2014 International Joint Conference on Neural Networks (IJCNN)
July 6-11, 2014, Beijing, China

# Adaptive Output Feedback Control for Cooperative Dynamic Positioning of Multiple Offshore Vessels

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***Abstract*—This paper considers cooperative dynamic positioning (CDP) of multiple offshore vessels in the presence of dynamical uncertainties, time-varying ocean disturbances and unmeasured velocity, aimed at collectively holding a relative formation and reaching a reference position. K-filter observers are first designed to estimate the unmeasured velocity information of each vessel, and then observer based CDP controllers are developed with the aid of dynamic surface control (DSC) technique, neural network and iterative learning approach. The formation among vehicles can be guaranteed if the graph induced by the vessels and the reference point contains a spanning tree. It is proved by Lyapunov analysis that the proposed control laws can ensure that all the signals in the closed-loop systems are uniformly ultimately bounded, and tracking errors converge to a small neighborhood of origin.**

## I. Introduction

Dynamic positioning (DP) system is defined as a set of components used to keep a floating structure at a specific position or to make it follow a pre-determined operation trajectory by means of active thrusters. Several offshore operations use DP systems, such as oil gas exploration, underwater pipe-laying, diving support, mineral drilling, etc. Currently, there are more than 2000 DP vessels operated in different kinds of offshore operations worldwide now [1].

Generally, the studies of DP controllers have experienced three stages. The first generation of DP systems was originated in 1960s, where single-input single-output PID control algorithms in combination with notch filters or low pass filters were used. In 1970s, control techniques based on optimal control and Kalman filter theory were proposed as the second generation of DP systems in [2], and this work was later modified and improved in many literatures such as [3], [4], [5]. In 1990s, with the application of nonlinear control theories, various control strategies were developed for DP systems. The proposed design techniques are ranging from nonlinear feedback linearizable control [6], backstepping control [7], nonlinear sliding model control [8], [9], fuzzy control [10], hybrid control [11], [12] to acceleration feedback control [13]. However, the above techniques are all devoted to single DP systems. To execute more challenging tasks, multiple DP systems are urgently needed in ocean engineering [15], [16]. Relevant applications include mobile nodes of sensor networks, platforms for oil gas exploitations and salvage operations.

DP systems possess many uncertainties in their dynamics such as payload variations, unmodeled hydrodynamics, and time-varying disturbances induced by wind, waves and ocean currents [17]. Therefore, different kinds of adaptive control methods have been suggested [18], [19], [20], [21]. In [20], a robust controller is developed for DP systems with parametric uncertainties. In [21], an exponential observer is devised to compensate for the constant unknown ocean currents. Both the adaptive control methods in [20] and [21] work efficiently under the parametric uncertainties, however, they cannot deal with the time-varying ocean disturbances. On the other hand, in most DP applications, only the position and heading information are available for feedback design [22]. This leads to the study of observers to estimate the unmeasured states which are required to feedback into the control laws, and the literatures on this problem is also rich. Some examples include the Luengberger observer designed in [23], a nonlinear observer proposed in [13], and a passivity-based scheme considered in [24].

Motivated by the above observations, this paper considers the CDP problem of multiple offshore vessels in the presence of dynamical uncertainties, time-varying ocean disturbances and unmeasured velocity, for the purpose of collectively holding a relative formation and reaching a reference position. K-filter observers are first used to tackle the unmeasured velocity information of each vessel, and observer based CDP controllers are proposed by exploiting DSC technique, neural network and iterative learning approach. The designed controllers are obviously different from the traditional DP controllers as only a portion of vessels have access to the reference signals. The stability properties of the closed-loop systems are proved by Lyapunov analysis, and tracking errors converge to a small neighborhood of origin.

The main contributions of the proposed scheme is three-fold. Firstly, K-filter observers are first designed to cope with the unmeasured velocity information of each vessel. Secondly, the problem of "explosion of complexity" inherent in the traditional backstepping method is avoided by applying the DSC technique into the controller design. Thirdly, neural network with iterative learning adaptive laws are proposed to compensate for the model uncertainties and time-varying ocean disturbances, and are easier to implement on digital processors since continuous update laws are replaced by discrete update laws.

This paper is organized as follows. Section II introduces some necessary preliminaries and formulates the problem of CDP in the presence of dynamical uncertainties and time-varying ocean disturbances. Section III proposes K-filter observers to estimate the unmeasured velocity. Section IV gives the design of the observer based CDP controllers. Section V presents the stability analysis of the closed-loop systems. Section VI concludes this paper.

## II. Preliminaries and Problem Formulation

### A. Preliminaries

*1) Notation:* The following notations are used in this paper. $\mathbb{R}^n$ is the n-dimensional Euclidean Space. $||\cdot||$ represents the Euclidean norm. $(\cdot)^T$ and $(\cdot)^{-1}$ denote transpose and inverse, respectively. $diag\{a_i\}$ is a block-diagonal matrix with $a_i$ being the $i$th diagonal element. $\lambda_{min}(\cdot)$ represents the smallest eigenvalue of a square matrix $(\cdot)$. $I_n$ denotes a n-dimension identity matrix. $\otimes$ denotes the Kronecker product.

*2) Graph theory:* Some basic concepts and results on graph theory are introduced. A graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ consists of a node set $\mathcal{V} = \{n_1, ..., n_N\}$ and an edge set $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V}\}$ with $(n_i, n_j)$ describes the communication from node $i$ to node $j$. An adjacency matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ where $a_{ij} = 1$, if $(n_j, n_i) \in \mathcal{E}$; and $a_{ij} = 0$, otherwise. If $a_{ij} = a_{ji}$, the digraph is undirected; otherwise is directed. If $a_{ij} = 1$ then $j \in \mathcal{N}_i$. Define a Laplacian matrix $L$ as $L = D - \mathcal{A}$, where $D = diag\{d_i\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. A *directed path* in the graph is an ordered sequence of nodes such that any two consecutive nodes in the sequence are an edge of the graph. A digraph has a *spanning tree*, if there is a node called as the root, such that there is a directed path from the root to every other node in the graph. Further, define a leader adjacency matrix $\mathcal{A}_0 = diag\{a_{i0}\}$ given by $a_{i0} = 1$, if and only if the $i$th vehicle has access to the information of the leader; and $a_{i0} = 0$, otherwise. Finally, define $H = L + \mathcal{A}_0$.

### B. Problem formulation

Consider a group of $N$ vessels governed by a three degree-of-freedom dynamic model which can be found in [17], and consists of the kinematics

$$\dot{\eta}_i = R(\psi_i)\nu_i, \tag{1}$$

and kinetics

$$M_i\dot{\nu}_i = -C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - g_i(\nu_i) + \tau_i + \tau_{iw}(t), \tag{2}$$

where

$$R(\psi_i) = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{3}$$

$\eta_i = [x_i, y_i, \psi_i]^T \in \mathbb{R}^3$ represents the earth fixed position and heading; $\nu_i = [u_i, v_i, r_i]^T \in \mathbb{R}^3$ represents the vessel-fixed velocity; $M_i = M_i^T \in \mathbb{R}^{3\times3}, C_i(\nu_i) \in \mathbb{R}^{3\times3}, D_i(\nu_i) \in \mathbb{R}^{3\times3}$ are the inertia matrix, coriolis/centripetal matrix, and damping matrix, respectively; $g_i(\nu_i) = [g_{iu}, g_{iv}, g_{ir}]^T \in \mathbb{R}^3$ is the unmodeled dynamics; $\tau_i = [\tau_{iu}, \tau_{iv}, \tau_{ir}]^T \in \mathbb{R}^3$ denotes the control input; $\tau_{iw} = [\tau_{iwu}, \tau_{iwv}, \tau_{iwr}]^T \in \mathbb{R}^3$ denotes the disturbance vector caused by unknown wind, waves and ocean currents.

Consider a reference point $\eta_r \in \mathbb{R}^3$ which acts as a virtual vessel (labeled as $n_0$), then the communication among the $N+1$ vessels is described by an augmented directed graph $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ with $\bar{\mathcal{V}} = \{n_0, n_1, ..., n_N\}$ and $\bar{\mathcal{E}} = \{(n_i, n_j) \in \bar{\mathcal{V}} \times \bar{\mathcal{V}}\}$. Before controllers design, we have the following assumption:

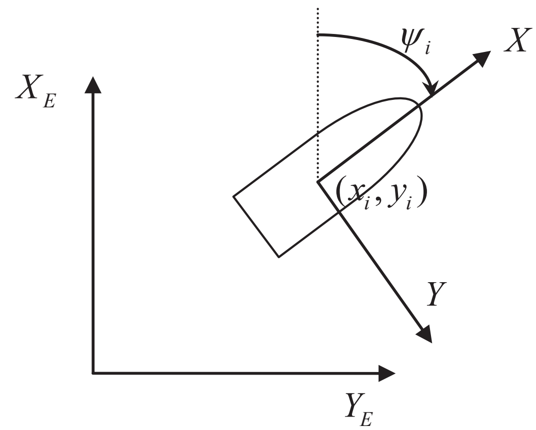


Fig. 1. Reference frames: Earth-fixed and body-fixed

*Assumption 1:* The augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with the root node being the leader node $n_0$.

The *control objective* of this paper is to design an adaptive control law $\tau_i$ for each vessel (1) (2), for the purpose that they can hold a relative formation and track the reference signal $\eta_r$, such that

$$\eta_i - \Delta_i \to \eta_r, i = 1, .., N, \tag{4}$$

with bounded errors. $\Delta_i = [\Delta_{ix}, \Delta_{iy}, \Delta_{i\psi}]^T \in \mathbb{R}^3$ is the desired relative deviation between the $i$th vessel and the reference point.

## III. Observer Design

In order to facilitate the observer design, we first introduce a coordinate transformation

$$\eta_{i1} = R_i^T\eta_i, \tag{5}$$

whose time derivative with (1) is given by

$$\dot{\eta}_{i1} = \nu_i - r_iS\eta_{i1}, \tag{6}$$

where

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

It follows that

$$\begin{cases} \dot{\eta}_{i1} &= \nu_i - r_iS\eta_{i1}, \\ \dot{\nu}_i &= M_i^{-1}[-C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i \\ & \quad -g_i(\nu_i) + \tau_{iw}(t)] + M_i^{-1}\tau_i, \end{cases} \tag{8}$$

which can be reshaped into a matrix form

$$\begin{cases} \dot{X}_i &= A_iX_i + \Phi_i + e_2M_i^{-1}\tau_i, \\ Y_i &= e_1X_i, \end{cases} \tag{9}$$

where

$$X_i = \begin{bmatrix} \eta_{i1} \\ \nu_i \end{bmatrix}; Y_i = \eta_{i1}; A_i = \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix}; \tag{10}$$

$$\Phi_i = \begin{bmatrix} -r_i S \eta_{i1} \\ f_{i1}(\nu_i, t) \\ f_{i2}(\nu_i, t) \\ f_{i3}(\nu_i, t) \end{bmatrix}; e_1 = [I_3, 0]; e_2 = \begin{bmatrix} 0, \\ I_3 \end{bmatrix}. \tag{11}$$

Neural network is employed to approximate the unknown functions, and the following assumption is required.

*Assumption 2:* The unknow functions $f_{in}$ can be linearly parameterized by neural network as

$$f_{in} = W_{in}^{*T}(t)\beta_{in}^*(\rho_i), \forall \rho_i \in \mathcal{D} \tag{12}$$

where $\rho_i = [\hat{\nu}_i, 1]$, and $\hat{\nu}_i$ are the estimation of unknow states, which will be constructed later; $W_{in}^*(t) \in \mathbb{R}^m$ is an unknown weight vector satisfying $||W_{in}^*(t)|| \le M_1$; $\beta_{in}^* : \mathbb{R}^4 \to \mathbb{R}^m$ is a known Lipschitz continuous basis vector of the form $\beta_{in}^*(\rho_i) = [\beta_{in1}^*(\rho_i), \beta_{in2}^*(\rho_i), ..., \beta_{inm}^*(\rho_i)]^T$ satisfying $||\beta_{in}^*|| \le M_2$; $\mathcal{D}$ is a sufficiently large domain.

*Remark 1:* Unlike the DP system considered in [20] which only contains linearly parameterized uncertainty, we consider a more complex situation where the uncertain parts of the nonlinear system is completely unknown. To deal with the modeling uncertainties and time-varying ocean disturbances, a neural network with iterative learning approach is introduced in this paper.

By substituting (12) into (9), system (9) can be expressed in the following form

$$\begin{cases} \dot{X}_i &= A_i X_i + \beta_i^T W_{i0} + e_2 M_i^{-1}\tau_i, \\ Y_i &= e_1 X_i, \end{cases} \tag{13}$$

where

$$\beta_i^T = \begin{bmatrix} -S\eta_{i1} & & & \\ & \beta_{i1}^{*T} & & \\ & & \beta_{i2}^{*T} & \\ & & & \beta_{i3}^{*T} \end{bmatrix}, W_{i0} = \begin{bmatrix} r_i \\ W_{i1}^* \\ W_{i2}^* \\ W_{i3}^* \end{bmatrix},$$

and we suppose that there exists a positive constant $M_0$ such that $\|\dot{W}_{i0}\| \le M_0$. Note that the velocity information cannot be measured directly, the following K-filters are proposed

$$\begin{cases} \dot{\delta}_i = A_{i0}\delta_i + w_i \eta_{i1}, \\ \dot{\Omega}_i = A_{i0}\Omega_i + \beta_i^T, \\ \dot{\vartheta}_i = A_{i0}\vartheta_i + e_2 M_i^{-1}\tau_i, \end{cases} \tag{14}$$

where $\delta_i = [\delta_{i1}^T, \delta_{i2}^T]^T$, $\Omega_i = [\Omega_{i1}^T, \Omega_{i2}^T]^T$, $\vartheta_i = [\vartheta_{i1}^T, \vartheta_{i2}^T]^T$, $w_i = [w_{i1}^T, w_{i2}^T]^T$, and $A_{i0} = A_i - w_i e_1^T$ is Hurwitz by choosing suitable $w_i$.

With the above filters, the designed state estimation is

$$\hat{X}_i = \delta_i + \Omega_i W_{i0} + \vartheta_i. \tag{15}$$

Denote state estimation error vector as

$$\epsilon_i = \hat{X}_i - X_i, \tag{16}$$

where $\epsilon_i = [\epsilon_{i1}^T, \epsilon_{i2}^T]^T$. The time derivation of (16) is given by

$$\dot{\epsilon}_i = A_{i0}\epsilon_i + \Omega_i \dot{W}_{i0}. \tag{17}$$

We now divide the error $\epsilon_i$ into two parts

$$\epsilon_i = \epsilon_{ia} + \epsilon_{ib}, \tag{18}$$

where $\epsilon_{ia}$ satisfies

$$\dot{\epsilon}_{ia} = A_{i0}\epsilon_{ia}, \tag{19}$$

with $\epsilon_{ia}(0) = \epsilon_i(0)$, and $\epsilon_{ib} = \int_0^t e^{A_{i0}(t-s)}(\Omega_i \dot{W}_{i0})ds$. It can be shown that

$$\begin{aligned} ||\epsilon_{ib}|| &\le \int_0^t ||e^{A_{i0}(t-s)}|| \cdot ||\Omega_i|| \cdot ||\dot{W}_{i0}||ds \\ &\le M_0 \int_0^t ||e^{A_{i0}(t-s)}|| \cdot ||\Omega_i||ds \\ &\le M_0 \int_0^t e^{-\lambda_\epsilon(t-s)} k_\epsilon \cdot ||\Omega_i||ds, \end{aligned} \tag{20}$$

where $\lambda_\epsilon$ and $k_\epsilon$ are chosen positive parameters such that

$$k_\epsilon e^{-\lambda_\epsilon t} \ge ||e^{A_0 t}||, \forall t > 0 \tag{21}$$

Thus $\epsilon_{ib}$ satisfies that

$$|\epsilon_{ib}| \le h_i(t) M_0, \tag{22}$$

where $h_i(t)$ is generated by

$$\dot{h}_i = -\lambda_\epsilon h_i + k_\epsilon(||\Omega_i||^2 + \tfrac{1}{4}). \tag{23}$$

To move on, the following donations are needed.

$$\bar{\epsilon}_i = \bar{\epsilon}_{ia} + \bar{\epsilon}_{ib} = \begin{bmatrix} \epsilon_{ia} \\ \epsilon_{1a} \\ \epsilon_{2a} \\ \vdots \\ \epsilon_{Na} \end{bmatrix} + \begin{bmatrix} \epsilon_{ib} \\ \epsilon_{1b} \\ \epsilon_{2b} \\ \vdots \\ \epsilon_{Nb} \end{bmatrix},$$

$$\bar{A}_{i0} = \begin{bmatrix} A_{i0} & & & & \\ & A_{10} & & & \\ & & A_{20} & & \\ & & & \ddots & \\ & & & & A_{N0} \end{bmatrix}.$$

Suppose $P_i$ is a positive definite matrix satisfying $P_i \bar{A}_{i0} + \bar{A}_{i0}^T P_i \le -2I$, and let

$$V_0 = \sum_{i=1}^N \bar{\epsilon}_{ia}^T P_i \bar{\epsilon}_{ia}. \tag{24}$$

From (19), the derivative of $V_0$ is given as

$$\dot{V}_0 \le -2\sum_{i=1}^N ||\bar{\epsilon}_{ia}||^2. \tag{25}$$

From the above filters design, the system (13) can be represented as

$$\begin{cases} \dot{\eta}_{i1} &= \delta_{i2} + \Omega_{i2} W_{i0} + \vartheta_{i2} - \epsilon_{i2} + \beta_{i(1)}^T W_{i0}, \\ \dot{\vartheta}_{i2} &= -w_{i1}\vartheta_{i2} + M_i^{-1}\tau_i, \end{cases} \tag{26}$$

where $\beta_{i(1)}^T$ is the first three rows of $\beta_i^T$.

## IV. Observer Based CDP Controller design

In this section, we present the observer based CDP controller design in two steps as follows.

*Step 1:* Define a neighborhood-based dynamic surface tracking error

$$z_{i1} = \sum_{j=1}^{N} a_{ij}[\eta_i - \Delta_{ij} - \eta_j] + a_{i0}[\eta_i - \Delta_i - \eta_r], \quad (27)$$

where $a_{ij}$ and $a_{i0}$ are defined in section II. Considering (1) and differentiating $z_1$ with respect to time, we obtain

$$\begin{aligned}\dot{z}_{i1} &= a_{id}R_i\dot{\eta}_{i1} - \sum_{j=1}^{N} a_{ij}R_j\dot{\eta}_{j1} - a_{i0}\dot{\eta}_r \\ &= a_{id}R_i(\delta_{i2} + \vartheta_{i2}) - \sum_{j=1}^{N} a_{ij}R_j(\delta_{j2} + \vartheta_{j2}) \\ &\quad - a_{i0}\dot{\eta}_r + \Theta_i W_i - \Upsilon_i\bar{\epsilon}_{i2}. \quad (28)\end{aligned}$$

where $a_{id} = d_i + a_{i0}$, $R_i = R(\psi_i)$, and $R_j = R(\psi_j)$; $\Theta_i = [a_{id}R_i(\Omega_{i2} + \beta_{i(1)}), -a_{i1}R_1(\Omega_{12} + \beta_{1(1)}), -a_{i2}R_2(\Omega_{12} + \beta_{2(1)}), ..., -a_{iN}R_N(\Omega_{12} + \beta_{N(1)})]$; $W_i = [W_{i0}^T, W_{10}^T, W_{20}^T, ..., W_{N0}^T]^T$; $\Upsilon_i = [a_{id}R_i, -a_{i1}R_1, -a_{i2}R_2, ..., -a_{iN}R_N]$; $\bar{\epsilon}_{i2} = [\epsilon_{i2}^T, \epsilon_{12}^T, \epsilon_{22}^T, ..., \epsilon_{N2}^T]^T$. Choose a virtual control law $\alpha_i$ as follows

$$\begin{aligned}\alpha_i = \frac{R_i^T}{a_{id}}\Big\{ &- k_{i1}z_{i1} - a_{id}R_i\delta_{i2} + \sum_{j=1}^{N} a_{ij}R_j(\vartheta_{j2} + \delta_{j2}) \\ &+ a_{i0}\dot{\eta}_r - \Theta_i\hat{W}_i - z_{i1}h^2M_0^2\Big\}, \quad (29)\end{aligned}$$

where $\hat{W}_i(t)$ is the estimate of $W_i(t)$, and the iterative learning update law is given by

$$\hat{W}_i(t) = \Gamma_i\hat{W}_i(t - t_d) + \kappa_i\Theta_i^T z_{i1}, \quad (30)$$

where $t_d > 0$, $\kappa_i > 0$, and $\Gamma_i$ satisfying $0 \leq \Gamma_i^T\Gamma_i < \varsigma I_l, 0 < \varsigma < 1/(1+\varepsilon), \varepsilon > 0$.

*Remark 2:* Compared with integrator-based update laws, the developed iterative learning update law possesses several advantages. Firstly, it can deal with those time-varying parameters without existence of derivatives at some time instants. Secondly, it can be easier to implement in digital processors due to the fact that the differentiation need great computational costs.

For the convenience of analysis, define

$$\hat{\varrho}_i(t) = \kappa_i\Theta_i^T z_{i1}, \quad (31)$$

and then we have

$$\varrho_i(t) = W_i(t) - \Gamma_i W_i(t - t_d), \quad (32)$$

where $||\varrho_i(t)|| \leq \delta^*$, $\delta^* = \sigma_i(1+||\Gamma_i(t)||)$ with $\sigma_i > 0$. Define the estimate error as

$$\tilde{W}_i(t) = \hat{W}_i(t) - W_i(t). \quad (33)$$

Substituting (30) and (32) into (33) gives

$$\tilde{W}_i(t) = \Gamma_i\tilde{W}_i(t - t_d) + \varrho_i(t) - \hat{\varrho}_i(t). \quad (34)$$

Let $\alpha_i$ pass through a low-pass first order filter with a time constant $\gamma_i > 0$ to obtain the filtered control signal $\nu_{id}$ as follows

$$\gamma_i\dot{\nu}_{id} + \nu_{id} = \alpha_i, \nu_{id}(0) = \alpha_i(0). \quad (35)$$

*Remark 3:* The addition of the low-pass first order filter allow the algorithm to be implemented without differentiating any model nonlinearities and suffering from the problem of the explosion terms. When the filter time is small enough and the surface gain is selected reasonably, the controller is easy and prone to be practiced.

*Step 2:* Define the second surface tracking error as

$$z_{i2} = \vartheta_{i2} - \nu_{id}, \quad (36)$$

whose time derivative is given by

$$M_i\dot{z}_{i2} = -w_{i2}M_i\vartheta_{i2} + \tau_i - M_i\dot{\nu}_{id}. \quad (37)$$

An adaptive control law is constructed as follows

$$\tau_i = -k_{i2}z_{i2} + w_{i2}M_i\vartheta_{i2} + M_i\dot{\nu}_{id}. \quad (38)$$

Substituting (38) into (37) yields

$$M_i\dot{z}_{i2} = -k_{i2}z_{i2}. \quad (39)$$

Therefore, the closed-loop network can be expressed by

$$\begin{cases} \dot{z}_{i1} = & -k_{i1}z_{i1} + a_{id}R_i(z_{i2} + q_i) \\ & -\Theta_i\tilde{W}_i - \Upsilon_i\bar{\epsilon}_{i2} - z_{i1}h^2M_0^2, \\ M_i\dot{z}_{i2} & = -k_{i2}z_{i2}, \end{cases} \quad (40)$$

where $q_i = \nu_{id} - \alpha_i$. The time derivative of $q_i$ is

$$\dot{q}_i = -\frac{q_i}{\gamma_i} + B_i(z_{i1}, z_{i2}, q_i, \hat{W}_i, \hat{\tau}_i, \hat{\tau}_j, \eta_r, \dot{\eta}_r, \ddot{\eta}_r), \quad (41)$$

where $i = 1, ..., N, j \in \mathcal{N}_i$, and $B_i(\cdot)$ is a continuous function expressed as $B_i(\cdot) = \frac{\dot{R}_i^T}{a_{id}}\Big\{ - k_{i1}z_{i1} - a_{id}R_i\delta_{i2} + \sum_{j=1}^{N} a_{ij}R_j(\vartheta_{j2} + \delta_{j2}) + a_{i0}\dot{\eta}_r - \Theta_i\hat{W}_i - z_{i1}h^2M_0^2\Big\} + \frac{R_i^T}{a_{id}}\Big\{ - k_{i1}\dot{z}_{i1} - a_{id}\dot{R}_i\delta_{i2} - a_{id}R_i\dot{\delta}_{i2} + \sum_{j=1}^{N} a_{ij}[\dot{R}_j(\vartheta_{j2} + \delta_{j2}) + R_j(\dot{\vartheta}_{j2} + \dot{\delta}_{j2})] + a_{i0}\ddot{\eta}_r - \dot{\Theta}_i\hat{W}_i - \Theta_i\dot{\hat{W}}_i - h^2M_0^2\dot{z}_{i1} - 2h\dot{h}M_0^2z_{i1}\Big\}$.

*Remark 4:* In contrast to the traditional dynamic positioning controllers, the realization of the proposed CDP controller is based on the information of neighboring vessels. Individuals in the system exchange information through a directed network and only a small portion of vessels know the reference signals.

## V. Stability Analysis

The main result of this paper is stated as follows.

*Theorem 1:* Consider a networked system consisting of $N$ marine vessels governed by the dynamics (1) (2) with Assumption 1 and 2 satisfied. Select the control law (38) with the update law (30), together with the filters (14). For bounded initial conditions $V(0) \leq \varpi$ where $\varpi$ is a positive constant and $V$ is defined in (42), all signals in the system are uniformly ultimately bounded and the positioning tracking errors can be made in a very small neighborhood of origin.

*Proof:* Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{N}\Big\{z_{i1}^T z_{i1} + z_{i2}^T M_i z_{i2} + q_i^T q_i + \int_{t-t_d}^{t}\tilde{W}_i^T(s)\tilde{W}_i(s)^T ds\Big\} + V_0, \tag{42}$$

whose time derivative is given by

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-\lambda_{min}(K_1)z_{i1}^T z_{i1} - \Upsilon_i\bar{\epsilon}_{ia2}z_{i1}^T - \Upsilon_i\bar{\epsilon}_{ib2}z_{i1}^T \\ &- h^2M_0^2 z_{i1}^2 - z_{i1}^T\Theta_i[\Gamma_i\tilde{W}_i(t-t_d) + \varrho_i(t) - \hat{\varrho}_i(t)] \\ &+ a_{id}z_{i1}^T R_i(z_{i2}+q_i) - \lambda_{min}(K_2)z_{i2}^T z_{i2} - \tfrac{q_i^T q_i}{\gamma_i} \\ &+ q_i^T B_i(\cdot) - \xi_i\tilde{W}_i^T(t)\tilde{W}_i(t) + \vartheta_i\tilde{W}_i^T(t)\tilde{W}_i(t) \\ &- \tilde{W}_i^T(t-t_d)\tilde{W}_i(t-t_d)\Big\} + V_0,\end{aligned} \tag{43}$$

where $K_1 = diag(k_{11},...,k_{N1})$, $K_2 = diag(k_{12},...,k_{N2})$, $\xi_i > 0$, and $\vartheta_i = 1+\xi_i$. Expanding the term $\vartheta_i\tilde{W}_i^T(t)\tilde{W}_i(t)$ with (34) produces

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-\lambda_{min}(K_1)z_{i1}^T z_{i1} - \Upsilon_i\bar{\epsilon}_{ia2}z_{i1}^T - \Upsilon_i\bar{\epsilon}_{ib2}z_{i1}^T \\ &- h^2M_0^2 z_{i1}^2 - z_{i1}^T\Theta_i[\Gamma_i\tilde{W}_i(t-t_d) + \varrho_i(t) - \hat{\varrho}_i(t)] \\ &+ a_{id}z_{i1}^T R_i(z_{i2}+q_i) - \lambda_{min}(K_2)z_{i2}^T z_{i2} - \tfrac{q_i^T q_i}{\gamma_i} + q_i^T B_i \\ &- \xi_i\tilde{W}_i^T(t)\tilde{W}_i(t) - \tilde{W}_i^T(t-t_d)\tilde{W}_i(t-t_d) \\ &+ \vartheta_i\tilde{W}_i^T(t-t_d)\Gamma_i^T\Gamma_i\tilde{W}_i(t-t_d) + \vartheta_i\hat{\varrho}_i^T(t)\hat{\varrho}_i(t) \\ &+ \vartheta_i\varrho_i^T(t)\varrho_i(t) - 2\vartheta_i\hat{\varrho}_i^T(t)\Gamma_i\tilde{W}_i(t-t_d) \\ &+ 2\vartheta_i\tilde{W}_i^T(t-t_d)\Gamma_i^T\varrho_i(t) - 2\vartheta_i\hat{\varrho}_i^T(t)\varrho_i(t)\Big\} + V_0.\end{aligned} \tag{44}$$

Using Young's inequality yields the following inequalities

$$\begin{cases} |z_{i1}^T R_i z_{i2}| \le \frac{1}{2}\|z_{i1}\|^2 + \frac{1}{2}\|z_{i2}\|^2, \\ |z_{i1}^T R_i q_i| \le \frac{1}{2}\|z_{i1}\|^2 + \frac{1}{2}\|q_i\|^2, \\ |q_i^T B_i(\cdot)| \le \frac{1}{2\zeta_{i1}}\|q_i\|^2\|B_i(\cdot)\|^2 + \frac{\zeta_{i1}}{2}, \\ -|\Upsilon_i\bar{\epsilon}_{ia2}z_{i1}^T| \le \frac{1}{4}\|\Upsilon_i\|^2\|z_{i1}\|^2 + \|\bar{\epsilon}_{ia}\|^2, \\ -|\Upsilon_i\bar{\epsilon}_{ib2}z_{i1}^T| \le \frac{1}{4}\|\Upsilon_i\|^2 + h^2M_0^2\|z_{i1}\|^2, \end{cases} \tag{45}$$

where $\zeta_{i1} > 0$, and then we have

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-[\lambda_{min}(K_1) - a_{id} - \tfrac{\|\Upsilon_i\|^2}{4}]z_{i1}^T z_{i1} - [\lambda_{min}(K_2) \\ &- \tfrac{a_{id}}{2}]z_{i2}^T z_{i2} - [\tfrac{1}{\gamma_i} - \tfrac{\|B_i(\cdot)\|^2}{2\zeta_{i1}} - \tfrac{a_{id}}{2}]q_i^T q_i + \tfrac{\zeta_{i1}}{2} \\ &- z_{i1}^T\Theta_i[\Gamma_i\tilde{W}_i(t-t_d) + \varrho_i(t) - \hat{\varrho}_i(t)] \\ &- \xi_i\tilde{W}_i^T(t)\tilde{W}_i(t) - \tilde{W}_i^T(t-t_d)\tilde{W}_i(t-t_d) \\ &+ \vartheta_i\tilde{W}_i^T(t-t_d)\Gamma_i^T\Gamma_i\tilde{W}_i(t-t_d) + \vartheta_i\hat{\varrho}_i^T(t)\hat{\varrho}_i(t) \\ &+ \vartheta_i\varrho_i^T(t)\varrho_i(t) - 2\vartheta_i\hat{\varrho}_i^T(t)\Gamma_i\tilde{W}_i(t-t_d) \\ &+ 2\vartheta_i\tilde{W}_i^T(t-t_d)\Gamma_i^T\varrho_i(t) - 2\vartheta_i\hat{\varrho}_i^T(t)\varrho_i(t) \\ &+ \|\bar{\epsilon}_{ia}\|^2 + \tfrac{\|\Upsilon_i\|^2}{4}\Big\} + V_0.\end{aligned} \tag{46}$$

Note the following inequality

$$\begin{aligned}&2\vartheta_i\tilde{W}_i^T(t-t_d)\Gamma_i^T\varrho_i(t) \le \zeta_{i2}\tilde{W}_i^T(t-t_d)\Gamma_i^T\Gamma_i \\ &\tilde{W}_i(t-t_d) + \tfrac{\vartheta_i^2}{\zeta_{i2}}\varrho_i^T(t)\varrho_i(t),\end{aligned} \tag{47}$$

where $\zeta_{i2} > 0$. Substituting (47) into (46) and using (31) with $\kappa_i = -\frac{1}{2\vartheta_i}$, it follows that

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-[\lambda_{min}(K_1) - a_{id} - \tfrac{\|\Upsilon_i\|^2}{4}]z_{i1}^T z_{i1} - [\lambda_{min}(K_2) \\ &- \tfrac{a_{id}}{2}]z_{i2}^T z_{i2} - [\tfrac{1}{\gamma_i} - \tfrac{\|B_i(\cdot)\|^2}{2\zeta_{i1}} - \tfrac{a_{id}}{2}]q_i^T q_i + \tfrac{\zeta_{i1}}{2} \\ &- \xi_i\tilde{W}_i^T(t)\tilde{W}_i(t) + (\vartheta_i + \tfrac{\vartheta_i^2}{\zeta_{i2}})\varrho_i^T(t)\varrho_i(t) \\ &- \tilde{W}_i^T(t-t_d)(I_l - (\vartheta_i+\zeta_{i2})\Gamma_i^T\Gamma_i)\tilde{W}_i(t-t_d) \\ &+ \|\bar{\epsilon}_{ia}\|^2 + \tfrac{\|\Upsilon_i\|^2}{4}\Big\} + V_0.\end{aligned} \tag{48}$$

Letting $\mu_i = \frac{1}{\vartheta_i+\zeta_{i2}} < 1$, we obtain

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-[\lambda_{min}(K_1) - a_{id} - \tfrac{\|\Upsilon_i\|^2}{4}]z_{i1}^T z_{i1} - [\lambda_{min}(K_2) \\ &- \tfrac{a_{id}}{2}]z_{i2}^T z_{i2} - [\tfrac{1}{\gamma_i} - \tfrac{\|B_i(\cdot)\|^2}{2\zeta_i} - \tfrac{a_{id}}{2}]q_i^T q_i - \xi_i\tilde{W}_i^T(t)\tilde{W}_i(t) \\ &- \tilde{W}_i^T(t-t_d)(I_l - \mu_i^{-1}\Gamma_i^T\Gamma_i)\tilde{W}_i(t-t_d) - \|\bar{\epsilon}_{ia}\|^2\Big\} + c,\end{aligned} \tag{49}$$

where $c = \sum_{i=1}^{N}[\frac{\zeta_{i1}}{2} + (\vartheta_i + \frac{\vartheta_i^2}{\zeta_{i2}})\delta^{*2} + \frac{\|\Upsilon_i\|^2}{4}]$. Since for $\varpi > 0$, the set $\Xi = \sum_{i=1}^{N}[z_{i1}^T z_{i1} + z_{i2}^T M_i z_{i2} + q_i^T q_i + \int_{t-t_d}^{t}\tilde{W}_i^T(s)\tilde{W}_i(s)ds] \le 2\varpi$ is compact. Therefore, there exists positive constants $B_i^*$ such that $\|B_i(\cdot)\| \le B_i^*$.

Choose suitable parameters satisfying

$$\begin{cases} \hbar_{i1} &= \lambda_{min}(K_1) - a_{id} - \frac{\|\Upsilon_i\|^2}{4} > 0, \\ \hbar_{i2} &= \lambda_{min}(K_2) - \frac{a_{id}}{2} > 0, \\ \hbar_{i3} &= \frac{1}{\gamma_i} - \frac{\|B_i^*\|^2}{2\zeta_{i1}} - \frac{a_{id}}{2} > 0, \\ \hbar_{i4} &= \xi_i > 0, \\ \hbar_{i5} &= \lambda_{min}(I_l - \mu_i^{-1}\Gamma_i^T\Gamma_i) > 0, \end{cases} \tag{50}$$

and then (49) can be expressed in a compact form

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Big\{ &-\hbar_{i1}||z_{i1}||^2 - \hbar_{i2}||z_{i2}||^2 - \hbar_{i3}||q_i||^2 \\ &- \hbar_{i4}||\tilde{W}_i(t)||^2 - \hbar_{i5}||\tilde{W}_i(t-t_d)||^2 - \|\bar{\epsilon}_{ia}\|^2\Big\} + c.\end{aligned} \tag{51}$$

Either $||z_{i1}|| > \sqrt{c/\hbar_{i1}}$ or $||z_{i2}|| > \sqrt{c/\hbar_{i2}}$ or $||q_i|| > \sqrt{c/\hbar_{i3}}$ or $||\tilde{W}(t)|| > \sqrt{c/\hbar_{i4}}$ or $||\tilde{W}(t-t_d)|| > \sqrt{c/\hbar_{i5}}$ renders $\dot{V} < 0$. Therefore, all signals in the closed-loop network are uniformly ultimately bounded. Furthermore, $||z_{i1}||$ is bounded by $||z_{i1}|| \le \sqrt{c/\hbar_{i1}}$.

Define the absolute positioning tracking error as

$$e_i = \eta_i - \Delta_i - \eta_r. \tag{52}$$

Letting $z_1 = [z_{i1}^T,...,z_{iN}^T]^T$ and $e = [e_1^T,...,e_N^T]^T$, it follows that

$$z_1 = (H \otimes I_3)e, \tag{53}$$

where $H$ is defined in section II. The following inequality is satisfied under Assumption 1

$$||e|| \le \frac{||z_1||}{\underline{\varrho}(H)}, \tag{54}$$

where $\underline{\varrho}(H)$ denotes the minimal singular value of $H$. Then, $||e||$ is bounded by

$$||e|| \leq \frac{\sqrt{c_1}}{\underline{\varrho}(H)\sqrt{\hbar_{i1}}}. \tag{55}$$

By appropriately increasing the parameter $\hbar_{i1}$, the absolute positioning tracking errors can be made in a very small neighborhood of origin. The proof is complete.

## VI. Conclusions

This paper considered the CDP problem of multiple offshore vessels in the presence of dynamical uncertainties, time-varying ocean disturbances and unmeasured velocity. K-filters are first designed to estimate the unmeasured velocity of each vessel, and the observer based CDP controllers are proposed by making use of DSC technique, neural network, and adaptive iterative learning approach. The designed controllers can guarantee that a relative formation among vessels is reached if the graph induced by the vessels and the reference point contains a spanning tree. It is proved by Lyapunov stability analysis that all signals in the closed-loop systems are uniformly ultimately bounded.


## Acknowledgment

This work was in part supported by the National Nature Science Foundation of China under Grants 61273137, 51209026, 61074017, and in part by the Scientific Research Fund of Liaoning Provincial Education Department under Grant L2013202, and in part by the Fundamental Research Funds for the Central Universities under Grant 3132014047.


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