

Neurodynamics-based Model Predictive Control of Autonomous Underwater Vehicles in Vertical Plane

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Abstract—This paper presents a model predictive control (MPC) method based on a recurrent neural network for control of autonomous underwater vehicles (AUVs) in a vertical plane. Both kinematic and dynamic models are considered in the set-point control of the AUV. A one-layer recurrent neural network called the general projection neural network is applied for real-time optimization to compute optimal control variables. Simulation results are discussed to demonstrate the effectiveness and characteristics of the proposed model predictive control method.

I. INTRODUCTION

AUTONOMOUS underwater vehicles (AUVs) have attracted much attention in recent years. There has been a considerable interest over the last few years for marine vehicle motion control; e.g., point stabilization (also known as set-point control), trajectory tracking, and path-following control. The point stabilization refers to the problem of steering a vehicle to a final target position with a desired orientation. The set-point control of underwater vehicles is one of the most important parts in AUV control [1].

In several studies, the AUV's behaviors are often modeled by multivariable linear systems [2]–[6]. In the diving control problems, two main assumptions are made on the AUV's dynamics. One assumption is that the pitch angle of the vehicle is small in diving behaviors, and the other one is that the pitch motion dynamics could be modeled by a linear equation. The problem of controlling an AUV in the dive plane is addressed in [7]. AUVs control issues in output feedback form are discussed in [8]–[12]. In [13], an effective depth control is proposed in the presence of the parameter uncertainties and control fin deflection constraints. In [14], a Lyapunov tracking control law using backstepping approach is proposed. In [15] and [16], nonlinear neural network adaptive controllers are used for diving control of AUVs. A set-point controller for AUV is proposed by Herman in [1]. A simplified model is proposed for pitch dynamics that takes into account the buoyancy of the AUV in [17]. Model predictive control of an unmanned submarine is developed with movement only in the X-Z plane [18]. In [19], a method using model predictive control of the yaw angle of an AUV is proposed, but dealing with SISO system only. In [20] and

[21], results on model predictive control of AUVs is also presented for with disturbances and constraints.

Model predictive control (MPC) is an optimization-based advanced control method and entails extensive online computation of real-time solutions to formulated optimization problems [22]. For large-scale and realtime optimization problems, recurrent neural networks emerged as promising computational models for real-time optimization problems. For example, in [23], a one layer general projection neural network is presented for solving convex optimization problems. In [24], another one-layer neural network was presented for pseudoconvex optimization problems. These recurrent neural network models are shown to perform well in terms of convergence property and model complexity. Some studies on MPC based on recurrent neural networks were carried out. In [25]–[30], the simplified dual network is applied for solving real-time quadratic optimizations in various MPC approaches. In [31] and [32], a two-layer recurrent neural network is applied for solving reformulated minimax optimization problems of robust MPC approaches. These neurodynamics-based MPC approaches are developed to improve the computational efficiency and control performance substantially.

The rest of the paper is organized as follows. Section II formulates the set-point control problem for an AUV in the vertical plane. In Section III, the MPC method for AUV control is formulated as a time-varying quadratic programming problem and a recurrent neural network is applied for solving the quadratic programming problem. The performance of the control system proposed is demonstrated using simulation results in Section IV. Finally, Section V contains the conclusions and describes some problems that warrant further research.

II. PROBLEM FORMULATION

In this section, the kinematic and vertical dynamic models of the Taipan-2 AUV are presented, and the formulation of driving the vehicle in the vertical plane to a set-point is stated. The mathematical model of an AUV in six DOF can be described as follows [33]:

$$\begin{aligned}\dot{\eta} &= J(\eta) \nu \\ M\dot{\nu} + C(v)\nu + D(v)\nu + g(\eta) + \tau_d &= \tau \\ y &= \eta\end{aligned}\quad (1)$$

where $\eta = [x \ y \ z \ \phi \ \theta \ \varphi]^T$ denotes the vehicle location and orientation in the earth-fixed frame, $\nu = [u \ v \ w \ p \ q \ r]^T$ is the vehicle's velocity and angular rate vector expressed in the body-fixed frame, y is the output of the system, $J(\eta)$ is

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the kinematic transformation matrix expressing the transformation from the body-fixed frame to earth-fixed frame and

$$\begin{aligned} J(\eta) &= \begin{bmatrix} J_1(\eta) & 0 \\ 0 & J_2(\eta) \end{bmatrix} \\ J_1(\eta) &= \begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\phi - s\varphi c\phi & s\varphi s\phi + c\varphi s\theta c\phi \\ s\varphi c\theta & c\varphi c\phi + s\phi s\theta s\varphi & -c\varphi s\phi + s\theta s\varphi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \\ J_2(\eta) &= \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \end{aligned} \quad (2)$$

where $s = \sin(\cdot)$, $c = \cos(\cdot)$ and $t = \tan(\cdot)$. In (1) and (2), a complete kinematic model of the AUV is given. and in this paper, we consider the AUV kinematic model in the vertical plane only which can be expressed as follows:

$$\begin{aligned} \dot{x} &= u \cos \theta + w \sin \theta \\ \dot{z} &= -u \sin \theta + w \cos \theta \\ \dot{\theta} &= q \end{aligned} \quad (3)$$

The Taipan-2 vehicle dynamic model has been theoretically estimated using classic methods in [33] and [34]. In this research, for the control design purpose, and as done in [35], we simplify the full model by neglecting the stable roll motion. Then the simplified vertical plane dynamic model can be written as:

$$\begin{aligned} \dot{u} &= \frac{F_u - d_u}{m_u} \\ \dot{w} &= \frac{F_w - m_{uq}uq - d_w}{m_w} \\ \dot{q} &= \frac{\Gamma_q - m_{pr}pr - d_q}{m_q} \end{aligned} \quad (4)$$

where F_u and F_w are the force along the x and z axis, Γ_q is the torque acting on the pitch angle θ ; $m_u = m - X_{\dot{u}}$, $m_w = m - Z_{\dot{w}}$, $m_q = I_{yy} - M_{\dot{q}}$, $m_{uq} = -m$, $m_{pr} = -I_{zz}$, $d_u = -X_{uu}u|u| + m(qw - vr + z_g(pr))$, $d_w = -Z_{ww}w|w| - Z_{uw}uw - mz_g(p^2 + q^2)$, $d_q = -M_{qq}q|q| - M_{uq}uq - M_{uw}uw + (z_gmg - z_bbg) \sin \theta + mz_g(wq - vr)$; X , Z , and M represent the dynamic derivative coefficients of the vertical plane dynamics of Taipan-2; the terms m , b and I are the mass, buoyancy, and moments of inertia of the vehicle, respectively; z_g and z_b are the location of the center of gravity and the center of buoyancy along the z_B axis with respect to the axis of propulsion. All the coefficients involved here are listed in Table I [36].

For the Taipan-2 AUV, considering both its kinematics and dynamics in the vertical plane, we define the state vector $\mathbf{x} = [x \ z \ \theta \ u \ w \ q]^T$ and the input vector $\mathbf{u} = [F_u \ F_w \ \Gamma_q]^T$.

By using Euler discretization, the AUV model can be transformed into a discrete-time model in the following form:

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (5)$$

TABLE I
HYDRODYNAMIC DIMENSIONAL COEFFICIENTS OF THE TAIPAN AUV

$X_{uu} = -4.00kg \ m^{-1}$	$X_{\dot{u}} = -5.070kg$
$Z_{uq} = -37.327kg \ rad^{-1}$	$Z_{ww} = -350.00kg \ m^{-1}$
$Z_{uu\delta_s} = -4.4913kg \ m^{-1} \ rad^{-1}$	$Z_{uw} = -40.750kg \ m^{-1}$
$Z_{uu\delta_b} = 4.4913kg \ m^{-1} \ rad^{-1}$	$Z_{\dot{w}} = -50.700kg$
$M_{uw} = 10.280kg$	$M_{uq} = -34.192kgmrad^{-1}$
$M_{qq} = -200.00kg \ m^2 \ rad^{-2}$	$M_{\dot{q}} = -18.020kg \ m^2 \ rad^{-1}$
$M_{uu\delta_s} = -16.874kg \ rad^{-1}$	$M_{uu\delta_b} = -8.4729kg \ rad^{-1}$
$z_g = 0.01757m$	$I_{yy} = 10.900kg \ m^2$
$z_b = 0.00316m$	$m = 50.7kg$
$g = 9.81m \ s^{-1}$	$b = 50.9kg$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the input vector; $y(k) \in \mathbb{R}^p$ is the output vector; $f(\cdot)$ and $g(\cdot)$ are nonlinear functions, and $C \in \mathbb{R}^{p \times n}$. The system is subject to the constraints:

$$\begin{aligned} u_{\min} &\leq u(k) \leq u_{\max}, \\ \Delta u_{\min} &\leq \Delta u(k) \leq \Delta u_{\max}, \\ x_{\min} &\leq x(k) \leq x_{\max}, \\ y_{\min} &\leq y(k) \leq y_{\max}, \end{aligned} \quad (6)$$

MPC is an iterative optimization technique: at each sampling time k , measure or estimate the current state, then obtain the optimal input vector by solving a real-time optimization problem. For model (5), the following cost function is commonly used in MPC for calculation:

$$\begin{aligned} J(k) &= \sum_{j=1}^N \|r(k+j|k) - y(k+j|k)\|_Q^2 \\ &\quad + \sum_{j=0}^{N_u-1} \|\Delta u(k+j|k)\|_R^2 \end{aligned} \quad (7)$$

where $r(k+j|k)$ denotes the reference vector for output, $y(k+j|k)$ denotes the predicted output vector, and $\Delta u(k+j|k)$ denotes the input increment vector, $\Delta u(k+j|k) = u(k+j|k) - u(k-1+j|k)$, N and N_u are prediction horizon and control horizon ($N > N_u > 0$), respectively. Q and R are appropriate weighting matrices, $\|\cdot\|$ denotes the Euclidean norm of the corresponding vector. The first term in (7) represents the error between the predicted output and the reference output while the second term considers the control energy. Hence with appropriate N , N_u , Q and R , the cost function (7) can also guarantee closed-loop stability. According to model (5), future state $x(k+j|k)$, $j = 1, 2, \dots, N$ at sampling instant k can be predicted by using the optimal input obtained at previous time instant,

i.e., $u(k+j|k-1), j=1, 2, \dots, N_u$:

$$\begin{aligned} x(k+1|k) &= f(x(k|k-1)) + g(x(k|k-1)) \\ &\quad (u(k-1) + \Delta u(k|k)) \\ x(k+2|k) &= f(x(k+1|k-1)) + g(x(k+1|k-1)) \\ &\quad (u(k-1) + \Delta u(k|k) + \Delta u(k+1|k)) \\ &\vdots \\ x(k+N|k) &= f(x(k+N-1|k-1)) + g(x(k+N-1|k-1)) \\ &\quad (u(k-1) + \dots + \Delta u(k+N_u-1|k)) \end{aligned}$$

Define the following vectors:

$$\begin{aligned} \bar{x}(k) &= [x(k+1|k) \dots x(k+N|k)]^T \in \mathbb{R}^{Nn} \\ \bar{u}(k) &= [u(k|k) \dots u(k+N_u-1|k)]^T \in \mathbb{R}^{N_u m} \\ \bar{y}(k) &= [y(k+1|k) \dots y(k+N|k)]^T \in \mathbb{R}^{Np} \\ \bar{r}(k) &= [r(k+1|k) \dots r(k+N|k)]^T \in \mathbb{R}^{Nn} \\ \Delta \bar{u}(k) &= [\Delta u(k|k) \dots \Delta u(k+N_u-1|k)]^T \in \mathbb{R}^{N_u m} \end{aligned} \quad (8)$$

Then the predicted output $\bar{y}(k)$ can be expressed in the following form:

$$\bar{y}(k) = \tilde{C} (G \Delta \bar{u}(k) + \tilde{f} + \tilde{g}) \quad (9)$$

where

$$\begin{aligned} \tilde{C} &= \begin{bmatrix} C & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C \end{bmatrix} \in \mathbb{R}^{Np \times Nn} \\ G &= \begin{bmatrix} g(x(k|k-1)) & \dots & 0 \\ g(x(k+1|k-1)) & \dots & 0 \\ \vdots & \ddots & \vdots \\ g(x(k+N-1|k-1)) & \dots & g(x(k+N-1|k-1)) \end{bmatrix} \in \mathbb{R}^{Nn \times N_u m} \\ \tilde{f} &= \begin{bmatrix} f(x(k|k-1)) \\ f(x(k+1|k-1)) \\ \vdots \\ f(x(k+N-1|k-1)) \end{bmatrix} \in \mathbb{R}^{Nn} \\ \tilde{g} &= \begin{bmatrix} g(x(k|k-1))u(k-1) \\ g(x(k+1|k-1))u(k-1) \\ \vdots \\ g(x(k+N-1|k-1))u(k-1) \end{bmatrix} \in \mathbb{R}^{Nn} \end{aligned}$$

hence, the original optimization problem (7) becomes:

$$\begin{aligned} \min \quad & \left\| \bar{r}(k) - \tilde{C} (G \Delta \bar{u}(k) + \tilde{f} + \tilde{g}) \right\|_Q^2 + \|\Delta \bar{u}(k)\|_R^2 \\ \text{s.t.} \quad & \Delta \bar{u}_{\min} \leq \Delta \bar{u}(k) \leq \Delta \bar{u}_{\max}, \\ & \bar{u}_{\min} \leq \bar{u}(k-1) + \tilde{I} \Delta \bar{u}(k) \leq \bar{u}_{\max}, \\ & \bar{x}_{\min} \leq \tilde{f} + \tilde{g} + G \Delta \bar{u}(k) \leq \bar{x}_{\max}, \\ & \bar{y}_{\min} \leq \tilde{C} (\tilde{f} + \tilde{g} + G \Delta \bar{u}(k)) \leq \bar{y}_{\max}, \end{aligned} \quad (10)$$

where

$$\tilde{I} = \begin{bmatrix} I & 0 & \dots & 0 \\ I & I & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \dots & I \end{bmatrix} \in \mathbb{R}^{N_u m \times N_u m}.$$

Problem (7) can be rewritten as a time-varying quadratic programming (QP) problem:

$$\begin{aligned} \min \quad & \frac{1}{2} \Delta \bar{u}^T W \Delta \bar{u} + c^T \Delta \bar{u}, \\ \text{s.t.} \quad & l \leq E \Delta \bar{u} \leq h. \end{aligned} \quad (11)$$

where the coefficients are:

$$\begin{aligned} W &= 2 \left(G^T \tilde{C}^T Q \tilde{C} G + R \right) \in \mathbb{R}^{N_u m \times N_u m} \\ c &= -2 G^T \tilde{C}^T Q \left(\bar{r}(k) - \tilde{C} \tilde{g} - \tilde{C} \tilde{f} \right) \in \mathbb{R}^{N_u m} \\ E &= \begin{bmatrix} -\tilde{I} & \tilde{I} & -G & G & -\tilde{C} G & \tilde{C} G & I \end{bmatrix}^T \\ &\in \mathbb{R}^{(3N_u m + 2Np) \times N_u m} \\ b &= \begin{bmatrix} -\bar{u}_{\min} + \bar{u}(k-1) \\ \bar{u}_{\max} - \bar{u}(k-1) \\ -\bar{x}_{\min} + \tilde{f} + \tilde{g} \\ \bar{x}_{\max} - \tilde{f} - \tilde{g} \\ -\bar{y}_{\min} + \tilde{C} \tilde{f} + \tilde{C} \tilde{g} \\ \bar{y}_{\max} - \tilde{C} \tilde{f} - \tilde{C} \tilde{g} \end{bmatrix} \in \mathbb{R}^{2N_u m + 2Np} \\ l &= \begin{bmatrix} -\infty \\ \Delta \bar{u}_{\min} \end{bmatrix} \in \mathbb{R}^{3N_u m + 2Np} \\ h &= \begin{bmatrix} b \\ \Delta \bar{u}_{\max} \end{bmatrix} \in \mathbb{R}^{3N_u m + 2Np}. \end{aligned}$$

The solution to the QP problem (11) gives optimal control increment vector $\Delta \bar{u}(k)$ whose first element $\Delta u(k)$ can be used to calculate the optimal control input.

III. NEURODYNAMIC OPTIMIZATION

In [23], a one-layer projection neural network is developed for solving convex optimization problems. It has been shown good performance and low computational complexity. Its dynamic equation can be described as follows:

$$\varepsilon \frac{du}{dt} = \Lambda [P_X (G(u) - F(u)) - G(u)] \quad (12)$$

where $u \in \mathbb{R}^{N_u \times m}$ is the input vector, $F(u)$ and $G(u)$ are continuously differentiable vector-valued functions, P_X is a projection operator, ε is a positive constant and Λ is a scaling matrix. According to the piecewise formulation in [14], we define $\Phi = EW^{-1}E^T$, $q = -EW^{-1}c$, $\Psi = W^{-1}E^T$, $a = -W^{-1}c$. Furthermore, $G(u) = \Phi \Psi^+(u-a) + q$, $F(u) = \Psi^+(u-a)$ and $\Lambda = \Psi^+$, where Ψ^+ is the pseudo-inverse of Ψ . Thus (12) can be expressed as:

$$\varepsilon \frac{du}{dt} = -\Lambda [\Phi \Psi^+(u-a) - h(\Phi \Psi^+(u-a) - \Psi^+(u-a) + q) + q]$$

where

$$h(x_i) = \begin{cases} l_i, & x_i < l_i; \\ x_i, & l_i \leq x_i \leq h_i; \\ h_i, & x_i > h_i; \end{cases} \quad (13)$$

According to the convergence analysis in [23], it is Lyapunov stable and globally convergent to the optimal solution of any strictly convex QP problem and compared with the simplified neural network, the number of neurons in the general projection neural network is much smaller. So we employ the general projection neural network here for solving (11) repetitively. The MPC scheme of the AUV control based on the general projection neural network is summarized as follows:

- 1) Let $k = 1$, Set control time terminal T , prediction horizon N , control horizon N_u , sample period T_s , weight matrices Q and R ;
- 2) Calculate process model matrices $G, \tilde{f}, \tilde{g}, \tilde{C}$ and neural network matrices $W, c, E, b, \Phi, \Psi, q, a$;
- 3) Solve the convex quadratic minimization problem (11) by using neural network (12) to obtain the optimal control action $\Delta \bar{u}(k)$;
- 4) Calculate the optimal input vector $\bar{u}(k)$ and implement the first element $u(k|k)$;
- 5) If $k < T$, set $k = k + 1$, go to Step 2; otherwise the iteration ends.

IV. SIMULATION RESULTS

In this section, simulation results are discussed to demonstrate the effectiveness of the proposed MPC scheme for the AUV control based on both its kinematics and dynamics. The AUV is supposed to move in the vertical plane to a set-point $(x, z) = (6, 3)$ with the orientation $\theta = \pi/6$, and the other three state variables (u, w, q) become zero when the vehicle arrives the desired point, thus the control input (F_u, F_w) returns to zero and Γ_q maintains a certain value to keep the vehicle's orientation. The initial inputs are $(F_u, F_w, \Gamma_q) = (50, 30, -2.6)$ and the initial position and orientation are $(x, z, \theta, u, w, q) = (1, 1, \pi/12, 1, 1, 1)$, the output matrix is $C = I$. Both prediction horizon N and the control horizon N_u are 10, $Q = 10I$, $R = 0.1I$ and the sample time $T_s = 0.1s$. For the discrete-time model (5), we have the following $g(x)$ and $f(x)$:

$$g(x) = T_s \begin{bmatrix} \mathbf{0} & 0 & 0 \\ 0.01793 & 0 & 0 \\ 0 & 0.0098 & 0 \\ 0 & 0 & 0.3458 \end{bmatrix} \in \mathbb{R}^{6 \times 3}$$

$$f(x) = \begin{bmatrix} x(k) \\ z(k) \\ \theta(k) \\ u(k) \\ w(k) \\ q(k) \end{bmatrix}^T + T_s \begin{bmatrix} u(k) \cos \theta(k) + w(k) \sin \theta(k) \\ -u(k) \sin \theta(k) + w(k) \cos \theta(k) \\ q(k) \\ u_1 u(k) |u(k)| - u_2 u(k) w(k) \\ w_1 u(k) q(k) + w_2 w(k) |w(k)| + w_3 u(k) w(k) + w_4 q(k)^2 \\ q(k) (q_1 |q(k)| + q_2 u(k) + q_3 w(k)) + q_4 u(k) w(k) + q_5 \sin \theta(k) \end{bmatrix}^T$$

the coefficients involved in $f(x)$ are shown in Table II. The control results are depicted in Figs. 1-3 and the control

TABLE II
THE COEFFICIENTS

u_x	w_x	q_x
		$q_1 = -6.9156$
	$w_1 = 0.5$	$q_2 = -1.1823$
	$w_2 = -3.4518$	$q_3 = -0.0308$
$u_1 = -0.0717$	$w_3 = -0.4019$	$q_4 = 0.3554$
$u_2 = 0.909$	$w_4 = 0.0087$	$q_5 = -0.2476$

actions are plotted in Fig. 4. The results show that with a proper input, the AUV can reach a set-point with a satisfactory precision.

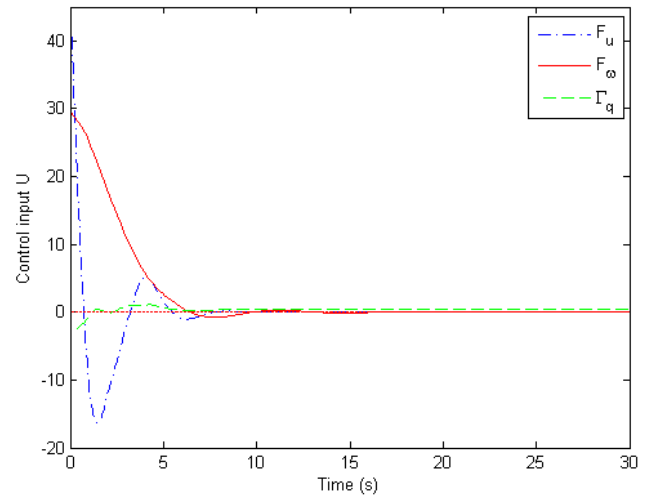


Fig. 1. Control force

V. CONCLUSIONS

This paper presents an MPC approach to steering a class of autonomous underwater vehicles in the vertical plane to a certain point. Based on an AUV model in the vertical plane, the MPC problem is formulated as a time-varying quadratic optimization problem which can be repeatedly solved by using a single-layer globally convergent recurrent neural network called the projection neural network. Simulation results show that the proposed method is able to control the AUV in the vertical plane with a good performance. The three-dimensional control of AUVs deserves further investigations.

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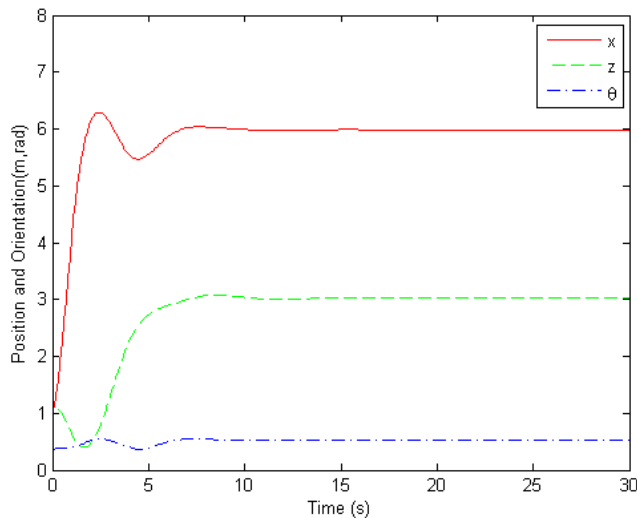


Fig. 2. Position and Orientation

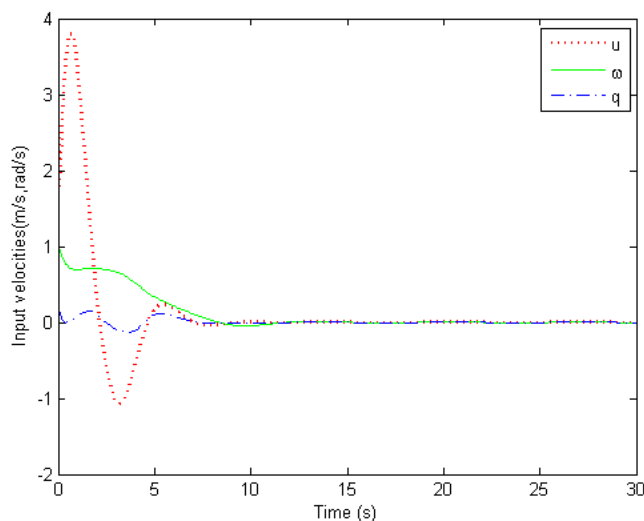


Fig. 3. Translational and angular velocities

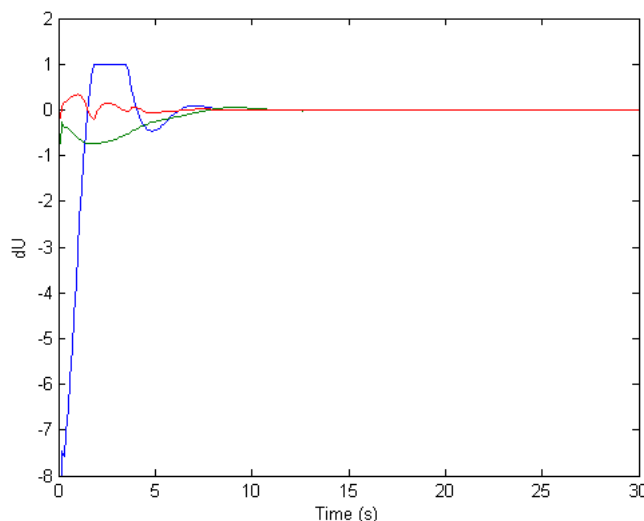


Fig. 4. Input action

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