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Discrete-Time Polynomial Fuzzy Observer Designs via a Sum of Squares Approach

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***Abstract*—In this paper, a sum of squares(SOS) method is proposed to design observers for discrete-time polynomial fuzzy systems. The proposed SOS approach has two improved and innovative results for the existing linear matrix inequality (LMI) method to Takagi-Sugeno(T-S) discrete fuzzy observer designs. Firstly, a polynomial discrete fuzzy model is developed, which is a generation of the well-known T-S fuzzy system. Secondly, the conditions in the proposed approach are obtained in terms of SOS, which is the extension of the LMI method. Therefore, the conditions given in this paper are more general than the existing LMI approaches to T-S fuzzy systems. An example is given to show the effectiveness, which also demonstrate the SOS approaches are more relaxed than the existing LMI approaches. Finally, a conclusion is given to complete the paper.**

I. Introduction

The fuzzy logic has been found extensive applications in the field of industrial systems and consumer products since the idea of it was proposed by Zadeh in 1965[1]. The fuzzy control method have obtained rich achievements to tackle the control problems [2]-[11]. Especially, T-S fuzzy control is one of the most popular and promising research method in the model-based fuzzy control [8]-[11]. The T-S fuzzy control becomes more natural, simpler and more effective to complement other nonlinear control methods [12] that require special and rather involved knowledge. LMI-based designs have been discussed numerously in the T-S fuzzy control areas by researchers [9]-[11]. The obtained conditions can be solved numerically and efficiently by LMI solvers. But not all the problems can be reformed to LMIs, some kinds of control design conditions [13]-[15] for polynomial fuzzy systems reduce to SOS problems. Obviously, the problems in them cannot be solved by interior point algorithms, e.g. by LMI solvers, but they can be solved via the recent developed SOSTOOLS [16] and an SDP solver [17]. This is a different approach from the existing LMI approaches. The SOS approach [13]-[15] presents that it's an extensive representation of LMIs. Otherwise, the existing results about the nonlinear system control still have great conservation. In this paper, we consider the observer design of the discrete-time polynomial fuzzy system via an SOS approach.

The authors [9] presented fuzzy observer designs for both continuous and discrete systems based on LMI constraint conditions. The paper [14] presents a sum of squares approach to polynomial fuzzy observer designs for continuous polynomial fuzzy systems. However, observer-based designs for discrete-time polynomial fuzzy systems have not been addressed in the literature. This motivates us to do this work. The main contribution of the paper lies in: 1) the design of the observer is an extension to the discrete-time polynomial fuzzy system. 2) the stability conditions given in this paper are more relaxed than the existing LMI approaches.

The rest of the paper is organized as follows: some foundational results for the later developments are recalled in section II. The polynomial fuzzy system and SOS are also described in this section. The main results are presented in section III. A simulation is given in section IV. Finally, a conclusion is given in Section V .

II. Problem Formulation and Preliminaries

In this section, we recall the T-S fuzzy model, the fuzzy controller design, the polynomial fuzzy model and the SOS-TOOLS.

First of all, consider a class of nonlinear plant as follows:

$$x(k+1) = f(x(k), u(k)) \tag{1}$$

where f is a smooth nonlinear function such that $f(0,0) = 0$. $x(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T \in \mathbf{R}^n$ is the system state vector, and $u(k) = [u_1(k), u_2(k), \dots, u_m(k)]^T \in \mathbf{R}^m$ is the system input. The main feature of a T-S fuzzy model is the consequent of each IF-THEN rules is a linear system model and T-S fuzzy model can be regarded as a universal approximator of most general nonlinear system, we can represent the nonlinear system (1) with the following T-S fuzzy form:

Plant Rule i: IF $z_1(k)$ is M_{i1}, $\cdots$, $z_p(k)$ is M_{ip}, THEN

$$x(k+1) = A_i x(k) + B_i u(k) \tag{2}$$

where $z_j(k)$ is the premise variable, M_{ij} is the fuzzy set associated with the ith model rule and the jth premise variable component, $A_i \in \mathbf{R}^{n\times n}$ and $B_i \in \mathbf{R}^{n\times m}$ are constant matrices, $i = 1, 2, \dots, r$, and r is the number of IF-THEN rules, $j = 1, 2, \dots, p$. The defuzzification process of the model (2) can be represented as bellow:

$$x(k+1) = \frac{\sum_{i=1}^{r} \omega_i(z(k)(A_i x(k) + B_i u(k))}{\sum_{i=1}^{r} \omega_i(z(k))} \tag{3}$$

where it is assumed that $\omega_i(z(k)) = \prod_{j=1}^{p} M_{ij}(z_j(k))$, $\omega_i(z(k)) \geq 0$, $\sum_{i=1}^{r} \omega_i(z(k)) > 0$, $i = 1, 2, \dots, r$.

The system (3) can be represented as below for brevity:

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$$x(k+1)=\sum_{i=1}^{r}h_i(z(k))\{A_ix(k)+B_iu(k)\} \quad (4)$$

where $h_i(z(k))=\omega_i(z(k))/\prod_{i=1}^{r}\omega_i(z(k))$, $h_i(z(k))\geq 0$, $\sum_{i=1}^{r}h_i(z(k))=1$, $i=1,2,\ldots,r$. The fuzzy controller for the nonlinear plant represented by (3) is designed to share the same IF parts with the plant as follows:

Control Rule i: IF $z_1(k)$ is M_{i1}, $\cdots$, $z_p(k)$ is M_{ip}, THEN

$$u(k)=-F_ix(k). \quad (5)$$

where $F_i\in\mathbf{R}^{m\times n}$ is a constant matrix.

The defuzzification process of the model (5) can be represented as bellow:

$$u(k)=\sum_{i=1}^{r}h_i(z(k))\{-F_ix(k)\}. \quad (6)$$

The polynomial fuzzy system is a fuzzy model with a polynomial model consequence. The following defuzzification process of the polynomial fuzzy system will be considered in this paper:

$$x(k+1)=\sum_{i=1}^{r}h_i(z(k))\{A_i(x(k))x(k)+B_i(x(k))u(k)\}. \quad (7)$$

where $A_i(x(k))\in\mathbf{R}^{n\times n}$ and $B_i(x(k))\in\mathbf{R}^{n\times m}$ are polynomial matrices in $x(k)$, $i=1,2,\ldots,r$.

To stabilize the fuzzy system (7), a polynomial fuzzy controller will be designed as follows:

$$u(k)=\sum_{i=1}^{r}h_i(z(k))\{-F_i(x(k))x(k)\}. \quad (8)$$

where $F_i(x(k))\in\mathbf{R}^{m\times n}$ is a polynomial matrix in $x(k)$, $i=1,2,\ldots,r$.

Remark 1: If the matrix $F_i(x(k))$ is a constant one, the model (8) will reduce to (6).

SOSTOOLS is a free toolbox for solving sum of squares programs. The techniques behind it are based on the sum of squares decomposition for multivariate polynomials, which can be efficiently computed using semi-definite programming. SOSTOOLS is developed as a consequence of the recent interest in sum of squares polynomials, partly due to the fact that these techniques provide convex relaxations for many hard problems such as global, constrained, and boolean optimization [16]. For more interests about it one can see the manual of SOSTOOLS.

Definition 1[16]: A multivariate polynomial $p(x)$, $x\in\mathbf{R}^n$, is a sum of squares(SOS), if there exist polynomials $f_1(x)$, $\cdots$, $f_m(x)$ such that

$$p(x)=\sum_{i=1}^{m}f_i^2(x). \quad (9)$$

Definition 2[16]: The SOS condition (9) is equivalent to the existence of a positive semi-definite matrix Q, such that

$$p(x)=Z^T(x)QZ(x), \quad (10)$$

where $Z(x)$ is some properly chosen vector of monomials.

Before deriving the main results, some preliminary lemmas are given in the following:

Lemma 1: If $P>0$ such that

$$P-M^T(x)PM(x))>0,$$
$$P-N^T(x)PN(x))>0,$$

where $M(x)\in\mathbf{R}^{n\times n}$ and $N(x)\in\mathbf{R}^{n\times n}$ are polynomial matrices in x, $P\in\mathbf{R}^{n\times n}$ is a constant matrix, then

$$2P-M^T(x)PN(x))-N^T(x)PM(x)>0.$$

Proof:

$$\begin{aligned}&2P-M^T(x)PN(x)-N^T(x)PM(x)\\&=(M(x)-N(x))^TP(M(x)-N(x))+2P-\\&\quad M^T(x)PM(x))-N^T(x)PN(x))\end{aligned}$$

Since $P>0$, $(M(x)-N(x))^TP(M(x)-N(x))\geq 0$,

$$2P-M^T(x)PN(x))-N^T(x)PM(x)>0.$$

Lemma 2: For any polynomial matrix $M_i(x)\in\mathbf{R}^{n\times n}$, $i=1,2,\cdots,r$, and $P>0$ with appropriate dimensions, we have

$$\sum_{i=1}^{r}M_i^T(x)PM_i(x)-\sum_{i=1}^{r}\sum_{j=1}^{r}M_i^T(x)PM_j(x)\geq 0.$$

Proof: The procedure is similar to Lemma 1.

III. Observer Design

In industry control problems, not all the states of a system can be measured. The polynomial fuzzy observer design is proposed based on SOS conditions in this section.

Polynomial fuzzy observers are required to satisfy the following conditions:

$$\lim_{t\to\infty}e=0$$

where $e=x-\hat{x}$, $\hat{x}$ denotes the state vector estimated by a polynomial fuzzy observer. In this part, we assume that $A_i(x(k))$ and $B_i(x(k))$ in (7) are measurable matrices. Under the assumption, we replace the polynomial fuzzy model (7) with

$$\begin{aligned}x(k+1)=\sum_{i,j=1}^{r}&h_i(z(k))h_j(z(k))\{A_i(\xi(k))x(k)+\\&B_i(\xi(k))u(x(k))\}.\end{aligned} \quad (11)$$

where $\xi(k)$ is a measurable vector that could be outputs, time, both of them or others. And the output for the polynomial fuzzy model is defined as

$$y(k)=\sum_{i=1}^{r}h_i(z(k))C_ix(k) \quad (12)$$

Then the polynomial fuzzy observer is proposed:

$$\hat{x}(k+1) = \sum_{i=1}^{r} h_i(z(k))\{A_i(\xi(k))\hat{x}(k) + B_i(\xi(k))u(k) + L_i(\xi(k))(y-\hat{y})\} \quad (13)$$

$$\hat{y}(k) = \sum_{i=1}^{r} h_i(z(k))C_i\hat{x}(k) \quad (14)$$

where $L_i(\xi) \in \mathbf{R}^{n\times q}$ is the polynomial observer gain. The following controller needs to be developed:

$$u(k) = \sum_{i=1}^{r} h_i(z(k))\{-F_i(\xi(k))\hat{x}(k)\} \quad (15)$$

Theorem 1: The equilibrium of the overall control system consisting of (11)-(15) is asymptotically stable in the large and the steady error between the real state and the estimated state converges to zero if there exist $Q_1 \in \mathbf{R}^{n\times n}$, $Q_2 \in \mathbf{R}^{n\times n}$, and polynomial matrices $M_i(\xi(k)) \in \mathbf{R}^{m\times n}$, $N_i(\xi(k)) \in \mathbf{R}^{n\times q}$ and $R_i(\xi(k)) \in \mathbf{R}^{n\times n}$ satisfying the following conditions:

$$\zeta_1^T(Q_1 - r_1 I)\zeta_1 \quad is \quad SOS. \quad (16)$$

$$\zeta_1^T(Q_2 - r_2 I)\zeta_1 \quad is \quad SOS. \quad (17)$$

$$\eta_i^T \begin{bmatrix} X_{1ii} & * \\ X_{2ii} & X_{1ii} \end{bmatrix} \eta_i \quad is \quad SOS. \quad (18)$$

$$\eta_i^T \begin{bmatrix} 2Q & * \\ X_{2ij}+X_{2ji} & 2Q \end{bmatrix} \eta_i \quad is \quad SOS, \ i<j. \quad (19)$$

where

$$X1_{ii} = \begin{bmatrix} Q_1 - r_{3i}I & 0 \\ 0 & Q_2 - r_{4i}I \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$$

$$X_{2ii} = \begin{bmatrix} \Xi_{ii11} & \Xi_{ii12} \\ 0 & \Xi_{ii22} \end{bmatrix}, \quad X_{2ij} = \begin{bmatrix} \Xi_{ij11} & \Xi_{ij12} \\ 0 & \Xi_{ij22} \end{bmatrix}$$

$$\Xi_{ii22} = A_i(\xi(k))Q_2 - N_i(\xi(k)),$$

$$\Xi_{ij22} = A_i(\xi(k))Q_2 - R_j(\xi(k)),$$

$$\Xi_{ii11} = A_i(\xi(k))Q_1 - B_i(\xi(k))M_i(\xi(k)),$$

$$\Xi_{ij11} = A_i(\xi(k))Q_1 - B_i(\xi(k))M_j(\xi(k)),$$

$$\Xi_{ii12} = B_i(\xi(k))M_i(\xi(k)), \ \Xi_{ij12} = B_i(\xi(k))M_j(\xi(k)),$$

$\zeta_1 \in \mathbf{R}^n$ is a vector that is independent of x, $\eta_i \in \mathbf{R}^{4n}$ is a vector that is also independent of x, r_1 and r_2 are positive values, r_{3i} and r_{4i} are nonnegetive polynomial fuctions about $\xi(k)$ such that $r_{3i} > 0$ and $r_{4i} > 0$ for $\xi(k) \neq 0$, $i, \ j = 1, 2, \cdots, r$. Moreover, the gains can be obtained $F_i(\xi(k)) = M_i(\xi(k))Q_1^{-1}$, $L_i(\xi(k)) = N_i(\xi(k))Q_2^{-1}C_i^-$, C_i^- is the generalized inverse matix of C_i.

Proof: First, the augmented system (20) consisting of (11)-(15)is obtained:

$$\tilde{x}(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))G_{ij}(\xi(k))\tilde{x}(k). \quad (20)$$

where

$$\tilde{x}(k) = [x \quad e]^T, \ G_{ij11} = A_i(\xi(k)) - B_i(\xi(k))F_j(\xi(k)),$$

$$G_{ij22} = A_i(\xi(k)) - L_i(\xi(k))C_j, \ G_{ij21} = 0,$$

$$G_{ij12} = B_i(\xi(k))F_j(\xi(k)), G_{ij}(\xi(k)) = \begin{bmatrix} G_{ij11} & G_{ij12} \\ G_{ij21} & G_{ij22} \end{bmatrix}.$$

Next, a candidate of a Lyapunov function is proposed:

$$V(\tilde{x}(k)) = \tilde{x}^T(k)P\tilde{x}(k). \quad (21)$$

where $P = Q^{-1} > 0$. Then

$$\Delta V(\tilde{x}(k)) = V(\tilde{x}(k+1)) - V(\tilde{x}(k))$$
$$= \tilde{x}^T(k+1)P\tilde{x}(k+1) - \tilde{x}^T(k)P\tilde{x}(k)$$

In consideration of clear expression, we will drop the notation with respect to time k and variable $z(k)$ in the following process of the proof, e.g. h_i, ξ and x will be used to instead of $h_i(z(k))$, $\xi(k)$ and $x(k)$, respectively.

$$= \tilde{x}^T(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j G_{ij}^T(\xi))P(\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j G_{ij}(\xi))\tilde{x} - \tilde{x}^T P\tilde{x}$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l \tilde{x}^T(G_{ij}^T(\xi)PG_{ij}(\xi) - P)\tilde{x}$$

$$= \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l \tilde{x}^T((G_{ij}(\xi)+G_{ji}(\xi))^T \cdot P(G_{kl}(\xi)+G_{lk}(\xi)) - 4P)\tilde{x}$$

Based on lemma 2,

$$\Delta V(\tilde{x}) < \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{x}^T(G_{ij}(\xi)+G_{ji}(\xi))^T \cdot P(G_{ij}(\xi)+G_{ji}(\xi)) - 4P)\tilde{x}$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \tilde{x}^T(\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2})^T \cdot P(\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2}) - P)\tilde{x}$$

$$= \sum_{i=1}^{r} h_i^2 \tilde{x}^T(G_{ii}^T(\xi)PG_{ii}(\xi) - P)\tilde{x} + 2\sum_{i<j}^{r} h_i h_j \tilde{x}^T \cdot ((\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2})^T p(\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2}) - P)\tilde{x}$$

If the following conditions are fixed, $\Delta V(\tilde{x}) < 0$ at $\tilde{x} \neq 0$,

$$G_{ii}(\xi)^T PG_{ii}(\xi) - P < 0 \quad (22)$$

$$(\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2})^T p(\frac{G_{ij}(\xi)+G_{ji}(\xi)}{2}) - P \leq 0 \quad (23)$$

Pre-multiplying and post-multiplying (22) by Q, using schur complete theorem, the following inequality is obtained:

$$\left[\begin{array}{cc} Q & * \\ G_{ii}(\xi)Q & Q \end{array}\right] > 0 \tag{24}$$

If the condition (18) holds, the inequation (24) could be obtained. Pre-multiplying and post-multiplying (23) by Q, using schur complete theorem, the following inequality is obtained:

$$\left[\begin{array}{cc} 2Q & * \\ (G_{ij}(\xi)+G_{ji}(\xi))Q & 2Q \end{array}\right] \geq 0 \tag{25}$$

If the condition (19) holds, the inequation (25) could be obtained. The theorem is proved.

IV. Simulation Results

In this section, a design example is proposed to verify the validity of the results. Consider the following system:

$$\begin{cases} x_1(k+1) = 0.5x_1(k), \\ x_2(k+1) = -x_2^2(k)x_1(k) + sin(x_2(k)). \end{cases} \tag{26}$$

This nonlinear system has a polynomial term $-x_2^2(k)$ and a nonlinear term $sin(x_2(k))$. Assume the range of $x_2(k)$, i.e. $x_2(k) \in [-a,\ a]$, where a is a positive value. We can get the following fuzzy system using the sector nonlinearity[9]:

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(z(k))\{A_i x(k) + B_i u(k)\}, \\ y(k) = \sum_{i=1}^{r} h_i(z(k))C_i x(k). \end{cases} \tag{27}$$

where

$$A_1 = \left[\begin{array}{cc} 0.5 & 0 \\ -a^2 & 1 \end{array}\right], \quad A_2 = \left[\begin{array}{cc} 0.5 & 0 \\ -a^2 & -0.2172 \end{array}\right],$$

$$A_3 = \left[\begin{array}{cc} 0.5 & 0 \\ 0 & 1 \end{array}\right], \quad A_4 = \left[\begin{array}{cc} 0.5 & 0 \\ 0 & -0.2172 \end{array}\right],$$

$$B_1 = B_2 = B_3 = B_4 = \left[\begin{array}{c} 0 \\ 1 \end{array}\right],$$

$$C_1 = C_2 = C_3 = C_4 = [\ 0 \quad 1\ ],$$

$$h_1(z(k)) = \frac{x_2^2(k)}{a^2}\frac{sin(x_2(k)) + 0.2172x_2(k)}{1.2172x_2(k)},$$

$$h_2(z(k)) = \frac{x_2^2(k)}{a^2}\frac{x_2(k) - sin(x_2(k))}{1.2172x_2(k)},$$

$$h_3(z(k)) = \frac{a^2 - x_2^2(k)}{a^2}\frac{sin(x_2(k)) + 0.2172x_2(k)}{1.2172x_2(k)},$$

$$h_4(z(k)) = \frac{a^2 - x_2^2(k)}{a^2}\frac{x_2(k) - sin(x_2(k))}{1.2172x_2(k)}.$$

The LMI design conditions [9] based on T-S fuzzy systems are derived as

$$Q > 0, \qquad \left[\begin{array}{cc} Q & * \\ X_{2ii} & Q \end{array}\right] > 0, \tag{28}$$

$$\left[\begin{array}{cc} 2Q & * \\ X_{2ij} + X_{2ji} & 2Q \end{array}\right] \geq 0, \quad i < j = 1, 2, \cdots, r. \tag{29}$$

where $X_{2ij} = A_i(x)Q - B_i(x(k))M_j(x(k)$. For a larger range $a \in [-10^9, 10^9]$, the LMI conditions (28)-(29) are infeasible. This means that the existing T-S fuzzy controller design method for the nonlinear system is not valid. Conversely, the SOS design method based on the polynomial fuzzy systems realizes that the polynomial fuzzy controller stabilizes the system and the estimated states converge to the real states.

Assume that x_2 is measurable and x_1 is estimated by the polynomial fuzzy observer. The nonlinear system (26) can be represented as the system (7) and (12), where

$$A_1(y(k)) = \left[\begin{array}{cc} 0.5 & 0 \\ -y^2(k) & 1 \end{array}\right],$$

$$A_2(y(k)) = \left[\begin{array}{cc} 0.5 & 0 \\ -y^2(k) & -0.2172 \end{array}\right],$$

$$B_1(y(k)) = B_2(y(k)) = \left[\begin{array}{c} 0 \\ 1 \end{array}\right],$$

$$C_1 = C_2 = [\ 0 \quad 1\ ],$$

$$h_1(z(k)) = \frac{sin(y(k)) + 0.2172y(k)}{1.2172y(k)},$$

$$h_2(z(k)) = \frac{y(k) - sin(y(k))}{1.2172y(k)}.$$

By solving the SOS conditions in theorem 1, we have Q_1, Q_2, polynomial matrices $M_i(x(k))$ and $N_i(x(k))$, where $i = 1, 2$. e^p means 10^p, where p is an integer.

$$Q_1 = \left[\begin{array}{cc} 0.70 & 0.13e^9 \\ 0.13e^9 & 0.72 \end{array}\right],$$

$$Q_2 = \left[\begin{array}{cc} 0.63 & 0.66e^9 \\ 0.13e^9 & 0.72 \end{array}\right],$$

$$M_1(y(k)) = \\ [\ -0.27e^9 - 0.70y^2(k) \quad 0.65 - 0.13e^9 y^2(k)\ ],$$

$$M_2(y(k)) = \\ [\ 0.60e^9 - 0.69y^2(k) \quad -0.14 - 0.13e^9 y^2(k)\ ],$$

$$N_1(y(k)) = \left[\begin{array}{c} -0.22e^8 - 0.67e^{16}y^2(k) \\ 0.90 + 0.2e^{40}y^2(k) \end{array}\right],$$

$$N_2(y(k)) = \left[\begin{array}{c} 0.24e^{10} + .12e^{16}y^2(k) \\ -0.20 + 0.2e^{40}y^2(k) \end{array}\right].$$

The polynomial feedback gains $F_i(y(k))$ and $L_i(y(k))$ are given as bellow:

$$F_1(y(k)) = [\ -0.56e^9 - y^2(k) \quad 0.90\ ],$$

$$F_2(y(k)) = [\ 0.91e^9 - y^2(k) \quad -0.19\ ],$$

$$L_1(y(k)) = \left[\begin{array}{c} -0.22e^8 - 0.67e^{16}y^2(k) \\ 0.90 + 0.2e^{40}y^2(k) \end{array}\right],$$

$$L_2(y(k)) = \left[\begin{array}{c} 0.24e^{10} + 0.12e^{16}y^2(k) \\ -0.20 + 0.2e^{40}y^2(k) \end{array}\right].$$

In comparison with Fig. 1, which represents the system behavior for the initial condition $x(0) = [0.5 \quad 0.5]^T$, Fig. 2 shows the controlled system behavior, which illustrates the efficiency of the designed fuzzy regulator and the fuzzy observer via SOS approach for the same initial condition. Fig. 3 shows the control and estimation results by the designed polynomial fuzzy observer.

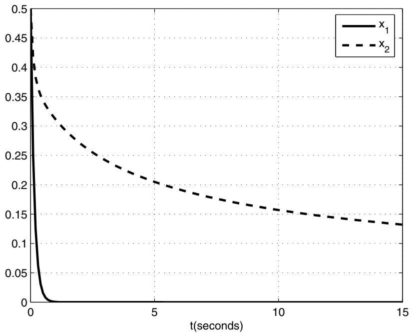

Fig. 1: System response without the input.

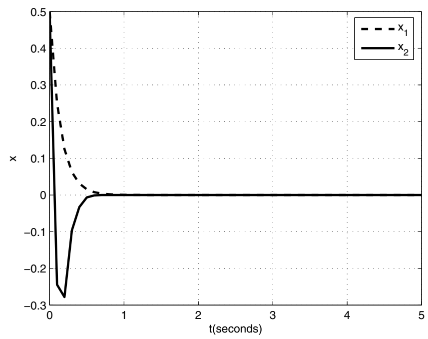

Fig. 2: System response with the polynomial fuzzy observer.

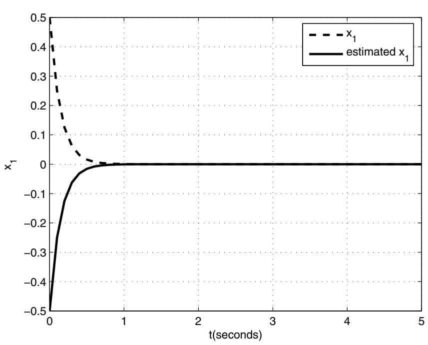

Fig. 3: Control and estimation results.

V. Conclusion

In this paper, a sum-of-squares(SOS) approach has been presented to design the polynomial fuzzy observer for the discrete polynomial fuzzy system. Some theoretical foundation with respect to the polynomial fuzzy system have been recalled. The parallel distributed compensation has been employed to design polynomial fuzzy regulators and polynomial fuzzy observers. The SOS-based design method for polynomial fuzzy regulators and polynomial fuzzy observers have been proposed. An example has been designed to demonstrate the effectiveness of the proposed approach.

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