech, G. Garcia, J. M. Gomes da Silva Jr., and I. Queinnec. *Stability and satbilization of linear systems with saturating actuators*. Springer, 2011.

[15] L. Wu, X. Su, P. Shi, and J. Qiu. A new approach to stability analysis and stabilization of discrete-time T-S fuzzy time-varying delay systems. *IEEE Trans. on Systems, Man, and Cybernetics – Part B*, 41(1):273–286, February 2011.

[16] S. Xu, G. Feng, Y. Zou, and J. Huang. Robust controller design of uncertain discrete time-delay system with input saturation and disturbances. *IEEE Transactions on Automatic Control*, 57(10):2604–2609, October 2012.

[17] X. Zhang, C. Wang, D. Li, X. Zhou, and D. Yang. Robust stability of impulsive Takagi-Sugeno fuzzy systems with parametric uncertainties. *Information Sciences*, 181(23):5278–5290, December 2011.

[18] X.-L. Zhu and G.-H. Yang. Jensen inequality approach to stability analysis of discrete-time systems with time-varying delay. In *American Control Conference - ACC*, pages 1644–1649, Seattle, Washington, USA, June 2008.

[19] L. Wu, X. Su, P. Shi, and J. Qiu. Stability analysis and controller synthesis for discrete-time delayed fuzzy systems via small gain theorem. *Information Sciences*, 41(1):273–286, February 2013.

[20] X. Su, P. Shi, L. Wu, and Y-D. Song. A novel control design on discrete-time Takagi-Sugeno fuzzy system with time-varying delays. *IEEE Trans. Fuzzy Systems*, 21(4):655–671, August 2013.

[21] X. Yang, L. Wu, H-K. Lam, and X. Su. Stability and stabilization of discrete-time T-S fuzzy systems with stochastic perturbation and time-varying delay. with time-varying delays. *IEEE Trans. Fuzzy Systems*, 22(1):124–138, February 2014.