Decoupling Control for Five-Phase Fault-Tolerant Permanent-Magnet Motor By using SVM Inverse System Method

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Abstract—This paper presents a new decoupling control of five-phase fault-tolerant permanent-magnet (FTPM) motor drives, in which support vector machine (SVM) and inverse system theory are incorporated. The inverse system is constructed to compensate the original system into a pseudo-linear system, while SVM is utilized to obtain the inverse system without knowledge of accurate motor model. The proposed FTPM motor drive is verified in Matlab/Simulink environment, showing that the d-axis current and speed of five-phase FTPM motor system are successfully decoupled. Additionally, the proposed motor drive offers fast speed response and high control accuracy.

I. INTRODUCTION

ERMANENT magnet synchronous motors (PMSMs) **P** are widely used because of their advantages such as high torque to current ratio, high efficiency, high power density, and low noise. However, in some high performance applications like electric vehicles and aerospace applications, high reliability and continued operation are required. So, fault-tolerant permanent-magnet (FTPM) motor has been investigated and used [1]. FTPM motors has not only the common features of permanent magnet motor, but also the characteristic of physical isolation, thermal isolation, magnetic separation, electrical isolation and inhibition of short circuit current. Therefore, the safety and reliability of the system has been improved. Compared with traditional three-phase one, five-phase FTPM motor has the advantages of high fault tolerance and fault isolation capabilities, high power density, and low torque ripple.

Recently, many control strategies have been investigated for FTPM motor drives [2-5]. It is well known that a PMSM drive is a strong nonlinear system. So, decoupling and linearization are the key issues of control. However, due to the nonlinear nature and the coupling variables of the nonlinear system, linear control methods are inappropriate for them. Hence, nonlinear system control strategies have been presented [6].

The feedback linearization method is one of the nonlinear system control methods. It contains differential geometry method and inverse system method. The differential geometry method mainly relies on precise mathematical equations. It is hard to promote in practical applications. Similar to differential geometry, the control law of inverse system also bases on the dynamic equations. Nevertheless, it is intuitive, simple and more direct than differential geometry method. So, the inverse system method is widely used for decoupling control of nonlinear system. However, the main drawback of the inverse system method is that the parameters of the controlled subject are required. The accurate parameters are very difficult to know in practice. Therefore, intelligent control has been proposed to solve the uncertainty and complexity of the controlled subject.

Neural network, as one of intelligent control methods, has been discussed in many literatures [7], [8]. By introducing the neural network method into inverse system, a new control method, called neural network inverse (NNI), is proposed to compensate the nonlinear system into a linear controllable system. Then linear control strategies can be applied. So far, many of the reported works have investigated back propagation (BP) neural network. However, BP neural network still exist some problems, e.g., over learning, local minimum and slow convergence speed. In order to solve these problems, support vector machine (SVM) which applies the structural risk minimization (SRM) is established [9]. It exhibits a good performance in solving small sample, nonlinear and high dimensional pattern recognition. SVM can also be applied to other machine learning problems such as function fitting. Therefore, a method based on SVM inverse system is developed. The method uses SVM to replace BP neural network and approach the inverse system. Then, by putting the inverse system and the original system in series, a pseudo-linear subsystem, which can be controlled using linear control strategies, will be built. SVM inverse system method synthesizes the merits both of inverse system and SVM. It has been successfully used in three-phase surface-mounted PMSM system [10].

In this paper, a new decoupling control for five-phase FTPM motor will be proposed based on SVM and the inverse system theory. The proposed control method can decouple the d-axis current and speed, offering fast speed response and high control accuracy. The mathematical model of five-phase FTPM motor will be introduced in Section II. Then the inverse system will be deduced based on the inverse theory and interactor algorithm. In Section III, the foundation theory of the SVM will be described. Then, the construction of SVM inverse system will be given to compensate the original system into a pseudo-linear system. Finally, computer

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simulation will be performed to verify of the proposed method.

II. MATHEMATICAL MODEL AND INVERSION SYSTEM

Fig. 1 shows the cross section of a five-phase FTPM. It consists of rotor, stator, permanent-magnet, coil, armature tooth and fault-tolerant tooth (FTT). The back-EMF of the motor is sinusoidal.



Fig. 1. Cross section of FTPM motor.

By using the multiple-reference-frame transformation which is extended from Park's transformation in three phase motors, the mathematical model of the motor can be derived. The multiple reference frame transformation is given by

$$T(\theta) = \frac{2}{5} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{5}) & \cos(\theta - \frac{4\pi}{5}) & \cos(\theta + \frac{4\pi}{5}) & \cos(\theta + \frac{2\pi}{5}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{5}) & -\sin(\theta - \frac{4\pi}{5}) & -\sin(\theta + \frac{4\pi}{5}) & -\sin(\theta + \frac{2\pi}{5}) \\ 1 & \cos\frac{2\pi}{5} & \cos\frac{4\pi}{5} & \cos\frac{4\pi}{5} & \cos\frac{2\pi}{5} \\ 0 & \sin\frac{2\pi}{5} & \sin\frac{4\pi}{5} & -\sin\frac{4\pi}{5} & -\sin\frac{2\pi}{5} \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$
(1)

where θ is the electrical rotor angle.

Then, the five-phase FTPM motor mathematical model can be derived as following:

$$\begin{cases} \frac{di_d}{dt} = \frac{1}{L_d} (u_d - R_s i_d + \omega_e i_q L_q) \\ \frac{di_q}{dt} = \frac{1}{L_q} (u_q - R_s i_q - \omega_e i_d L_d - \omega_e \varphi_m) \\ \frac{d\omega_e}{dt} = \frac{n_p}{J} [\frac{5n_p \varphi_m}{2} i_q + \frac{5n_p}{2} (L_d - L_q) i_d i_q - T_L] \end{cases}$$

$$(2)$$

where i_d and i_q are the stator currents in corresponding rotating frames; ω_e is the electrical rotor speed; u_d and u_q are the d- and q- axis stator voltage, respectively; L_d and L_q are inductances in the rotating frames; R_s is the stator resistance; φ_m is the permanent magnet flux linkage; n_p is the number of pole pairs; J is the moment of inertia of rotating parts; and T_L is the load torque.

Due to the existence of coupling effect between d-axis stator current and the rotational speed, i_d and ω_e are chosen to be the output of the FTPM motor system. The output variables are $\mathbf{y} = [y_1, y_2]^{\mathrm{T}} = [i_d, \omega_e]^{\mathrm{T}}$. The input variables are $\mathbf{u} = [u_1, u_2]^{\mathrm{T}} = [u_d, u_q]^{\mathrm{T}}$. Moreover, the state variables are $\mathbf{x} = [x_1, x_2, x_3]^{\mathrm{T}} = [i_d, i_q, \omega_e]^{\mathrm{T}}$. Consequently, (2) can be expressed as:

$$\dot{x} = \begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \dot{x}_{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_{d}} (u_{1} - R_{s}x_{1} + x_{3}x_{2}L_{q}) \\ \frac{1}{L_{q}} (u_{2} - R_{s}x_{2} - x_{3}x_{1}L_{d} - x_{3}\varphi_{m}) \\ \frac{5n_{p}^{2}\varphi_{m}}{2J}x_{2} + \frac{5n_{p}^{2}}{2J} (L_{d} - L_{q})x_{1}x_{2} - \frac{n_{p}}{J}T_{L} \end{bmatrix}$$
(3)

According to the inverse system theory, the corresponding Jacobi matrix can be described as:

$$J(x,u) = \begin{bmatrix} \frac{\partial \dot{y}_1}{\partial u_1} & \frac{\partial \dot{y}_1}{\partial u_2} \\ \frac{\partial \ddot{y}_2}{\partial u_1} & \frac{\partial \ddot{y}_2}{\partial u_2} \end{bmatrix}$$
(4)
$$= \begin{bmatrix} \frac{1}{L_d} & 0 \\ \frac{5n_p^2 (L_d - L_q)i_q}{2JL_d} & \frac{5n_p^2 [\varphi_m + (L_d - L_q)]i_d}{2JL_q} \end{bmatrix}$$

Then the rank of the Jacobi matrix can be calculated as:

$$Det(J(x,u)) = \frac{5n_p^2}{2JL_dL_q} [\varphi_m + (L_d - L_q)i_d]$$
(5)

Obviously, (5) is not equal to zero. The relative order of the system is $\boldsymbol{\alpha} = (\alpha_1, \alpha_2) = (1, 2)$, and the order of the system is n = 3. Since the relative order is equal to the order of the system, the inverse system is existent and can be described as:

$$u = \phi(\dot{y}_1, y_1, \ddot{y}_2, \dot{y}_2, y_2) \tag{6}$$

III. DESIGN OF SVM INVERSE SYSTEM CONTROL

A. SVM Algorithm

Although the inverse system has been obtained by the above-mentioned method, it is still hard to obtain the equations exactly. So, the investigation for effectively constructing the inverse system is significant.

SVM is a kernel-based learning machine and plays an important role in classification problems. The basic theory has been described in [11]. SVM has the advantages of simple structure, strong robustness and better generalization.

It has been known that the linear un-separable samples in low dimensional space can be linear separable in high by using nonlinear mapping. Then a linear regression function can be built. The linear regression function in high dimensional space is given as:

$$f(x) = (w, \varphi(x)) + b \tag{7}$$

where w is the weight vector in high dimensional space; b is the threshold.

The given training sample set is $\{x_i, y_i\}$, $i = 1, 2, \dots, n$. The purpose of (7) is searching for the optimal approximation in high dimensional space. According to the SRM principle, (7) should guarantee the structural risk function minimize. By introducing the relaxation factors ξ_i^* and ξ_i , the solution of (7) can be turned into solving the following constrained optimization problem:

$$\min J = \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n (\xi_i^* + \xi_i)$$
s.t.
$$\begin{cases} y_i - (w, \varphi(x_i)) - b \le \varepsilon + \xi_i^* \\ b + (w, \varphi(x_i)) - y_i \le \varepsilon + \xi_i \\ \xi_i^*, \xi_i \ge 0, \quad i = 1, \cdots, n \end{cases}$$
(8)

where C is the penalty parameter. After using the lagrange function, duality principle and kernel function theory, the problem transforms into a quadratic programming problem. It can be rewritten as:

$$\min J = \varepsilon \sum_{i=1}^{n} (\alpha_i^* + \alpha_i) - \sum_{i=1}^{n} y_i (\alpha_i^* + \alpha_i) + \frac{1}{2} \sum_{i,j=1}^{n} (\alpha_i^* - \alpha_i) (\alpha_j^* - \alpha_j) K(x_i, y_j)$$
(9)
s.t.
$$\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) = 0, 0 \le \alpha_i, \alpha_i^* \le C/n, i = 1, \cdots, n$$

where α_i^* and α_i are lagrange multipliers; $K(\cdot, \cdot)$ is an arbitrary symmetric function. It should be mentioned that Guass function is chosen to be the kernel function in this work. Finally, the regression function can be expressed as:

$$f(x) = \sum_{i=1}^{m} (\alpha_i - \alpha_i^*) K(x_i, x) + b$$
 (10)

where *m* is the number of support vectors. The kernel function is used to map the vectors selected from the original dimensional space into a component in high feature space.

B. SVM Inverse System

In Section II, the existence of the inverse system has been proved. Then the SVM inverse system is established. It consists of some integral elements used as dynamic compensation and one SVM applied as nonlinear mapping. So, SVM inverse system can realize dynamic decoupling of i_d and ω_e . Then, the original system will be decoupled into two subsystems, current subsystem and speed subsystem. The developed method does not need the accurate model of the original system. The pseudo-linear system is shown in Fig. 2.



IV. VERIFICATION

In order to verify the effectiveness of the proposed method, a Matlab/Simulink-based simulation model has been developed. The complete simulation block diagram is developed in Fig. 3. The FTPM motor parameters are shown in Table I. SVM inverse system and BP-NNI system has been compared when the FTPM motor operates under the same condition. The training samples of SVM and BP-NN are 20000. According to the effectiveness of training approximation and generalization, the parameters of SVM and BP-NN are adjusted to obtain best results. The *C* and the insensitive coefficient ε of SVM are chosen to be 200 and 0.02.



Fig. 3. Diagram of the complete simulation system.

MOTOR PARAMETERS		
Symbol	Signification	Value
L_d	d-axis inductance	$381 \times 10^{-6} \text{ H}$
L_q	q-axis inductance	$956 \times 10^{-6} \text{ H}$
R_s	stator resistance	0.21Ω
J	moment of inertia	0.015Kg·m ²
$\varphi_{\scriptscriptstyle m}$	permanent-magnet flux linkage	0.043Wb
n_p	number of pole-pairs	4

TABLE I



Fig. 4. Speed response for step changes from 100 to 350 rpm.



Fig. 5. Speed responses of the proposed scheme. (a) D-axis current; (b) Rotor speed.



Fig. 6. Speed responses of BP-NNI scheme. (a) D-axis current; (b) Rotational speed.

In all simulated results, the dash line means the reference value, while the solid line stands for the response value.

Fig. 4 shows the simulated speed response for step changes input by the SVM inverse control method. The reference speed is switching between 100 and 350 rpm. It can be seen that the proposed scheme follows the reference speed rapidly. Additionally, there is no overshoot, undershoot, and steady state error.

Figs. 5 and 6 show the decoupling effectiveness of SVM inverse control scheme and BP-NNI scheme. D-axis current changes at 8 s and rotational speed changes at 16 s. It can be seen from Figs. 5 (b) and 6 (b) that the speed response time of the proposed control strategy is faster than BP-NNI control method. Additionally, comparing Figs. 5 (a) and 6 (a), the

d-axis current of the proposed control method is more steady when the speed changes.

I. CONCLUSION

A SVM inverse system control strategy for five-phase FTPM motor has been presented. The proposed method and BP-NNI control method has been compared. From the simulation results, it can be concluded that the proposed control strategy successfully decouples the d-axis current and rotor speed of five-phase FTPM motor system. Also, speed and current can follow the reference value accurately. Moreover, SVM inverse control method provides faster and more stable speed response than BP-NNI control strategy. The whole system has good dynamic and static performances.

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