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STDP Learning Rule Based on Memristor with STDP Property

Ling Chen, Chuandong Li, Tingwen Huang, Xing He, Hai Li and Yiran Chen

***Abstract*—Spike-timing-dependent plasticity (STDP) learning ability has been observed in physical memristors, but whether the STDP is caused by the neuron or the memristor is unclear. In this paper, we proved the STDP property in the model for both symmetric and asymmetric memristor. We also employed the symmetric/asymmetric memristors with STDP property and the simplified neurons to perform the STDP learning ability. At last, the sequence learning experiment of the memritive neural network (MNN) with the symmetric memristor synapse further verifies the STDP learning ability of the memristor.**

I. Introduction

A nanoscale, two-terminal device emulating plasticity and energy efficiency of biological synapses is a critical element for realizing brain-inspired computational systems and real-time brain simulators. And memristor, as a nanoscale nonvalatile memory element, is a natural device for synapse in artificial neural network. There have been many groups working on the use of memristor to build electronic synapses which implement synaptic plasticity with picojoule level energy consumption. In [1], Persin et al. developed a memristor emulator to build a simple memristive neural network (MNN) to complete the classical Pavlov experiment and proved the associate memory of the MNN. In [2], Kim et al. built a memristor bridge synapse to improve the traditional synapse circuit structure and exhibit the ability of image learning of the MNN. In [3] Liu et al. took memristor as synapse in BP neural network for noise eliminating training and pattern recall rate improving. Besides serving as synapse, memristor has a much more widely application due to their special memory and nonlinear properties. In [4], Kim et al. built a physical memristor crossbar array to store binary image and 10-level gray image, and the result verified the ability of memristor serving as a new type of memory material. In [5] and [6], memristor as the forth basic circuit element device, brought a new perspective for the oscillating circuit, it also brought new blood for the dynamical of nonlinear systems because of the nonlinear variation behavior as [7] exhibited.

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However, since the first physical memristor was found by the HP labs in 2008[8], memristor has not yet been popular even though there are more and more researchers working on it, this is because the properties of memristors are highly affected by the material, size and fabricating process, for example, the behavior of TiO2 memristor[9] is major different from the WOx memristor[10]. It is difficult to produce the uniform memristor in industry, so memristor is still a device stays in lab. But along with the found of memristor with different materials, more and more distinctive properties of memristor are observed, such as the short-term/long-term memory[11], the bipolar/unipolar property[12], and the Spike-timing-dependent plasticity (STDP) learning ability[13]. These new properties make memristor become hotter and hotter, but also more and more complex for investigation[14].

In general, the STDP learning rule, which adjusts the connection strengths between neurons based on the relative timing, serves as a supplement learning rule to the classical Hebb learning rule. There are many different forms for STDP learning rule as Fig.1 shows[15], in (b) and (d) the weight increases or decreases only deterred by the relative time, which is similar with the HEBB learning rule but is described “fire more closely, wire more strongerly” , not “fire together, wire together” . (a) and (c) is a kind of more general form for STDP , the weight increases or decreases are deterred by both the relative firing time of the presynaptic and post-synaptic neurons and the firing sequence of them. For example, under the STDP process, if the pre-synaptic neuron fires earlier than the post-synaptic neuron, the connection will get stronger (Long-term potentiation), otherwise, the connection will get weaker (Long-term depression), hence, it’s “spike-timing-dependent plasticity”. In [16], Masquelier et al. compared the STDP learning rule with the HEBB learning rule for image learning and feature extraction, and proved the validation of the STDP learning rule. In [17], the STDP phenomenon similar with (a) and (c) is also observed in memristor. In this paper, we focus on the STDP property of memristor and analysis the difference of the STDP rule in symmetric and asymmetric memristors. We prove the STDP property of a memristor model and apply it to a MNN for the sequence learning ability for forecasting function.

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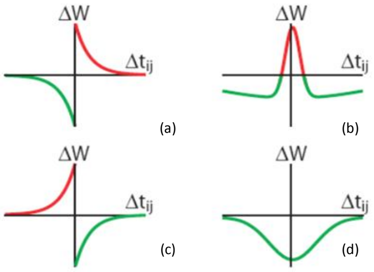


Fig. 1 The different forms of SDTP learning rule, image comes from [15].

II. SYMMETRIC/ASYMMETRIC MEMRISTOR WITH STDP PROPERTY

A. The memristor model

Memristor is firstly proposed by Leon Chua based on the completeness of circuit theory in 1971[18], it is a memory device that record the history of the passing through current, it defined the relationship between charge and flux as $R_m = dq/d\varphi$; and it is expanded as memristive system to be $x = f(\varphi, q)$, $i = f(v)$. For describing different properties of memristor, different models are proposed[1][7][9][10][19]-[22], for example, the HP memristor model describes the nonlinear property of memristor[9], the threshold memristor model describes the threshold property of memristor[1], the WOx model describes the asymmetric property of memristor[10], the compact model describes all the nonlinear, threshold, and asymmetric properties, but it is too complex for analysis and application[19]. Here a simplified model of memristor is improved from [20], compared with the previous models, it may not be so powerful in fitting with the datas of physical memristors, but it can also describe the nonlinear, threshold and symmetric/asymmetric properties of memristor, what's more, it can exhibit the STDP property. The model of memristor is described as:

$$\dot{x} = \begin{cases} -av^2 f(x), v > v_{th} \\ 0, -v_{th} < v < v_{th} \\ bv^2 f(x), v < -v_{th} \end{cases} , \tag{1}$$

where $a > 0, b > 0, f(x)$ is the window function , x is the inner state which belongs to (0,1). Since the memristance is dependent on a tunneling effect, which is highly nonlinear, any change in the tunnel barrier width (the inner state x) changes the memristance, and is assumed to change in an exponential manner. The relationship between memristance R_m and the inner state x becomes:

$$R_m = R_{on} e^{\lambda x} , \tag{2}$$

where λ is a fitting parameter, R_{on} and R_{off} are the equivalent effective resistance at the bounds, similar to the notation in the linear ion drift model, and satisfy $\lambda = \log(R_{off}/R_{on})$, note that here x increases while memristance increases, decreases while memristance decreases, so the positive voltage will decrease both x and memristance R_m while the negative voltage will increase both. The current i passes through the memristor is:

$$i = v/R_m \quad . \tag{3}$$

We build the Simulink model to perform the behavior of the memristor, it consists of the voltage source, the integrator and the window function as shown in Fig. 2:

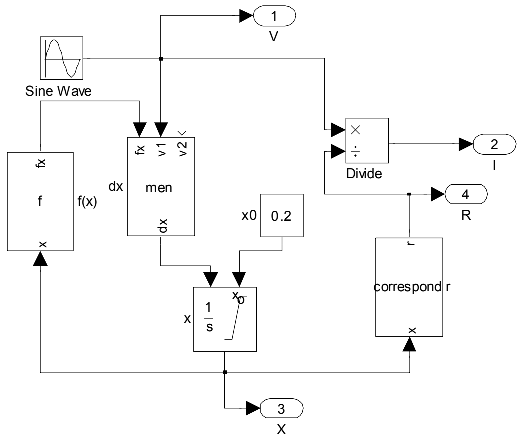
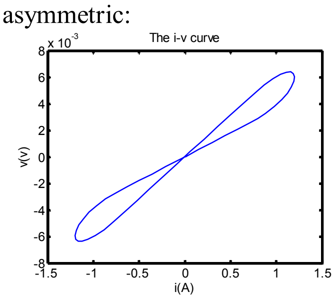
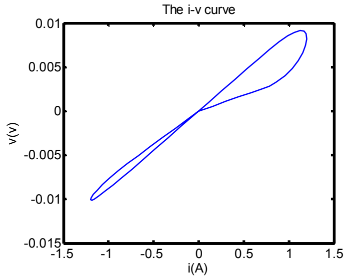


Fig. 2 The Simulink of the memristor model

Set the initial inner state value $x_o = 0.2$, $R_{on} = 100\Omega$, $R_{off} = 16000\Omega$, $f(x) = 1 - \left((x-0.5)^2 + 0.75\right)^2$, $v_{th} = 0.8$, $a = 0.5, b = 0.5$ for the symmetric memristor and $a = 0.8, b = 0.5$ for the asymmetric memristor. Apply a sinusoidal voltage with amplitude $v = 1.2\text{V}$, frequency $w = 0.25\pi$ to the memristor, the simulation results of the classical hysteresis loop are shown in Fig. 3(a) and Fig. 3(b) respectively, it is obvious that the i-v curve in Fig. 3(b) is asymmetric:



(a)



(b)

Fig. 3 (a) The simulation result of the symmetric memrsitor. (b) The simulation result of the symmetric memrsitor.

B. The STDP property of the memristor

In this section, we prove the STDP property of the memristor model proposed in this paper. Assume the relative spiking time between the pre-synaptic and post synaptic neurons are:

$$t = \left| t_{post} - t_{pre} \right|. \tag{4}$$

Divide the relative time t into three parts, that is:

$$\begin{cases} (0, t_1), v > v_{th} \\ (t_1, t_2), -v_{th} < v < v_{th} \\ (t_2, t), v < -v_{th} \end{cases}. \tag{5}$$

Integrate (1), assume $v^2 f(x)$ as a constant value and get:

$$\begin{aligned} x &= \int_0^{t_1} a v^2 f(x) dt + \int_{t_1}^{t_2} 0 dt + \int_{t_2}^{t} b v^2 f(x) dt \\ &= a' t_1 + b'(t - t_2) \\ &= \begin{cases} a' t_1, if, t_{pre} < t_{post} \\ b'(\Delta t - t_2), if, t_{pre} > t_{post} \end{cases} \end{aligned}. \tag{6}$$

$a' \cong a v^2 f(x), b' \cong b v^2 f(x)$. (2) can be rewritten as:

$$R_m = \begin{cases} R_{on} e^{\lambda a' t_1}, \text{if}, t_{pre} < t_{post} \\ R_{on} e^{\lambda b'(t - t_2)}, \text{if}, t_{pre} > t_{post} \end{cases}. \tag{7}$$

The derivative of R_m is:

$$\begin{aligned} \dot{R}_m &= \begin{cases} a' R_{on} e^{a' t_1}, if, t_{pre} < t_{post} \\ b' R_{on} e^{b'(t - t2)}, if, t_{pre} > t_{post} \end{cases} \\ &\cong \begin{cases} A_+ e^{(\Delta t / \tau_+)}, if, t_{pre} < t_{post} \\ A_- e^{(-\Delta t / \tau_-)}, if, t_{pre} > t_{post} \end{cases} \end{aligned}, \tag{8}$$

where $A_+ = a' R_{on}, A_- = b' R_{on}, 1/\tau_+ = a', 1/\tau_- = b'$. Both t_1 and t_2 are closely related to the relative time Δt, so the variation of memristance will depend on the relative spiking time.

III. NEURON NETWORK FOR THE STDP

A. The neuron model

Neuron and synapse are the basic components of artificial neural network. Neuron is excited by the input signal of synapses. It processes and transmits information through the electrical signal. A typical neuron possesses a cell body, multiple dendrites and an axon which may branch hundreds of times before it terminates. Electrical signals (the pre-spike and the post-spike) are sent from the axon of one pre-synaptic neuron to a dendrite of one postsynaptic neuron as Fig. 4 shows.

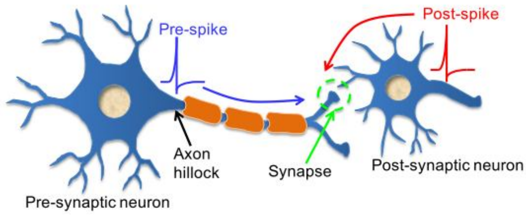


Fig. 4 The structure between neurons, image comes from [25].

The neuron will transmit an action potential when membrane potential surpass a threshold, but the exact shape of action potential, among neuroscientists, is difficult to measure precisely since the experimental setup influences strongly. Furthermore, different action potential shapes have been recorded for different types of neurons, although in general they all display a certain resemblance. For our discussion, it suffices to assume a generic action potential shape with the following properties. During spike on-set, membrane voltage increases to a positive pulse amplitude F_+. After this, it changes quickly to a negative pulse amplitude F_- and returns to its resting potential. A shape of the type shown in Fig. 5 (N1 and N2) can be expressed mathematically, as

$$\begin{cases} F_+\left(\sum i\right) = \frac{\gamma}{1 + e^{\sum i}} \\ F_-\left(\sum i\right) = -\frac{\sigma}{1 + e^{\sum i}} \end{cases}. \tag{9}$$

B. The STDP behavior of the MNN

The mechanism for STDP learning rule is less well understood, in [23], Duygu Kuzum et al. studied the STDP property of memristor itself and claimed that the asymmetric memristor has the ability of the STDP learning ability while the symmetric not. In some references[13][24], the STDP property is explained as the overlap of the forward pulse of the pre-synaptic neuron and the feedback voltage of the postsynaptic neuron. In this case, whatever the type of memristor is, they will be always with the STDP property. In the above, we have proved the intrinsic STDP property in both symmetric and asymmetric memristor. Here in order to make the intrinsic STDP property of memristor controlled by the neuron, we combined the memristor with STDP property and the neuron with overlap signal together to study the STDP behavior of the MNN.

Because both the pre-synaptic action potential and the post synaptic action potential are worked on the synapse memristor, we assume that the positive voltage works on memristor is v_+, the negative voltage works on memristor is v_-, thus, the dynamic of memristor will become:

$$\dot{x} = \begin{cases} a v_+^2 f(x), v > v_{th} \\ 0, -v_{th} < v < v_{th} \\ b v_-^2 f(x), v < -v_{th} \end{cases} . \quad (10)$$

It can be described by the signal of the S in Fig. 5(a) where N1 is the signal transited from the pre-synaptic neuron, N2 is the signal transited from the postsynaptic neuron. From (10), we can find that the dynamic of memristor is affected by both the memristor itself and the voltage working on it. Therefore, we can control the STDP learning of the MNN by adapting the amplitude of the action potentials of pre-synaptic neuron and post synaptic neuron as Fig. 5(b) shows. Besides, we can obtain the assumptions as follow:

- 1, The symmetric memristor will leads to symmetric STDP learning rule, if $a = b$, $v_+ = -v_-$.
- 2, The asymmetric memristor will leads to asymmetric STDP learning rule, if $a \neq b$, $v_+ = -v_-$.
- 3, The asymmetric memristor will leads to symmetric STDP learning rule, if $a \neq b$, $v_+ \neq -v_-$, and $a v_+^2 = b v_-^2$.

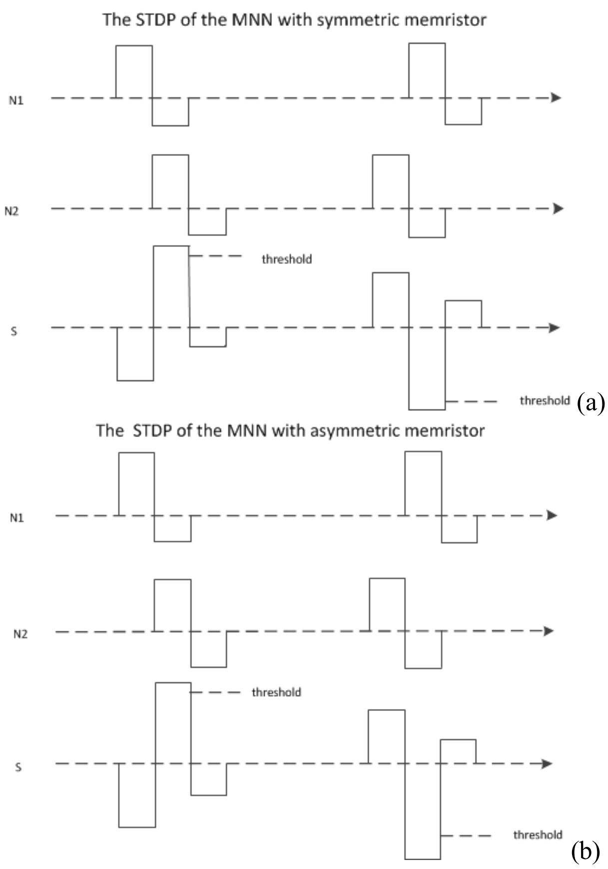


Fig. 5 The STDP mechanism (a) The STDP of the MNN with symmetric memristors. (b) The STDP of the MNN with asymmetric memristors.

C. The Simulink of the STDP of the MNN

We simulate the MNN to verify the conclusions in the above as shown in Fig. 6. It consists of the memristor proposed in Section 2 and the action potentials of pre- and post- synaptic neurons.

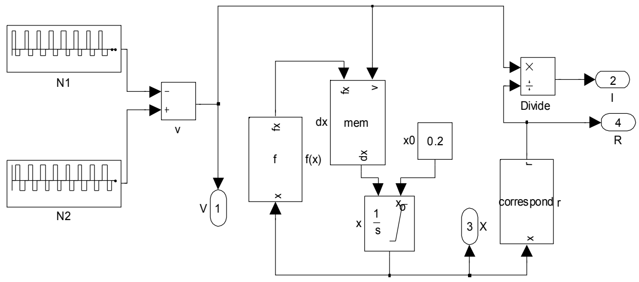


Fig. 6 The Simulink of the STDP of the MNN

When the MNN is with symmetric memristor used in the above and symmetric neuron which $v_+ = -v_- = 0.9\mathrm{V}$ (both pre- and post- synaptic neuron are with $F_+ = 0.6\mathrm{V}$, $F_- = -0.3\mathrm{V}$, the period of the action potential is 1s) , the inner state x increases and decrease as shown in Fig. 7(a), and we can see that the corresponding STDP form in Fig. 7(b) is ideal symmetric form like the third one in Fig. 1. When the MNN is with asymmetric memristor and symmetric neuron which $v_+ = -v_- = 0.9\mathrm{V}$, the corresponding STDP form is not ideal symmetric form as shown in Fig. 7(c), but if we replace the symmetric neuron with asymmetric neurons that $v_+ = 0.9, v_- = -1.1\mathrm{V}$ ($F_+ = 0.7\mathrm{V}$, $F_- = -0.3\mathrm{V}$ in presynaptic neuron, $F_+ = 0.6\mathrm{V}$, $F_- = -0.4\mathrm{V}$ in postsynaptic neuron), the STDP form can be recovered to be symmetric one as Fig. 7(d) shows. The simulation results of Fig. 7 verify the assumptions in Part B.

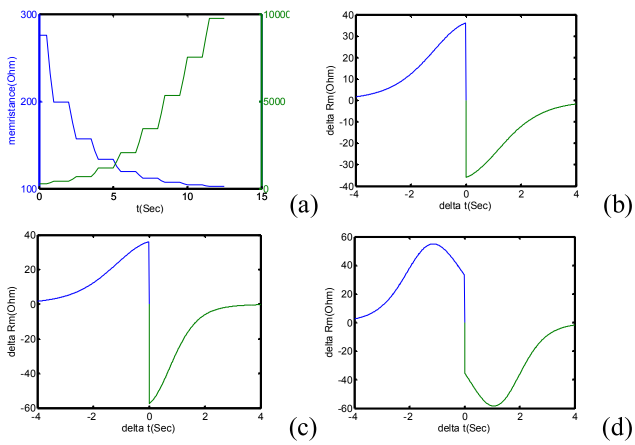


Fig. 7 (a) The plasticity of memristance. (b) The symmetric STDP form of the MNN with symmetric memristor and symmetric neuron. (c) The asymmetric STDP form of the MNN with asymmetric memristor and symmetric neuron. (d) The symmetric STDP form of the MNN with asymmetric memristor and asymmetric neuron.

IV. THE APPLICATION OF THE MNN WITH THE STDP

In this section, we build a Hopfield-like recurrent MNN with the STDP learning rule for Sequence learning[25][26]. In the MNN, N neurons are full connected with each other by $N \times N$ memristor synapses as shown in Fig. 8[25]. Here $N = 25$, and the synaptic weight matrix (SWM) is randomly initialized. We divide all the neurons into five groups G1 to G5, every five neurons is a group. G1 to G5 are stimulated with 0.05s timing difference. Fig. 9 is the change of the SWM after 10 training epochs in the network of 25 neurons and 625 synapses. Fig. 9(a) and Fig. 9(b) are the SWMs with symmetric and asymmetric memristor synapse respectively. The color corresponds to the relative change in synaptic strength. The red means the increase memristance, and the blue is corresponding to the decrease memristance. From Fig. 9, it is obvious that synaptic weights are adjusted according to stimulation sequence, for example, synapse connections between G1-G2 get stronger and stronger (the blue one) while G1-G3, G1-G4, G1-G5 do not, synapse connections between G2-G1 get weaker and weaker while G2-G3, G2-G4, G2-G5 do not. The same phenomenon is also occurred in G2-G3, G3-G4, G4-G5. The connections between other nonadjacent groups will not be affected by the input signals. Thus, a trained MNN can forecast the sequence of the next group neuron if a group of signals is input, i.e., if we input signals in G1, the MNN will active G2 as a forecast. Both the learning results of the MNN with symmetric and asymmetric memristor are verified the effectiveness of the MNN consisting with memristor with STDP property, the only difference is that the synaptic strength of the MNN with asymmetric memristor in Fig. 9(b) will get stronger than the MNN with symmetric memristor in Fig. 9(a), which is good for strengthening associating connections. Because of the limitation of Hopfiled-like neural network, all the neurons here are the same kind, that is, symmetric ones, so the STDP learning rule of the memristor can't be controlled by the neurons. If we change the type of the MNN, such as a feed forward neural network, we may use the asymmetric neurons to control the learning effect of the SWM to get a better performance, and this is what we will do in the future.

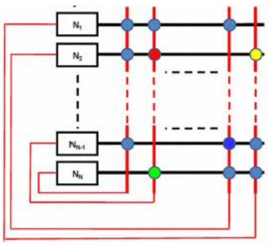
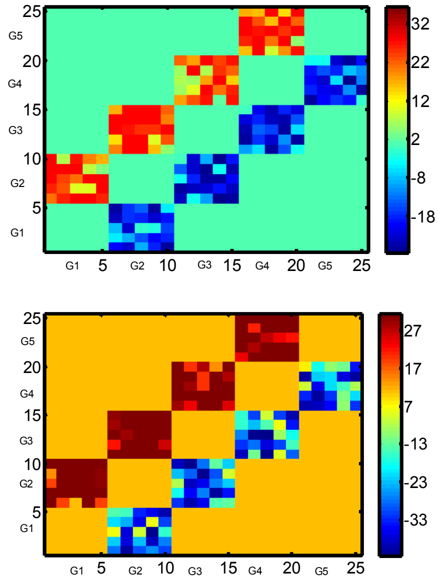


Fig. 8 The structure of the MNN, the different color of synaspes means different memristance. Image comes from [25].



(a)

(b)

Fig. 9 The sequence learning. (a) The MNN with symmetric memristor. (b) The MNN with asymmetric memristor.

V. CONCLUSION

In this paper, we simplified and proved a memristor model with STDP property for both symmetric and asymmetric memristor. Based on the developed memristor model, we studied the STDP learning rule and built a MNN with sequence learning ability to verify the function of the MNN consisting of the memristor with STDP property. However, because all the neuron in the Hopfield neural network is the symmetric neurons, the function of the MNN with asymmetric neuron needs another kind of experiment to support it, and we will do it in our next work.

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REFERENCES

[1] Y. V. Pershin, M. D. Ventra, "Experimental demonstration of associative memory with memristive neural networks," Neural Networks, vol. 23, no. 7, pp.881–886. May. 2010.

[2] H. Kim, and M.P. Sah, and C. Yang, and T. Roska, and L.O. Chua, "Memristor Bridge Synapses" Proceedings of the IEEE, vol.10, pp.2061-2070, 2012.

[3] B. Y. Liu, M. Hu, H. Li, Z. H. Mao, Y. Chen, T. Huang, W. Zhang, "Digital-assisted Noise-eliminating Training for Memristor

Crossbar-based Analog Neuromorphic Computing Engine," Design Automation Conference, pp.1-6, 2013.

[4] K. H. Kim, S. Gaba, D. Wheeler, J. M. Cruz-Albrecht, T. Hussain, N. Srinivasa, and W. Lu, "A Functional Hybrid Memristor Crossbar-Array/CMOS System for Data Storage and Neuromorphic Applications," Nano Letters, vol. 12, no. 3, pp.389-395, Dec. 2011.

[5] M. Itoh, L. O. Chua, "Memristor oscillators," International Journal, vol. 18, no. 11, pp. 3183-3206, 2008.

[6] L. Chen, C. Li, T. Huang, X. Wang, "Quick noise-tolerant learning in a circuit memristive neural network," neurocomputing, http://dx.doi.org/10.1016/j.neucom.2013.05.043. ,2013,

[7] L. Chen, C. Li, T. Huang, Y. Chen, S. Wen, J. Qi, "A synapse Memristor Model with Forgetting Effect," Physics Letters A, vol. 377, pp. 3260-3265, 2013.

[8] D. B. Strukov, G. S. Snider, D. R. Stewart and R. S. Williams, "The missing memristor found," Nature, vol. 452, no. 7191, pp. 80-83, May 2008.

[9] Z. Biolek, D. Biolek, V. Biolkova, "SPICE model of memristor with nonlinear dopant drift," Radio Engeering, vol. 18, no.2, pp. 210–214, 2009.

[10] T. Chang, S. J. Kim, P. Sheridan, S. Gaba, W. Lu, "Synaptic behaviors and modeling of a metal oxide memristive device," Applied Physics, vol. 102, pp. 857-863, 2011.

[11] T. Ohno, T. Hasegawa, T. Tsuruoka, K. Terabe, J. Gimzewski, "Short-term plasticity and long-term potentiation mimicked in single inorganic synapses". Nature Materials. Vol. 10, no.8, pp. 591-595, 2011.

[12] S. Buhm, S. Hyoung, H. Keun, *et al*, "Reversible changes between bipolar and unipolar resistance-switching phenomena in a Pt / SrTiOx / Pt cell," Current Applied Physics, vol. 12, no. 6, pp. 1515-1517, 2012.

[13] Z. Q. Wang, H. Y. Xu, X. H. Li, et al, "Synaptic Learning and Memory Functions Achieved Using Oxygen Ion Migration / Diffusion in an Amorphous InGaZnO Memristor," Advanced Function Material, vol. 22, pp. 2759-2765, 2012.

[14] S. Wen, Z. G. Zeng, T. W. Huang, "Exponential stability analysis of memristor-based recurrent neural networks with time-varying delays," Neurocomputing, vol. 97 pp. 233-240, 2012.

[15] Neurosci J, "Spike timing dependent plasticity – STDP," Test. 1998.

[16] T. Masquelier and S. J. Thorpe, "Unsupervised Learning of Visual Features through Spike Timing Dependent Plasticity," PLoS computational biology, vol. 3, no. 2, e31, pp. 247-257, Feb. 2007.

[17] T. Chang, S. Jo, W. Lu, "Short-Term Memory to Long-Term Memory Transition in a Nanoscale Memristor," ACS Nano, vol. 5, no. 9, pp. 7669-7676, 2011.

[18] L. O. Chua, "Memristor: the missing circuit element," IEEE Transactionson Circuit Theory, vol.18, no. 5, pp. 507-519, Sep. 1971.

[19] C. Yakopcic, S. Member, T. M. Taha, et al, "A Memristor Device Model," IEEE Electron device letters, vol. 32, no. 10, pp. 1436-1438, Oct. 2011.

[20] S. Kvatinsky, E. Friedman, A. Kolodny, U. Weiser. "TEAM: ThrEshold Adaptive Memristor Model". IEEE Transactions on Circuits and Systems, 2013;60(1):211-221.

[21] M. Hu, H. Li, Y. Chen, X. Wang, R.E. Pino, "Geometry Variations Analysis of TiO2 Thin-Film and Spintronic Memristors", Proceedings of the 16th Asia and South Pacific Design Automation Conference,25-30,2011

[22] D. Wang, Z. Hu, X. Yu, J. Yu, "A PWL Model of Memristor and Its Application Example," vol. pp.932-934, 2009.

[23] A. Williamson, L. Schumann, L. Hiller, *et al*, "Synaptic behavior and STDP of asymmetric nanoscale memristors in biohybrid systems," Nanoscale, vol. 5, pp. 7297-7303, 2013.

[24] C. Z. Ramos, L. A. C. Mesa, J. A. P. Carrasco, et al, "On spike-timing-dependent-plasticity, memristive devices , and building a self-learning visual cortex," Frontiers in Neuroscience, vol. 5, no. 26, pp. 1-22, 2011.

[25] D. Kuzum, R. G. Jeyasingh, H. P. Wong, "Energy Efficient Programming of Nanoelectronic Synaptic Devices for Large-Scale Implementation of Associative and Temporal Sequence Learning," IEDM, pp. 693-696, 2011.

[26] B. L. Jackson, B. Rajendran, G. S. Corrado, M. Breitwisch, "Nanoscale electronic synapses using phase change devices," ACM Journal on Emerging Technologies in Computing, vol. 9, no. 12, article 12, 2013.