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Coordinated Pattern Tracking of Multiple Marine Surface Vehicles with Uncertain Kinematics and Kinetics

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***Abstract*—This paper considers the coordinated pattern tracking of multiple marine surface vehicles in the presence of uncertain kinematics and kinetics. Distributed pattern tracking controllers depending on the information of neighboring vehicles are derived based on a backstepping technique, neural networks and an identifier. Specifically, the identifier is devised to precisely estimate the time-varying ocean currents at the kinematic level. Neural networks together with adaptive filtering methods are employed to extract the low frequency content of the model uncertainty and ocean disturbances at the kinetic level. The benefit of the proposed design results in adaptive pattern tracking controllers over any undirected connected graphs with guaranteed low frequency control signals, which facilitates practical implementations. The stability properties of the multi-vehicle systems are established via Lyapunov analysis, and the pattern tracking errors converge to an adjustable neighborhood of origin. An example is given to show the performance of the proposed approach.**

I. Introduction

In recent years, cooperative control of multi-vehicle systems has drawn significant attention from control communities [1], [2]. Applications of multi-vehicle systems can be found everywhere; in space, in the air, on land and at sea. Examples include formation flight of satellites, coordinated control of aerial vehicles, formation control of mobile robots, cooperative control of marine vehicles. In particular, there has been considerable attention drawn to formation control of multiple marine surface vehicles (MSVs). Various approaches have been reported, ranging from virtual structure framework [3], behavioral approach [4], leader-follower mechanisms [5], [6], [7], to synchronized path following framework [8]. Apparently, these control strategies only result in low-level cooperative behaviors. However, to execute more challenging missions, it requires the use of multiple vehicles working together to achieve a collective objective [1], [2], [9], [10], [11]. For example, a group of MSVs are required to achieve coverage in a sensor network, where the coverage center can be only known by a portion of vehicles for security reasons. They exchange their knowledge by communicating with a subset of nearby vehicles, in order to achieve the coverage. Obviously, such motion control scenario cannot be completed by those formation control strategies mentioned above.

A major constraint in a networked system is that the information flow can be severely restricted. This situation is getting worse when a large lumber of vehicles are involved. Consequently, centralized controllers based on the information gathered by all agents are generally impractically to implement. Therefore, distributed control strategies based on local information have been widely explored in literature [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23]. These results correspond to first-order systems [10], [11], [12], [13], second-order systems [14], [15], [16], [17], high-order systems [18] and general linear systems [19], [20], [21], [22], [23], which may be not adequate to describe the practical dynamics of MSVs as they undergo maneuvers at sea. Hopefully, the results shed some light onto the formation control of multiple MSVs discussed in this paper.

MSV possesses many uncertainties in its dynamics such as payload variations, unmodeled hydrodynamics, and time-varying ocean disturbances [24]. To overcome such problem, adaptive control methods have been suggested [25], [26], [27], [28], [29], [30], [31]. In [25], a projection-based adaptive controller is developed for ship with parametric uncertainty and unknown ocean disturbances. In [26], adaptive update laws are devised to estimate the unknown model parameters and bounded disturbances. In [25], [26], the uncertainty is assumed to be parametric. By designing the neural adaptive controllers, references [27], [28], [29], [30], [31] investigated the control problem of surface vehicles with unmodeled dynamics and ocean disturbances. It is well known that the ocean disturbances including wind, waves and ocean currents not only contain low frequency content, but also high frequency content. In particular, the adaptive methods given in [25], [26], [27], [28], [29], [30], [31] try to learn the vehicle uncertainty at arbitrary accuracy. However, from a practical perspective, only low frequency content can be compensated because the high frequency content is surely outside the bandwidth of actuators [32]. Therefore, it is of practical importance to derive an adaptive controller capable of extracting the low frequency content of vehicle uncertainties.

This paper considers the coordinated pattern tracking of networked MSVs in the presence of uncertain kinematics and kinetics induced by wind, waves and ocean currents. Distributed pattern tracking controllers depending on the information of neighboring vehicles are derived based on a backstepping technique, neural networks and an identifier. Specifically, the identifier is proposed to estimate the time-varying ocean currents at the kinematic level. Neural networks

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together with adaptive filtering methods are employed to extract the low frequency content of the model uncertainty and ocean disturbances at the kinetic level. Lyapunov analysis demonstrate that all signals in the closed-loop network are uniformly ultimately bounded (UUB), and the pattern tracking errors converge to an adjustable neighborhood of origin. An illustrative example is given to show the effectiveness and performance of proposed scheme.

Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean Space. $||\cdot||$ denotes the Euclidean norm. $\lambda(\cdot)$, $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the eigenvalue, the smallest eigenvalue and the largest eigenvalue of a square matrix $(\cdot)$, respectively. $\underline{\sigma}(\cdot)$ denotes the smallest singular value of a given matrix. $diag\{\Lambda_1, ..., \Lambda_N\}$ represents a block-diagonal matrix with matrixes $\Lambda_i, i = 1, ..., N$, on its diagonal.

II. Preliminaries and problem formulation

A. Preliminaries

Consider a system consisting of N vehicles and a leader. Each vehicle is assumed to know its own state and have access to the state information from a subset of the vehicle group called the neighbor set denoted by $\mathcal{N}_i \subseteq \{1, ..., N\} \setminus \{i\}$. If each vehicle is considered as a node, the neighbor relation can be described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{n_1, ..., n_N\}$ is a node set and $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V}\times\mathcal{V}\}$ is an edge set with the element (n_i, n_j) that describes the communication from node i to node j. Further, define the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with the diagonal entries $a_{ii} = 0$, and the non-diagonal entries $a_{ij} = 1$, if $(n_j, n_i) \in \mathcal{E}$; $a_{ij} = 0$, otherwise. Define the Laplacian matrix $L = [l_{ij}]$ with $l_{ij} = -a_{ij}$, if $j \neq i$, and $l_{ij} = \sum_{k=1}^{N} a_{ik}$, otherwise. If $a_{ij} = a_{ji}\ \forall i, j$; then the graph $\mathcal{G}$ is undirected. If there is a path between any two nodes of an undirected network, then the graph $\mathcal{G}$ is connected. Finally, define a diagonal matrix $B = diag\{b_1, ..., b_N\}$ to be a leader adjacency matrix, where $b_i > 0$ if and only if the ith vehicle is a neighbor of the leader; otherwise $b_i = 0$. For convenience, let $H = L + B$. The following lemmas play an important role in design and analysis of the proposed formation controllers.

Lemma 1 [10]. Let the graph $\mathcal{G}$ be undirected and connected, and at least one vehicle has access to the leader. Then the matrix H is positive definite.

Definition 1 [36]. Assume that an unknown $\theta^* \in \mathbb{R}^n$ exists $\|\theta^*\| \leq \theta^*_M$ with $\theta^*_M > 0$ and let θ be dented by its estimation. Then, the projection operator $\text{Proj} : \mathbb{R}^n \to \mathbb{R}^n$ is defined as

$$\text{Proj}(y) \triangleq \begin{cases} y - \frac{\phi'(\theta)\phi'^T(\theta)y}{\|\phi'(\theta)\|^2}\phi(\theta), & \text{if } \phi(\theta) \geq 0 \text{ and } \phi'(\theta)y < 0, \\ y, & otherwise, \end{cases} \tag{1}$$

where $\phi : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable convex function

$$\phi(\theta) = \frac{\theta^T\theta - \vartheta^2}{2\varepsilon_\theta\vartheta + \varepsilon_\theta^2}, \tag{2}$$

where ϑ and ε_θ are positive constants with $\vartheta = \theta^*_M$. $\phi'(\theta) = \partial\phi/\partial\theta$.

Given $\theta(0) \leq \vartheta$, the projection operator takes the following properties

$$\begin{aligned} &\|\theta(t)\| \leq \theta_M, \forall\ t \geq 0, \\ &\|\tilde{\theta}\| \leq \tilde{\theta}_M, \forall\ t \geq 0, \\ &\tilde{\theta}^T[\text{Proj}(y) - y] \leq 0, \end{aligned} \tag{3}$$

where $\tilde{\theta} = \theta - \theta^*$, $\theta_M = \vartheta + \varepsilon_\theta$, $\tilde{\theta}_M = 2\vartheta + \varepsilon_\theta$,

Moreover, the definition of the projection operator can be generalized to matrices as $\text{Proj}(Y)$, where $\Theta \in \mathbb{R}^{n\times m}$ and $Y \in \mathbb{R}^{n\times m}$. In this case, it follows from the property (3) that

$$tr[(\Theta - \Theta^*)^T(\text{Proj}(Y) - Y)] \leq 0, \Theta^* \in \mathbb{R}^{n\times m}, \tag{4}$$

where Θ^* denotes the true value of Θ.

B. Problem formulation

Two reference frames are used to describe the motion of MSV, namely, a local earth-fixed frame and a body-fixed frame. The components $\eta_i = [x_i, y_i, \psi_i]$ are the north-east positions (x_i, y_i) of the vehicle relative to the earth-fixed frame and the yaw angle ψ_i relative to the north. The components of the velocity vector $\nu_{ir} = [u_{ir}, v_{ir}, r_i]^T$ are the surge and sway velocities relative to ocean currents (u_{ir}, v_{ir}) and the yaw rate r_i. Here, the fluid is assumed to be irrotational. Consider a group of N MSVs governed by the following model [24] with kinematics

$$\dot{\eta}_i = R(\psi_i)\nu_{ir} + V_{ic}(t), \tag{5}$$

and kinetics

$$M_i\dot{\nu}_{ir} + C_i(\nu_{ir})\nu_{ir} + D_i(\nu_{ir})\nu_{ir} + g_i(\nu_{ir}) = \tau_i + \tau_{ien}(t), \tag{6}$$

where

$$R(\psi_i) = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{7}$$

where $M_i = M_i^T \in \mathbb{R}^{3\times3}, C_i(\nu_{ir}) \in \mathbb{R}^{3\times3}, D_i(\nu_{ir}) \in \mathbb{R}^{3\times3}$ denote the inertia matrix, coriolis/centripetal matrix, and damping matrix, respectively; $g_i(\nu_{ir}) = [g_{iu}, g_{iv}, g_{ir}]^T \in \mathbb{R}^3$ is unknown term including the restoring forces due to gravity and buoyancy forces, and other unmodeled dynamics; $\tau_i = [\tau_{iu}, \tau_{iv}, \tau_{ir}]^T \in \mathbb{R}^3$ denotes the control input; $\tau_{ien}(t) = [\tau_{ienu}(t), \tau_{ienv}(t), \tau_{ienr}(t)]^T \in \mathbb{R}^3$ is the resulting environmental force and moment vector due to wind and waves. $V_{ic}(t) = [\upsilon_{ix}(t), \upsilon_{iy}(t), 0]^T \in \mathbb{R}^3$ is the vector representing the time-varying ocean currents.

Definition 2. A desired geometric formation pattern is defined as $\mathcal{P} = \{\mathcal{P}_i\}$ where $\mathcal{P}_i = [p_{ix}, p_{iy}, p_{i\psi}]^T, i = 1, ..., N$, and $p_{ix}, p_{iy}, p_{i\psi}$ are constants.

Without lose of generality, assume that $\sum_{i=1}^{N} \mathcal{P}_i = [0, 0, 0]^T$, i.e., the center of the geometric pattern $\mathcal{P}$ is at the origin of the earth-fixed frame.

Given a reference point $\eta_r \in \mathbb{R}^3$, the *coordinated pattern tracking* problem is to achieve the formation pattern $\mathcal{P}$ with a desired reference point η_r, i.e.,

$$\lim_{t\to\infty} \|\eta_i - \eta_j - \mathcal{P}_{ij}\| \le \delta_1,\ i \ne j, \tag{8}$$

$$\lim_{t\to\infty} \|\sum_{i=1}^{N} \eta_i - \eta_r\| \le \delta_2, \tag{9}$$

where $P_{ij} = \mathcal{P}_i - \mathcal{P}_j$; δ_1 and δ_2 are small positive constants.

The following assumptions are made in the following controller design.

Assumption 1. The network $\mathcal{G}$ is undirected and connected.

Assumption 2 [32]. A nonlinear function $f_i(\chi_i, t)$ can be approximated by a neural network as

$$f_i(\chi_i, t) = W_i^T(t)\varphi_i(\chi_i) + \varepsilon_i(\chi_i),\quad \forall\chi_i \in \mathcal{D}, \tag{10}$$

where $W_i(t)$ is an unknown time-varying matrix satisfying $\|W_i(t)\|_F \le W_{iM}$ and $\|\dot{W}_i\|_F \le W_{iM}^d$ with $W_{iM} \in \mathbb{R}, W_{iM}^d \in \mathbb{R}$ positive constants; $\varphi_i(\chi_i) : \mathcal{D} \to \mathbb{R}^s$ is a known vector function of the form $\varphi_i(\chi_i) = [\varphi_{i1}(\chi_i), \varphi_{i2}(\chi_i), ..., \varphi_{is}(\chi_i)]^T$ satisfying $\|\varphi_i\| \le \varphi_{iM}$ with φ_{iM} a positive constant, and $\mathcal{D}$ is compact set; $\varepsilon_i(\chi_i)$ is the approximation error satisfying $\|\varepsilon_i(\chi_i)\| \le \varepsilon_{iM}$ with ε_{iM} a positive constant.

III. Coordinated pattern tracking under time-varying ocean currents

This section addresses the pattern stability under the time-varying ocean currents. At first, an identifier is developed to precisely identify the unknown time-varying ocean currents. The identifier is designed at the kinematic level and has a simple structure. However, extra effort should be made to derive the stability of the entire system by putting together the identifier and kinetic control law.

A. Identifier design

From (5), the position dynamics can be described by

$$\begin{cases} \dot{x}_i &= u_i\cos(\psi_i) - v_i\sin(\psi_i) + \upsilon_{ix}(t), \\ \dot{y}_i &= u_i\sin(\psi_i) + v_i\cos(\psi_i) + \upsilon_{iy}(t). \end{cases} \tag{11}$$

Let $\hat{\upsilon}_{ix}(t)$ and $\hat{\upsilon}_{iy}(t)$ be the estimate of $\upsilon_{ix}(t)$ and $\upsilon_{iy}(t)$, respectively, and then a local identifier is constructed as follows

$$\begin{cases} \dot{\hat{x}}_i &= u_i\cos(\psi_i) - v_i\sin(\psi_i) + \hat{\upsilon}_{ix}(t) - \kappa_{i1}\tilde{x}_i, \\ \dot{\hat{y}}_i &= u_i\sin(\psi_i) + v_i\cos(\psi_i) + \hat{\upsilon}_{iy}(t) - \kappa_{i2}\tilde{y}_i, \end{cases} \tag{12}$$

where $\tilde{x}_i = \hat{x}_i - x_i$ and $\tilde{y}_i = \hat{y}_i - y_i$ are observing errors; $\kappa_{i1} \in \mathbb{R}$ and $\kappa_{i2} \in \mathbb{R}$ are positive constants; $\hat{\upsilon}_{ix}(t)$ and $\hat{\upsilon}_{iy}(t)$ are updated as

$$\begin{cases} \dot{\hat{\upsilon}}_{ix}(t) &= \Gamma_{ix}\text{Proj}\{-\tilde{x}_i + k_x(\hat{\upsilon}_{ixf}(t) - \hat{\upsilon}_{ix}(t))\}, \\ \dot{\hat{\upsilon}}_{iy}(t) &= \Gamma_{iy}\text{Proj}\{-\tilde{y}_i + k_y(\hat{\upsilon}_{iyf}(t) - \hat{\upsilon}_{iy}(t))\}, \end{cases} \tag{13}$$

where $\hat{\upsilon}_{ixf}(t)$ and $\hat{\upsilon}_{iyf}(t)$ are low-pass filter weight estimates of $\hat{\upsilon}_{ix}(t)$ and $\hat{\upsilon}_{iy}(t)$ given by

$$\begin{cases} \dot{\hat{\upsilon}}_{ixf}(t) &= \Gamma_{ixf}\text{Proj}\{\hat{\upsilon}_{ix}(t) - \hat{\upsilon}_{ixf}(t)\}, \\ \dot{\hat{\upsilon}}_{iyf}(t) &= \Gamma_{iyf}\text{Proj}\{\hat{\upsilon}_{iy}(t) - \hat{\upsilon}_{iyf}(t)\}, \end{cases} \tag{14}$$

where $k_x \in \mathbb{R}, k_y \in \mathbb{R}, \Gamma_{ix} \in \mathbb{R}, \Gamma_{iy} \in \mathbb{R}, \Gamma_{ixf} \in \mathbb{R}, \Gamma_{iyf} \in \mathbb{R}$ are positive constants. The resulting errors dynamics of $\tilde{x}_i$ and $\tilde{y}_i$ can be described by

$$\begin{cases} \dot{\tilde{x}}_i &= -\kappa_{i1}\tilde{x}_i + \tilde{\upsilon}_{ix}, \\ \dot{\tilde{y}}_i &= -\kappa_{i2}\tilde{y}_i + \tilde{\upsilon}_{iy}. \end{cases} \tag{15}$$

where $\tilde{\upsilon}_{ix} = \hat{\upsilon}_{ix} - \upsilon_{ix}$, and $\tilde{\upsilon}_{iy} = \hat{\upsilon}_{iy} - \upsilon_{iy}$.

The following lemma plays an important role in establishing the stability of the closed-loop system.

Lemma 2. For kinematic dynamics (11) with the identifier (12) and the adaptive laws (13) (14) guarantee that the error signals $\tilde{x}_i$, $\tilde{y}_i$, $\tilde{\upsilon}_{ix}$, $\tilde{\upsilon}_{iy}$ are UUB.

Proof. Consider the following Lyapunov function candidate

$$\begin{aligned} \mathcal{V}_o = &\sum_{i=1}^{N}\Big\{\tilde{x}_i^2 + \tilde{y}_i^2 + \Gamma_{ix}^{-1}\tilde{\upsilon}_{ix}^2 + \Gamma_{iy}^{-1}\tilde{\upsilon}_{iy}^2 \\ &+ k_x\Gamma_{ixf}^{-1}\tilde{\upsilon}_{ixf}^2 + k_y\Gamma_{iyf}^{-1}\tilde{\upsilon}_{iyf}^2\Big\}, \end{aligned} \tag{16}$$

where $\tilde{\upsilon}_{ixf} = \hat{\upsilon}_{ixf} - \upsilon_{ix}$, and $\tilde{\upsilon}_{iyf} = \hat{\upsilon}_{iyf} - \upsilon_{iy}$. Its time derivative of which along (15) can be described by

$$\begin{aligned} \dot{\mathcal{V}}_o = &\sum_{i=1}^{N}\Big\{ - \kappa_{i1}\tilde{x}_i^2 - \kappa_{i2}\tilde{y}_i^2 + \tilde{\upsilon}_{ix}(\tilde{x}_i + \Gamma_{ix}^{-1}\dot{\hat{\upsilon}}_{ix}) \\ &+ k_x\tilde{\upsilon}_{ixf}\Gamma_{ixf}^{-1}\dot{\hat{\upsilon}}_{ixf} + \tilde{\upsilon}_{iy}(\tilde{y}_i + \Gamma_{iy}^{-1}\dot{\hat{\upsilon}}_{iy}) \\ &+ k_y\tilde{\upsilon}_{iyf}\Gamma_{iyf}^{-1}\dot{\hat{\upsilon}}_{iyf} - \tilde{\upsilon}_{ix}(\Gamma_{ix}^{-1} + k_x\Gamma_{ixf}^{-1})\dot{\upsilon}_{ix} \\ &- \tilde{\upsilon}_{iy}(\Gamma_{iy}^{-1} + k_y\Gamma_{iyf}^{-1})\dot{\upsilon}_{iy}\Big\}. \end{aligned} \tag{17}$$

Substituting the adaptive laws into (17) yields

$$\begin{aligned} \dot{\mathcal{V}}_o = &\sum_{i=1}^{N}\Big\{ - \kappa_{i1}\tilde{x}_i^2 - \kappa_{i2}\tilde{y}_i^2 - \tilde{\upsilon}_{ix}(\Gamma_{ix}^{-1} + k_x\Gamma_{ixf}^{-1})\dot{\upsilon}_{ix} \\ &- \tilde{\upsilon}_{iy}(\Gamma_{iy}^{-1} + k_y\Gamma_{iyf}^{-1})\dot{\upsilon}_{iy}\Big\}. \end{aligned} \tag{18}$$

Let $\kappa_1 = diag\{\kappa_{11}, ..., \kappa_{N1}\}$, $\kappa_2 = diag\{\kappa_{12}, ..., \kappa_{N2}\}$, $\Gamma_x = diag\{\Gamma_{1x}, ..., \Gamma_{Nx}\}$, $\Gamma_y = diag\{\Gamma_{1y}, ..., \Gamma_{Ny}\}$, $\Gamma_{xf} = diag\{\Gamma_{1xf}, ..., \Gamma_{Nxf}\}$, $\Gamma_{yf} = diag\{\Gamma_{1yf}, ..., \Gamma_{Nyf}\}$, $\tilde{x} = [\tilde{x}_1, ..., \tilde{x}_N]^T$, $\tilde{y} = [\tilde{y}_1, ..., \tilde{y}_N]^T$, $\tilde{\upsilon}_x = [\tilde{\upsilon}_{1x}, ..., \tilde{\upsilon}_{Nx}]^T$, $\tilde{\upsilon}_y = [\tilde{\upsilon}_{1y}, ..., \tilde{\upsilon}_{Ny}]^T$, and it follows that

$$\begin{aligned} \dot{\mathcal{V}}_o \le &- \tilde{x}^T\kappa_1\tilde{x} - \tilde{y}^T\kappa_2\tilde{y} - (\tilde{\upsilon}_x^T\Gamma_x^{-1} + k_x\tilde{\upsilon}_{fx}^T\Gamma_{xf}^{-1})\dot{\upsilon}_x \\ &- (\tilde{\upsilon}_y^T\Gamma_y^{-1} + k_y\tilde{\upsilon}_{fy}^T\Gamma_{yf}^{-1})\dot{\upsilon}_y. \end{aligned} \tag{19}$$

The projection operation leads to the following bound

$$\begin{aligned} |-(\tilde{\upsilon}_x^T\Gamma_x^{-1} + k_x\tilde{\upsilon}_{fx}^T\Gamma_{xf}^{-1})\dot{\upsilon}_x| \le &[\lambda_{max}(\Gamma_x^{-1}) \\ &+ k_x\lambda_{max}(\Gamma_{xf}^{-1})]\tilde{\upsilon}_{xM}\upsilon_{xM}^d \end{aligned} \tag{20}$$

$$\begin{aligned} |-(\tilde{\upsilon}_y^T\Gamma_y^{-1} + k_y\tilde{\upsilon}_{fy}^T\Gamma_{yf}^{-1})\dot{\upsilon}_y| \le &[\lambda_{max}(\Gamma_y^{-1}) \\ &+ k_y\lambda_{max}(\Gamma_{yf}^{-1})]\tilde{\upsilon}_{yM}\upsilon_{yM}^d \end{aligned} \tag{21}$$

where $\tilde{\upsilon}_{xM} \in \mathbb{R}$, $\tilde{\upsilon}_{yM} \in \mathbb{R}$, $\upsilon_{xM}^d \in \mathbb{R}$, $\upsilon_{yM}^d \in \mathbb{R}$ are positive constants. Finally, one has

$$\dot{\mathcal{V}}_o \le -\lambda_{min}(\kappa_1)\tilde{x}^2 - \lambda_{min}(\kappa_2)\tilde{y}^2 + \epsilon_o,$$

with $\epsilon_o = [\lambda_{max}(\Gamma_x^{-1}) + k_x\lambda_{max}(\Gamma_{xf}^{-1})]\tilde{\upsilon}_{xM}\upsilon_{xM}^d + [\lambda_{max}(\Gamma_y^{-1}) + k_x\lambda_{max}(\Gamma_{yf}^{-1})]\tilde{\upsilon}_{yM}\upsilon_{yM}^d$. Note that $\tilde{x} >$

$\sqrt{\epsilon_o/\lambda_{min}(\kappa_1)}$ and $\tilde{y} > \sqrt{\epsilon_o/\lambda_{min}(\kappa_1)}$ renders $\dot{\mathcal{V}}_o < 0$. It follows that $\tilde{x}$ and $\tilde{y}$ are UUB. The projection operator ensures that the weights $\hat{\upsilon}_x$ and $\hat{\upsilon}_y$ are contained in compact sets for all t, which implies that $\tilde{\upsilon}_x$ and $\tilde{\upsilon}_y$ are UUB. The proof is complete. □

Remark 1. In [33], an observe is proposed to identify constant ocean currents. In [34], a directed adaptive method is employed to identify the constant ocean currents. This paper, to our best knowledge, is the first to deal with time-varying ocean currents.

B. Pattern tracking controller design

Step 1. Define two variables

$$\begin{cases} z_{i1} = \eta_i - \mathcal{P}_i - \eta_r, \\ z_{i2} = \nu_{ir} - \alpha_{i1}, \end{cases} \tag{22}$$

where $\alpha_{i1} \in \mathbb{R}^3$ is a virtual control input. Take the time derivative of z_{i1}, and it follows that

$$\dot{z}_{i1} = R_i\alpha_{i1} + R_i z_{i2} + V_{ic}(t), \tag{23}$$

where $R_i = R(\psi_i)$.

Then, a distributed kinematic control law α_{i1} based on the local information is proposed as follows

$$\alpha_{i1} = -K_{i1}R_i^T s_i - R_i^T\hat{V}_{ic}(t). \tag{24}$$

where $\hat{V}_{ic} = [\hat{\upsilon}_{ix}, \hat{\upsilon}_{iy}, 0]^T$; $K_{i1} = diag\{k_{i11}, k_{i12}, k_{i13}\}$ is a diagonal matrix with $k_{i11} \in \mathbb{R}, k_{i12} \in \mathbb{R}, k_{i13} \in \mathbb{R}$ being positive constants; s_i is defined as

$$s_i = \sum_{j\in\mathcal{N}_i} a_{ij}(\eta_i - \eta_j - \mathcal{P}_{ij}) + b_i z_{i1}. \tag{25}$$

where a_{ij} and b_i defined in Section 2.1.1.

Substituting (24) into (23) yields

$$\dot{z}_{i1} = -K_{i1}s_i + R_i z_{i2} - \tilde{V}_{ic}(t), \tag{26}$$

where $\tilde{V}_{ic}(t) = \hat{V}_{ic}(t) - V_{ic}(t)$.

Let $z_1 = [z_{11}^T, ..., z_{N1}^T]^T$, $z_2 = [z_{12}^T, ..., z_{N2}^T]^T$, $s = [s_1^T, ..., s_N^T]^T$, $R = diag\{R(\psi_1), ..., R(\psi_N)\}$, $K_1 = diag\{K_{11}, ..., K_{N1}\}$, $\tilde{V}_c(t) = [\tilde{V}_{1c}^T(t), ..., \tilde{V}_{1c}^T(t)]^T$. Then, the N subsystem (23) with (26) can be expressed as

$$\dot{z}_1 = -K_1 s + Rz_2 - \tilde{V}_c(t), \tag{27}$$

Consider a Lyapunov function candidate

$$\mathcal{V}_{11} = \frac{1}{2}z_1^T(H \otimes I_3)z_1, \tag{28}$$

whose time derivative along (27) is given by

$$\dot{\mathcal{V}}_{11} = -s^T K_1 s + s^T R z_2 - s^T\tilde{V}_c(t). \tag{29}$$

Step 2. Taking the time derivative of z_{i2} yields

$$\begin{aligned} M_i\dot{z}_{i2} = &- C_i(\nu_{ir})\nu_{ir} - D_i(\nu_{ir})\nu_{ir} - g_i(\nu_{ir}) + \tau_i \\ &+ \tau_{ien}(t) - M_i\dot{\alpha}_{i1}. \end{aligned} \tag{30}$$

Then, consider the second Lyapunov function candidate

$$\mathcal{V}_{12} = \mathcal{V}_{11} + \tfrac{1}{2}z_2^T M z_2, \tag{31}$$

where $M = diag\{M_1, ..., M_N\}$. Its time derivative with (30) is

$$\begin{aligned} \dot{\mathcal{V}}_{12} = &- s^T K_1 s - s^T\tilde{V}_c(t) + \sum_{i=1}^N\{z_{i2}(-C_i(\nu_{ir})\nu_{ir} \\ &- D_i(\nu_{ir})\nu_{ir} - g_i(\nu_{ir}) + \tau_i + \tau_{ien}(t) - M_i\dot{\alpha}_{i1} + R_i^T s_i)\}. \end{aligned} \tag{32}$$

The desired kinetic control law τ_i is chosen as

$$\tau_i = -K_{i2}z_{i2} - R_i^T s_i + f_i(\chi_i, t), \tag{33}$$

where $f_i(\chi_i, t) = M_i\dot{\alpha}_{i1} + C_i(\nu_{ir})\nu_{ir} + D_i(\nu_{ir})\nu_{ir} + g_i(\eta_i, \nu_{ir}) - \tau_{ien}(t)$ with $\chi_i = [1, \eta_i, \eta_j, \nu_{ir}, \nu_{jr}]^T$, $j \in \mathcal{N}_i$; $K_{i2} = diag\{k_{i21}, k_{i22}, k_{i23}\} \in \mathbb{R}^{3\times3}$ with $k_{i21} \in \mathbb{R}, k_{i22} \in \mathbb{R}, k_{i23} \in \mathbb{R}$ being positive constants.

Note that without the explicit knowledge of $C_i, D_i, g_i, M_i, \tau_{ien}(t)$, the controller given in (33) cannot be available. Then, let $f_i(\chi_i, t)$ be approximated by the NN in (10).

In what follows, a practical kinetic control law is constructed as follows

$$\tau_i = -K_{i2}z_{i2} - R_i^T s_i + \hat{W}_i^T(t)\varphi_i(\chi_i), \tag{34}$$

where $\hat{W}_i(t)$ is an estimate of $W_i(t)$ that updated as

$$\dot{\hat{W}}_i(t) = \Gamma_{iW}\text{Proj}\{-\varphi_i(\chi_i)z_{i2}^T + k_W[\hat{W}_{if}(t) - \hat{W}_i(t)]\}, \tag{35}$$

where $W_{if}(t)$ is a low-pass filter weight estimate of $W_i(t)$ given by

$$\dot{\hat{W}}_{if}(t) = \Gamma_{if}\text{Proj}\{\hat{W}_i(t) - \hat{W}_{if}(t)\}, \tag{36}$$

where $k_W \in \mathbb{R}, \Gamma_{iW} \in \mathbb{R}, \Gamma_{if} \in \mathbb{R}$ are positive constants.

Substituting the control law (34) into (32) yields

$$\begin{aligned} \dot{\mathcal{V}}_{12} = &- s^T K_1 s - s^T\tilde{V}_c(t) - z_2^T K_2 z_2 \\ &+ \sum_{i=1}^N z_{i2}^T[\tilde{W}_i^T(t)\varphi_i(\chi_i) - \varepsilon_i], \end{aligned} \tag{37}$$

where $K_2 = diag\{K_{12}, ..., K_{N2}\}$ and $\tilde{W}_i(t) = \hat{W}_i(t) - W_i(t)$. The resulting closed-loop network system can be described by

$$\begin{cases} \dot{z}_{i1} = -K_{i1}s_i + R_i z_{2i} - \tilde{V}_{ic}(t), \\ M_i\dot{z}_{i2} = -K_{i2}z_{i2} - R_i^T s_i + \tilde{W}_i^T(t)\varphi_i(\chi_i) - \varepsilon_i, \\ \dot{\tilde{x}}_i = -\kappa_{i1}\tilde{x}_i + \tilde{\upsilon}_{ix}, \\ \dot{\tilde{y}}_i = -\kappa_{i2}\tilde{y}_i + \tilde{\upsilon}_{iy}. \end{cases} \tag{38}$$

C. Stability analysis

It is the position to state the result of this paper.

Theorem 1. Consider a networked system consisting of N MSVs governed by the dynamics (5) (6) with Assumptions 1 and 2 satisfied. Select the control laws (34) with the adaptive laws (35) (36). Then, all signals in the closed-loop system are UUB, and the pattern tracking errors $\eta_i - \eta_j - \mathcal{P}_{ij}$ satisfy (8) (9) for some constants δ_1 and δ_2.

Proof. Take the following Lyapunov function candidate $\mathcal{V}_3 = \mathcal{V}_1 + \mathcal{V}_o$, whose time derivative along (37) can be put into

$$\begin{aligned}\dot{\mathcal{V}}_3 &= -s^T K_1 s - z_2^T K_2 z_2 - s^T \tilde{V}_c - z_2^T \varepsilon \\ &- tr[(\tilde{W}^T \Gamma_W^{-1} + k_W \tilde{W}_f^T \Gamma_f^{-1})\dot{W}] \\ &- \tilde{x}^T \kappa_1 \tilde{x} - \tilde{y}^T \kappa_2 \tilde{y} - (\tilde{\upsilon}_x^T \Gamma_x^{-1} + k_x \tilde{\upsilon}_{fx}^T \Gamma_{xf}^{-1})\dot{\upsilon}_x \\ &- (\tilde{\upsilon}_y^T \Gamma_y^{-1} + k_y \tilde{\upsilon}_{fy}^T \Gamma_{yf}^{-1})\dot{\upsilon}_y. \end{aligned} \tag{39}$$

Using Young's inequality, it is easy to verify that

$$\dot{\mathcal{V}}_3 \leq -\hbar_1 \|s\|^2 - \hbar_2 \|z_2\|^2 - \hbar_3 \|\tilde{x}\|^2 - \hbar_4 \|\tilde{y}\|^2 + \epsilon_s,$$

where $\hbar_1 = \lambda_{min}(K_1) - 1/2$; $\hbar_2 = \lambda_{min}(K_2) - 1/2$; $\hbar_3 = \lambda_{min}(\kappa_1) - 1/2$; $\hbar_3 = \lambda_{min}(\kappa_2) - 1/2$; $\epsilon_s = \frac{1}{2}\|\varepsilon_M\|^2 + [\lambda_{max}(\Gamma_W^{-1}) + k_W \lambda_{max}(\Gamma_f^{-1})]\tilde{W}_M W_M^d + \epsilon_o$.

Noting that either $\|s\| > \sqrt{\epsilon_s/\hbar_1}$, or $\|z_2\| > \sqrt{\epsilon_s/\hbar_2}$, or $\|\tilde{x}\| > \sqrt{\epsilon_s/\hbar_3}$, or $\|\tilde{y}\| > \sqrt{\epsilon_s/\hbar_4}$ renders $\dot{\mathcal{V}}_3 < 0$, it follows that s, z_2, $\tilde{x}$, $\tilde{y}$ are UUB. Noting that $s = (H \otimes I_3) z_1$ and the fact H is positive definite By Lemma 1, it follows that

$$\|z_{i1}\| \leq \|z_1\| \leq \sqrt{\frac{\epsilon}{\lambda_{min}(H)\hbar_1}}, \tag{40}$$

implying (8) with δ_2 taken as

$$\delta_1 = 2\sqrt{\frac{\epsilon}{\lambda_{min}(H)\hbar_1}}.$$

Also, note that

$$\|\sum_{i=1}^{N} \frac{\eta_i}{N} - \eta_r\| \leq \frac{\sum_{i=1}^{N} \|q_{i1}\|}{N}, \tag{41}$$

which leads to (8) with δ_3 taken as

$$\delta_2 = \sqrt{\frac{\epsilon}{\lambda_{min}(H)\hbar_1}}.$$

This completes the proof. □

IV. An Example

Consider a system consisting of five vehicles with the information exchange topology given in Figure 1. The model parameters can be found in [35]. The control parameters are set to $K_{i1} = diag\{0.2, 0.2, 0.2\}$, $K_{i2} = diag\{75, 22, 68.4\}$, $\Gamma_{iW} = 1000$, $\Gamma_{if} = 2$, $k_W = 0.1$. The desired pattern is chosen as $\mathcal{P}_1 = [-1.5, 0, 0]^T$, $\mathcal{P}_2 = [-1.5\cos(72°), 1.5\sin(72°), 0]^T$, $\mathcal{P}_3 = [-1.5\cos(72°), -1.5\sin(72°), 0]^T$, $\mathcal{P}_4 = [1.5\cos(36°), 0.7\sin(36°), 0]^T$, $\mathcal{P}_5 = [1.5\cos(36°), -1.5\sin(36°), 0]^T$. The desired formation center is set to $\eta_r = \{(2, 1, 0)^T, (4, 1, 45°)^T\}$.

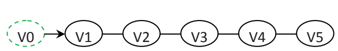

Fig. 1. Communication topology

Simulation results are shown in Figures 2-4. Figure 2 shows that the formation pattern cannot be stabilized due to the time-varying ocean currents. By contrast, Figure 3 demonstrates the formation is well maintained by the proposed identifier-based pattern controller. Figure 4 verifies that the time-varying ocean currents can be identified accurately by the proposed identifier.

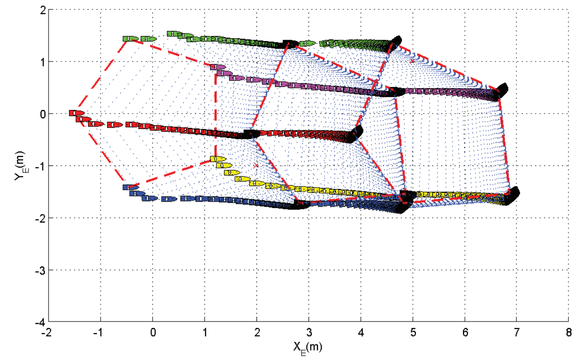

Fig. 2. Formation trajectories without identifier (t=120s)

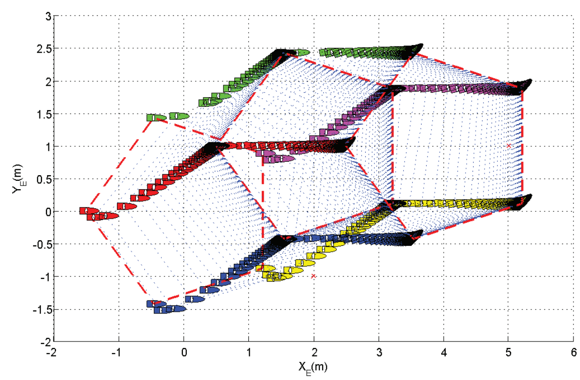

Fig. 3. Formation trajectories with identifier (t=120s)

V. Conclusions

This paper considered the coordinated pattern tracking problem of multiple marine surface vehicles with uncertain kinematics and kinetics. Neural networks, identifier and back-stepping techniques are employed to devise the distributed pattern tracking controllers, under which a stationary formation can be reached for any undirected connected graphs. Lyapunov stability analysis demonstrate that all signals in the closed-loop systems are uniformly ultimately bounded. The main advantage lies in the fact the proposed control scheme leads to adaptive pattern controllers with guaranteed low frequency control signals, which facilities the practical implementations under hazardous sea environment. Simulation results showed the efficacy of the proposed cooperative controllers.

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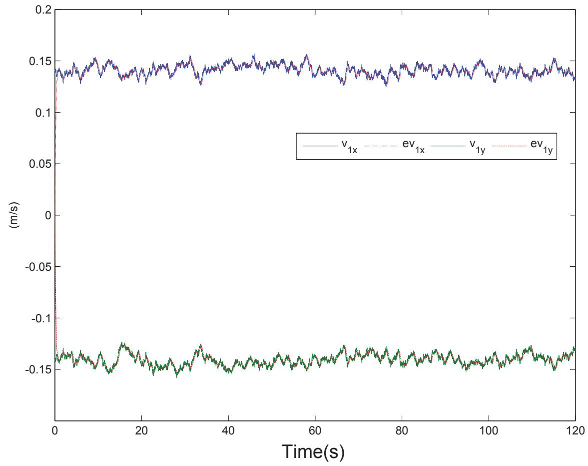

Fig. 4. Estimation of ocean currents (ev_{1x} denotes the estimate of v_{1x}; ev_{1y} denotes the estimate of v_{1y}.)

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