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Neurodynamics-based Robust Eigenstructure Assignment for Second-order Descriptor Systems

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***Abstract*— In this paper, a neurodynamic optimization approach is proposed for robust eigenstructure assignment problem of second-order descriptor systems via state feedback control. With a novel robustness measure serving as the objective function, the robust eigenstructure assignment problem is formulated as a pseudoconvex optimization problem. Two coupled recurrent neural networks are applied for solving the optimization problem with guaranteed optimality and exact pole assignment. Simulation results are included to substantiate the effectiveness of the proposed approach.**

I. Introduction

Eigenstructure assignment is a vitally important problem in linear control systems design. Since poles (eigenvalues) and their associated eigenvectors of a closed-loop system greatly impact on the control performance such as the stability condition and the convergence speed, pole assignment is an effective approach to place poles of the close-loop system at any desired locations on the complex plane via a state feedback law with appropriate gains. In practice, robust control is more desirable as the systems cannot be precisely modeled or the systems are subject to parameter uncertainties. The robust pole assignment problem is to find the feedback gains such that the robustness of the eigensystem is optimized. Kautsky et al. [20] first formulated the robust pole (eigenstructure) assignment by means of minimizing the spectral condition number of the eigenvector matrix. Alternative robustness measures and various optimization approaches in linear control systems design were widely investigated [22], [24]–[26], [32], [35], [41].

Second-order linear systems constitute an important class of systems, as they can capture the dynamic behaviors of many natural phenomena. There exist numerous applications in various fields, such as vibration and structural analysis, spacecraft control and robotics control [1], [2], [7]. Furthermore, as second-order systems can be viewed as special cases of high-order systems, synthesis approach to second-order systems may be applied for higher-order systems. A few results on robust pole assignment in second-order linear systems are available in the literature [3], [5], [8], [9], [23], [30]. In specific, [30] proposed a robustness measure for second-order control by solving a generalized linear eigenvalue assignment problem subject to structured perturbations. However, most existing algorithms cannot guarantee the achievement of global optimality due to the complexity and nonconvexity of the applied measures. In addition, most proposed methods are not applicable for on-line computing.

Neurodynamic optimization based on recurrent neural networks is competent for solving optimization problems in real time. The essence of neurodynamic optimization lies in its parallel and distributed information processing capability. Various neurodynamic optimization approaches have been widely developed with guaranteed optimality, expended applicability, improved convergence properties, and reduced model complexity, e.g., [10], [11], [14], [15], [18], [21], [27]–[29], [34], [36]–[38]. There have been some investigations on developing neurodynamic optimization approaches to robust pole assignment [12], [13], [16], [17], [19], [26]. Especially, [12] achieved robust approximate pole assignment for second-order systems using neural network computation.

This paper focuses on robust pole assignment in second-order linear systems via proportional-plus-derivative coordinate control. The robust pole assignment problem is formulated as a pseudoconvex optimization problem with a novel robustness measure as the objective to be minimized. Different from existing results, the proposed neurodynamic optimization approach is able to solve the problem on-line with guaranteed optimality and exact pole assignment. The rest of this paper is organized as follows. In Section II, the optimization problem is formulated. In Section III, a neural network approach used in the optimization of robust control is presented. Section IV shows the simulation results. Finally, Section V concludes the paper.

II. Problem Formulation

A. Second-order Descriptor System

Consider the following second-order descriptor linear control system:

$$M\ddot{x} + D\dot{x} + Kx = Bu, \tag{1}$$

where $x \in \Re^n$ and $u \in \Re^r$ are respectively state vector and input vector, and $M, D, K \in \Re^{n\times n}$, $B \in \Re^{n\times r}$ are system coefficient matrices. M may be singular or nonsingular. The system (1) is assumed to satisfy the following conditions:

$$\begin{aligned}&\mathrm{rank}(M) = n_0, 0 < n_0 \le n,\\ &\mathrm{rank}[s^2M + sD + KB] = n, \text{ for all } s \in \mathbb{C}.\end{aligned}$$

As usual, the following control law can be applied to control the states of the system:

$$u = F_0x + F_1\dot{x},\ F_0, F_1 \in \Re^{r\times n}. \tag{2}$$

The closed-loop system via state feedback is then as follows:

$$M\ddot{x} + (D - BF_1)\dot{x} + (K - BF_0)x = 0. \tag{3}$$

The dynamics of this closed-loop system are governed by the eigenvalues and eigenvectors of the closed-loop quadratic pencil:

$$P_c(\lambda) = \lambda^2 M + \lambda(D - BF_1) + (K - BF_0). \quad (4)$$

The generalized eigenvalues of the quadratic polynomial are given by the $n + n_0$ values of $\lambda \in \mathbb{C}$ for which $\det(\mathrm{P_c}(\lambda)) = 0$. The corresponding right and left eigenvectors are defined, respectively, to be nonzero vectors z and w satisfying

$$(\lambda^2 M + \lambda(D - BF_1) + (K - BF_0))z = 0, \quad (5)$$
$$w^H(\lambda^2 M + \lambda(D - BF_1) + (K - BF_0)) = 0. \quad (6)$$

There exist full rank matrices $Z,\ W \in \Re^{(n+n_0)\times n}$ that simultaneously satisfy

$$MZ\Lambda^2 + (D - BF_1)Z\Lambda + (K - BF_0)Z = 0, \quad (7)$$
$$\Lambda^2 W^H M + \Lambda W^H(D - BF_1) + W^H(K - BF_0) = 0, \quad (8)$$

where the columns of Z and W are the right and left eigenvectors, respectively, and $\Lambda \in \mathbb{C}^{(n+n_0)\times(n+n_0)}$ is in Jordan canonical form with the eigenvalues of $P(\lambda)$ on the diagonal; i.e., $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_{n+n_0})$.

Open-loop system (1) and closed-loop system (3) can be rewritten in the first-order state space model:

$$M_c\dot{z} = A_c z + B_c u, \quad (9)$$

where

$$M_c = \begin{bmatrix} I_n & 0 \\ 0 & M \end{bmatrix}, A_c = \begin{bmatrix} 0 & I_n \\ -K & -D \end{bmatrix},$$
$$B_c = \begin{bmatrix} 0 \\ B \end{bmatrix}, z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}.$$

The closed-loop control system is then as follows:

$$M_c\dot{z} = A_{cc}z, \quad \text{where } A_{cc} = \begin{bmatrix} 0 & I_n \\ -K + BF_0 & -D + BF_1 \end{bmatrix}. \quad (10)$$

According to the conclusion in [8], for the closed-loop system (3), the right normal eigenvector matrix Z_∞ associated with the infinite eigenvalues is denoted by $[0\ Z_\infty]^T$, where Z_∞ satisfies:

$$MZ_\infty = 0;\ \mathrm{rank}(Z_\infty) = n - n_0. \quad (11)$$

Then, right eigenvectors $\tilde{Z}$ for the first-order system can be express as:

$$\tilde{Z} = \begin{bmatrix} Z & 0 \\ Z\Lambda & Z_\infty \end{bmatrix}. \quad (12)$$

Given an arbitrary parameter matrix $G_\infty \in \Re^{n\times(n-n_0)}$, the left eigenvalue matrix W^H can be expressed as follows:

$$W^H = [I_{n+n0}\ 0]\begin{bmatrix} Z & Z_\infty \\ MZ\Lambda & -DZ_\infty + BG_\infty \end{bmatrix}^{-1}. \quad (13)$$

Pole assignment for second-order system is to find feedback matrix F_0 and F_1 satisfying (7) and (8), which can be found as follows:

$$\begin{aligned} &[F_0\ F_1] = G\begin{bmatrix} Z \\ Z\Lambda \end{bmatrix}^\dagger, \\ &MZ\Lambda^2 + DZ\Lambda + K = BG, \end{aligned} \quad (14)$$

B. Robustness Measure

For linear control systems, several robustness measures have been investigated [6]. Among these measures, Kautsky et al. [20] first proposed a robust measure using the spectral condition number of the eigenvector matrix, as the closed-loop poles move at a rate no greater than the condition number per unit change in the norm of the variation of the closed-loop system matrix. Later, Lam and Yan [22] used the Frobenius norm to replace the spectral norm in the condition number and its additive substitutes. For second-order system, [30] proposed a measure as follows:

$$c(\lambda) = \frac{\sqrt{|\lambda|^4 + |\lambda|^2 + 1}\|w^H M\|_2\|z\|_2}{|\lambda||w^H(2\lambda M - D)z|}. \quad (15)$$

Due to the complexity and nonconvexity of (15), it may not be applied widely. Meanwhile, a robustness measure is proposed in [33] for descriptor systems inspired by the idea in linear control systems. A robustness measure is given as follows:

$$J := \frac{1}{2}(\|Z\|_2^F + \|Z^{-1}\|_2^F + \|W\|_2^F + \|W^{-1}\|_2^F). \quad (16)$$

As a measure of the sensitivity of closed-loop eigenvalues, we use the condition numbers $\kappa_2(Z)$ and $\kappa_2(W)$ of Z and W with respect to the spectral norm. In view of the fact that W can be represented by Z, $\kappa_2(\tilde{Z})$ can be considered as a suitable robustness performance index for computational convenience, where

$$\tilde{Z} = \begin{bmatrix} Z & 0 \\ Z\Lambda & Z_\infty \end{bmatrix}.$$

Therefore, the robust pole assignment can be formulated as follows:

$$\begin{aligned} \min \quad & J := \kappa_2^2(\tilde{Z}) \\ \text{s.t.} \quad & MZ\Lambda^2 + DZ\Lambda + K = BG, \\ & MZ_\infty = 0, \end{aligned} \quad (17)$$

where $\kappa_2^2(\tilde{Z}) = \lambda_{\max}/\lambda_{\min}$, and $\lambda_{\max}$ and $\lambda_{\min}$ represent the nonzero largest and the smallest eigenvalues of $\tilde{Z}^T\tilde{Z}$.

According to [4] (Theorem 3. 2. 10), any rational function is pseudoconvex if its numerator is nonnegative (positive) and convex and its denominator is positive and concave. As $\lambda_{\max}(\tilde{Z}^T\tilde{Z})$ is positive and convex, and $\lambda_{\min}(\tilde{Z}^T\tilde{Z})$ is positive and concave, the objective function $\kappa_2^2(\tilde{Z})$ is shown to be pseudoconvex. (17) can be reformulated as follows with a pseudoconvex objective function and a linear constraint $\tilde{A}z = 0$.

$$\begin{aligned} \min \quad & \bar{\kappa}(z) \\ \text{s.t.} \quad & \tilde{A}z = 0, \end{aligned} \quad (18)$$

where $z = [\mathrm{vec}(Z)^T\ \mathrm{vec}(G)^T]^T \in \Re^{n(n+n_0+m)}$, $\tilde{A} = [\Lambda^2 \otimes M + \Lambda \otimes D + I \otimes K \mid I \otimes B] \in \Re^{n(n+n_0)\times n(n+n_0+m)}$; $\bar{\kappa}(z)$ is equal to to $\kappa_2^2(\tilde{Z})$ in terms of z.

III. Neurodynamic Approach

A. Condition Number Minimization

Neurodynamic optimization approaches were successfully applied for convex optimization problems. It was until recent years that several recurrent neural networks were developed for solving pseudoconvex or nonconvex optimization problems [11], [15], [18], [27]–[29]. In particular, a one-layer recurrent neural network [11] is suitable for solving linearly constrained pseudoconvex optimization problems such as the problem (18) formulated in the preceding section:

$$\epsilon_1 \frac{dz}{dt} = -(I - \tilde{A}^T(\tilde{A}\tilde{A}^T)^{-1}\tilde{A})\nabla\bar{\kappa}(z) - \tilde{A}^T g(\tilde{A}z), \quad (19)$$

where ϵ_1 is a positive scaling constant, $\nabla\bar{\kappa}(z)$ is the gradient of the given objective function $\bar{\kappa}(z)$, $g(y)$ is a vector valued discontinuous activation function with its components defined as

$$g(y) = \begin{cases} 1, & y > 0; \\ 0, & y = 0; \\ -1, & y < 0. \end{cases}$$

It is proved in [11] that the state vector z of the recurrent neural network in (19) is globally convergent to the feasible region $S = \{z|\tilde{A}z = 0\}$ in finite time t_S and stays there thereafter, where t_S is given by

$$t_S = \frac{\epsilon_1\|\tilde{A}z_0\|_1}{\lambda_{\min}(\tilde{A}\tilde{A}^T)}, \quad (20)$$

where z_0 is the initial state vector. It is also proved in [11] that the recurrent network is globally convergent to the unique optimal solution of a pseudoconvex optimization problem with linear equality constraints.

The gradient of the objective function $\bar{\kappa}(z)$ can be expressed as:

$$\nabla\bar{\kappa}(z) = \mathrm{vec}(\partial\kappa_2^2(\tilde{Z})/\partial Z), \quad (21)$$

according to the chain rule:

$$\begin{aligned}\frac{\partial\kappa_2^2(\tilde{Z})}{\partial Z} &= \frac{\partial\lambda_{\max}(\tilde{Z}^T\tilde{Z})/\lambda_{\min}(\tilde{Z}^T\tilde{Z})}{\partial\tilde{Z}^T\tilde{Z}}\frac{\partial\tilde{Z}^T\tilde{Z}}{\partial\tilde{Z}}\frac{\partial\tilde{Z}}{\partial Z} \\ &= 2\frac{\partial\tilde{Z}}{\partial Z}\tilde{Z}\frac{\lambda_{\min}v_{\max}v_{\max}^T - \lambda_{\max}v_{\min}v_{\min}^T}{\lambda_{\min}^2}, \quad (22)\end{aligned}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the largest and smallest eigenvalues of $\tilde{Z}^T\tilde{Z}$, respectively; $v_{\max}$ and $v_{\min}$ are corresponding eigenvectors of $\lambda_{\max}$ and $\lambda_{\min}$, respectively. It is found that

$$\frac{\partial\mathrm{vec}(\tilde{Z}^T)}{\partial\mathrm{vec}(Z^T)} = [I\ I\otimes\Lambda], \quad (23)$$

$\nabla\bar{\kappa}(z)$ is then expressed as

$$\begin{aligned}\nabla\bar{\kappa}(z) =& 2\mathrm{vec}([I\ I\otimes\Lambda]\cdot \\ & \mathrm{vec}(\tilde{Z}\frac{\lambda_{\min}v_{\max}v_{\max}^T - \lambda_{\max}v_{\min}v_{\min}^T}{\lambda_{\min}^2}]^T))^T. \quad (24)\end{aligned}$$

B. Eigenvector Computation

In (24), $\nabla\bar{\kappa}(z)$ contains the eigenvalues and eigenvectors of $\tilde{Z}^T\tilde{Z}$. Noting that explicitly computing the eigenvalues and eigenvectors is intractable, it is desirable to apply a neural network to compute the eigenvalues and eigenvectors in real time. Recently, several neurodynamic approaches were developed for computing eigenvalues and eigenvectors of symmetric positive definite matrices; e.g., [31], [39], [40]. In particular, the state and output equations of a simple and concise model for computing the largest and smallest eigenvalues and corresponding eigenvectors can be applied as follows [40].

$$\epsilon_2\frac{d}{dt}\begin{pmatrix} u_{\max} \\ u_{\min}\end{pmatrix} = \begin{pmatrix} \tilde{Z}^T\tilde{Z}u_{\max} - u_{\max}^T u_{\max}u_{\max} \\ -(\tilde{Z}^T\tilde{Z} - \lambda_{\max}I)u_{\min} - u_{\min}^T u_{\min}u_{\min}\end{pmatrix}, \quad (25)$$

where $u_{\max}, u_{\min} \in \Re^n$ are state vectors corresponding to the eigenvectors of maximum and minimum eigenvalues. ϵ_2 is a positive scaling constant such that $\epsilon_2 \ll \epsilon_1$. RNN_1 is supposed to converge more rapidly than the control system in a smaller time scale, whereas RNN_2 is supposed to converge more rapidly than RNN_1 in an even smaller time scale. The convergence of the recurrent neural networks (RNN_1 and RNN_2) can be proportionally expedited by using small time constants ϵ_1 and ϵ_2. The multiple time-scales characteristics will be well illustrated in the illustrative example. The output equation of (25) are

$$\lambda_{\max} = \bar{u}_{\max}^T\bar{u}_{\max}, \quad v_{\max} = \frac{\bar{u}_{\max}}{\sqrt{\lambda_{\max}}}, \quad (26)$$

$$\lambda_{\min} = -\bar{u}_{\min}^T\bar{u}_{\min} + \lambda_{\max}, \quad v_{\min} = \frac{\bar{u}_{\min}}{\sqrt{\lambda_{\min}}}, \quad (27)$$

where $\bar{u}_{\max}$ and $\bar{u}_{\min}$ are respectively the equilibrium of $u_{\max}$ and $u_{\min}$. According to [40], the convergence of the recurrent neural network can be guaranteed with any nonzero $u_{\max}(0)$ and $u_{\min}(0)$. The robust pole assignment processes for synthesizing second-order descriptor control system is delineated in Fig. 1, where one recurrent neural network (RNN_1) described in (19) is responsible for conditioning optimization and another recurrent neural network (RNN_2) described in (25) is used for computing the largest and smallest eigenvalues and corresponding eigenvectors.

IV. Simulation Results

In this section, the simulation results of an illustrative example will be discussed in detail to demonstrate the effectiveness and characteristics of the proposed method. Consider a system with the following system parameters

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 2.5 & -1 & 0 \\ -1 & 2.5 & -2 \\ 0 & -2 & 2 \end{bmatrix},$$

$$K = \begin{bmatrix} 10 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \\ 0 & 1 \end{bmatrix}.$$

The objective is to synthesize a robust state feedback controller such that the closed-loop system poles are -1, -2, -3, -4, and

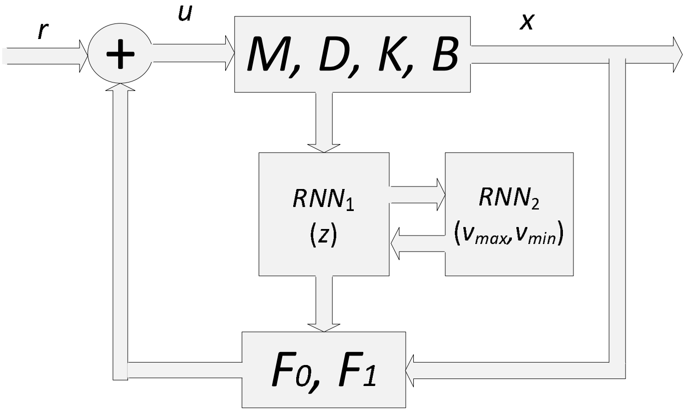

Fig. 1. Block diagram of the neurodynamics-based second-order descriptor control system via robust pole assignment.

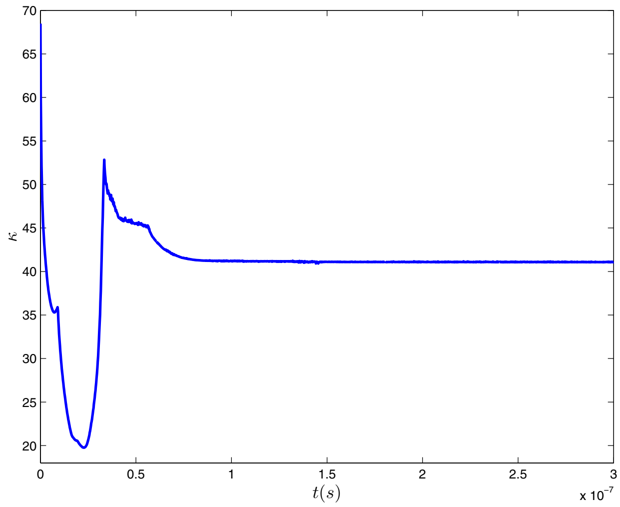

Fig. 2. Transient behavior of the condition number κ in the second-order descriptor system.

-5. So,

$$\Lambda = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix},$$

Let ϵ_1 be 10^{-4} for RNN_1 and ϵ_2 be 10^{-10} for RNN_2 . The minimum condition number of the eigensystem is 41.07. Define $Z_\infty = [0\ 0\ 1]^T$. Fig. 2 depicts the transient behavior of the spectral condition number from random initiate states. Fig 3 illustrates the convergence of the constraint norm $\|MZ\Lambda^2 + DZ\Lambda + K - BG_1\|_2$, which substantiates that the exact pole assignment is achieved. The convergent values of Z and G

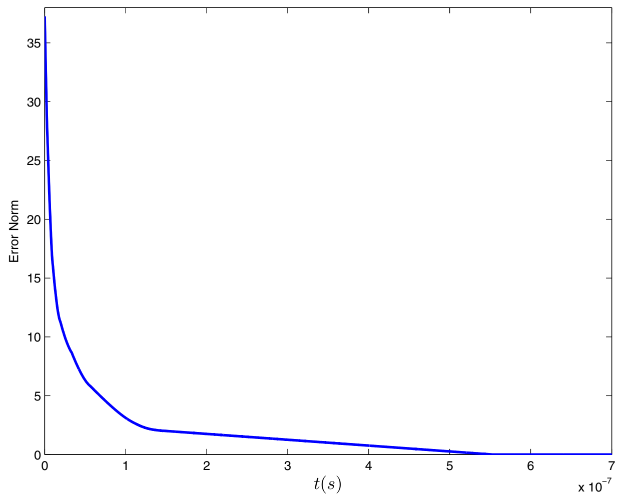

Fig. 3. Transient behavior of the constraint error norm in the second-order descriptor system.

and corresponding F_0 and F_1 are presented as follows:

$$Z = \begin{bmatrix} 0.2882 & 0.1668 & -0.3661 & 0.2788 & 0.0367 \\ 0.2868 & 0.2829 & -0.0036 & 0.0009 & 0.0664 \\ 0.3023 & 0.3827 & 0.0847 & -0.0674 & 0.2398 \end{bmatrix},$$

$$G = \begin{bmatrix} 1.1595 & 0.3698 & -4.1973 & 4.4576 & 0.6597 \\ 0.2800 & 1.5960 & 1.2359 & -0.8191 & 1.7343 \end{bmatrix},$$

$$F_0 = \begin{bmatrix} -1.0539 & 2.1755 & 3.5529 \\ -4.4340 & 0.8462 & 2.1895 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -4.6870 & 6.3693 & -0.7977 \\ -0.6921 & -0.7588 & -0.7814 \end{bmatrix},$$

Fig. 4 depicts the transient behaviors of state variables $u_{\max}$ and $u_{\min}$ in RNN_2 in second-order descriptor system, which substantiates that the recurrent neural networks RNN_2 will converge within 0.1 nanoseconds. Figs. 5, 6, and 7 illustrate the transient behaviors of the state vector z, G of RNN_1 and the corresponding feedback gain matrix F_0 and F_1 in the second-order descriptor system, respectively. Fig. 8 represents the transient behaviors of the states x and derivative states $\dot{x}$ in second-order descriptor system.

V. Concluding Remarks

In this paper, a novel neurodynamic optimization approach is proposed for synthesizing second-order descriptor control systems via robust pole assignment. A novel robustness measure is defined and optimized. By minimizing the spectral condition number of the eigensystem in real time, the proposed approach is shown to be capable of making exact pole assignment as well as obtaining the global optimal solution regardless of initial conditions. In addition to guaranteed global convergence, the proposed approach can compute the solutions in real time, which renders its online tuning capability. Further investigations of neurodynamics-based robust pole assignment will be aimed at the extension of the present results for synthesizing high-order systems and linear parameter-varying systems.

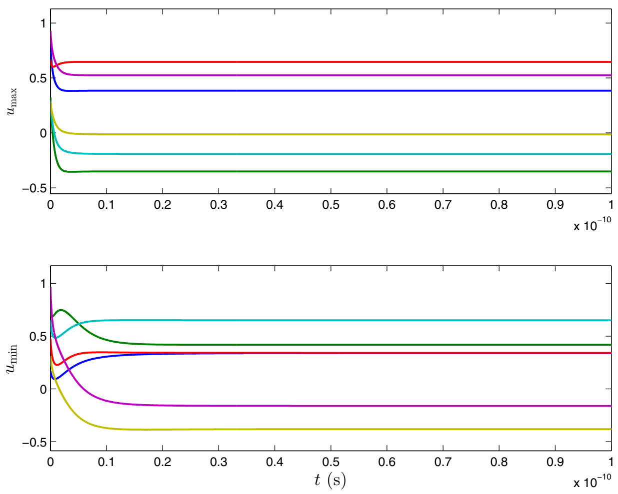

Fig. 4. Transient behaviors of state variables $u_{\max}$ and $u_{\min}$ in RNN_2 in second-order descriptor system.

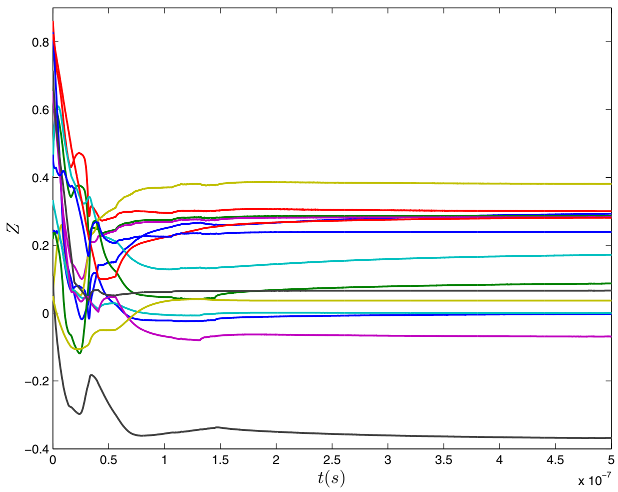

Fig. 5. Transient behaviors of the state vector Z in the second-order descriptor system.

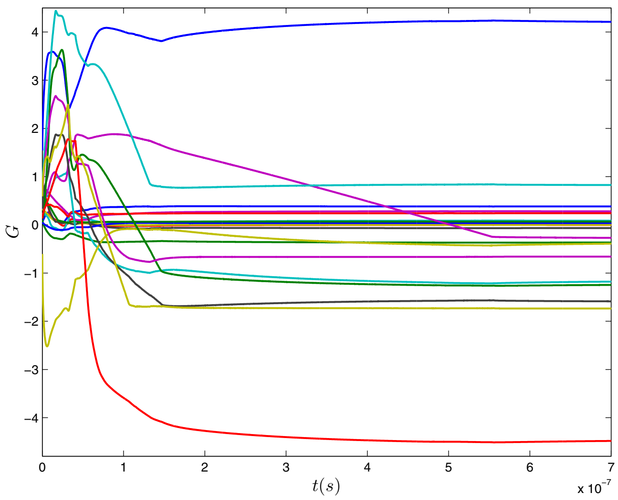

Fig. 6. Transient behaviors of the state vector G in the second-order descriptor system.

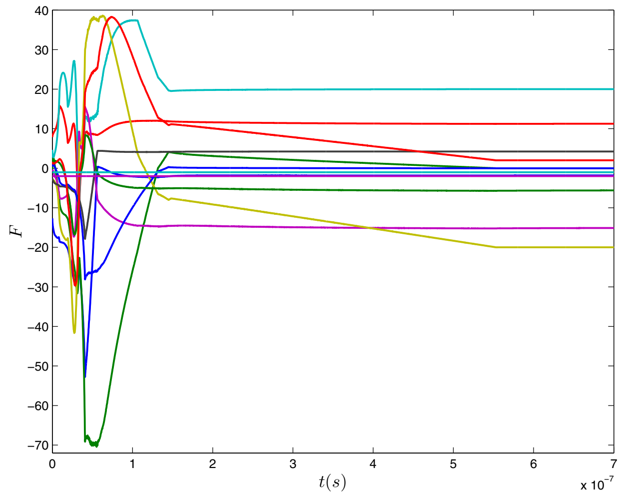

Fig. 7. Transient behaviors of the state feedback variables F_0 and F_1 in the second-order descriptor system.

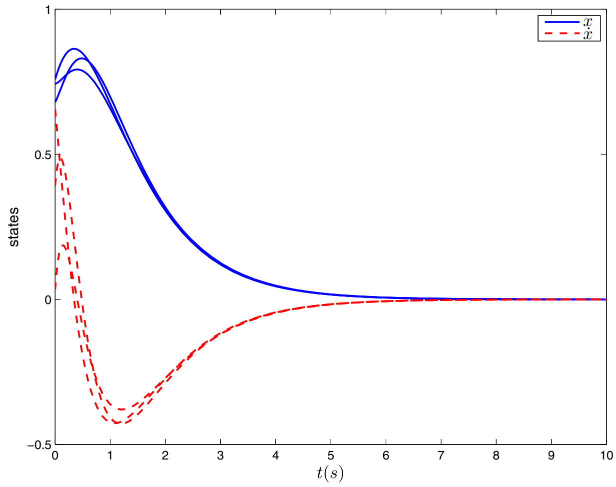

Fig. 8. Transient behaviors of the states x and derivative states $\dot{(x)}$ in the second-order descriptor system.

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