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# FUZZY MULTI-OBJECTIVE RELIABILITY-REDUNDANCY ALLOCATION PROBLEM

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*Abstract*— **Reliability is the measure of the result of the quality of the system over a long run. The reliability-redundancy allocation problem (RRAP) aims to ensure high systems reliability in the presence of optimally redundant systems components. This is one of the most important design considerations for the systems designers. Several researchers have addressed this important issue during last few decades. However, due to the embedded uncertainty in the parameters of the system components, reliability as well as the costs of the whole system fits very well to be modeled as fuzzy quantity. We therefore modeled this problem as a fuzzy multi-objective optimization problem (MORRAP) that is addressed using the popular multi-objective evolutionary algorithm, NSGA-II (non-dominated sorting genetic algorithm-II). We have considered the based MORRAP with fuzzy type-2 uncertainty. As far as we know, no research has been reported where MORRAP was considered under type-2 fuzzy uncertainty. A typical numerical example is included and results are compared showing that our approach outperforms other recently reported results.**

**Keywords:** Reliability, Redundancy, Reliability-Redundancy Allocation Problem, NSGA-II, Fuzzy Reliability, Fuzzy Cost

## I. INTRODUCTION

The probability that a system shall perform its desired functions without any intermediate failure for a certain amount of time under defined working conditions is known as reliability of the system. The reliability of a system can mainly be increased by the addition of additional components or through the increase in the reliability of the individual components [1]. However, the addition of surplus components causes the increase of the cost, weight and volume etc. of the designed system. Therefore, it is imperative to find a balance between the reliability of system and added redundancy of the components. This little background is at the centre of evolution of a well-known research problem called, 'reliability redundancy allocation problem (RRAP)''. As the problem is computationally intractable [2], it has drawn immense attraction from research community over the last few decades. This problem has been solved both as a single-objective optimization problem and multiple-objective optimization problem using a number of approaches viz. dynamic programming, mixed integer programming, non-linear programming [3], integer programming [4], heuristic algorithms [5], meta-heuristic algorithms, evolutionary algorithms [6] etc.

For real-life applications, the reliability optimization problem that considers the maximization of reliability function as an only objective function is not very suitable. Therefore, researchers formulated the reliability redundancy allocation problem (RRAP) as a multi objective optimization problem (MORRAP) in which reliability becomes very high and designing cost gets minimized. The multi-objective reliability-redundancy allocation problem (MORRAP) has recently been addressed by a significant number of researchers considering the objectives of reliability maximization and redundancy minimization under different design constraints. However, a very remarkable feature of the problem remained unnoticed until recently that is nothing other than the embedded uncertainties in the reliability and the costs of the components. The fact is that consideration of these uncertainties plays a significant role in the computation of the overall system reliability and cost.

In this paper, we address the multi-objective reliability-redundancy allocation problem (MORRAP) with uncertainties in the reliability and cost of the components. These uncertainties in the reliability and cost are as modelled as fuzzy uncertainties. We therefore addressed the problem with the consideration of the system components having reliabilities and costs modelled with type-1 fuzzy parameter. We have considered model the problem as fuzzy type-2 uncertainty based MORRAP. This has been augmented by the fact that during the system designing time, all the information on the reliabilities and the cost of the components are designers' approximation. Different designers may have different opinions about the membership grades of the associated uncertainties in the cost and reliabilities of the system components. Thus, it is highly appropriate to model these parameters with type-2 fuzzy sets. As far as we know, no research has been reported till date where MORRAP has been considered under type-2 fuzzy uncertainty. This problem is then very suitably reduced to the type-1 fuzzy MORRAP with the application of a suitable type-reduction method [28-30]. We therefore solved the Fuzzy MORRAP with a well-known evolutionary algorithms viz. NSGA-II (non-dominated sorting genetic algorithm-II). Suitable numerical examples are included to demonstrate that our approach outperforms the other recently reported results.

The rest of the paper is organized as follows. Section II gives a brief literature survey of the related research of the RRAP. In the Section III, the mathematical formulation of the problem is given. The solution and numerical example is discussed in the Section IV. The paper is concluded in the Section V with a discussion of the results.

## II. LITERATURE REVIEW

In reliability theory, one of the core issues is the problem of finding the optimal allocation of component redundancy under maximized systems' reliability because of the non-polynomial computational complexity of the problem. In his seminal research paper [8], M.-S. Chern proved that the reliability-redundancy allocation problem is NP-hard. This prompted researchers to look-for heuristic based approaches rather than algorithmic solutions. To analyze and determine the optimal design configuration of the series-parallel systems, D. W. Coit et. al. [9] proposed a genetic algorithm based solution approach by modeling the problem as a linear optimization problem with a dynamic penalty function. A very significant research work on the reliability-redundancy allocation problem has been carried out by Kuo and Prasad [10]. In this paper authors have discussed the problem thoroughly with an overview of the researches reported over a decade and highlighted all the methods that have been developed. They have argued that exact solutions for this problem are not essentially looked-for as it is difficult to obtain precise solutions. In [11], J. E. R-Marquez et. al. formulated the redundancy allocation problem for systems with multi-state reliability behaviour under system-level performance constraints and solved the problem heuristically to minimize the design cost. The authors considered multi-state series-parallel systems having binary components which were capable of providing varying performance levels. H. A. Taboada et. al. [12] considered the multi-objective reliability design optimization problems and solved the problem using the non-dominated sorting genetic algorithm (NSGA). For multi-state series-parallel systems Z. Tian et. al. [13] investigated the reliability-redundancy allocation problem and designed a genetic algorithm to solve it. Z. Wang et al. [14] addressed the multi-objective Reliability Redundancy Allocation Problem (MORAP) with two objective functions, maximize the reliability and minimize the design cost with a Multi-objective Evolutionary Algorithm, namely Non-dominated Sorting Genetic Algorithm II (NSGA-II) under a number of constraints. W.-C. Yeh et al. [8] proposed a particle swarm optimization based on the Monte-Carlo Simulation problem technique to solve not only the traditional series-parallel system for evaluating the reliability optimization problem also can solve the complex network. V. Ebrahimipour et al. [16] develop a fuzzy multi-objective optimization problem for MORRAP with consideration of triangular fuzzy number. The expected value concept was used to convert developed model to a crisp model and to solve the crisp model Multi- objective particle swarm optimization is used. K. K.- Damghani, et. al. [17] proposed a particle swarm optimization based dynamic self-adaptive method for solving the multi-objective reliability redundancy allocation problem. The authors used an efficient version of the epsilon-constraint method together with non-dominated sorting genetic algorithm method and a customized time-variant multi-objective particle swarm optimization method in their approach to test the solutions. C. Dingzhou et. al. [18] proposed a decomposition-based approach to solve the Multi-objective Redundancy Allocation Problem for series-parallel systems. The authors treated redundancy allocation problem as a multi-objective problem and decompose the original problem into several multi-objective sub-problems then solve each sub-problems and then combine the solutions. H. Garg et. al. [19] formulated a Fuzzy Multi-objective Reliability Redundancy Allocation Problem for the original Multi-objective Reliability Redundancy Allocation Problem of a series-parallel system. With the help of crisp optimization a fuzzified MOOP is constructed then particle swarm optimization is applied to solve this problem and results are compare with genetic algorithm (GA). B Xing et. al. [20] addressed the reliability optimization problem by a computational intelligence method for globally optimal solutions. In [21], H. Garg et. al. used intuitionistic fuzzy programming technique in the multi-objective reliability redundancy allocation problem to avoid the conflict between objectives.

## III. PROBLEM FORMULATION

In the design of series-parallel system with m independent subsystems arranged in series and in each subsystem have i components, arranged in parallel which are equivalent in functionality [3]. A subsystem can work properly if at least one of its components is operational. The typical structure of a series-parallel system is illustrated in Fig. 1. Increasing the number of redundant components will increase the system reliability, but that also increases its cost, weight and others constrains. The aim is to optimize the redundant components while balancing the other objectives.

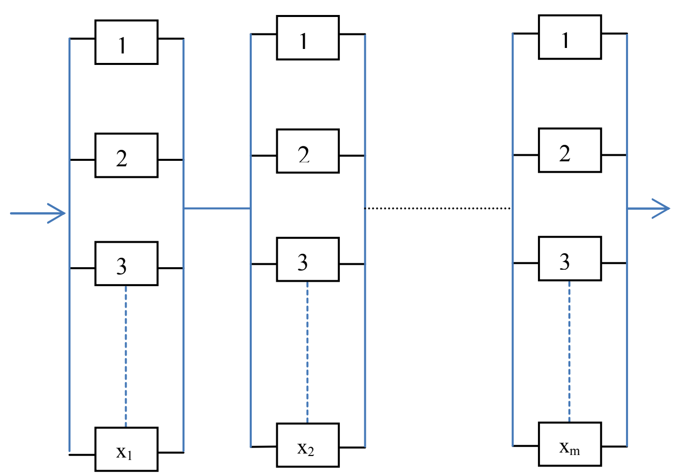



Fig-1: Typical Series-Parallel Systems with **m** subsystems

The basic assumptions for the multi-objective reliability-redundancy allocation problem (MORRAP) are as follows:

a) All components are assumed to be non-repairable.
b) All components are assumed to have binary states i.e. a component can only in the working state or in the failed state.
c) The functioning of all the components is known, deterministic, and time-independent.

d) Physical properties e.g. reliability, volume, weight, and cost etc.
e) All the redundant components for individual subsystems are identical.
f) Failure of any redundant components are independent and do not damage the system.

The notations used in our mathematical model for the multi-objective reliability-redundancy allocation problem is given in the Table 1.

**Table 1 : Notations Used in Mathematical Model**

| Abbreviations | Details |
|---|---|
| **m** | Number of sub-systems |
| **i** | Index of sub-systems, i = 1,2,…,m |
| $n_i$ | the number of components in the ith sub-system |
| $r_i$ | reliability of each component in ith subsystem |
| $R_s$ | system reliability function of the reliability model |
| $C_s$ | system cost function of the reliability model |
| $c(r_i)$ | the cost of each component with reliability $r_i$ at subsystem i |
| $v_i$ | volume of each component in ith subsystem |
| $w_i$ | weight of each component in ith subsystem |
| $g_i$ | ith constrain of the system |
| **T** | operating temperature |
| **V** | allowed volume of the system |
| **W** | allowed weight of the system |
| **C** | Maximum allowed cost of the system |
| $\alpha_i$ | shaping factor of each component in ith subsystem |
| $\beta_i$ | scaling factor of each component in ith subsystem |

The system reliability of an m-stage parallel-series system is expressed by

$$R_s\left(x_1,x_2,x_{3,},\ldots,x_n\right)=\prod_{i=1}^{m}\left[1-\left(1-r_i\right)^{n_i}\right] \quad (1)$$

where $r_i$ is the component reliability and $n_i$ the number of components at the stage-i. The cost is an increasing function of $r_i$ or conversely a decreasing function of the component failure rate expressed by

$$c_i\left(r_i\right) = \alpha_i\lambda_i^{-\beta_i} \quad (2)$$

where $\alpha_i$, and $\beta_i$ are shaping factor and scaling factor representing the inherent characteristics of each component at stage-i, $\beta_i > 1$. If for all *i* each component follows the negative exponential failure law, i.e.,

$$r_i = e^{-\lambda_i t} \quad (3)$$

then, component cost at stage i is

$$c\left(r_i\right)=\alpha_i\left(-t/\ln(r_i)\right)^{\beta_i} \quad (4)$$

where t is the operating time during which the component at stage-i will not fail. Usually $\alpha_i$ and $\beta_i$ and t are given. Thus, $c(r_i)$ * $n_i$ give the cost of the components at stage-i as a function of $r_i$ and $n_i$. An additional cost $c(r_i)$ * $\exp(n_i/4)$ is included, as the cost for interconnecting parallel elements. Using the above equations the total cost function of the system can be determine as

$$C_S\left(x_1,x_2,x_3,\ldots,x_n\right)=\sum_{i=1}^{m}\alpha_i\left(-t/\ln(r_i)\right)^{\beta_i}.\left(n_i+\exp\left(n_i/4\right)\right) \quad (5)$$

The first constraint is a combination of weight and volume:

$$g_1\left(r,n\right)=\sum_{i=1}^{m}w_iv_i^2n_i^2 \le V \quad (6)$$

The second constraint is weight which similarly evaluated as the cost is evaluated earlier:

$$g_2\left(r,n\right)=\sum_{i=1}^{m}w_i.\left(n_i+\exp\left(\frac{n_i}{4}\right)\right)\le W \quad (7)$$

where, $w_i * n_i$ is the weight of all of the components at stage-i. Again an additional factor is added, which is exp (X1/4), due to the hardware for interconnecting the links. The weight constraint is not a function of component reliability.

### *A. Mathemathical Model of Multi-Obective Realiability redundancy Allocation Problem*

In most of the cases RRAP is assumed to be a single objective problem for maximizing the reliability of the system and minimizing the cost, weight, and volume. But now the interest of the researches is fast moving towards taking more than one objective. In terms of the RRAP, the main goal is to determine the optimal component reliabilities and the redundancy level of components in a system to maximize the system reliability subject to several resource constraints. In this model we maximize the reliability of system and also minimized cost simultaneously. The multi-objective reliability-redundancy allocation problems (MORRAPs) presented here is based on a single objective formulation with one or several constraints [kuo & Prasad, 2000]. The multi objective formulation was obtained by converting the cost constraints into an objective function.

Therefore, the MORRAP for the system with the consideration of two separate objectives viz. reliability and cost, may be stated mathematically as follows:

$$\textbf{Maximize } R_s\left(x_1,x_2,x_{3,},\ldots,x_m\right)=\prod_{i=1}^{m}\left[1-\left(1-r_i\right)^{n_i}\right] \quad (9)$$

$$\textbf{Minimize } C_S\left(x_1,x_2,x_3,\ldots,x_m\right)=\sum_{i=1}^{m}c\left(r_i\right).\left(n_i+\exp\left(n_i/4\right)\right) \quad (10)$$

subject to

$$g_1\left(r,n\right)=\sum_{i=1}^{m}u_in_i^2 \le V \quad (11)$$

$$g_2\left(r,n\right)=\sum_{i=1}^{m}w_i.\left(n_i+\exp\left(\frac{n_i}{4}\right)\right)\le W \quad (12)$$

where, $c\left(r_i\right)=\alpha_i\left(-T/\ln(r_i)\right)^{\beta_i}$ and $u_i = w_i * v_i^2$ .

*B. Establishing Fuzzy multi-objective optimization model for reliability*

We have formulated FMOOP for series parallel system corresponding to optimization problem discussed in section C. In order to maximize the reliability of the above model through fuzzy theory we fuzzily the reliability of individual subsystem (arrange in the series) and take its minimum value. Simultaneously to minimize the cost involved in keeping the required optimized reliability we compute the cost of corresponding subsystems and take its maximum.

Now to model our problem various types of membership functions are available such as linear, tangent type, exponential type etc. Here for the sake of simplicity we are using linear membership function as defined in the following Eqs. (19) & Eq. (22). We assumed that $R_t^l$ is the minimum value of the system reliability, that is, the necessarily requirements of the decision-maker for the system reliability; $R_t^u$ is that the decision-makers think the ideal reliability of the system to be; $\mu_{f_t}(r_i, n_i)$ is a linear membership function of system reliability objective function $R_S(x_i)$.Similarly in the second objective function $C_t^l$ is the upper value of the system cost, showing the maximum value of investment for system; $C_t^u$ is that lowest cost the decision-makers thing for the system ideal cost to be: $\{v_{f_t}(r_i, n_i)\}$ is a linear membership function of system reliability cost function $\tilde{C}_s(x_i)$.

The fuzzy MORRAP for the system is stated as

**Maximize** $\tilde{R}_s(x_1, x_2, x_3, \dots, x_m) =$

$$\{\tilde{R}_S(x_1), \tilde{R}_S(x_2), \tilde{R}_S(x_3), \dots, \tilde{R}_S(x_m)\} \quad (17)$$

where, $$\tilde{R}_s(x_i) = \{1 - \mu_{f_t}(r_i, n_i)\} \quad (18)$$

$$\mu_{f_t}(r_i, n_i) = \begin{cases} 1, & R_S(x_i) \ge R_t^l \\ \frac{R_S(x_i) - R_t^l}{R_t^u - R_t^l}, & R_t^l \le R_S(x_i) \le R_t^u \\ 0, & R_S(x_i) \le R_t^l \end{cases} \quad (19)$$

and

**Minimize** $\tilde{C}_s(x_1, x_2, x_3, \dots, x_m) =$

$$\{\tilde{C}_S(x_1), \tilde{C}_S(x_2), \tilde{C}_S(x_3), \dots, \tilde{C}_S(x_m)\} \quad (20)$$

where, $$\tilde{C}_s(x_i) = \{v_{f_t}(r_i, n_i)\} \quad (21)$$

$$v_{f_t}(r_i, n_i) = \begin{cases} 1, & C_S(x_i) \ge C_t^l \\ \frac{C_t^u - C_S(x_i)}{C_t^u - C_t^l}, & C_t^l \le C_S(x_i) \le C_t^u \\ 0, & C_S(x_i) \ge C_t^u \end{cases} \quad (22)$$

subject to, $$g_1(r, n) = \sum_{i=1}^{m} u_i n_i^2 \le V \quad (23)$$

$$g_2(r, n) = \sum_{i=1}^{m} w_i \cdot \left(n_i + \exp\left(\frac{n_i}{4}\right)\right) \le W \quad (24)$$

where, $c(r_i) = \alpha_i\left(-T/\ln(r_i)\right)^{\beta_i}$ and $u_i = w_i * v_i^2$.

*C. Representation of Fuzzy sets*

Reliability is very critical outcome of any real-world application. The timing parameters are very crucial for the effectiveness of the reliability. Also, when the cost minimization is the concerned with the component reliability, we have to consider the uncertainty in the parameters. There can be many approximation of one input which is concerned with the reliability. All of these approximations have to be dealt with care. So we propose to use the Type-2 fuzzy arithmetic in this paper which related to the two conflicting objectives of our problem. Some works have been done in these fields which have dealt with type-2 fuzzy for reliability. Very few have been done with regards to Type-2 Fuzzy sets (T2FS). Nobody has used the concept of T2FS in this field to model uncertainty as here we propose to use it. The reason for using T2FS is that type-1 may suffer from interpretability issues. The uncertainties are modeled with type-2 fuzzy numbers [24][25]. We have to first consider the definition of type-2 fuzzy sets, which is:

A type-2 fuzzy set [26], denoted by $\tilde{\tilde{A}}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.,

$$\tilde{\tilde{A}} = \{((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\}, 0 \le \mu_{\tilde{A}}(x, u) \le 1.$$

Here, x is a primary variable and its measurement domain is denoted by X; u is a secondary variable, $u \subseteq J_x$ at each $x \in X$; $J_x$ is a primary membership degree of x. $\tilde{\tilde{A}}$ can also expressed in discrete cases as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} J_x \subseteq [0, 1].$$

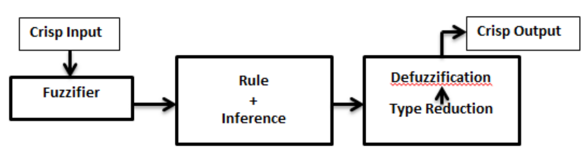



Fig-2: Type-2 Fuzzy Logic System

If $\underline{\mu}_{\tilde{A}}(x)$ is the Lower membership function (LMF) and $\overline{\mu}_{\tilde{A}}(x)$ is Upper membership function (UMF), then FOU is collection of LMF and UMF i.e. FOU= $[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]$. Type-2 fuzzy sets incorporate a Footprint of Uncertainty (FOU) which allows for better modeling of uncertainty. FOU is certainly the collection of all the embedded type-1 fuzzy sets. It offers more degree of freedom to the T2FS. Many publications have shown that T2FS can outperform their type-1 FSs counterparts in a variety of applications, ours one is specific to reliability and cost constraints. The structure of T2FLS is shown in the Figure-2. We have considered the T2FLS having $n$ inputs, $x_1 \in X_1,$, $x_2 \in X_2$,…,, $x_n \in X_n$ and one output, $y \in Y$, the rule-base is composed of $N$ rules, where the $i$th rule $R^i$ can be expressed as follows:

$\boldsymbol{R^i}$**: IF** $\boldsymbol{x_1}$ **is** $\tilde{\mathrm{A}}_1$ **and** $\boldsymbol{x_2}$ **is** $\tilde{\mathrm{A}}_2$**… and** $\boldsymbol{x_n}$**is** $\tilde{\mathrm{A}}_n$**. THEN y is** $\boldsymbol{B}$

The output of Fuzzifier is T2FS, which is then provided to the Rule & Inference Block and so on The firing strength $f^i$'s are defined as:

$$\underline{f}^i = Min \text{ (LMF's)}$$

$$\overline{f}^i = Min \text{ (UMF's)}$$

$\underline{\mathrm{f}}^{\mathrm{i}}$ is the Lower limit of firing-strength and $\overline{\mathrm{f}}^{\mathrm{i}}$ is the Upper limit of firing-strength.

Next we have to perform the type reduction and the defuzzification for our reliability and cost optimization. We have used the Nie-Tan Type-reduction method [27]. Vertical-slice representation is used in NT method. This method provides the output with less complexity. First the average $avg_i$ of the FOU of $\tilde{A}$ at each $x_i$ is calculated. $avg_i$ can be calculated as [28][29]:

$$avg_i = \frac{(\underline{\mu}_{\tilde{A}}(x_i) + \overline{\mu}_{\tilde{A}}(x_i))}{2}$$

With the help of this average $avg_i$, we can calculate the defuzzified value (avg) of the Type-2 fuzzy set as:

$$\text{avg} = \frac{\sum_{i=1}^{N} x_i avg_i}{\sum_{i=1}^{N} avg_i}$$

where $a_i$is computed from above. The defuzzified output is then used for our functional computation.

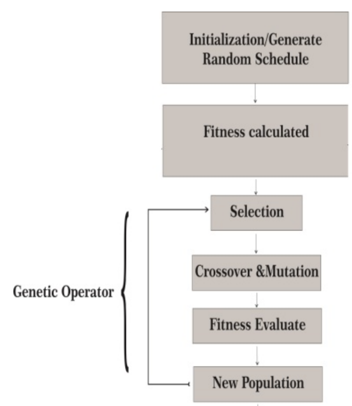
## IV. SOLUTION TECHNIQUE

In real world problem the chances are very least that we come-up with only a single-Objective problem. Our problem is to increase the reliability in the real-world application with the conflicting approach to minimize the overall cost. Moreover there is always an Up-down scale in the values of these two conflicting criteria's. One objective is compromised for the other. Practically these are called the multi-objective optimization problems. Multi-objective optimization problem are well addressed considered by the NSGA-II (non-dominated sorting genetic algorithm) algorithm [11], which is one of the most popular variants of the traditional genetic algorithmic approaches. NSGA-II is a fast and exclusive Multi-objective Genetic Algorithm. It's an extension to the genetic algorithm which has been widely used for its multi-objective optimization capability. We therefore solve the MORRAP problem using the techniques of NSGA-II. The basic operations of the NSGA-II algorithm is shown in the flow-chart given in the Fig. 3. The concept of domination sort is used where certain conditions are followed if solution from objective 1 dominates the solutions from objective 2. Below are given the conditions:

- First solution is no worse than the other solution
- First solution is strictly better than the other solution in at least one objective.
- If first solution dominates other solution ,then it can be said that
  - Other solution is dominated by First solution.
  - First solution is non-dominated by Other solution.
  - Other solution is inferior to First Solution.

The benefits of using NSGA-II is that it uses the improved sorting algorithm, incorporates elitism and sharing parameter are not needed to be chosen *a priori*. The initialization of population is same as used for Genetic algorithm. After the population initialization, it is sorted depending on the non-domination into each front. We will be having some fronts which depend on the procedure run on population. The first front is completely non-dominate done in the current population and the second front is dominated by the individuals in the population in the first front only. The front goes on of there are more than two fronts. The ranks which are the fitness values, the results of objective function, are assigned to each individual in the first front and so on. Individuals in first front are assigned a fitness value of 1 and individuals in second are given a fitness value as 2.

Fig 3: Flow-chart of the NSGA-II Algorithm




After the fitness values are calculated, a new parameter called *crowding distance* is calculated for each individual forming the front. Large average crowding distance will result in better diversity in the population. Parents are selected from the population by using binary tournament selection based on the rank and crowding distance. An individual is selected if the rank is lesser than the other or if crowding distance is greater than the other.

## V. EXAMPLE [20]

We now demonstrate our fuzzy type-2 based model for the multi-objective reliability-redundancy allocation problem (FT2MORRAP) with the help of a numerical example [20]. Considering the typical numerical parameters, the FT2MORRAP problem may be summarized as given below:

**Maximize** $$R_S = \prod_{i=1}^{5}\left[1-\left(1-r_i\right)^{n_i}\right]$$

**Minimize** $$C_S = \sum_{i=1}^{5} c\left(r_i\right).\left(n_i + \exp\left(n_i/4\right)\right)$$

subject to $g_1(r,n) = \sum_{i=1}^{5} u_i n_i^2 \le V$

$$g_2(r,n) = \sum_{i=1}^{5} w_i \cdot \left( n_i + \exp\left( \frac{n_i}{4} \right) \right) \le W$$

$0.5 \le r_i \le 0.95$, $1 \le n_i \le 3$ and $n_i \in Z^+$,
$C_s \le C$, $g_1(r_i,n_i) \le W$ and $g_2(r_i,n_i) \le V$.

We have solved this problem using the NSGA-II. The basic operations of the NSGA-II algorithm were explained in the Section IV. The input data for the NSGA-II are shown in the Table 2, which includes all the parameters related to the problem formulation. These parameters are already defined in the notations mentioned in the Table 1. We have considered here five different type of components and all of their related parameters are mentioned. Each row in the Table 2 represents the values for parameters of a single system component.

TABLE 2 INPUT PARAMETERS DATA

| Sr. | $10^5\alpha_i$ | $\beta_i$ | $w_i$ | $v_i$ | V | C | W | T(h) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.516724 | 1.5 | 6.0 | 4.0 | 289 | 553 | 483 | 1000 |
| 2 | 1.982908 | 1.5 | 8.0 | 4.0 | 289 | 553 | 483 | 1000 |
| 3 | 3.578225 | 1.5 | 5.0 | 3.0 | 289 | 553 | 483 | 1000 |
| 4 | 2.966955 | 1.5 | 10.0 | 4.0 | 289 | 553 | 483 | 1000 |
| 5 | 0.611360 | 1.5 | 9.0 | 4.0 | 289 | 553 | 483 | 1000 |

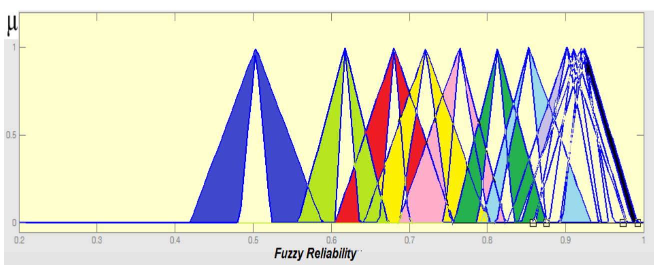



Fig. 4: Components Reliability with IT2 Fuzzy Membership Functions

TABLE 3 RESULTS FOR 1ST DATA INPUT (SR. 1)

| $r_i$ | $n_i$ | $R_s$ | $C_s$ | PF | CD |
|---|---|---|---|---|---|
| 0.50534 | 3 | 0.524629 | 50.46003 | 1 | 0.108837 |
| 0.601112 | 3 | 0.720466 | 78.35737 | 1 | 0.11412 |
| 0.732451 | 3 | 0.907839 | 163.7687 | 1 | 0.117802 |
| 0.789882 | 3 | 0.954469 | 248.3759 | 1 | 0.194419 |
| 0.847239 | 3 | 0.982302 | 421.5547 | 1 | 0.145055 |
| 0.86178 | 3 | 0.986866 | 495.925 | 1 | 0.160795 |
| 0.895154 | 3 | 0.994251 | 771.8787 | 1 | 0.215009 |
| 0.910143 | 3 | 0.996378 | 984.8456 | 1 | 0.168113 |
| 0.922074 | 3 | 0.997636 | 1231.268 | 1 | 0.11453 |
| 0.931168 | 3 | 0.998371 | 1493.982 | 1 | 0.133006 |
| 0.933756 | 3 | 0.998547 | 1585.628 | 1 | 0.179852 |
| 0.942644 | 3 | 0.999057 | 1982.036 | 1 | 0.171008 |
| 0.947185 | 3 | 0.999264 | 2251.076 | 1 | 0.11744 |

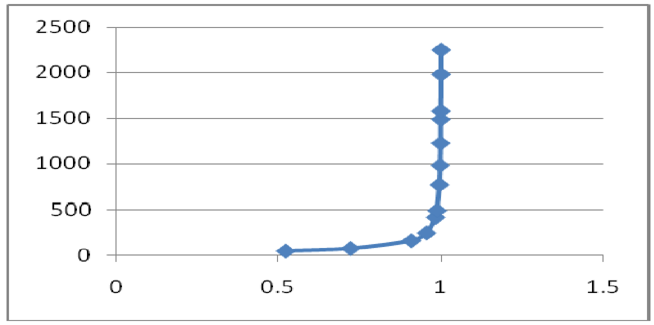



Fig 5: Graph for Table: 3

TABLE 4 RESULTS FOR 2ND DATA INPUT (SR. 2)

| $r_i$ | $n_i$ | $R_s$ | $C_s$ | PF | CD |
|---|---|---|---|---|---|
| 0.773784 | 3 | 0.943443 | 123.5239 | 1 | 0.156179 |
| 0.796249 | 3 | 0.958417 | 147.5145 | 1 | 0.154532 |
| 0.860127 | 3 | 0.986392 | 274.3004 | 1 | 0.145394 |
| 0.868663 | 3 | 0.988724 | 303.6568 | 1 | 0.119661 |
| 0.893924 | 3 | 0.994046 | 427.248 | 1 | 0.186499 |
| 0.910427 | 2 | 0.996412 | 558.0811 | 1 | 0.149224 |
| 0.925907 | 3 | 0.997968 | 751.111 | 1 | 0.197608 |
| 0.930285 | 3 | 0.998307 | 825.8492 | 1 | 0.142603 |
| 0.93609 | 3 | 0.998696 | 945.2496 | 1 | 0.130798 |
| 0.938534 | 3 | 0.998839 | 1004.107 | 1 | 0.123049 |

TABLE 5 RESULTS FOR 3RD DATA INPUT (SR. 3)

| $r_i$ | $n_i$ | $R_s$ | $C_s$ | PF | CD |
|---|---|---|---|---|---|
| 0.628132 | 3 | 0.767999 | 91.29915 | 1 | 0.236482 |
| 0.772984 | 3 | 0.942855 | 221.5626 | 1 | 0.287334 |
| 0.809015 | 3 | 0.965651 | 296.7143 | 1 | 0.106282 |
| 0.881524 | 3 | 0.991713 | 646.4972 | 1 | 0.107677 |
| 0.893704 | 3 | 0.994009 | 768.4545 | 1 | 0.125004 |
| 0.913061 | 3 | 0.996719 | 1055.44 | 1 | 0.219206 |
| 0.923904 | 3 | 0.997799 | 1300.158 | 1 | 0.123583 |
| 0.925978 | 3 | 0.997974 | 1357.451 | 1 | 0.116929 |
| 0.933053 | 3 | 0.998501 | 1587.145 | 1 | 0.104984 |

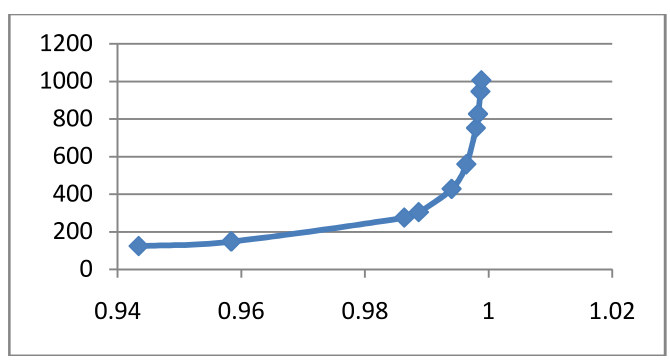



Fig. 6: Graph for Table: 4

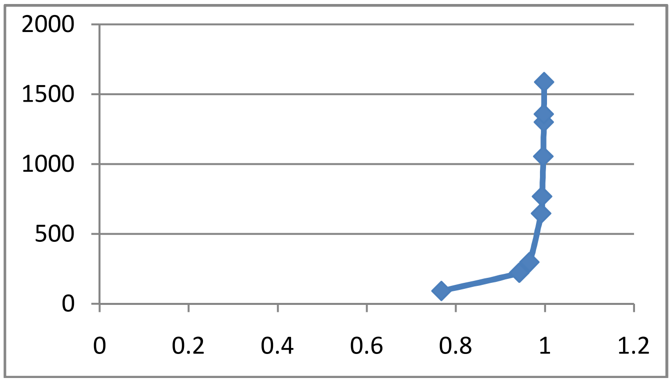



Fig. 7: Graph for Table: 5

TABLE 6 RESULTS FOR 4TH DATA INPUT (SR. 4)

| $r_i$ | $n_i$ | $R_s$ | $C_s$ | PF | CD |
|---|---|---|---|---|---|
| 0.745348 | 3 | 0.920115 | 150.6568 | 1 | 0.118939 |
| 0.810938 | 3 | 0.966664 | 250.2207 | 1 | 0.269634 |
| 0.883482 | 3 | 0.992115 | 550.5144 | 1 | 0.223342 |
| 0.894959 | 3 | 0.994219 | 649.296 | 1 | 0.142628 |
| 0.910324 | 3 | 0.996399 | 833.5246 | 1 | 0.145505 |
| 0.916995 | 3 | 0.997144 | 941.0721 | 1 | 0.086334 |
| 0.924049 | 3 | 0.997811 | 1081.284 | 1 | 0.101666 |
| 0.935059 | 3 | 0.998631 | 1379.663 | 1 | 0.125113 |
| 0.93775 | 3 | 0.998795 | 1473.207 | 1 | 0.08369 |

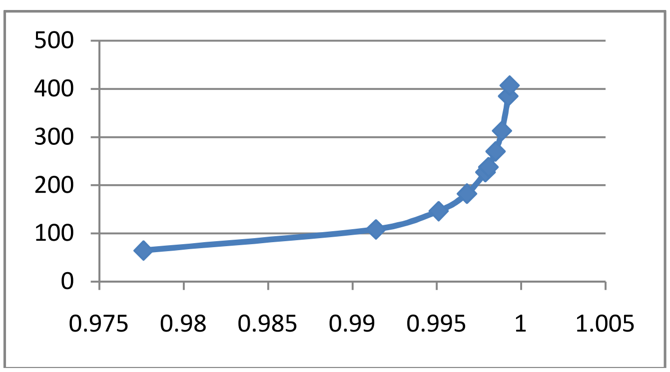



Fig. 8 Graph for Table: 6

TABLE 7 RESULTS FOR 5TH DATA INPUT (SR. 5)

| $r_i$ | $n_i$ | $R_s$ | $C_s$ | PF | CD |
|---|---|---|---|---|---|
| 0.834732 | 3 | 0.977633 | 64.42346 | 1 | 0.207145 |
| 0.880032 | 3 | 0.991397 | 108.2685 | 1 | 0.211845 |
| 0.900678 | 3 | 0.995111 | 146.1959 | 1 | 0.181189 |
| 0.913684 | 3 | 0.996789 | 182.3729 | 1 | 0.194021 |
| 0.924921 | 3 | 0.997886 | 226.8555 | 1 | 0.131713 |
| 0.927076 | 3 | 0.998063 | 237.3952 | 1 | 0.100346 |
| 0.932854 | 3 | 0.998487 | 269.926 | 1 | 0.104034 |
| 0.938927 | 3 | 0.998862 | 312.6706 | 1 | 0.137712 |
| 0.946569 | 3 | 0.999238 | 384.402 | 1 | 0.126636 |
| 0.948468 | 3 | 0.999316 | 406.4555 | 1 | 0.098218 |

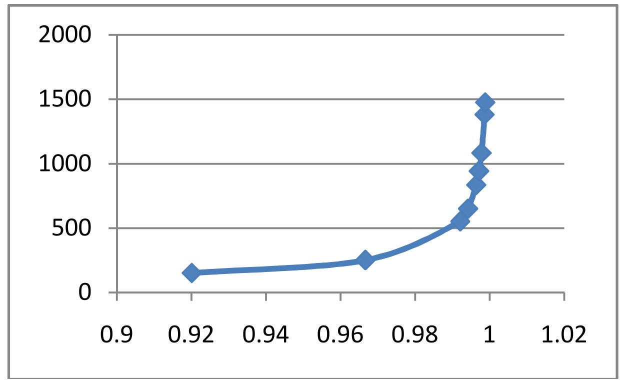



Fig. 9: Graph for Table: 7

As stated earlier and shown in the problem formulation, we have considered the component reliability and the cost as type-fuzzy fuzzy quantity. Fig. 4 shows the components reliability with type-2 fuzzy membership functions for a specific case. The type-2 fuzzy parameters are suitably subjected to the defuzzification process using the famous Karnik-Mendel approach [26], [29]. The defuzzified parameters are then used for running the NSGA-II algorithm. In the NSGA-II, we have used the crossover probability of 0.9, mutation probability of 0.2 and population size is varied from 50 to 100. The results for the components listed in the Sr. nos. 1 to 5 are shown in the Tables 3 to 7 respectively. The tables show the values for the optimized reliability and corresponding cost of the components, obtained Pareto-fronts and the optimized system reliability and amount of redundancy. Fig. 5 to Fig. 9, where the x axis and y axis represents the system reliability and designing cost in providing solutions respectively, illustrates the solutions obtain by the NSGA-II algorithm for different parameters. The Fig. 5 to 9 also shows the shapes of the Pareto-fronts for different components. We thus see that NSGA-II gives us a set of solutions which are non-dominated to each other. Thus the system designer can chose any of the solutions by considering the appropriate reliability and cost as per their own conveniences. The proposed model therefore provides a wider range of solutions to the system designers.

## VI. CONCLUSION

Reliability is the measure of the result of the quality of the system over a long run. The reliability-redundancy allocation problem (RRAP) aims to ensure high systems reliability in the presence of optimally redundant systems components. This is one of the most important design considerations for the systems designers. Several researchers have addressed this important issue during last few decades. However, due to the embedded uncertainty in the parameters of the system components, reliability as well as the costs of the whole system fits very well to be modeled as fuzzy quantity. We therefore modeled this problem as a fuzzy multi-objective optimization problem (MORRAP) that is addressed using a popular multi-objective evolutionary algorithm, viz. non-dominated sorting genetic algorithm-II. We have considered the based MORRAP with fuzzy type-2 uncertainty. As far as we know, no research has been reported where MORRAP was considered under type-2 fuzzy uncertainty. We have used the solved the problem with the well-known evolutionary algorithm, NSGA-II with the crossover probability of 0.9, mutation probability of 0.2 and population size is varied from 50 to 100. The results demonstrate that the problem can be addressed quite efficiently by this technique. A number of Pareto-optimal solutions are obtained from the NSGA-II. Fig. 5 to 9 show the shapes of the Pareto-fronts for different components. Therefore, a set of solutions which are non-dominated to each other are obtained from NSGA-II. Hence, the designers of the systems enjoys flexibilities in choosing a particular solution by selecting the favorable values of the reliability and cost. The proposed model therefore provides a wider range of solutions to the system designers. The typical numerical example considered here is there very nicely demonstrates our new FT2MORRAP model. In future we wish to continue research in this area with the consideration of some other objectives viz. component weight, time duration etc. in the same model. The problem might pose some interesting results as NSGA-II may have some limitations in handling the MORRAP problem in the presence of more than two objectives.

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