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Direct Adaptive Control of a Four-rotor Helicopter Using Disturbance Observer

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***Abstract*—In this paper, a stable multivariable model reference adaptive control (MRAC) scheme is proposed for a four-rotor helicopter with unknown external disturbance. Firstly, the disturbance observer is designed to well monitor the unknown disturbance. And then, the adaptive controller is developed based on the disturbance observer to compensate the external disturbance and track the desired system states. Finally, the simulation results illustrate the effectiveness of the proposed adaptive control scheme.**

***Keywords* — Adaptive control; Disturbance observer; Four-rotor helicopter; MRAC**

I. Introduction

Aviation safety under external interference has attracted increasing research attention recently [1-5]. The unknown external disturbance can cause variations to system parameters, where adaptive control is a suitable solution to this problem [6-11], with the capacity of handling such system uncertainties. Model reference adaptive control (MRAC) is one of the most important adaptive control methods. It provides feedback controller structures and adaptive laws to ensure closed-loop signals bounded and asymptotic tracking the reference signals of reference system, and much effort has been done to develop it. In [6], a direct adaptive control scheme is proposed for a faulty helicopter using the outer-loop compensation technique. Reference [7] revisits the multivariable MRAC with state feedback for output tracking, and a stable adaptive control scheme is developed based on LDS decomposition of high frequency gain matrix, which ensures closed-loop stability and asymptotic output tracking. In [8], a Nussbaum gain based multivariable MRAC design is proposed based on LDS decomposition, to relax the assumption on the sign information of the high frequency gain matrix.

Over the past few years, a lot of attention has been paid to the design of disturbance observer to approximate external disturbance and unknown model uncertainties [12-15] . Applications have shown that disturbance observers can enhance the disturbance attenuation and performance robustness. In [12], robust tracking controller is designed based on the output of disturbance observer for uncertain nonlinear system. A general framework for nonlinear systems subject to disturbance using disturbance observer based control (DOBC) techniques is presented in [13]. References [14-15] propose fuzzy disturbance observers and their applications.

In this paper, we develop an adaptive control scheme with state feedback for state tracking based on the disturbance observer. The structure of the paper is organized as follows. Section 2 presents the system model of a four-rotor helicopter system and the control problems. Section 3 details the design of the disturbance observer. The adaptive scheme for updating the controller parameters and its stability analysis are developed in Section 4. The simulation results are presented in Section 5, followed by some conclusions in Section 6.

II. Description of the System Model and Control Problems statement

Four-rotor helicopter is an underactuated, dynamic vehicle with four input forces and 6 DOF motion. The helicopter has four propellers installed in a cross configuration as shown in Fig. 1. Ignoring the gyroscopic effect, bearing friction and atmospheric disturbance on the propellers, a simple model consists of three differential equations can be get based on the force condition of the system,. When a positive voltage is applied to a motor, a positive thrust is generated and this causes the corresponding propeller assembly to rise. The thrust force generated by the front, back, left and right propellers is denoted as V_f , V_b , V_l and V_r respectively. The thrust force generated by the front and back motors primarily actuate motions about the pitch axis while the right and left motors primarily move the hover about its roll axis.

A. Pitch-axis Model

We define the Y axis of the body as pitch axis and the rotation angle generated by the motion of the body rounding the pitch axis as the pitch angle, and a positive front motor voltage results in a positive pitch angle. Assuming the roll angle is zero, the pitch axis torque is controlled by the lift of the front and rear propellers. According to moment of momentum theorem, the motion equation of the pitch angle is

$$\ddot{p} = l\frac{K_f}{J_p}(V_f - V_b)\,, \tag{1}$$

where p is the pitch angle, l is the distance between the motors and the propeller centre, K_f is the thrust-force constant and J_p is the moment of inertia about the pitch axis.

B. Roll-axis Model

We define the X axis of the body as roll axis and the rotation angle generated by the motion of the body rounding the roll axis as the roll angle, and a positive right motor voltage results in a positive roll angle. Similar to the pitch-axis model, the motion equation of the roll angle is

$$\ddot{r} = l\frac{K_f}{J_r}(V_r - V_l)\,, \tag{2}$$

where r is the roll angle, J_r is the moment of inertia about the roll axis.

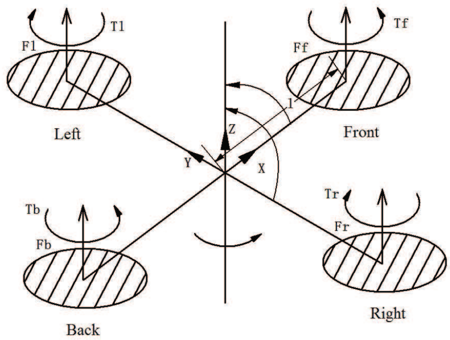

Fig. 1. Mechanic model of the four-rotor helicopter.

C. Yaw-axis Model

We define the Z axis of the body as yaw axis and the rotation angle generated by the motion of the body rounding the yaw axis as the yaw angle, and the yaw angle is defined as positive for a counter-clockwise rotation (when looking down on the system from above). The motion of the yaw is a function of the sum of the torques generated by the four propellers

$$\ddot{y} = \frac{K_{t,c}}{J_y}(V_f + V_b) + \frac{K_{t,n}}{J_y}(V_r + V_l)\,, \tag{3}$$

where y is the yaw angle, $K_{t,c}$ is the counter-clockwise thrust-torque constant and $K_{t,n}$ is the normal thrust-torque constant, J_r is the moment of inertia about the yaw axis.

From Eqs. (1)-(3), we can get the state-space equations of the three degrees of freedom quad-rotor helicopter as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{4}$$

with the sate $x = [y, p, r, \dot{y}, \dot{p}, \dot{r}]^T$, control input $u = [V_f, V_b, V_r, V_l]^T$ and output $y = [y, p, r]^T$.

In this paper, we consider the four-rotor helicopter associated with external disturbance, which can be shown as

$$\dot{x}(t) = Ax(t) + Bu(t) + B_\omega \omega(t) \tag{5}$$

where $x(t)$ is the state space vector, $u(t)$ is the control input, $\omega(t)$ represents the bounded piecewise continuous external disturbance. $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $B_\omega \in R^{n\times n_\omega}$ are parameter matrices.

The control objective is to design an adaptive feedback control law for the plant (5) such that all closed-loop signals are bounded and the plant state vector $x(t)$ asymptotically tracks a reference state vector $x_m(t)$, which generated from the reference model

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t)\,, \tag{6}$$

where $A_m \in R^{n\times n}$, $B_m \in R^{n\times l}$ are known constant matrices and all the eigenvalues of A_m are in the left-half complex plane, all columns of B_m are independent, and $r(t) \in R^l$ is bounded and has piecewise continuous components.

The following standard assumptions are given:

Assumption 1. There exists a matrix $K_1^* \in R^{n\times m}$ satisfying

$$A + BK_1^{*T} = A_m\,, \tag{7}$$

and a matrix $K_2^* \in R^{l\times m}$ satisfying

$$BK_2^* = B_m\,. \tag{8}$$

Assumption 2. There is a known matrix $S \in R^{m\times l}$ such that $K_2^* S$ is symmetric and positive definite:

$$M_s = K_2^* S = (K_2^* S)^T = S^T K_2^{*T} > 0. \tag{9}$$

Assumption 3. The change rate of the external disturbance is much less than the disturbance observer dynamic's, that is to say

$$\dot{\omega} \approx 0\,. \tag{10}$$

III. Nonlinear Disturbance Observer

The disturbance observer [12-15] will be designed for the system (5) in this section. Define

$$\bar{\omega}(t) = B_\omega \omega\,, \tag{11}$$

then from Eq. (5), we have

$$\bar{\omega}(t) = \dot{x}(t) - Ax(t) - Bu(t)\,. \tag{12}$$

The nonlinear observer is designed as following first:

$$\dot{\hat{\bar{\omega}}}(t) = -L(x)\hat{\bar{\omega}} + L(x)[\dot{x} - Ax(t) - Bu(t)]\,, \tag{13}$$

where $L(x)$ is the gain of the nonlinear observer to be designed. Define

$$\tilde{\bar{\omega}} = \bar{\omega} - \hat{\bar{\omega}}\,, \tag{14}$$

differentiating Eq. (14) and substituting Eqs. (12) and (13), we have

$$\dot{\tilde{\bar{\omega}}} = \dot{\bar{\omega}} - \dot{\hat{\bar{\omega}}} = -L(x)\tilde{\bar{\omega}}\,. \tag{15}$$

If $L(x)$ is designed to make the equation

$$\dot{\tilde{\bar{\omega}}} + L(x)\tilde{\bar{\omega}} = 0 \tag{16}$$

exponentially stable for all x, then $\hat{\bar{d}}$ can approach $\bar{d}$ exponentially as $t \to \infty$.

Since $\dot{x}$ is not available, the observer described by Eq. (13) can not be realized. Here, an auxiliary variable is introduced,

$$z = \hat{\bar{\omega}} - p(x), \quad (17)$$

where $p(x)$ should meet the following equation:

$$\dot{p}(x) = L(x) \cdot \dot{x}. \quad (18)$$

Differentiating Eq. (17) and substituting Eqs. (13) and (18), the error dynamic of the observer can be expressed as following:

$$\begin{aligned} \dot{z} &= \dot{\hat{\bar{\omega}}} - \dot{p}(x) \\ &= -L(x)\hat{\bar{\omega}} - L(x)[Ax(t) + Bu(t)] \\ &= -L(x)[z + p(x) + Ax(t) + Bu(t)] \end{aligned} \quad (19)$$

Following Eq. (17), we get

$$\hat{\bar{\omega}} = z + p(x). \quad (20)$$

Therefore, the designed disturbance observer consisted of Eqs. (19) and (20) can make $\hat{\bar{\omega}}$ follow $\bar{\omega}$ if $L(x) = \partial p(x) / \partial x$ is chosen such that $\tilde{\bar{\omega}}$ is globally exponentially convergent for all $x \in R^n$ in Eq. (16).

Remark 1: When $L(x) = \partial p(x) / \partial x > 0$ for all $x \in R^n$, then $\tilde{\bar{\omega}}$ is globally exponentially convergent.

IV. Adaptive Controller

The system control chart is shown in Fig. 2.

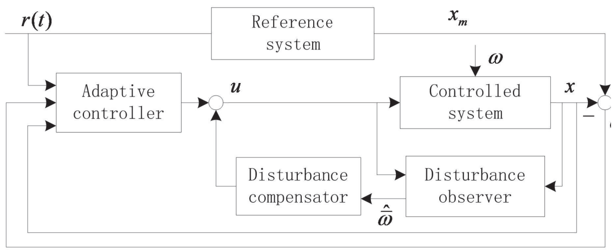

Fig. 2. The chart of direct adaptive control with disturbance observer

The controller is designed using direct adaptive control with state feedback for state tracking in this paper, and its structure is

$$u(t) = K_1^T(t)x(t) + K_2(t)r(t) + K_3(t), \quad (21)$$

where $K_1(t)$ and $K_2(t)$ are the estimates of K_1^*, K_2^*, which are the constant matrices of the control laws when parameters of A and B are known. $K_3(t)$ is used to compensate the external disturbance, which can be designed based on the above disturbance observer:

$$K_3 = -(B)^+ \hat{\bar{\omega}}, \quad (22)$$

where $(\cdot)^+$ denotes the pseudo-inverse operation. To design adaptive update laws for $K_1(t)$ and $K_2(t)$ which ensure the stability of the closed-loop system, we first define the parameter errors

$$\tilde{K}_1(t) = K_1(t) - K_1^*, \quad (23)$$

$$\tilde{K}_2(t) = K_2(t) - K_2^*, \quad (24)$$

and the tracking error

$$e(t) = x(t) - x_m(t). \quad (25)$$

Then the dynamics of the tracking error can be written in the form

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{x}_m(t) \\ &= Ax(t) + B[K_1^T(t)x(t) + K_2(t)r(t) + K_3] \\ &\quad + B_\omega \omega(t) - A_m x_m(t) - B_m r(t) \\ &= A_m x(t) + B_m r(t) + B[\tilde{K}_1^T(t)x(t) + \tilde{K}(t)r(t)] \\ &\quad - A_m x_m(t) - B_m r(t) + BK_3 + B_\omega \omega(t) \\ &= A_m e(t) + B_m[K_2^{*-1}\tilde{K}_1^T(t)x(t) + K_2^{*-1}\tilde{K}_2(t)r(t)] \\ &\quad + BK_3 + B_\omega \omega(t). \end{aligned} \quad (26)$$

Here, the adaptive laws for $K_1(t)$ and $K_2(t)$ can be chosen as [11]:

$$\dot{\tilde{K}}_1^T(t) = \dot{K}_1^T(t) = -S^T B_m^T P e(t) x^T, \quad (27)$$

$$\dot{\tilde{K}}_2(t) = \dot{K}_2(t) = -S^T B_m^T P e(t) r^T(t), \quad (28)$$

where $P \in R^{n \times n}$ is such that $P = P^T > 0$ and satisfies

$$PA_m + A_m^T P = -Q \quad (29)$$

for some constant matrix $Q \in R^{n \times n}$ such that $Q = Q^T > 0$, $M_s = M_s^T > 0$ satisfy the assumption 2 and $K_1(0)$, $K_2(0)$ being arbitrary. Then we have the following theorem:

Theorem 1. The reconfiguration controller (21), with the disturbance observer (19)-(20), control law (22) and the adaptive laws (27)-(28), applied to system (5) with external disturbance ω, guarantees that all closed-loop signals are bounded and $\lim_{t\to\infty}(x(t) - x_m(t)) = 0$.

Proof:

Let $e_c(t) = (e^T(t), \tilde{k}_{11}^T(t), \cdots, \tilde{k}_{1n}^T(t), \tilde{k}_{21}^T(t), \cdots, \tilde{k}_{2m}^T(t))^T$, where $\tilde{k}_{1i}(t) \in R^m$ is the ith column of $\tilde{K}_1^T(t), i = 1, 2, \cdots, n$, and $\tilde{k}_{2j}(t) \in R^l$ is the jth column of $\tilde{K}_2^T(t)$, $j = 1, 2, \cdots, m$, firstly, we define a Lyapunov function candidates as:

$$V(e_c) = e^T P e + tr[\tilde{K}_1 M_s^{-1} \tilde{K}_1^T] + tr[\tilde{K}_2^T M_s^{-1} \tilde{K}_2]. \quad (30)$$

Then, according to Eqs. (26) and (29), the time derivative of V_p is

$$\begin{aligned} \dot{V} &= 2e^T(t)P\dot{e}(t) + 2\sum_{i=1}^{n} \tilde{k}_{1i}^T(t) M_s^{-1} \dot{\tilde{k}}_{1i}(t) \\ &\quad + 2\sum_{j=1}^{m} \tilde{k}_{2j}^T(t) M_s^{-1} \dot{\tilde{k}}_{2j}(t) \\ &= 2e^T(t)P\dot{e}(t) + 2tr[\tilde{K}_1(t) M_s^{-1} \dot{\tilde{K}}_1^T(t)] \\ &\quad + 2tr[\tilde{K}_2^T(t) M_s^{-1} \dot{\tilde{K}}_2(t)] \\ &= 2e^T(t)P\{A_m e(t) \\ &\quad + B_m[K_2^{*-1}\tilde{K}_1^T(t)x(t) + K_2^{*-1}\tilde{K}_2(t)r(t)] \\ &\quad + BK_3 + B_\omega \omega(t)\} + 2tr[\tilde{K}_1(t) M_s^{-1} \dot{\tilde{K}}_1^T(t)] \\ &\quad + 2tr[\tilde{K}_2^T(t) M_s^{-1} \dot{\tilde{K}}_2(t)] \end{aligned}$$

$$
\begin{aligned}
&= -e^T(t)Qe(t) + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_1^T(t)x(t) \\
&\quad + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_2^T(t)r(t) \\
&\quad + 2e^T(t)P\,[BK_3 + B_\omega \omega(t)] \\
&\quad + 2tr[\tilde{K}_1(t)M_s^{-1}\dot{\tilde{K}}_1^T(t)] + 2tr[\tilde{K}_2^T(t)M_s^{-1}\dot{\tilde{K}}_2(t)].
\end{aligned} \tag{31}
$$

Substituting Eqs. (11) and (22) in (31), we have

$$
\begin{aligned}
\dot{V} &= -e^T(t)Qe(t) + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_1^T(t)x(t) \\
&\quad + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_2^T(t)r(t) \\
&\quad + 2e^T(t)P\,[-B(B)^+\hat{\bar{\omega}} + \bar{\omega}] \\
&\quad + 2tr[\tilde{K}_1(t)M_s^{-1}\dot{\tilde{K}}_1^T(t)] + 2tr[\tilde{K}_2^T(t)M_s^{-1}\dot{\tilde{K}}_2(t)].
\end{aligned} \tag{32}
$$

Ignore the dynamic response, according to the design process of the disturbance observer, Eq. (32) can be written as

$$
\begin{aligned}
\dot{V} &= -e^T(t)Qe(t) + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_1^T(t)x(t) \\
&\quad + 2e^T(t)PB_m K_2^{*-1}\tilde{K}_2^T(t)r(t) \\
&\quad + 2tr[\tilde{K}_1(t)M_s^{-1}\dot{\tilde{K}}_1^T(t)] + 2tr[\tilde{K}_2^T(t)M_s^{-1}\dot{\tilde{K}}_2(t)] \\
&= -e^T(t)Qe(t) + 2tr[\tilde{K}_1(t)M_s^{-1}S^T B_m^T Pe(t)x^T(t)] \\
&\quad + 2tr[\tilde{K}_2^T(t)M_s^{-1}S^T B_m^T Pe(t)r^T(t)] \\
&\quad + 2tr[\tilde{K}_1(t)M_s^{-1}\dot{\tilde{K}}_1^T(t)] + 2tr[\tilde{K}_2^T(t)M_s^{-1}\dot{\tilde{K}}_2(t)].
\end{aligned} \tag{33}
$$

By Eqs. (27)-(28), we have

$$
\dot{V}_p \le -e^T Qe \le -q_m \|e(t)\|^2 \le 0, \tag{34}
$$

where $q_m > 0$ is the minimum eigenvalue of Q. Hence the equilibrium state $e_c = 0$ of the closed-loop system consisting of Eqs. (19)-(20) and Eqs. (26)-(28) is uniformly stable and its solution $e_c(t)$ is uniformly bounded. Furthermore Eq. (34) implies $e(t) \in L^2$ and so $\lim_{t\to\infty} e(t) = 0$.

V. Numerical Simulation

The coefficient matrices of system (5) are

$$
A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0327 & -0.0327 & 0.0327 & 0.0327 \\ 0.424 & -0.424 & 0 & 0 \\ 0 & 0 & 0.424 & -0.424 \end{bmatrix},
$$

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.
$$

Here, the values of A_m, B_m in Eq. (6) are chosen as:

$$
A_m = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -8.005 & 0 & 0 & -4.0025 & 0 & 0 \\ 0 & -94.8064 & 0 & 0 & -23.4048 & 0 \\ 0 & 0 & -94.8093 & 0 & 0 & -23.4048 \end{bmatrix}
$$

$B_m = 10 * b_1$,

where A_m is from an LQR design. b_1 is the first column of the plant actuation matrix B.

The simulations are given with the following parameters:

$B_\omega = I_{4\times 4}$, $\omega(t) = [0, 0, 0, 0.8, 1.4, 2.2]^T$, $t \ge 40$, $r = 10$, $Q = 10 * I_6$, $S = [10, 10, 10, 10]^T$.

The initial values are set to:

$x(0) = x_m(0) = [0,0,0,0,0,0]^T$, $k_{1i}(0) = [0, 0, 0, 0, 0, 0]^T$, $k_{2i}(0) = [5, 5, -5, -5]$,.

With the initial values, the attitude tracking errors of the four-rotor helicopter are shown in Fig. 3.

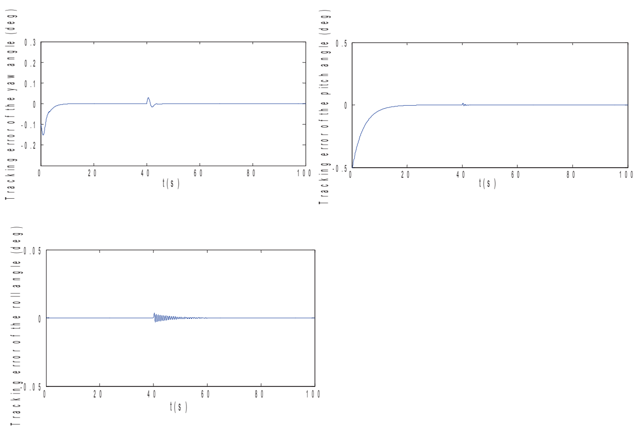

Fig. 3. The simulation results of the helicopter system with external disturbance

From the simulation results, it can be seen that there are transient tracking errors of the attitude angles at the time instants at 40s, but with the effect of the designed controller, the tracking errors can go to zero asymptotically in a very short time.

VI. Conclusions

In this paper, an adaptive control scheme is proposed for the MIMO flight control system with unknown disturbance based on a disturbance observer, which is proposed to approximate the external disturbance. Due to the fact that the direct adaptive control has some limitations at dealing with the modeling error and external disturbance, the disturbance observer we introduced can greatly improve the system's anti-interference capability. Simulation results also demonstrate the effectiveness of the proposed scheme in this paper.

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