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Adaptive Fault-Tolerant Control for a Class of Uncertain Nonlinear MISO Discrete-Time Systems in Triangular Forms with Actuator Failures

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Abstract—**This paper investigates the adaptive actuator failure compensation control for a class of uncertain multi input single out (MISO) discrete time systems with triangular forms. The systems contain the actuator faults of both loss of effectiveness and lock-in-place. With the help of radial basis function neural networks (RBFNN) to approximate the unknown nonlinear functions, an adaptive RBFNN fault-tolerant control (FTC) scheme is designed. Compared with some exist result in which solving linear matrix inequality (LMI) is required, we introduce the backstepping technique to achieve the FTC task. It is proved that the proposed control approach can guarantee that all the signals of the closed-loop system are bounded and that the output can successfully track a reference signal in the presence of the actuator failures. Finally, simulation results are provided to confirm the effectiveness of the control approach.**

I. Introduction

In the recent, the FTC technique has been obtained many applications and has been attracted much attention to researchers. Due to the growing demand for reliability, maintainability, and survivability in industrial processes, it is increasingly important to ensure their safety and reliability. As is well known, faults may cause control system performance deterioration, and lead to instability and even catastrophic accidents. This has motivated researchers to concentrate on FTC, which is capable in both of automatically compensating for the effects of faults and of maintaining the performance of the controlled system, at some acceptable level, even in the presence of faults. Generally speaking, fault tolerance can be achieved either passively by using feedback control laws that are robust to possible system faults, or actively by means of faults diagnosis and accommodation architecture. Survey papers [1] provided excellent overviews of the research work on FTC.

To handle the problem of system with actuator or sensor faults, many FTC approaches had been developed. The authors in [2] presented adaptive fault-tolerant controllers for linear systems with both losses of effectiveness and lock-in-place actuator faults. Other FTC strategies for nonlinear systems were developed in [3]-[4]. Adaptive fault-tolerant controllers for a class of single input single output (SISO) nonlinear systems were developed in [3] with the same actuator faults as in [2]. Adaptive actuator failure compensation for nonlinear multi input multi output (MIMO) systems was proposed in [4] and this compensator was successfully applied in an aircraft system. One passive FTC law and two active FTC laws were designed in [5] to ensure the controlled synchronization of the complex interconnected neural networks in the presence of sensor faults.

In the above results, the FTC approaches were obtained for nonlinear systems in the continuous time forms. It is well known that the discrete time systems can be more veritably to describe the practical problem in control systems than the continuous time systems. To this end, some researchers had devoted so many efforts to study the FTC problem of nonlinear discrete-time systems. In [6], the authors studied the stochastic stability and controller design for the nominal discrete time FTC system, and then, [7] extended the results to include norm bounded parameter uncertainties in the modeling of discrete time FTC system. Parameter uncertainties [8] and stochastic stability with state delays [9] were studied for a class of discrete time fault tolerant control system, respectively. By using a newly constructed Lyapunov functional and the average dwell time scheme, a FTC method for a class of discrete-time switched systems with time-varying delay and actuator saturation is investigated in [10]. However, these researches [6-10] carried the analysis utilizing a less unified approach that lead to stability results in terms of large matrix equations that are difficult to solve and to test. In addition to this, the above studies [6-10] did not centralized where the block triangular forms were needed.

It is worth pointing out that the problems of actuator or sensor faults in the discrete time systems with triangular forms are more general and common. This dynamic will face more difficulties to arrive the FTC object. Some previous achievements were presented to overcome the control problems of nonlinear continues time systems with a triangular structure such as based on SISO continues time systems [11], based on MIMO continues time systems [12] and based on fault diagnosis [13]. This motivated us to extend the scheme in discrete time systems for this study.

This paper focuses on the adaptive actuator failure compensation control for a class of uncertain MISO discrete time systems with triangular forms. The actuator faults of both loss of effectiveness and lock-in-place are presented in the systems. By utilizing RBFNN to approximate the unknown nonlinear functions, an adaptive FTC scheme is designed. In contrast to some exist result in which solving

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LMI is needed, we make full use of the backstepping technique to achieve the FTC object. It is proved that the proposed control approach can guarantee that the output can successfully track a reference signal in the presence of the actuator failures and that all the signals of the closed-loop system are bounded. Finally, in order to confirm the effectiveness of the control approach, a simulation study is given.

II. Problem and Formulation

A. System description

Consider the following nonlinear MISO discrete time systems in block-triangular forms which may be subject to actuator faults described as

$$\begin{cases} x_i(k+1) = f_i(\bar{x}_i(k)) + g_i(\bar{x}_i(k))x_{i+1}(k) \\ \qquad i = 1,\dots,n-1 \\ x_n(k+1) = f_n(\bar{x}_n(k)) + \bar{b}_m \bar{u}(k) + d_1(k) \\ y_k = x_1(k) \end{cases} \tag{1}$$

where $\bar{x}_i(k) = [x_1(k),\cdots,x_i(k)]^T \in R^i, i = 1,\cdots,n,$ $\bar{u}(k) = [u_1, u_2,\cdots,u_m]^T \in R^m$ and $y_k \in R$ are the state variables, the inputs and the output of the systems, respectively; $f_i(\bar{x}_i(k))$ and $g_i(\bar{x}_i(k)), i = 1,\cdots,n-1$ are unknown nonlinear dynamics; $\bar{b}_m = [b_1, b_2,\cdots,b_m] \in R^m$ are positive constant vectors. $d_1(k)$ is the unknown but bounded disturbance with $\|d_1\| \le d_M$.

The actuator faults considered in this paper are either lock-in-place or loss of effectiveness, which are described in [14-15] in detail such as

Lock-in-place model:

$$\begin{aligned} &u_j^F(k) = u_j, k \ge k_j, \\ &j \in \{j_1, j_2, \cdots, j_p\} \subset \{1,2,\cdots,m\} \end{aligned} \tag{2}$$

where u_j stands for the place, where the $j-th$ actuator in the systems is stuck. k_j is the time instant at which the lock-in-place fault occurs.

Loss of effectiveness model:

$$\begin{aligned} &u_l^F(k) = \eta_l(k)v_l(k), k \ge k_l \\ &l \in \overline{\{j_1, j_2, \cdots, j_p\}} \cap \{1,2,\cdots,m\} \end{aligned} \tag{3}$$

where $v_l(k)$ is the $l-th$ applied control input of the systems, $\overline{\{j_1, j_2, \cdots, j_p\}}$ is the complementary set of the $\{j_1, j_2, \cdots, j_p\}$, and k_l is the time instant at which the loss of effectiveness fault takes place. $\eta_l(k) \in [\underline{\eta}_l, 1]$ is the effectiveness factor of the corresponding actuator $u_l^F(k)$, and $0 < \underline{\eta}_l \le 1$ is the lower bound of $\eta_l(k)$. Generally speaking, the $l-th$ applied control input is free of the actuator failures if $\underline{\eta}_l = 1$. Thus, taking (2) and (3) into account, the control input $u_s, s = 1,2,\cdots,m$ can be described as

$$u_s(k) = (1-\delta_s)\eta_s(k)v_s(k) + \delta_s u_s \tag{4}$$

where δ_s is the lock factor described as follows:

$$\delta_s = \begin{cases} 1 & \textit{if the } s-th \textit{ actuator} \\ & \textit{in the system is stuck} \\ 0 & \textit{otherwise} \end{cases} \tag{5}$$

Remark 1: The uniqueness of k_i indicates that a failure occurs only once on the $i-th$ actuator. Hence there exists a finite K_r denoting the time instant of the last failure. Such an assumption on the finite number of actuator failures can be found in many previous literature, see for example, [16-17].

B. Preliminaries and control objective

For the system (1) and the actuator failures (4), it needs to make the assumptions as follows.

Assumption 1: The signs of $g_i(\bar{x}_i(k)), i = 1,\dots,n$ are known and there exist the known constants $\underline{g}_i, \bar{g}_i > 0$ such that $\underline{g}_i \le |g_i(\bar{x}_i(k))| \le \bar{g}_i, \forall \bar{x}_n(k) \in \Omega \subset R^n$.

Assumption 2: The desired trajectory $y_d(k) \in \Omega_y, \forall k > 0$ is smooth and known where Ω_y is a bounded compact set.

Assumption 3 ([16]): The system (1) is so constructed that when any up to $p-1$ actuators stuck at some unknown places, the other(s) may lose effectiveness as (3), the closed-loop system can still be driven to achieve the following control objective.

The control objective is to design an adaptive FTC scheme for the plant (1) with actuator faults so that: 1) all the signals in the closed-loop system are bounded and 2) the system output tracks the desired reference signal $y_d(k)$ to a small compact set.

C. RBFNN Approximation Property

As we all known, RBFNN has been frequently used as function approximators. In this paper, RBFNNs is used to approximate the continuous function $f^{NN}(y)$

$$f^{NN}(y,\theta) = \phi^T(y)\theta \tag{6}$$

where $y \in R^q$ is the input variable of the RBFNNs, $\theta = [\theta_1,\cdots,\theta_l]^T$ is the weight vector with the RBFNNs node number l, $\phi(y)$ is the smooth basis vector to be $\phi(y) = [\phi_1(y),\cdots,\phi_l(y)]^T$ and $\phi_i(y)$ is chosen as the commonly used Gaussion functions

$$\phi_i(y) = \exp\left[\frac{-(y-\mu_i)^T(y-\mu_i)}{\tau_i^2}\right], i = 1,\cdots,l \tag{7}$$

where $\mu_i = [\mu_{i1},\cdots,\mu_{iq}]^T$ and τ_i are the center and the width of the Gaussion functions, respectively.

III. Adaptive Law NN Controller Designs

Consider the strict-feedback MISO nonlinear discrete-time system described in (1). Because Assumption 1 is only valid on the compact set Ω, it needs to guarantee the system states remaining in Ω for all time.

With the help of the transformation process in [18], the original strict feedback from (1) is equivalent to

$$\begin{cases} x_1(k+n) = F_1(\bar{x}_n(k)) + G_1(\bar{x}_n(k))x_2(k+n-1) \\ \quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ x_{n-1}(k+2) = F_{n-1}(\bar{x}_n(k)) + G_{n-1}(\bar{x}_n(k))x_n(k+1) \\ x_n(k+1) = f_n(\bar{x}_n(k)) + \bar{b}_m\bar{u}(k) + d_1(k) \\ y_k = x_1(k) \end{cases} \tag{8}$$

where

$$\begin{cases} F_i(\bar{x}_n(k)) = f_i\left(F_{i+1,i}^c(\bar{x}_n(k))\right) \\ G_i(\bar{x}_n(k)) = g_i\left(F_{i+1,i}^c(\bar{x}_n(k))\right) \\ F_{i+1,j}^c(\bar{x}_n(k)) = \left[f_{i+1,1}^c(\bar{x}_2(k)),\cdots,f_{i+1,j}^c(\bar{x}_{j+1}(k))\right]^T \\ f_{i+1,j}^c(\bar{x}_{j+1}(k)) = f_j\left(F_{i+2,j}^c(\bar{x}_{j+1}(k))\right) \\ \qquad\qquad + g_j\left(F_{i+2,j}^c(\bar{x}_{j+1}(k))\right)f_{i+2,j+1}^c(\bar{x}_{j+2}(k)) \\ i=1,\cdots,n-1,\ j=1,\cdots,i \end{cases}$$

For convenience in analysis and discussion, let $F_i(k)=F_i(\bar{x}_n(k))$, $G_i(k)=G_i(\bar{x}_n(k))$, $f_n(k)=f_n(\bar{x}_n(k))$ and $g_n(k)=g_n(\bar{x}_n(k))$, $i=1,\ldots,n-1$. It is clear that the value of $G_i(\bar{x}_n(k)), i=1,\cdots,n-1$ satisfy $\underline{g}_i \le G_i(\bar{x}_n(k)) \le \bar{g}_i$, $\forall \bar{x}_n(k)\in\Omega$ under Assumption 1.

Step 1: Define $z_1(k)=x_1(k)-y_d(k)$ and its nth difference is

$$\begin{aligned} z_1(k+n) &= x_1(k+n) - y_d(k+n) \\ &= F_1(k) + G_1(k)x_2(k+n-1) - y_d(k+n) \end{aligned} \tag{9}$$

By viewing $x_2(k+n-1)$ as a virtual control input in (9) and choosing

$$x_2(k+n-1) = \alpha_1^*(k) = -\frac{1}{G_1(k)}\left[F_1(k) - y_d(k+n)\right] \tag{10}$$

It can be known that $\eta_1(k+n)=0$. But $F_1(k)$ and $G_1(k)$ are unknown, $\alpha_1^*(k)$ can not be implemented. Therefore, with the aid of the approximation property of RBFNNs, $\alpha_1^*(k)$ can be approximated as follows

$$\alpha_1^*(k) = \Phi_1^T(S_1(k))\theta_1^* + \varepsilon_1(S_1(k)) \tag{11}$$

where $S_1(k)=\left[\bar{x}_n^T(k), y_d(k+n)\right]^T \in \Omega_1 \subset R^{n+1}$.

Construct the adaptive virtual control input as follows

$$\begin{aligned} x_2(k+n-1) &= \alpha_1(k) + z_2(k+n-1) \\ &= \Phi_1^T(S_1(k))\hat{\theta}_1(k) + z_2(k+n-1) \end{aligned} \tag{12}$$

Choose the following adaptation law for $\hat{\theta}_1(k)$

$$\begin{aligned} \hat{\theta}_1(k+1) &= \hat{\theta}_1(m_1) - \Gamma_1 \\ &\quad\times\left[\Phi_1(S_1(m_1))z_1(k+1) + \sigma_1\hat{\theta}_1(m_1)\right] \end{aligned} \tag{13}$$

with $m_1=k-n+1$ and $\Gamma_1=\Gamma_1^T>0$ being diagonal constant matrix and $\sigma_1>0$ is the design parameter.

Substituting (10)-(12) into (9) yields

$$\begin{aligned} z_1(k+n) &= G_1(k)\Phi_1^T(S_1(k))\tilde{\theta}_1(k) \\ &\quad + G_1(k)\left[z_2(k+n-1) - \varepsilon_1(S_1(k))\right] \end{aligned} \tag{14}$$

From (14), we have

$$\begin{aligned} z_1(k+1) &= G_1(m_1)\Phi_1^T(S_1(m_1))\tilde{\theta}_1(m_1) \\ &\quad + G_1(m_1)\left[z_2(k) - \varepsilon_1(S_1(m_1))\right] \end{aligned} \tag{15}$$

Thus, it can be obtained from (15) that

$$\begin{aligned} \Phi_1^T(S_1(m_1))\tilde{\theta}_1(m_1) &= z_1(k+1)/G_1(m_1) \\ &\quad - z_2(k) + \varepsilon_1(S_1(m_1)) \end{aligned} \tag{16}$$

Choose the Lyapunov function candidate

$$V_1(k) = \frac{1}{\underline{g}_1}z_1^2(k) + \sum_{j=0}^{n-1}\tilde{\theta}_1^T(m_1+j)\Gamma_1^{-1}\tilde{\theta}_1(m_1+j) \tag{17}$$

The first difference of (17) is

$$\begin{aligned} \Delta V_1 &= \frac{1}{\underline{g}_1}\left[z_1^2(k+1) - z_1^2(k)\right] + \tilde{\theta}_1^T(k+1)\Gamma_1^{-1} \\ &\quad\times\tilde{\theta}_1(k+1) - \tilde{\theta}_1^T(m_1)\Gamma_1^{-1}\tilde{\theta}_1(m_1) \end{aligned} \tag{18}$$

Substituting (16) into (18), the above equation becomes

$$\begin{aligned} \Delta V_1 &\le -\frac{1}{\bar{g}_1}z_1^2(k+1) + 2z_2(k)z_1(k+1) \\ &\quad -\frac{1}{\underline{g}_1}z_1^2(k) - 2\varepsilon_1(S_1(m_1))z_1(k+1) \\ &\quad + \Phi_1^T(S_1(m_1))\Gamma_1\Phi_1(S_1(m_1))z_1^2(k+1) \\ &\quad + 2\sigma_1\Phi_1^T(S_1(m_1))\Gamma_1\hat{\theta}_1(m_1)z_1(k+1) \\ &\quad - 2\sigma_1\tilde{\theta}_1^T(m_1)\hat{\theta}_1(m_1) + \sigma_1^2\Gamma_1\left\|\hat{\theta}_1(m_1)\right\|^2 \end{aligned} \tag{19}$$

$\Phi_1(z_1(m_1))$ is a vector which is composed of Gaussian functions. It is obvious that $\Phi_1^T(S_1(m_1))\Phi_1(S_1(m_1)) \le l_1$ where l_1 is the node number of the RBFNNs. Then, one has

$$\Phi_1^T(S_1(m_1))\Gamma_1\Phi_1(S_1(m_1))z_1^2(k+1) \le l_1\lambda_1^* z_1^2(k+1)$$

where λ_1^* is the maximum eigenvalue of the matrix Γ_1.

Using the Young's inequality, we have

$$-2\varepsilon_1(S_1(m_1))z_1(k+1) \le \frac{\lambda_1^* z_1^2(k+1)}{\bar{g}_1} + \frac{\bar{g}_1\bar{\varepsilon}_1^2}{\lambda_1^*}$$

$$2z_2(k)z_1(k+1) \le \frac{\lambda_1^* z_1^2(k+1)}{\bar{g}_1} + \frac{\bar{g}_1 z_2^2(k)}{\lambda_1^*}$$

$$\begin{aligned} &2\sigma_1\Phi_1^T(S_1(m_1))\Gamma_1\hat{\theta}_1(m_1)z_1(k+1) \\ &\qquad \le \frac{\lambda_1^* l_1 z_1^2(k+1)}{\bar{g}_1} + \sigma_1^2\bar{g}_1\lambda_1^*\left\|\hat{\theta}_1(m_1)\right\|^2 \end{aligned}$$

$$2\sigma_1\tilde{\theta}_1^T(m_1)\hat{\theta}_1(m_1)=\sigma_1\left(\left\|\tilde{\theta}_1(m_1)\right\|^2+\left\|\hat{\theta}_1(m_1)\right\|^2-\left\|\theta_1^*\right\|^2\right)$$

Based on the above facts, (19) can be rewritten as

$$\Delta V_1(k)\le-\frac{\rho_1}{\bar{g}_1}z_1^2(k+1)+\beta_1+\frac{\bar{g}_1 z_2^2(k)}{\lambda_1^*}-\frac{1}{\bar{g}_1}z_1^2(k)$$
$$-\sigma_1\left(1-\sigma_1\lambda_1^*-\bar{g}_1\sigma_1\lambda_1^*\right)\left\|\hat{\theta}_1(m_1)\right\|^2 \qquad (20)$$

where $\rho_1=1-2\lambda_1^*-\lambda_1^* l_1-\bar{g}_1 l_1\lambda_1^*$, $\beta_1=\frac{\bar{g}_1\bar{\varepsilon}_1^2}{\lambda_1^*}+\sigma_1\left\|\theta_1^*\right\|^2$.

Step $\boldsymbol{i\,(2\le i<n)}$**:** Define $z_i(k)=x_i(k)-\alpha_{i-1}(m_{i-1})$ with $m_{i-1}=k-n+i-1$ and its $(n-i+1)th$ difference can be expressed as

$$z_i(k+n-i+1)=F_i(k)+G_i(k)$$
$$\times x_{i+1}(k+n-i)-\alpha_{i-1}(k) \qquad (21)$$

Similar to the procedure of step 1, designing the following virtual control

$$\alpha_i(k)=\Phi_i^T(S_i(k))\hat{\theta}_i(k) \qquad (22)$$

Then, one get

$$\Phi_i^T(S_i(m_i))\tilde{\theta}_i(m_i)=z_i(k+1)/G_i(m_i)$$
$$-z_{i+1}(k)+\varepsilon_i(z_i(m_i)) \qquad (23)$$

Choose the following adaptation law for $\hat{\theta}_i(k)$

$$\hat{\theta}_i(k+1)=\hat{\theta}_i(m_i)$$
$$-\Gamma_i\times\left[\Phi_i(S_i(m_i))z_i(k+1)+\sigma_i\hat{\theta}_i(m_i)\right] \qquad (24)$$

where $m_i=k-n+i$, $\Gamma_i=\Gamma_i^T>0$ is a diagonal constant matrix and $\sigma_i>0$ is the design parameter.

Choose the Lyapunov function candidate

$$V_i(k)=\frac{1}{\bar{g}_i}z_i^2(k)+\sum_{j=0}^{n-i}\tilde{\theta}_i^T(m_i+j)\Gamma_i^{-1}\tilde{\theta}_i(m_i+j) \qquad (25)$$

The first difference of (25) is

$$\Delta V_i=\frac{1}{\bar{g}_i}\left[z_i^2(k+1)-z_i^2(k)\right]$$
$$+\tilde{\theta}_i^T(k+1)\Gamma_i^{-1}\tilde{\theta}_i(k+1)-\tilde{\theta}_i^T(m_i)\Gamma_i^{-1}\tilde{\theta}_i(m_i) \qquad (26)$$

Using the Young's inequality, one has

$$\Phi_i^T(S_i(m_i))\Gamma_i\Phi_i(S_i(m_i))z_i^2(k+1)\le l_i\lambda_i^* z_i^2(k+1)$$

$$-2\varepsilon_i(S_i(m_i))z_i(k+1)\le\frac{\lambda_i^* z_i^2(k+1)}{\bar{g}_i}+\frac{\bar{g}_i\bar{\varepsilon}_i^2}{\lambda_i^*}$$

$$2z_{i+1}(k)z_i(k+1)\le\frac{\lambda_i^* z_i^2(k+1)}{\bar{g}_i}+\frac{\bar{g}_i z_{i+1}^2(k)}{\lambda_i^*}$$

$$2\sigma_i\Phi_i^T(S_i(m_i))\Gamma_i\hat{\theta}_i(m_i)z_i(k+1)$$
$$\le\frac{\lambda_i^* l_i z_i^2(k+1)}{\bar{g}_i}+\sigma_i^2\bar{g}_i\lambda_i^*\left\|\hat{\theta}_i(m_i)\right\|^2$$

$$2\sigma_i\tilde{\theta}_i^T(m_i)\hat{\theta}_i(m_i)=\sigma_i\left(\left\|\tilde{\theta}_i(m_i)\right\|^2+\left\|\hat{\theta}_i(m_i)\right\|^2-\left\|\theta_i^*\right\|^2\right)$$

where λ_i^* is the maximum eigenvalue of the matrix Γ_i.

Based on the above facts, one gets

$$\Delta V_i(k)\le-\frac{\rho_i}{\bar{g}_i}z_i^2(k+1)-\frac{1}{\bar{g}_i}z_i^2(k)+\bar{g}_i z_{i+1}^2(k)/\lambda_i^*$$
$$+\beta_i-\sigma_i\left(1-\sigma_i\lambda_i^*-\bar{g}_i\sigma_i\lambda_i^*\right)\left\|\hat{\theta}_i(m_i)\right\|^2 \qquad (27)$$

where $\rho_i=1-2\lambda_i^*-\lambda_i^* l_i-\bar{g}_i l_i\lambda_i^*$, $\beta_i=\frac{\bar{g}_i\bar{\varepsilon}_i^2}{\lambda_i^*}+\sigma_i\left\|\theta_i^*\right\|^2$.

Step $\boldsymbol{n}$**:** Define $z_n(k)=x_n(k)-\alpha_{n-1}(k-1)$ and its first difference is

$$z_n(k+1)=f_n(\bar{x}_n(k))+d_1(k)-\alpha_{n-1}(k)+\bar{b}_m\bar{u}(k) \qquad (28)$$

Similar to the scheme adopted in [4, 17], we utilize the proportional-actuation method as

$$v_s=\omega_s(\bar{x}_s)u_0 \qquad (29)$$

where $0<\underline{\omega}_s\le\omega_s(\bar{x}_s)\le\bar{\omega}_s$, $s\in\{1,\cdots,m\}$. $\underline{\omega}_s$ and $\bar{\omega}_s$ are the lower and upper bounds of $\omega_s(\bar{x}_s)$, respectively.

Thus, one obtains

$$\bar{b}_m\bar{u}(k)=\sum_{s=1}^{m}b_s u_s(k)=\sum_{s=j_1,\cdots,j_p}b_s u_s$$
$$+\sum_{s\ne j_1,\cdots,j_p}b_s\eta_s(k)\omega_s(\bar{x}_s)u_0 \qquad (30)$$

Denote ideal control input as

$$u_0^*=-(g_s)^{-1}\left(f_n(\bar{x}_n(k))-\alpha_{n-1}(k)+\sum_{s=j_1,\cdots,j_p}b_s u_s\right) \qquad (31)$$

where $g_s=\sum_{s\ne j_1,\cdots,j_p}b_s\eta_s(k)\omega_s(\bar{x}_s)>0$.

Note that the ideal control input in (31) is unavailable and it can be approximated as

$$u_0^*(k)=\Phi_n^T(S_n(k))\theta_n^*+\varepsilon_n(S_n(k)) \qquad (32)$$

where $S_n(k)=\left[\bar{x}_n^T(k),\alpha_{n-1}(k)\right]^T\in\Omega_n\subset R^{n+1}$.

Define $\tilde{\theta}_n(k)=\hat{\theta}_n(k)-\theta_n^*$. Construct the adaptation law as

$$\hat{\theta}_n(k+1)=\hat{\theta}_n(k)-\Gamma_n$$
$$\times\left[\Phi_n(S_n(k))z_n(k+1)+\sigma_n\hat{\theta}_n(k)\right] \qquad (33)$$

where $\Gamma_n=\Gamma_n^T>0$ is a diagonal constant matrix and $\sigma_n>0$ is a design parameter.

Construct the following control law

$$u_0=\Phi_n^T(S_n(k))\hat{\theta}_n(k) \qquad (34)$$

Thus, (28) becomes

$$z_n(k+1)=g_s\left[\Phi_n^T(S_n(k))\tilde{\theta}_n-\varepsilon_n(S_n(k))\right]+d_1(k) \qquad (35)$$

Then, it follows that

$$\Phi_n^T(S_n(k))\tilde{\theta}_n(k)=z_n(k+1)/g_s$$
$$-d_1(k)/g_s+\varepsilon_n(S_n(k)) \qquad (36)$$

Choose the Lyapunov function candidate

$$V_n(k)=\frac{1}{g_s}z_n^2(k)+\tilde{\theta}_n^T(k)\Gamma_n^{-1}\tilde{\theta}_n(k) \qquad (37)$$

Its first difference is

$$\Delta V_n = \frac{1}{g_s}\left[z_n^2(k+1) - z_n^2(k)\right]$$
$$+\tilde{\theta}_n^T(k+1)\Gamma_n^{-1}\tilde{\theta}_n(k+1) - \tilde{\theta}_n^T(k)\Gamma_n^{-1}\tilde{\theta}_n(k) \qquad (38)$$

From (33), it can be obtained

$$\tilde{\theta}_n(k+1) = \tilde{\theta}_n(k) - \Gamma_n \times\left[\Phi_n(S_n(k))z_n(k+1) + \sigma_n\hat{\theta}_n(k)\right] \qquad (39)$$

Substituting (36) and(39) into (38) leads to

$$\Delta V_n = -\frac{1}{g_s}z_n^2(k+1) - \frac{1}{g_s}z_n^2(k)$$
$$-2\varepsilon_n z_n(k+1) + 2d_1(k)z_n(k+1)/g_s$$
$$+\Phi_n^T(S_n(k))\Gamma_n\Phi_n(S_n(k))z_n^2(k+1)$$
$$+2\sigma_n\Phi_n^T(S_n(k))\Gamma_n\hat{\theta}_n(k)z_n(k+1)$$
$$-2\sigma_n\tilde{\theta}_n^T(k)\hat{\theta}_n(k) + \sigma_n^2\hat{\theta}_n(k)^T\Gamma_n\hat{\theta}_n(k) \qquad (40)$$

Obviously, $\Phi_n^T(S_n(k))\Phi_n(S_n(k)) \le l_n$ where l_n is the node number of the RBFNN.

Using the Young's inequality as

$$\Phi_n^T(S_n(k))\Gamma_n\Phi_n(S_n(k))z_n^2(k+1) \le l_n\lambda_n^* z_n^2(k+1)$$
$$-2\varepsilon_n z_n(k+1) \le \frac{\lambda_n^* z_n^2(k+1)}{g_s} + \frac{g_s\bar{\varepsilon}_n^2}{\lambda_n^*}$$
$$2\sigma_n\Phi_n^T(S_n(k))\Gamma_n\hat{\theta}_n(k)z_n(k+1)$$
$$\le \frac{\lambda_n^* l_n z_n^2(k+1)}{g_s} + \sigma_n^2 g_s\lambda_n^*\left\|\hat{\theta}_n(k)\right\|^2$$
$$-2\sigma_n\tilde{\theta}_n^T(k)\hat{\theta}_k(k) = -\sigma_n\left(\left\|\tilde{\theta}_n(k)\right\|^2 + \left\|\hat{\theta}_n(k)\right\|^2 - \left\|\theta_n^*\right\|^2\right)$$
$$\frac{2d_1(k)z_n(k+1)}{g_s} \le \frac{d_M^2}{\lambda_n^* g_s} + \frac{\lambda_n^* z_n{}^2(k+1)}{g_s}$$

where λ_n^* is the maximum eigenvalue of the matrix Γ_n.

Combining the above inequation with (40) results in

$$\Delta V_n(k) \le -\frac{\rho_n}{g_s}\eta_n^2(k+1) + \beta_n$$
$$-\frac{1}{g_s}\eta_n^2(k) - \sigma_n\left(1 - \sigma_n\lambda_n^*\right.$$
$$\left.-g_s\sigma_n\lambda_n^*\right)\left\|\hat{\theta}_n(k)\right\|^2 \qquad (41)$$

where

$$\rho_n = 1 - 2\lambda_n^* - \lambda_n^* l_n - g_s l_n\lambda_n^*$$

and

$$\beta_n = \frac{g_s\varepsilon_n^2}{\lambda_n^*} + \sigma_n\left\|\theta_n^*\right\|^2 + \frac{d_M^2}{\lambda_n^* g_s}.$$

Theorem 1: Consider the system (1) subject to actuator faults. Under Assumptions 1-3, by designing the virtual control (12), (22) and the actual control input (34), and constructing the adaptation laws (13), (24) and (33), the presented methods guarantees that all the signals in the closed loop system is bounded and the system output tracks the reference signal to a compact set.

IV. Simulation Example

To verify the effectiveness of the proposed approach, consider a class of nonlinear MISO systems as

$$x_1(k+1) = \frac{1.4x_1^2(k)}{1+x_1^2(k)} + \left(0.1 + 0.005\cos\left(x_1(k)\right)\right)x_2(k)$$
$$x_2(k+1) = \frac{x_1(k)}{1+x_1^2(k)+x_2^2(k)} + 0.2 + \bar{b}_m\bar{u}(k) \qquad (42)$$
$$y_k = x_1(k)$$

where $m=2$, $\bar{b}_2 = [b_1, b_2] = [0.8, 0.8]$, $\bar{u}(k) = [u_1, u_2]^T$.

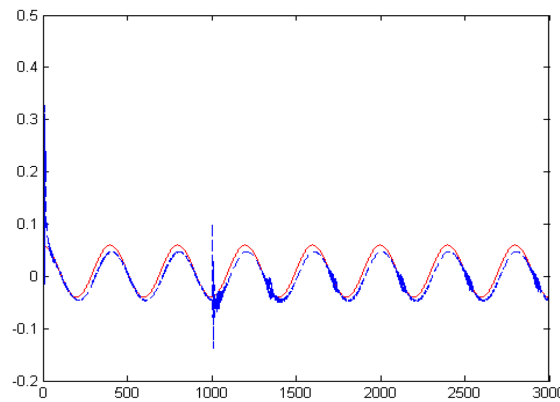

Fig.1.The curves of $x_1(k)$ (blue) and y_d (red)

The FTC objective is to make the output y_k follow a desired reference signal $y_d(k) = 0.05\sin x_1$ and all the signals of the closed-loop system are bounded.

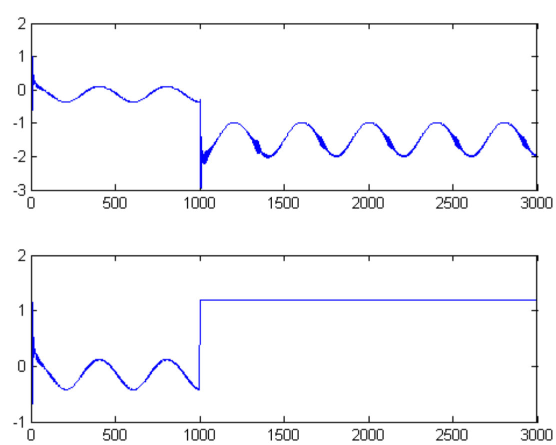

Fig.2.The curves of u_1 and u_2

In the simulation, the actuator faults are chosen as $u_1 = \omega_1 u_0 = 0.7u_0, u_2 = 1.2$, when $k \ge 3000s$. The basis vector of the neural networks is chosen as the commonly used Gaussion functions. The RBFNN $\Phi_1^T(S_1(k))\hat{\theta}_1(k)$ contains 25 nodes, with centers $\mu_i^1, i = 1, \cdots, 25$ evenly spaced in $[-5,5]\times[-5,5]\times[-5,5]$, and width $\tau_{i1} = 2$ .The RBFNN

$\Phi_2^T(S_2(k))\hat{\theta}_2(k)$ contains 25 nodes, with centers $\mu_i^2, i=1,\cdots,25$ evenly spaced in $[-4,4]\times[-4,4]\times[-4,4]$, and widths $\tau_{i2}=1.5$. The design parameters of the proposed control approach are chosen as $\Gamma_1=0.03I$, $\sigma_1=0.02$, $\Gamma_2=0.01I$, $\sigma_2=0.5$. The initial values for θ_1 and θ_2 are given as $\theta_1=\theta_2=0.5$, and the initial condition for the system states is chosen as $x_1(0)=0.2, x_2(0)=-0.5$.

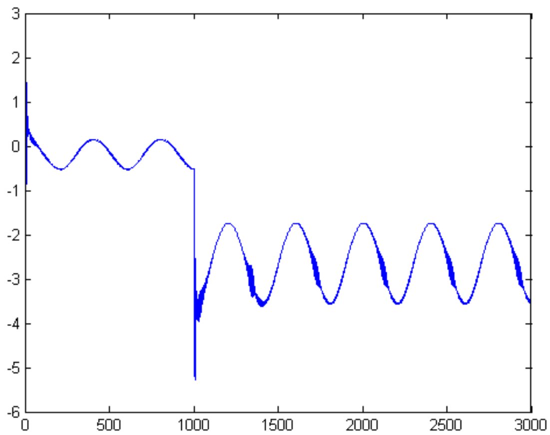

Fig.3. The curves of u_0

Figs. 1-3 illustrate the simulation results which are obtained by applying the proposed FTC approach. The tracking performance is given in Fig. 1 and it can be seen that a good tracking trajectory is achieved. Figs. 2 shows the trajectories of the control inputs u_1 and u_2. The trajectories of u_0 is described in Fig. 3. It can be observed from Figs. 1-3 that these variables are bounded.

V. Conclusion

In this paper, an adaptive FTC scheme was developed to solve the tracking problem for a class of uncertain nonlinear MISO discrete time systems in the presence of triangular forms. RBF neural networks are used to approximate the unknown functions. Backstepping design procedure is employed to construct the control input and the adaptation laws. Based on Lyapunov stability theory, it is proven that all the signals of the resulting closed-loop system are stale and the tracking error can be reduced to a small compact set. A simulation example is studied to verify the effectiveness of the proposed approach.

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