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Trotting Gait Planning for a Quadruped Robot with High Payload Walking on Irregular Terrain

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***Abstract*—Walking on irregular terrain is usually a common task for a quadruped robot. It is however difficult to control the robot in this situation as undesirable impulse force by collision between the foot of robot and obstacles makes the robot unstable. This paper presents a Posture Feedback Compensation Controller (PFCC) for a quadruped robot with high payload walking on irregular terrain. In order to make the robot walk stably and fast on irregular terrain, we choose trotting gait for walking. The foot trajectory is scheduled based on the Bezier curve method in order to improve the stability of quadruped robot. Simulations of walking on irregular terrain have been performed. The results have verified that the proposed methods have better stability and higher speed for walking on the irregular terrain.**

Keywords*—quadruped robot, gait planning, trot, trajectory planning

I. INTRODUCTION

We study the problem of gait planning for a quadruped robot walking on irregular terrain in this paper. The research on walking on uneven terrain is very crucial as this is the case for robots to walk in most circumstances, especially in disaster relief sites. It is however difficult to control the robot in this situation as undesirable impulse force by collision between the foot of robot and an obstacle makes the robot unstable. Conventional position control can not be used to solve this problem as the position control only utilize the foot trajectory designed in advance to control the robot's motion. When walking on even terrain, the robot is stable. But when walking on uneven terrain, the robot is unstable because the designed trajectory of robot's foot will be interrupted by obstacles or irregular terrain.

The work described here is in pursuit of cyclic locomotion of robots walking on irregular terrain stably. Many researches refer to gait planning of quadruped robots walking on even terrain [1], [2], [3], [4]. The methods in these papers are not applicable when the robots run into an obstacle or walking on uneven terrain. Quadruped walking on irregular terrain has been pursued recently by several labs using the LittleDog robotic platform [5], [6]. The main goal of these projects was always path planning and foot placement on large-scale obstacles. Many of the circumstances traversed by these robots were also modeled in advance, and full state of robots was given to the robot through offboard sensing. Therefore the autonomy of robot is weak. A force threshold-based position controller was designed in [7]. The method only used the force feedback signal to adjust the length of the robot's legs. So the robot may not be stable in highly complex terrain.

In order to solve the problems above, we design a controller to counteract the disturbance produced by the collision between the foot of robot and the obstacle. The biggest challenge we face in this problem is that the obstacle's size is random and unknown. So the controller we design should be robust to adjust the compensation value in real-time in the light of the obstacle's size.

The robot we study should have the ability of high payload as it is always used to carry many goods compared with other robots. Therefore another challenge we face is the robot we design should exhibit higher stability than usual ones to ensure goods safety. That is to say we should make the roll angle, pitch angle of the robot as small as possible. For instance, a robot without high payload on it may be considered to be stable when the roll angle and pitch angle of it are all less than 10°. But in our case, we have to make those angles less than 5°.

The paper is structured as follows. The kinematicmodel of the robot is presented in Section 2. Section 3describes the trajectory planning method and the reasons for choosing the trotting gait. The designed PFCC Controller is given is in Section 4. The simulations are proposed in Section 5. Finally, the conclusion is given in Section 6.

II. KINEMATIC MODEL OF THE ROBOT

The quadruped robot used in this paper consists of a body frame and four legs as show in Fig. 1. Each leg of the robot we designed has three degrees of freedom, one on on the Coronal Plane (CP) and two along the Sagittal Plane (SP).

Next, the forward kinematics (FK) and inverse kinematics

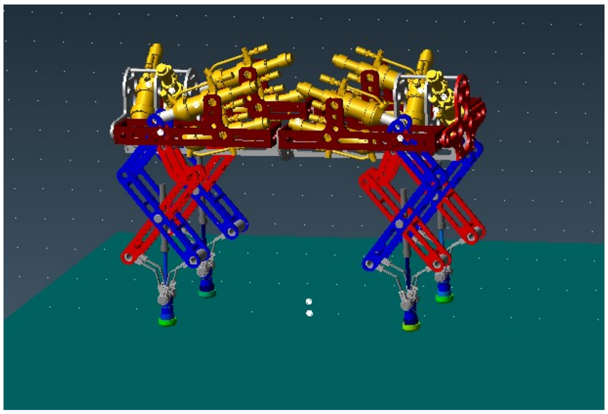

Fig. 1. Quadruped robot system.

(IK) equations of single leg will be built. The Coronal Plane and Sagittal Plane of the kinematic structure of the robot's right front leg are shown in Fig.2 and Fig.3 respectively.

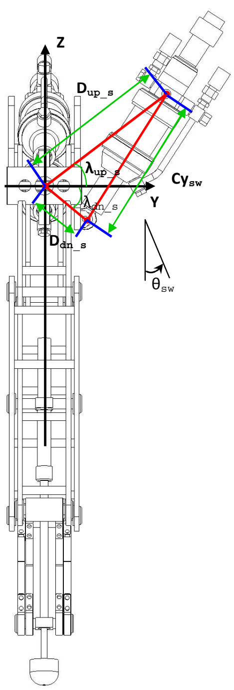

Fig. 2. Coronal plane of the right front leg.

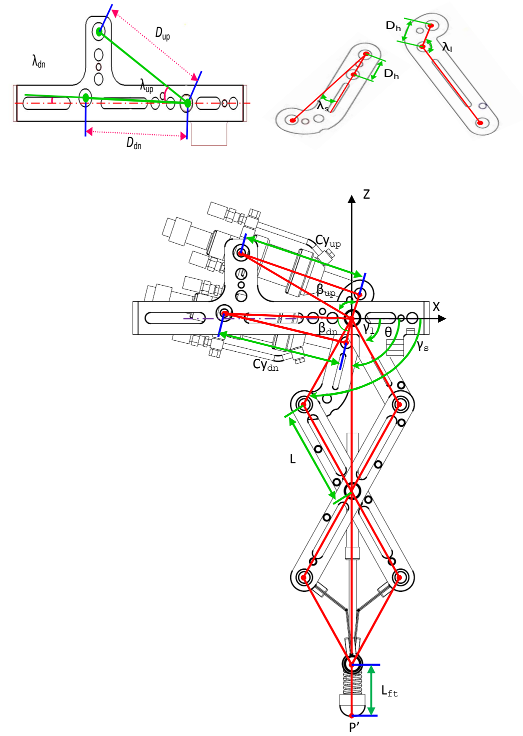

Fig. 3. Sagittal plane of the right front leg.

A. The solution of inverse kinematics

Given the foot tip position $P_b(x_p, y_p, z_p)$, the coordinate of the foot tip on the sagittal plane can be derived from the transform matrix below:

$$\theta_{sw} = -\arctan\left(\frac{y_p}{z_p}\right), \tag{1}$$

$$P(x, 0, z) = R_x(\theta_{sw}) \cdot P_b(x_p, y_p, z_p), \tag{2}$$

In which, R_x is the transform matrix from the frame fixed with the body to the frame rotate with the sagittal plane.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{sw} & \sin\theta_{sw} \\ 0 & -\sin\theta_{sw} & \cos\theta_{sw} \end{bmatrix}, \tag{3}$$

On the sagittal plane in Fig.5, we can get the equations below easily based on simple geometric relationships.

$$L_p = \sqrt{x^2 + z^2}, \tag{4}$$

$$L_{p'} = L_p - L_{ft}, \tag{5}$$

$$\theta = \frac{\pi}{2} - \arctan\frac{x}{z}, \tag{6}$$

$$\alpha = \arccos\left(\frac{L_{p'}}{4L}\right), \tag{7}$$

$$\begin{cases} \gamma_l = \theta - \alpha \\ \gamma_s = \theta + \alpha, \end{cases} \tag{8}$$

$$\beta_{up} = \pi - (\lambda_l - \gamma_l) - \lambda_{up}, \tag{9}$$

$$\beta_{dn} = \pi + \lambda_s - \gamma_s + \lambda_{dn}, \tag{10}$$

$$\beta_{sw} = \lambda_{up\_s} + \lambda_{dn\_s} - \theta_{sw}, \tag{11}$$

Then, we can get the length of the cylinder as follows:

$$Cy_{up}^{\ 2} = D_{up}^{\ 2} + D_h^{\ 2} - 2D_{up}D_h \cos(\beta_{up}), \tag{12}$$

$$Cy_{dn}^{\ 2} = D_{dn}^{\ 2} + D_h^{\ 2} - 2D_{dn}D_h \cos(\beta_{dn}), \tag{13}$$

$$Cy_{sw}^{\ 2} = D_{dn\_s}^{\ 2} + D_{up\_s}^{\ 2} - 2D_{dn\_s}D_{up\_s} \cos(\beta_{sw}), \tag{14}$$

So we can know that, by inputting the foot-tips value, the length of the cylinder will be solved by the aid of IK equations.

B. The solution of forward kinematics

Given the length of the three cylinders $Cy_{up}, Cy_{dn}, Cy_{sw}$ we can obtain β_{up} , β_{dn} , β_{sw} of the Fig.3 by simple geometric relationships.

$$\beta_{up} = \arccos\left(\frac{D_{up}^{\ 2} + D_h^{\ 2} - Cy_{up}^{\ 2}}{2D_{up}D_h}\right), \tag{15}$$

$$\beta_{dn} = \arccos\left(\frac{D_{dn}^{\ 2} + D_h^{\ 2} - Cy_{dn}^{\ 2}}{2D_{dn}D_h}\right), \tag{16}$$

$$\beta_{sw} = \arccos\left(\frac{D_{dn\_s}^{\ 2} + D_{up\_s}^{\ 2} - Cy_{sw}^{\ 2}}{2D_{dn\_s}D_{up\_s}}\right), \tag{17}$$

We can also obtain the equations by simple geometric relationships as follows:

$$\gamma_l = -\pi + \lambda_{up} + \lambda_l + \beta_{up}, \tag{18}$$

$$\gamma_s = \pi + \lambda_{dn} + \lambda_s - \beta_{dn}, \tag{19}$$

$$\theta_{sw} = \lambda_{up\_s} + \lambda_{dn\_s} - \beta_{sw}, \tag{20}$$

$$\theta = \frac{\gamma_l + \gamma_s}{2}, \tag{21}$$

$$\alpha = \theta - \gamma_l, \tag{22}$$

$$L_p = 4L\cos\alpha + L_{ft}, \tag{23}$$

$$\begin{cases} x = L_p \cos\theta \\ y = 0 \\ z = L_p \sin\theta, \end{cases} \tag{24}$$

Then we can get the foot tip position $P_b(x_p, y_p, z_p)$:

$$P_b(x_p, y_p, z_p) = R_x^{\ T}(\theta_{sw}) \cdot P(x, 0, z). \tag{25}$$

So we know that, by formulating the FK equations and knowing the foot tips position of the quadruped robot, it will be able to construct the pattern of gait planning for trotting.

III. Gait and Foot Trajectory Planning

A. Trotting gait planning

Gait is a pattern of discrete foot placements performed in a given sequence. The gaits of the quadruped robots are classified into static gaits and dynamics gaits. Static gaits which contain crawl and wave mean that the vertical projection of center of mass always remains inside the polygon formed by the supporting legs of quadruped robot. Dynamic gaits which include trot, pace and gallop occur when the vertical projection of center of mass is not necessary to remain inside the polygon formed by the supporting legs of quadruped robot with the dynamic balance to be maintained.

In this paper, we choose trotting gait for the quadruped robot as its high energy efficiency over a wide range of running speed and its wide use in nature [8]. The trot is a symmetric gait during which the diagonal front leg and behind leg move in unison, ideally contacting and leaving the ground simultaneously.

When describing the gait, the support phase of a leg is the period in which the foot is on the ground while the swing phase of a leg is the period in which the foot is not on the ground. The sequence diagram of trot is shown in Fig.4. We set the duty factor here is 0.85.

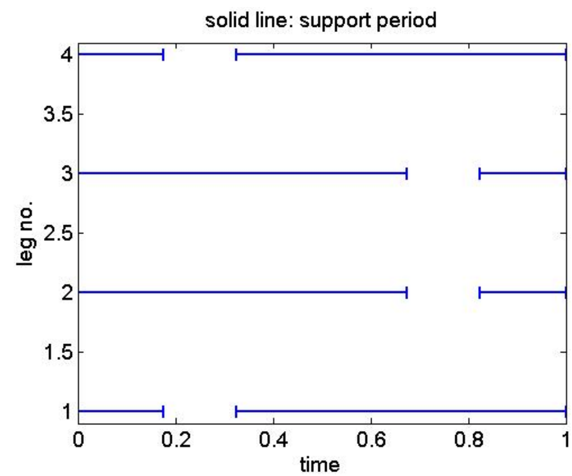

Fig. 4. Sequence diagram of trot gait.

The trotting gait is a more practical way than other dynamic walking gaits. The reasons are as follows:

The robot's diagonal legs have the same motion phase. They strike and leave the ground simultaneously. Therefore, the symmetry trotting gait can implement the symmetry motion of the quadruped robot which can reduce the complexity of attitude control and keep the self-stability. Even if the two support diagonal legs overturned, the other swing diagonal legs can also prevent the robot overturning with the help of touching the ground quickly.

The trotting gait is adaptation to the static gait of crawling which make it transform to crawly easily and vice versa.

The trotting gait has a high energy efficiency and greater range of adaptation of speed.

B. Foot trajectory planning

Two factors should be considered to design the foot trajectory. The first is that the foot trajectory can't make a great impact when the robot's feet land on the ground. The second is that the shape of foot trajectory should be similar to trajectory of an animal.

In the Fig.4, If we let the T be the time of one whole period it shows a leg will take only 0.15T to complete its own motion.

In order to have a high stability, the foot trajectory of quadruped robot must meet the requirements as follows: the foot trajectory should be continuous; the velocity and acceleration of foot must be zero when foot starts to leave and land on the floor. Therefore, we use Bezier curve to design the trajectory of foot which can satisfy the requirements above perfectly. There are also some other methods to plan the trajectory of quadruped robot like Cubic Trajectory and Sinusoidal Trajectory and so on. However, none of these trajectories take the acceleration into account which may result in a fierce collision when the foot of robot lands on the ground. The equation of Bezier curve is defined as [9].

$$B(t)=\sum_{i=0}^{n} C_{n}^{i} P_i (1-t)^{n-i} t^i, \tag{26}$$

Then we can get a 5 order Bezier curve to define the foot trajectory on X orientation from equation (26) as follows:

$$X(t)=P_0(1-t)^5+5P_1(1-t)^4 t+10P_2(1-t)^3 t^2 +10P_3(1-t)^2 t^3+5P_4(1-t)t^4+P_5 t^5, \tag{27}$$

It must meet the following requirements:

$$\text{Position: } \begin{cases} X(0)=X_0 \\ X(0.15T)=X_0+\Delta X, \end{cases} \tag{28}$$

$$\text{Velocity: } \begin{cases} \dot{X}(0)=0 \\ \dot{X}(T)=0, \end{cases} \tag{29}$$

$$\text{Acceleration: } \begin{cases} \ddot{X}(0)=0 \\ \ddot{X}(T)=0, \end{cases} \tag{30}$$

where X_0 is the start position of robot's foot on X orientation while ΔX is the step pitch on X orientation. So we can get the coefficient P_0 to P_5 from equations (26) to (30). Let X_0 =0.5m, ΔX =0.33m, T=0.65s , then we have $X(t)$ as follows:

$$X(t)=0.5+3560.41t^3-54775.55t^4+224720.2t^5 \quad (0\le t\le 0.0975), \tag{31}$$

Similarly, let Z_0 =0, ΔZ =0.2m , where Z_0 is the start position of robot's foot on Z orientation while ΔZ is the step height on Z orientation. Then we can get the foot trajectory $Z(t)$ as follows:

$$Z(t)=\begin{cases} 17262.6t^3-531156.84t^4+4358209.93t^5 \\ \quad (0\le t\le 0.04875) \\ -17262.6(t-0.04875)^3+531156.84(t-0.04875)^4 \\ -4358209.93(t-0.04875)^5+0.2 \\ \quad (0.04875\le t\le 0.0975) \end{cases} \tag{32}$$

The X , Y and Z direction displacements curves and velocity curves of robot's four legs are in Fig. 5 and Fig.6 respectively.

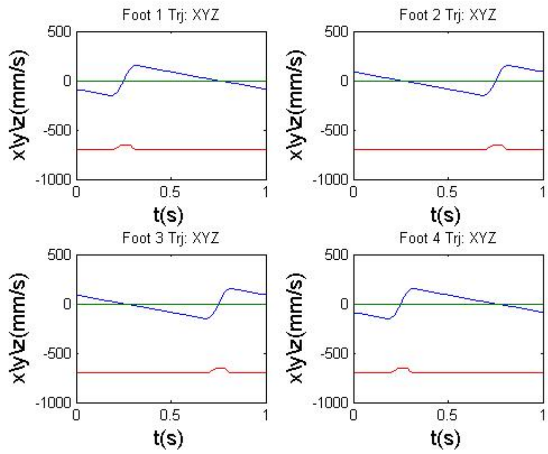

Fig. 5. Position curves of planning trajectory.

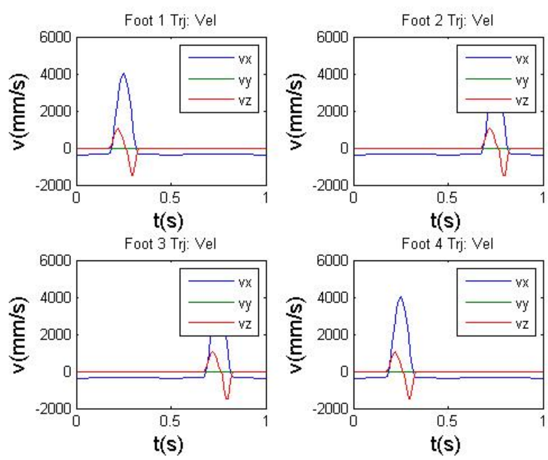

Fig. 6. Velocity curves of planning trajectory.

IV. Posture Feedback Compensation Controller

Fig.7 shows the model of quadruped robot walking on an obstacle. As shown in the figure, we suppose the robot's heading direction is to the right, and the right front leg runs into the obstacle firstly. In the Fig.7, θ_{roll} , θ_{pitch} , θ_{yaw} is the roll angle, pitch angle and yaw angle of the robot respectively, which represents the rotation angel about X-axis, Y-axis, Z-axis respectively, and the counterclockwise direction is positive. L_{rf} , L_{rb} , L_{lf} , L_{lb} represents right front leg, right back leg, left front leg and left back leg respectively.

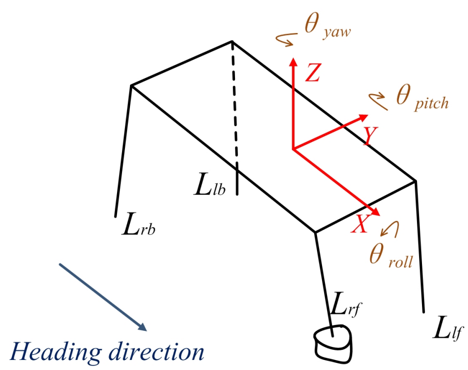

Fig. 7. Quadruped robot waling on an obstacle.

The PFCC controller detects $\theta_{roll}, \theta_{pitch}, \theta_{yaw}$ in real-time. In this paper, we let the robot walk a straight line, so the θ_{yaw}=0. We set θ_1 and θ_2 is the threshold of θ_{pitch} and θ_{roll} respectively, $\theta_1 > 0$, $\theta_2 > 0$. When $|\theta_{pitch}| < \theta_1$ and $|\theta_{roll}| < \theta_2$ happens simultaneously, the robot walks very stably, so we regard the terrain is even. When $|\theta_{pitch}| \geq \theta_1$ or $|\theta_{roll}| \geq \theta_2$ happens, the robot walks unstably, so we regard the terrain is uneven. θ_1, θ_2 are always set with experience and practical requirement. Considering our robot with high payload, in order to make the goods on the robot never drop, we usually set θ_1=5°, θ_2=5°.

The design of PFCC controller is divided into two cases as follows:

Case 1 When $|\theta_{pitch}| < \theta_1$, $|\theta_{roll}| < \theta_2$, the quadruped robot walking on even terrain or the relief amplitude is very low. In this case, we just utilize equations (31) and (32) as the foot trajectory curve of the robot. We don't use the feedback signal to adjust the length of the robot's foot here because in this case, the pitch angle and roll angle are all very small, the robot is very stable. If we use the feedback compensation, we can just improve the stability of robot by decreasing the pitch angle and roll angle a little, but it will cost more time to compute the trajectory curve.

Case 2 When $|\theta_{pitch}| \geq \theta_1$, $|\theta_{roll}| \geq \theta_2$, in this case, there are four subcases:

Subcase 1 $\theta_{pitch} < -\theta_1$, $\theta_{roll} < -\theta_2$, in this subcase, the robot's right front foot runs into an obstacle or robot's left back foot drops into a hole.

Subcase 2 $\theta_{pitch} > \theta_1$, $\theta_{roll} > \theta_2$, in this subcase, the robot's right front foot drops into a hole or robot's left back foot runs into an obstacle.

Subcase 3 $\theta_{pitch} < -\theta_1$, $\theta_{roll} > \theta_2$, in this subcase, the robot's left front foot drops into a hole or robot's right back runs into an obstacle.

Subcase 4 $\theta_{pitch} > \theta_1$, $\theta_{roll} < -\theta_2$, in this subcase, the robot's left front foot runs into an obstacle or robot's right back runs into a hole.

In above four subcases, when robot's one foot running into an obstacle, if we still just utilize the equations (31) and (32) as the foot trajectory curve of the robot, that leg of the robot will go on extending, then the foot unilateral will leave the ground, and the robot will become unstable or might even tumble. For instance, as shown in Fig.7, when the right front foot runs into the obstacle, the right leg of the robot will go on extending, then the left front foot of the robot will leave the ground. When robot's one foot runs into a hole, if we still just utilize the equations (31) and (32) as the foot trajectory curve of the robot, that leg of the robot will stop extending which can cause that the foot hang in the air, then the robot will become unstable or might even tumble. In order to maintain the stability of the robot, we adjust every leg of the robot. Set the length of the $L_{rf}, L_{rb}, L_{lf}, L_{lb}$ is l_{Lrf}, l_{Lrb}, l_{Llf}, l_{Llb} respectively, then we adjust every leg of the robot as follows:

$$l'_{Lrf} = l_{Lrf} + \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (33)$$

$$l'_{Lrb} = l_{Lrb} + \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (34)$$

$$l'_{Llf} = l_{Llf} - \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (35)$$

$$l'_{Llb} = l_{Llb} - \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (36)$$

Where, δ_1, δ_2 are PD gains on joint controllers, We adjust them based on experience.

We don't consider the case $|\theta_{pitch}| > \theta_1$, $|\theta_{roll}| < \theta_2$, and $|\theta_{pitch}| < \theta_1$, $|\theta_{roll}| > \theta_2$, because when walking on irregular terrain, $|\theta_{pitch}| > \theta_1$, $|\theta_{roll}| > \theta_2$ usually happen simultaneously.

The control architecture is shown in Fig. 8.

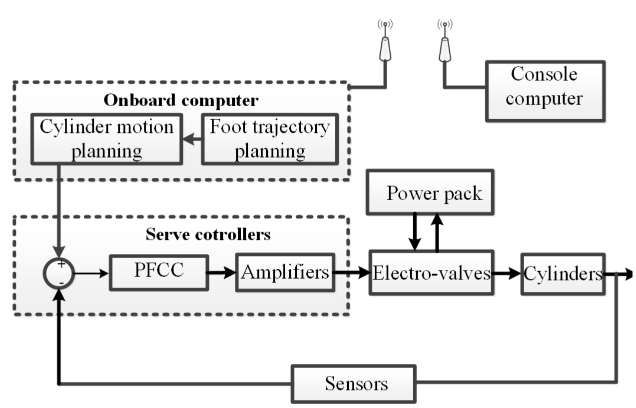

Fig. 8. Control architecture of the quadruped robot.

V. Simulation Results

To test the effectiveness and superiority of the PFCC controller we designed, we make several simulations in RecurDyn. Combined simulation is divided into two steps. Firstly, building a quadruped robot model in RecurDyn. Secondly, the corresponding control strategy is built by simulation which can be used to drive the model in RecurDyn. Then the trotting gait of the quadruped robot walking on irregular terrain is realized.

We designed the terrain with random two humps. One is 4cm tall and the other is 6cm tall. Then we made a simulation that the quadruped robot traversed them with trotting gait. The simulation is shown in Fig.9 (b) which is the snapshots of simulation in RecurDyn.

In order to verify the superiority of PFCC, we also made a simulation of robot without PFCC. The body pitch angle and body roll angle are shown in Fig. 10 and Fig. 11.

We also made a simulation that the quadruped robot

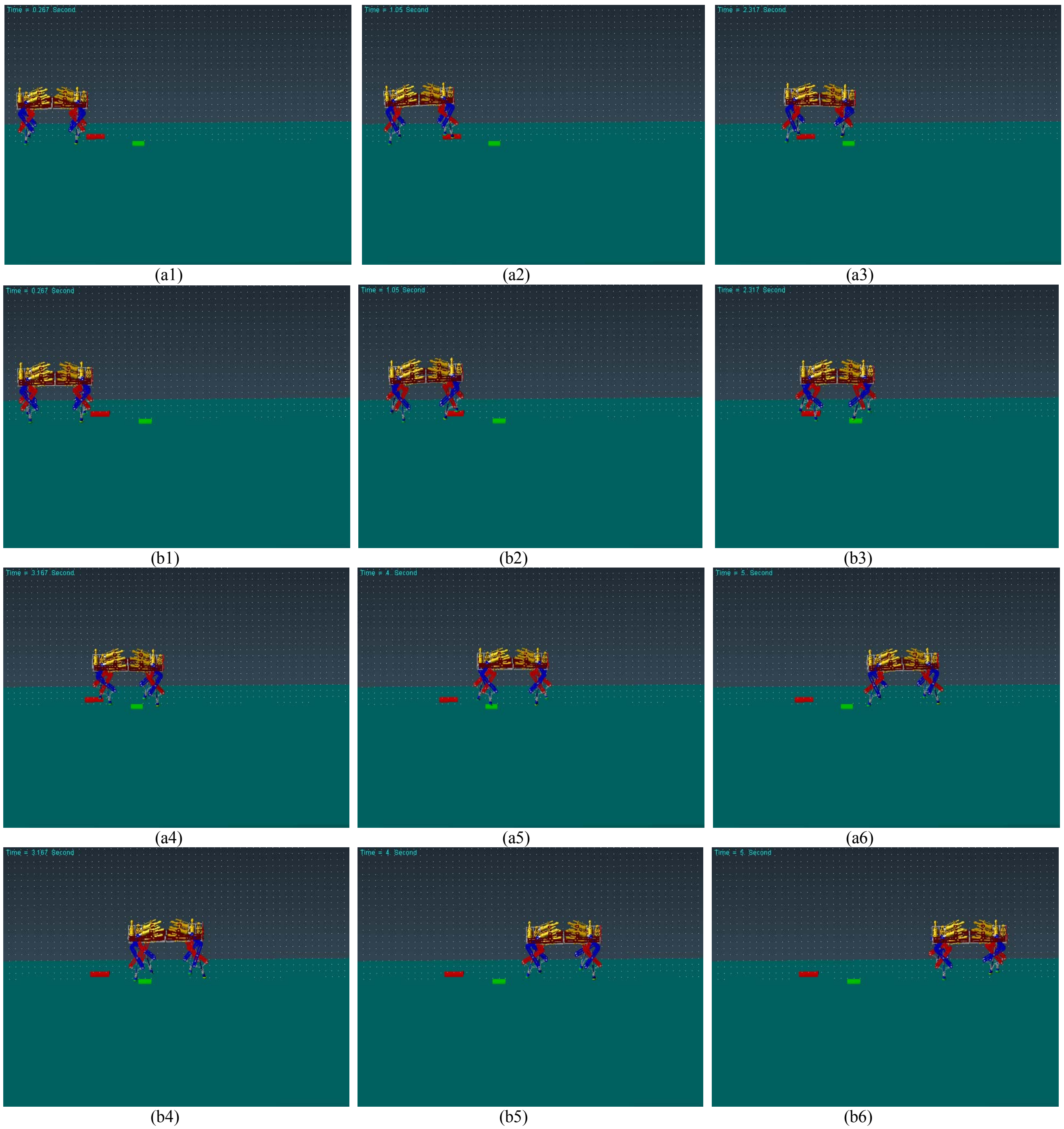

Fig. 9. Traversing a complex terrain.

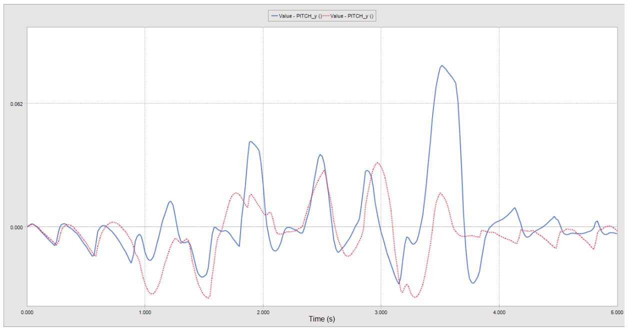

Fig.10. The body pitch angle of robot with and without PFCC.

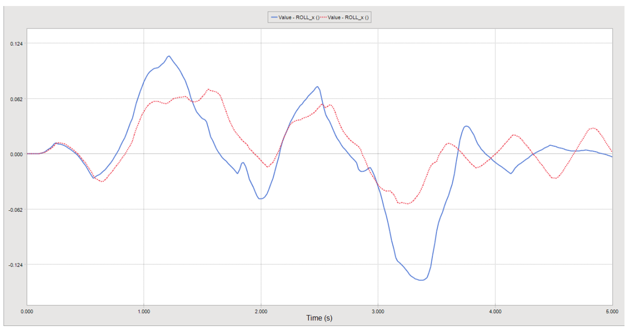

Fig. 11. The body roll angle of robot with and without PFCC.

traversed them with crawling gait. The simulation is shown in Fig.9 (a) which is the snapshots of simulation in RecurDyn.

From Fig.9 (b), we can see that the quadruped with PFCC controller has traversed a complex terrain very stably.

In Fig.10 and Fig.11, the dotted red line is the pitch angle and roll angle of robot with FPFC while the solid blue line is the pitch angle and roll angle of robot without PFCC respectively. We can see from it that, when traversing a complex terrain, the peak value of pitch angle of robot with PFCC is about approximately 1.7°, while that of robot without PFCC is approximately 5.3°. And the peak value of pitch angle of robot with PFCC is approximately 4.1°, while that of robot without PFCC is approximately -8.6°. So PFCC controller can improve the stability of the robot substantially when traversing a complex terrain.

From Fig. 9 (a) and (b), we can see that under the same initial condition, when traversing a complex train, the robot using trotting gait we propose has a more high speed than that using crawling gait. The time of corresponding snapshots of (a) and (b) are same.

VI. CONCULSION

In this paper, we design a PFCC controller to realize stable walking of quadruped robot on irregular terrain. We utilize the feedback signals detected by gyroscope to adjust every leg of robot. Then we can improve the stability of robot by decreasing the pitch angle and roll angle of robot. We design trotting gait for walking, which can improve the robot's speed. We have made several simulations under different environment, which verify the effectiveness and superiority of PFCC controller. The results are also verified that the proposed methods have better stability and higher speed for walking on the irregular terrain.

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