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An Optimal Real-time Pricing for Demand-side Management: A Stackelberg Game and Genetic Algorithm Approach

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***Abstract*—This paper proposes a real-time pricing scheme for demand response management in the context of smart grids. The electricity retailer determines the retail price first and announces the price information to the customers through the smart meter systems. According to the announced price, the customers automatically manage the energy use of appliances in the households by the proposed energy management system with the aim to maximize their own benefits. We model the interactions between the electricity retailer and its customers as a 1-leader, N-follower Stackelberg game. By taking advantage of the two-way communication infrastructure, the sequential equilibrium can be obtained through backward induction. At the followers' side, given the electricity price information, we develop efficient algorithms to maximize customers' satisfaction. At the leader's side, we develop a genetic algorithms based real-time pricing scheme by considering the expected customers' reactions to maximize retailer's profit. Experimental results indicate that the proposed scheme can not only benefit the retailers but also the customers.**

I. Introduction

With the increasing demand of electricity from residential customers in recent years, the traditional electricity grid is faced with many existing and potential problems such as blackouts. In addition, the average household electricity load has the potential to double with the deployment of plug-in hybrid electric vehicles (PHEVs), which will further endanger the existing grid [1]. Although the electricity prices in the wholesale market fluctuate accordingly to the generation cost, almost all end customers nowadays are charged flat retail electricity prices. As a result, the customers do not have the incentives to shift their electricity use from peak periods to off-peak periods [2].

Instead of building more power plants to meet the peak demand of customers, demand response (DR) is a better choice for solving the above problems. Demand Response is defined as a tariff or program established to motivate changes in electric use by customers in response to changes in the price of electricity over time [3]. Real-time pricing (RTP) is one of the most important DR strategies, where the prices announced by retailers change typically hourly to reflect variations of the price in the wholesale market over time. Generally, customers are notified of RTP prices the day before or a few hours before the delivery time. One of the most typical types of RTP is day-ahead RTP, in which customers receive the prices for the next 24 hours [4].

Both customers and utility companies can benefit from real-time pricing. For customers, they have the incentive to shift their electricity use from high price periods (peak time) to low price periods (off-peak time) in order to reduce their utility bills. For utility companies, they can obtain a low peak-to-average load ratio, which can help them reduce their generation cost and improve the grid reliability [5].

In general, the existing research on demand response and real-time pricing can be divided into three parts. Firstly, the work of [6][7] deals with how customers respond to the real-time prices, i.e., they provide customers with an efficient energy scheduling scheme by utilizing the benefits of smart grids and smart meters, which provides a new picture of demand response research in the context of smart grid. Secondly, [1][8][9] are concerned with how the retailers set the real-time pricing where they analytically models the customers' preferences and customers' electricity consumption patterns in the form of utility functions. Thirdly, work of [2][10][11][12][13] deals with how the retailers determine the real-time pricing based on the expected responses of customers. [10] propose a decision-making scheme for electricity retailers based on Stackelberg game. They model the customers' preference and satisfaction as utility functions. [12] present an optimal demand response scheduling with Stackelberg game approach. Similar to [10], they model the customers' behaviour patterns as utility functions. [13] proposes a dependable demand response management scheme via Stackelberg game in the context of smart grids where they also model the electricity use patterns of customers as utility functions. Rather than model customers' behaviours as utility functions, [11][2] presents Stackelberg game based models, in which the follower level problems are modelled with appliance-level details.

Our results and analysis in this paper differ from the related work in several aspects. For customer-side problems, the difference between our work and [6][7] lies in that we consider all categories of the home appliances including non-shfitable appliances, interruptible appliances, non-interruptible appliances and curtailable appliances, i.e., our models are more accurate and comprehensive. For retailer-side problems, existing research [1][8][9] have not given the explicit form of

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the customers' utility functions and thus are unable to help the customers to find the best usage and scheduling scheme to maximize their benefits. To overcome this weakness, the approach given in this paper aims to provide the best and usable solution for customers to achieve their maximal benefits. When determining the real-time prices for retailers based on the expected responses of customers, the existing research [2][10][11][12][13] are similar to ours in some aspects, i.e., we all model the interactions between an energy retailer and its customers as a Stackelberg game. However, the difference between our work and [10], [12] and [13] lies in that we model the follower level problem (customers level problem) with appliance-level details, which is more practical. Although [11] and [2] model the follower level problem with appliance-level details, they do not consider all types of appliances. In this regard, our models which consider all four different types of appliances and include all possible types of applications, are most comprehensive and accurate.

In this paper, we propose a decision making scheme based on Stackelberg game for a given electricity retailer and its customers. We firstly design an efficient energy management system to schedule the electricity use of appliances to achieve the maximal benefits for customers. Furthermore, we design an efficient real-time pricing algorithm for the retailer with the aim to maximize its profits by considering the rational responses from customers. The main contributions of this paper are summarized as follows:

• We propose an efficient energy management system for customers. We divide the appliances into non-shiftable appliances, interruptible appliances, non-interruptible appliances and curtailable appliances according to their load types. For different category of appliances, different models have been proposed in this paper. To the best of our knowledge, this is the first literature to consider all four types of appliances and give relevant mathematical models.

• We model the interactions between the retailer and its electricity customers as a 1-leader, N-follower Stackelberg game and genetic algorithms have been adopted to obtain Stackelberg solutions.

The rest of this paper is organized as follows. The system model is proposed and the problem formulation is introduced in Section II. In Section III, the system model is analysed, and the optimal solution is given. Experimental results are provided and discussed in Section IV. The paper is concluded in Section V.

II. System Model and Problem Formulation

In this section, we will first introduce the concepts of Stackelberg games with one leader and multi-followers, then analyse how the optimal real-time pricing problem for demand-side management can be modelled as a Stackelberg game problem.

In a Stackelberg game with one leader and N followers, the rule of playing is as follows: Firstly one player called the leader (such as the retailer) chooses and announces its strategy (such as price) u_L from its strategy space U_L; After knowing the leader's strategy, the other N players (such as customers) called the followers will decide their best response (such as energy consumption) strategies $u^*_{F_i} = R_i(u_L)(i = 1, 2, ..., N)$ from their strategy spaces $U_{F_i}(i = 1, 2, ..., N)$ respectively. Assume that the followers' goal is to minimise their objective (such as bill) functions, then the followers' best response strategies are defined as

$$u^*_{F_i} = R_i(u_L) = \arg \min_{u_{F_i} \in U_{F_i}} J_{F_i}(u_L, u_{F_i}) \quad i = 1, 2, ..., N \tag{1}$$

Taking into account the followers' reaction functions $R_i(u_L)(i = 1, 2, ..., N)$, if the leader's goal is to find its best strategy which maximises its objective (such as profit) function $J_L(u_L, u_{F_1}, ..., u_{F_N})$. That is, find

$$u^*_L = \arg \max_{u_L \in U_L} J_L[u_L, R_1(u_L), ..., R_N(u_L)] \tag{2}$$

Definition 1. In the above Stackelberg game with one leader and N followers, an optimal Stackelberg strategy is a strategy vector $\left(u^S_L, u^S_{F_1}, ..., u^S_{F_N}\right)$ satisfying the following conditions:

$$u^S_{F_i} = R_i(u^S_L) = \arg \min_{u_{F_i} \in U_{F_i}} J_{F_i}(u^S_L, u_{F_i}) \quad i = 1, 2, ..., N \tag{3}$$

$$u^S_L = \arg \max_{u_L \in U_L} J_L[u_L, R_1(u_L), ..., R_N(u_L)] \tag{4}$$

In the following, we consider a micro-grid shown as Fig.1 which consists of a retailer and N customers [11]. It is assumed that each customer is equipped with a smart meter. The retailer procures electricity from the wholesale market, determines the retail prices and sends the price information to the customers (smart meters) via LAN. The smart meters then manage the electricity use of home appliances in response to the price signal and transmit the electricity demand information to the retailer. The interactions between retailer and its customers can be enabled through a two way communication infrastructure. As it can be seen from the above interaction process, this is a 1-leader, N-follower Stackelberg game, i.e., the customers make their best energy consumption decisions based on the prices announced by the retailer and in turn the retailer designs the best real-time prices by analyzing the customers' responses to its prices.

In this section, we provide a Stackelberg game representation of the considered two-stage decision making problems. The retailer plays the leader-level game and the customers play the follower level game. Firstly, our focus is to formulate the energy management problem in response to the real-time pricing in each household at the follower level. Secondly, we model the profit maximization problem for the retailer who will offer the 24 hours real-time prices to the customers at the leader level.

Throughout this paper, let $\mathcal{N} = \{1, 2, ..., N\}$ denote the considered set of customers, where $N \triangleq |\mathcal{N}|$ and $\mathcal{H} \triangleq \{1, 2, ..., H\}$ denote the scheduling horizon. Usually, $H = 24$.

We define the prices offered by the retailer as a price vector: $P = [p^1, p^2, ..., p^h, ...p^H]$, where p^h represents the electricity price at hour h .

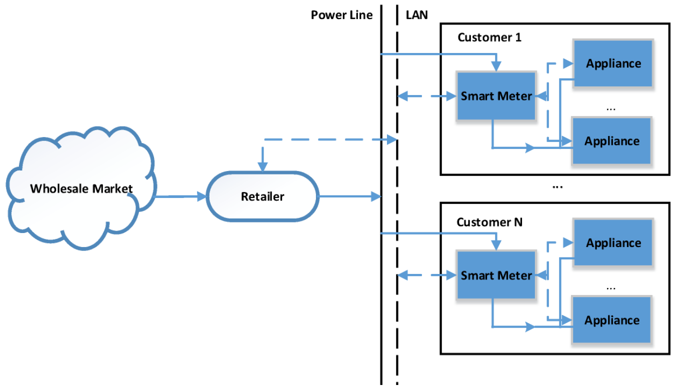

Fig. 1: Structure of a Micro-grid

A. *Customer-side Problem at the Follower Level*

We categorize the home appliances into non-shiftable appliances, shiftable appliances and curtailable appliances according to their load types.

For shiftable appliances, the operations of this part of appliances can be shifted from high price periods to low price periods in order to lower the customers' payment bills. However, the total energy consumption to accomplish the operations is fixed.

We further divide the shiftable appliances into interruptible and non-interruptible appliances. For the interruptible appliances such as PHEVs, the operations of these appliances can be interruptible, i.e., it is possible to charge the PHEV for one hour, then stop charging for one or several hours and then complete the charging after that. For non-interruptible appliances such as washing machines and dish washers, the operations of these appliances are non-interruptible, i.e., since such appliances start, they have to keep running till the completion.

For curtailable appliances such as air conditioning and space heater, the total energy consumption can be adjusted. For example, if a customer feels the price in a given hour is too high, he can reduce the usage or even stop the use of the appliance.

For each customer $n \in \mathcal{N}$, we denote respectively the whole set of appliances in each household as A_n, the set of non-shiftable appliances as NS_n, interruptible appliances as I_n, non-interruptible appliances as NI_n and curtailable appliances as C_n.

Since the non-shiftable appliances consume a fixed amount of energy at each hour during a fixed working period, there is no flexibility to adjust the energy consumption in response to the price and no mathematical model is needed.

1) Interruptible Appliances: This model improves some existing work [6] [2]. In [6] [2], an upper limit of hourly electricity usage is set for each household, but we do not have such constraints for the optimization problem at customers' side as there is no such usage limits in practice. Instead, we consider the total upper limit of hourly usage of all the customers served by the same retailer and put the constraint in the optimization problem at retailers' side. This is to represent the maximum supply capability by the retailer or the maximum load capacity of power networks. Therefore, the retailer can actually influence the hourly use of electricity of each household by properly determining the retail price, which is more feasible and realistic from an application point of view.

For each interruptible appliance $a \in I_n$, we define a scheduling vector of energy consumption over the scheduling horizon $\mathcal{H} = \{1, 2, ..., H\}$ as follows:

$$\mathrm{x}_{n,a} = [x^1_{n,a}, ..., x^h_{n,a}, ..., x^H_{n,a}] \tag{5}$$

where $x^h_{n,a} \geq 0$ represents the nth user's electricity consumption of appliance a at time h over the scheduling horizon.

The total electricity consumed during the scheduling window of appliance $a \in I_n$ is defined as $E_{n,a}$ and the scheduling window for each appliance a is set by each customer according to his/her preference and can be defined as $\mathcal{H}_{n,a} \triangleq \{\alpha_{n,a}, \alpha_{n,a}+1, ..., \beta_{n,a}\}$. Since the window $\mathcal{H}_{n,a}$ is consecutive, one only needs to specify the beginning scheduling time $\alpha_{n,a} \in \mathcal{H}$ and end time $\beta_{n,a} \in \mathcal{H}$. To meet the energy need of operations for each appliance a over the scheduling window, we have:

$$\sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x^h_{n,a} = E_{n,a} \tag{6}$$

As there is no operation needed for the hours outside the scheduling window, we have:

$$x^h_{n,a} = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a} \tag{7}$$

Furthermore, the energy consumption for each appliance a at each hour $h \in \mathcal{H}_{n,a}$ is subject to:

$$\gamma^{min}_{n,a} \leq x^h_{n,a} \leq \gamma^{max}_{n,a}, \forall h \in \mathcal{H}_{n,a}. \tag{8}$$

where $\gamma^{min}_{n,a}$ is the minimum power level of each appliance a and $\gamma^{max}_{n,a}$ is the maximum power level.

Finally, the payment minimization problem for an interruptible appliance can be modelled as follows:

$$\begin{aligned} &\min J_{I_n(a)} = \min_{x^h_{n,a}} \textstyle\sum_{h=1}^{H} p^h \times x^h_{n,a} \\ &s.t. \\ &\textstyle\sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x^h_{n,a} = E_{n,a}, \\ &\gamma^{min}_{n,a} \leq x^h_{n,a} \leq \gamma^{max}_{n,a}, \forall h \in \mathcal{H}_{n,a}, \\ &x^h_{n,a} = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}. \end{aligned} \tag{9}$$

2) Non-interruptible Appliances: As the operations of each non-interruptible appliance $a \in NI_n$ are consecutive, we define the length of the operations $L_{n,a}$. For non-interruptible appliance, the customers also need to set the scheduling window $\mathcal{H}_{n,a} \triangleq \{\alpha_{n,a}, \alpha_{n,a}+1, ..., \beta_{n,a}\}$ by specifying the beginning scheduling time and end time.

The optimization problem is to find the optimal start time $s^*_{n,a}$ that minimize the customer's payment. As a result, for each appliance a, to meet the energy need of accomplishing the operations, we have:

$$\sum_{h=s_{n,a}}^{s_{n,a}+L_{n,a}} x^h_{n,a} = E_{n,a}, \tag{10}$$

where $\alpha_{n,a} \leq s_{n,a} \leq \beta_{n,a} - L_{n,a}$.

Furthermore, the constraints Eq.(7) and Eq.(8) also apply to the non-interruptible appliances.

As a result, the payment minimization problem for a non-interruptible appliance can be defined as follows:

$$
\begin{aligned}
&\min J_{NI_n(a)} = \min_{s_{n,a}}\{\min_{x^h_{n,a}} \textstyle\sum_{h=s_{n,a}}^{s_{n,a}+L_{n,a}} p^h \times x^h_{n,a}\} \\
&s.t. \\
&\textstyle\sum_{h=s_{n,a}}^{s_{n,a}+L_{n,a}} x^h_{n,a} = E_{n,a}, \\
&\alpha_{n,a} \leq s_{n,a} \leq \beta_{n,a} - L_{n,a}, \\
&x^h_{n,a} = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}, \\
&\gamma^{min}_{n,a} \leq x^h_{n,a} \leq \gamma^{max}_{n,a}, \forall h \in \mathcal{H}_{n,a}.
\end{aligned} \tag{11}
$$

3) Curtailable Appliances: Similarly to shiftable appliances, we assume that for each appliance $a \in C_n$, the customers need to select the beginning hour $\alpha_{n,a} \in \mathcal{H}$ and end hour $\beta_{n,a} \in \mathcal{H}$ for a valid scheduling window. Compared to shiftable appliances, the window of curtailable appliances should be more strict and accurate because the appliances will be on for the whole window. For example, for a typical summer weekday, it is not reasonable to select the beginning hour $\alpha_{n,a} = 7$ PM and the end hour $\beta_{n,a} = 7$ AM (the next day) for the air conditioning, which instead is a perfect time window for PHEVs or washing machine. However, $\alpha_{n,a} = 7$ PM and $\beta_{n,a} = 12$ PM could be a reasonable time setting for the air conditioning.

We define the energy consumption at each typical hour h for each curtailable appliance $x^h_{n,a}$, the minimum acceptable consumption level $\underline{u}^h_{n,a}$ and maximum affordable consumption level $\overline{u}^h_{n,a}$. The minimum acceptable consumption level $\underline{u}^h_{n,a}$ and maximum affordable consumption level $\overline{u}^h_{n,a}$ differ among customers and can be set according to each individual customer's preference. Usually, $\gamma^{min}_{n,a} \leq \underline{u}^h_{n,a}$ and $\overline{u}^h_{n,a} \leq \gamma^{max}_{n,a}$, where $\gamma^{min}_{n,a}$ and $\gamma^{max}_{n,a}$ are defined in the previous section. Finally, we have:

$$
\underline{u}^h_{n,a} \leq x^h_{n,a} \leq \overline{u}^h_{n,a} \tag{12}
$$

By the customers' behaviour analysis, there are two types of optimization models for curtailable appliances.

For some customers, in particular more price sensitive ones, they prefer reducing the spending as much as possible subject to an acceptable comfortable level (for example, the air condition is turned on at least half an hour with each hour between 7:00pm and 10:00pm). For the other (and more) customers, they prefer a budget based comfortable maximization model in the sense that, for a given curtailable appliance such as air conditioning, they set up a daily budget (i.e., the maximum allowed daily spending) under which they choose to make their life as comfortable as possible under the given budget. To meet the above two types of customers' preferences, two types of optimization model for curtailable appliances are proposed as below and a customer can choose one of them dependent on his/her preference.

a) Minimize Bill Subject to an Acceptable Life Quality: The life quality for customers in terms of energy use can be represented as the total electricity consumed during the scheduling window. In this case, for each curtailable appliance, the electricity consumed during the operation period should not less than a minimum consumption level, i.e., there exists a life quality constraint for each appliance a:

$$
\sum_{h=\alpha_{n,a}}^{\beta n,a} x^h_{n,a} \geq U^{min}_{n,a} \tag{13}
$$

The optimization problem is to minimize the payment bill of customers subject to the life quality constraint shown as follows:

$$
\begin{aligned}
&\min J1_{C_n(a)} = \min_{x^h_{n,a}} \textstyle\sum_{h=\alpha_{n,a}}^{\beta n,a} p^h \times x^h_{n,a} \\
&s.t. \\
&\underline{u}^h_{n,a} \leq x^h_{n,a} \leq \overline{u}^h_{n,a}, \\
&\textstyle\sum_{h=\alpha_{n,a}}^{\beta n,a} x^h_{n,a} \geq U^{min}_{n,a}.
\end{aligned} \tag{14}
$$

b) Maximize Life Quality Subject to an Acceptable Financial Constraint: In this case, for each curtailable appliance, the money spent during the operation period should not exceed the given budget, i.e., there exists a financial cap for each appliance a:

$$
\sum_{h=\alpha_{n,a}}^{\beta n,a} p^h \times x^h_{n,a} \leq C^{max}_{n,a} \tag{15}
$$

Finally, the optimization problem is to maximize the life quality of customers subject to the budget constraint shown as follows:

$$
\begin{aligned}
&\max J2_{C_n(a)} = \max_{x^h_{n,a}} \textstyle\sum_{h=\alpha_{n,a}}^{\beta n,a} x^h_{n,a} \\
&s.t. \\
&\underline{u}^h_{n,a} \leq x^h_{n,a} \leq \overline{u}^h_{n,a}, \\
&\textstyle\sum_{h=\alpha_{n,a}}^{\beta n,a} p^h \times x^h_{n,a} \leq C^{max}_{n,a}.
\end{aligned} \tag{16}
$$

Since there are two types of optimization model for curtailable appliances, the optimization problem for customer n including all types of appliances has two different optimization objectives shown as Eqs.(17) and (18). The customers can choose one of them depending on their preferences.

$$
\begin{aligned}
&\min J1_n = \min\{\sum_{a \in I_n} J_{I_n(a)} + \sum_{a \in NI_n} J_{NI_n(a)} + \sum_{a \in C_n} J1_{C_n(a)}\} \\
&\text{subject to the constraints (6)-(8), (10) and (12)-(13).}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&\min J2_n = \min\{\sum_{a \in I_n} J_{I_n(a)} + \sum_{a \in NI_n} J_{NI_n(a)} - \sum_{a \in C_n} J2_{C_n(a)}\} \\
&\text{subject to the constraints (6)-(8), (10), (12) and (15).}
\end{aligned} \tag{18}
$$

B. Retailer-side Problem at the Leader Level

In this subsection, the energy cost model for a retailer will be discussed first, and then a profit maximization model will be proposed.

Let function $C_h(L_h)$ represent the cost of providing electricity by the retailer at each hour $h \in \mathcal{H}$, where L_h represents the amount of power consumed by all users at each hour of the day. If per unit (such as KW) cost at hour h is a_h, then the cost function can be given as follows:

$$C_h(L_h) = a_h L_h \tag{19}$$

where $a_h > 0$ and $h \in \mathcal{H}$.

For each hour $h \in \mathcal{H}$, by defining the minimum price that the retailer (utility company) can offer p^{min} and the maximum price p^{max}, we have:

$$p^{min} \leq p^h \leq p^{max}. \tag{20}$$

p^{min} and p^{max} are usually designed based on history prices, market competition, customers' acceptability and the wholesale price. It is reasonable to assume that the price the retailers can offer is greater than the wholesale price for each hour, and there exists a price cap for the retail prices due to retail market competition and regulation. Then p^{max} can be used to represent such a price cap.

Note that there is usually a maximum supply capacity, denoted as $E_h^{\max}$, at each hour. Thus, we have the following constraint:

$$\sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h \leq E_h^{\max}, \forall h \in \mathcal{H} \tag{21}$$

Furthermore, due to the political pressure and customers' acceptability, the retailer needs to set up a total revenue cap, denoted as R^{max}. As a result, we have the following constraint:

$$\sum_{h \in \mathcal{H}} p^h \times \sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h \leq R^{max} \tag{22}$$

Finally, the profit maximization problem can be modelled as follows:

$$\begin{aligned}
&\max_{p^h} \{ \sum_{h \in \mathcal{H}} p^h \times \sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h - \sum_{h \in \mathcal{H}} C_h(\sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h) \} \\
&s.t. \\
&p^{min} \leq p^h \leq p^{max}, \\
&\sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h \leq E_h^{\max}, \forall h \in \mathcal{H}, \\
&\sum_{h \in \mathcal{H}} p^h \times \sum_{n \in \mathcal{N}} \sum_{a \in A_n} x_{n,a}^h \leq R^{max}.
\end{aligned} \tag{23}$$

C. A Two Stage Stackelberg Game Model

We model the interactions between the retailer and the customers as a 1-leader, N-followers two stage Stackelberg game.

- Stage 1: The retailer determines the electricity prices $P = [p^1, p^2, ..., p^h, ...p^H]$ to offer to the customers.

- Stage 2: The customers determine their individual electricity demand to maximize their pay-off, given the price P.

III. Stackelberg Game Model Solutions

A. Existence of Stackelberg Strategy

In the previous section, we have shown that the optimal real-time pricing problem for demand-side management can be modelled as a Stackelberg game problem, in which the customers as the followers need to find the optimal energy consumption schemes or strategies to minimize their objective functions given in Eq.(17) or Eq.(18), whereas the retailer as the leader needs to find the optimal real-time price strategy to maximise its objective function given in Eq.(23). Before we develop the algorithms which can find the optimal energy consumption schemes for the customers and the optimal real-time price strategy for the retailer, we need to know whether such optimal consumption schemes and optimal real-time price strategy exist. In other words, whether the optimal Stackelberg strategy for the considered Stackelberg pricing game problem exists? The following theorem gives a positive answer to this question.

Theorem 1. *Consider the Stackelberg game with one leader and N followers as follows: 1) The leader's objective function is defined in Eq.(23) and its strategy space is defined by the constraints (20)-(22); 2) There are N followers, in which some followers' objective functions are defined by Eq.(17) and their strategy spaces are defined by the constraints (6)–(8), (10) and (12)–(13), whereas the other followers' objective functions are defined by Eq.(18) and their strategy spaces are defined by the constraints (6)–(8), (10), (12) and (15). Then the optimal Stackelberg strategy exists.*

Due to the space limitation, the proof for the above theorem is omitted in this paper. With the above theorem ensure the existence of the optimal Stackelberg strategy, the algorithms to find the optimal Stackelberg strategy are going to be developed and presented in the remaining part of this section.

B. Backward Induction

The proposed leader-follower Stackelberg game model consists of two-stage, sequential decision-making problems. The common solution concept for a multi-stage Stackelberg game is the sub-game perfect equilibrium(SPE). Backward Induction, which starts at the last action and reasons backwards, is a general method to determine the SPE [14]. We first starts from stage 2 and analyse how the customers (followers) adjust their energy consumption to maximize the pay-off, given the prices offered by the retailer (leader). Then it moves backwards in time to stage 1 to analyse how the retailer makes real-time pricing decisions based on the expected response of followers.

If the analytical solutions exist both at the followers' side problem and leader's side problem, we can obtain the optimal Stackelberg strategies for this game. However, in our case, there are no analytic solutions because the followers' reaction functions are not differentiable. As a result, genetic algorithms are adopted to obtain the Stackelberg solutions.

C. Solutions to Customer-side Problem in Stage 2

As the follower's optimization problem is the sum of three separable sub-optimization problems corresponding to interruptible, non-interruptible, and curtailable appliances respectively. As a result, the follower's optimization problem can be solved by each sub-optimization problem separately. As the result, in the following, we discuss how to solve each sub-optimization problem.

1) Interruptible Appliances: The mathematical model of interruptible appliances is shown as Eq.(9) which is a typical linear programming problem. Simplex method is a common method to this problem. In this paper, we adopt IBM CPLEX solver to solve this problem.

2) Non-interruptible Appliances: Eq.(11) shows the mathematical model of non-interruptible appliances. The optimization problem is to find the optimal start time $s^*_{n,a}$ for each non-interruptible appliance to minimize the customer's payment bill. We define the sub-problem of the original model Eq.(11) as follows by fixing the start time at $s'_{n,a} \in [\alpha_{n,a}, \beta_{n,a} - L_{n,a}]$.

$$\begin{array}{l}\min\limits_{x^h_{n,a}} \sum_{h=s'_{n,a}}^{s'_{n,a}+L_{n,a}} p^h \times x^h_{n,a} \\ s.t. \\ \sum_{h=s'_{n,a}}^{s'_{n,a}+L_{n,a}} x^h_{n,a} = E_{n,a}, \\ x^h_{n,a} = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_{n,a}, \\ \gamma^{min}_{n,a} \leq x^h_{n,a} \leq \gamma^{max}_{n,a}, \forall h \in \mathcal{H}_{n,a}.\end{array} \tag{24}$$

Eq.(24) is a linear programming problem and can be easily solved with IBM CPLEX solver. As a result, the original problem Eq.(11) can be solved in an iterative way.

3) Curtailable Appliances: The optimization problems Eq.(14) and Eq.(16) for curtailable appliances are linear programming problems and can be solved by existing solvers such as IBM CPLEX.

D. Solutions to Retailer-side Problem in Stage 1

We adopt genetic algorithms (GAs) to solve profit maximization problem at the retailer's side and show how the retailer finds the optimal electricity real-time prices taking into account the customers' responses.

GAs are computationally simple and powerful and are very good tools for non-linear optimization problems since they have more chances to find the global optimal solutions. Furthermore, GAs are often the only method available for some ill-defined optimization problem such as those involving with non-differential, discontinuous, or non-analytically definable functions. The optimization problem for the retailer's side is one of such cases.

In our genetic algorithm, binary encoding and deterministic tournament selection without replacement is adopted. For the crossover and mutation operations, we employ uniform crossover and bit flip mutation respectively.

Finally, the GA based decision-making algorithms are shown as Algorithm 1 and 2, which are implemented at the retailer-side and customer-side respectively. At the end, the most profitable prices for the retailer and the best usage patterns and schedules with the minimized bill and/or maximized life quality for each customer are found.

Algorithm 1 GA based pricing algorithm to Eq.(23) executed by the retailer

1: Population Initialization, i.e., generating a population of N chromosomes randomly; each chromosome denotes a strategy of the leader (retailer).
2: **for** i=1 to N **do**
3: The leader plays the strategy $u_{L,i}$, i.e., announces a set of 24-hour prices by decoding the ith chromosome to the smart meters (customers) via LAN;
4: Receive the optimal response of each follower n including the optimal energy consumption information: $u_{F_n,i}$.
5: Evaluate the fitness of strategy i based on $u_{L,i}$ and $u_{F_n,i}$ with the aim to solve the profit maximization problem shown as Eq.(23).
6: **end for**
7: A new generation of chromosomes is created by using the selection, crossover and mutation operations of the genetic algorithm.
8: Steps 2 - 7 are repeated until the stopping condition is reached.
9: The retailer announces the finalized price vector to the smart meters (customers) via LAN at the beginning of the scheduling horizon.

Algorithm 2 Energy management algorithm executed by each smart meter (customer)

1: Receive the prices information from the retailer.
2: The smart meter calculates the energy consumption in response to prices by solving the follow-side problem Eq.(17) or Eq.(18).
3: The smart meter sends back the total energy consumption at each hour to the retailer via LAN.

IV. Experimental Results

We simulate a neighbourhood consisting of 100 customers served by one energy retailer. It is assumed that each customer has 4 appliances: PHEV, dishwasher, washing machine, air-conditioning. The scheduling horizon is set from 8AM to 8AM (the next day). We assume that the end customers are homogeneous, i.e., $E_{n,a} = E_a$, $H_{n,a} = H_a$, $\gamma^{min}_{n,a} = \gamma^{min}_a$, etc. With the purpose of simulations, for air-conditioning (curtailable appliance), we assume that all the customers choose the second optimization model for curtailable appliances, i.e., maximize life quality subject to an acceptable financial constraint.

As a result, the parameter settings of each type of appliances are given in Table I, II, III.

For the cost of the energy provided to customers by retailer, the cost function is $C_h(L_h) = a_h L_h$, where L_h represents the amount of power consumed by all users at each hour of the day. The values of a_h from 8AM to 8AM (the next day) are set as $(4.5, 3, 2.5, 3, 4, 5, 5.5, 4.5, 4.5, 3, 2.5, 3, 4, 5, 5.5, 4.5, 3, 6, 10, 9.5, 10, 10, 9.6, 9, 8, 5, 3, 2, 2, 1.5, 5, 5)$.

In this section, we will firstly show the benefits to the retailer by employing our proposed real-time pricing scheme. Secondly, we will show the benefits to customers by adopting our proposed energy management scheme.

TABLE I: Parameters for each interruptible appliance

Appliance Name	E_a	H_a	γ_a^{min}	γ_a^{max}
PHEV	9.9kwh	8AM-8AM	0.1kwh	3.0kwh

TABLE II: Parameters for each non-interruptible appliance

Appliance Name	E_a	H_a	γ_a^{min}	γ_a^{max}	L_a
Dishwasher	2.5kwh	10PM-6AM	0kwh	1kwh	3hrs
Washing machine	1.94kwh	8AM-7PM	0kwh	1kwh	2hrs

TABLE III: Parameters for each curtailable appliance

Appliance Name	C_a^{max}	α_a	β_a	$\underline{u}_a^h$	$\overline{u}_a^h$
Air-conditioning	150 pence	7PM	11PM	0.5kwh	2KW

TABLE IV: Parameter settings of GA

Parameter Name	Symbol	Values
Chromosome Length	L	10
Population Size	N	50
Mutation Probability	P_m	0.005
Terminate Generation	T	30

A. Benefits to Retailer

In this section, we compare our proposed optimal real-time pricing scheme with optimal flat pricing scheme.

The parameter settings of our proposed genetic algorithm are shown in Table IV.

Since the customers have no incentives to change their energy consumption pattern when responding to flat pricing, we assume that, under the flat pricing, the customers start the operations of appliances right at the beginning of the scheduling window H_a and the appliances work at their typical power levels. We assume that, for each hour h, $a_h \le p^h \le 14$ holds. When calculating the optimal flat pricing, we use same parameters and model as those of optimal real-time pricing. As a result, the determination of optimal flat pricing becomes a linear programming problem.

Finally, the obtained optimal real-time prices and the optimal flat prices are given as Figure 2.

We begin to look at the peak-to-average ratio (PAR) [7] in the residential load when using our proposed optimal real-time pricing scheme. Clearly, the retailers are interested in having a more balanced load demand with a lower PAR.

The total energy consumption of the customers under optimal real-time prices and optimal flat prices are given as Figure 3. Compared with the flat pricing, we can see from Figure 3 that the real-time pricing can flatten the load demand curve and reduce the peak-to-average ratio. The PAR under flat pricing is 3.94 while the PAR under real-time pricing is 3.45, which means a 12% reduction.

Furthermore, the details of revenue, cost and profit under optimal RTP and optimal flat prices can be found in Table V.

From Table V, we can see that, to make the same revenue which means the total bills for all customers are the same, the cost of the retailer under optimal real-time pricing is 132.63 pounds and the cost under optimal flat pricing is higher (155.18 pounds). This is due to the increase of peak demand and thus the increase of peak-time cost. Furthermore, the profit under optimal real-time pricing (67.37 pounds) is much higher than the profit under optimal flat pricing (44.82 pounds). The example shows a very important potential for the real-time pricing and our proposed approach: the real-time pricing enables to significantly increase the retailer's profit without increasing customers' expenses.

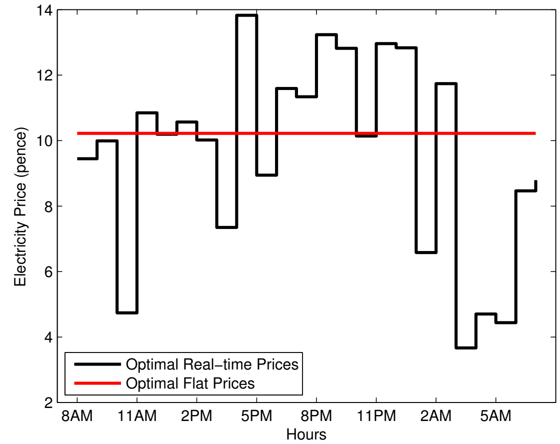

Fig. 2: Obtained optimal real-time prices and optimal flat prices

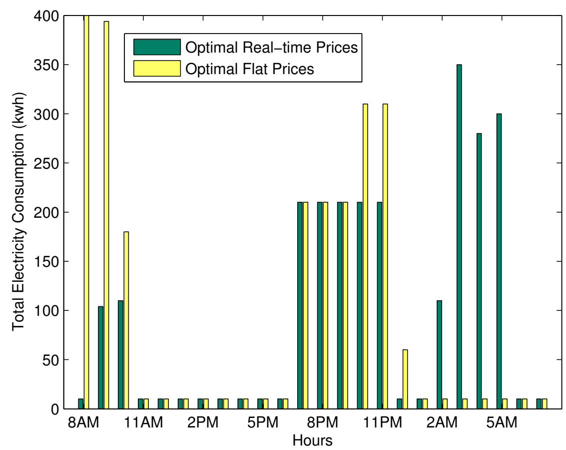

Fig. 3: Total energy consumption under real-time prices and flat prices at each hour of the day

TABLE V: Comparison of revenue, cost and profit under different price settings

Price setting	Revenue (pounds)	Cost (pounds)	Profit (pounds)
Optimal Real-time Pricing	200	132.63	67.37
Optimal Flat Pricing	200	155.18	44.82

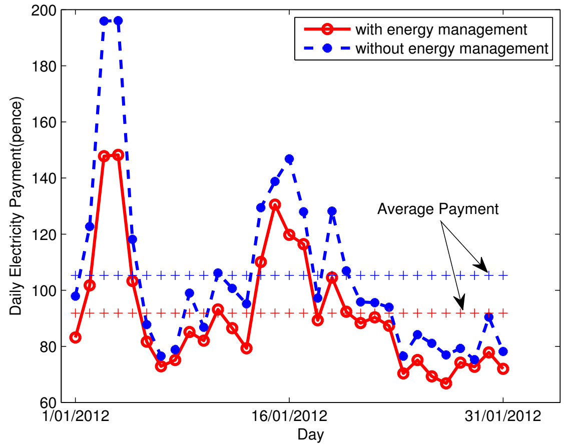

Fig. 4: Comparison of daily electricity payment of each customer over one month

B. Benefits to Customers

In this subsection, we show the effectiveness of the proposed energy management scheme based on public real-time price data. We use the actual Real Time Price (RTP) data adopted by ISO New England from January 1, 2012 to January 31, 2012, which is available to the public online at [15]. With the purpose to design a benchmark, we assume, without our proposed energy management scheme, the appliances start operations right at the beginning of the time horizon H_a and work at their typical power levels.

The simulation result is shown as Figure 4 where we can easily find that, after adopting energy management scheme, the daily payment bills are always reduced. More specifically, the average daily payment of one customer over one month can be reduced from 105.31 pence to 91.87 pence, which means a more than 13% save on the bill.

V. Conclusion

In this paper, by utilizing the benefits of smart girds, we model the interactions between the retailer and its customers as a 1-leader, N-follower Stackelberg game. Firstly, according to the load types, we categorize home appliances into non-shiftable, interruptible, non-interruptible and curtailable appliances. For different category of appliances, different appliance-level optimization models are given. Secondly, genetic algorithms have been adopted to obtain Stackelberg solutions. Since the experimental results show that both the retailer and its customers can benefit from the proposed scheme, it has great potential to improve the implementation of current energy pricing programs, help customers to reduce the increasing energy bills, and change their energy usage patterns.

Our future work will focus on Stackelberg game problems with imperfect information. Our existing work takes the assumption that there exists a two-way communication infrastructure between the retailer and its customers. However, in reality, there may not exist this two-way communication either at retailer-side or at customer-side, i.e., the retailers are unable to obtain the expected response (energy consumption information) from customers. The above problem can be modelled as a Stackelberg game with imperfect information, i.e., the leader does not know its followers' reaction function. As a result, the leader has to learn the reaction function from history data.

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