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Time Series Forecasting via Weighted Combination of Trend and Seasonality Respectively with Linearly Declining Increments and Multiple Sine Functions

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***Abstract*—In this paper, a novel weighted-combination-of-components (WCC) method is proposed for modeling and forecasting trend and seasonal time series, and such a method is based on decomposition model which regards the time series as the weighted combination of trend, seasonality and other components. Specifically, the Holt's two-parameter exponential smoothing (HTPES) method is improved (for short, the IHTPES method) to evaluate the trend with linearly declining increments; and the multiple sine functions decomposition (MSFD) method is developed to evaluate the seasonality. Then the weighted combination of the evaluations is obtained to estimate the global time series. Numerical experiment results substantiate the effectiveness and superiority of the proposed WCC method in terms of modeling and forecasting time series from the NN3 competition.**

I. Introduction

Time series modeling and forecasting is important and useful for applications in numerous different fields [1]–[5] such as business, economics, and science. Note that many time series exhibit the characteristics of trend and/or seasonality [6]. In some cases, trend and seasonality even drive the evolutions in the quarterly or monthly time series [5][6]. In other words, trend and seasonal components have a significant impact on the process of modeling and forecasting time series [7]. Thus, effective methods for modeling and forecasting time series with trend and/or seasonality is especially meaningful for practical applications.

One of traditional approaches [7]–[9] to model and forecast the time series with trend and seasonality is based on the classic decomposition model which regards the time series as the combination of trend, seasonality and other components. Generally, the traditional methods first estimate the seasonal influence, and then remove it from the data before the trend is estimated, which ignores the interaction of the trend and the seasonal components.

In this paper, a novel method termed WCC (weighted combination of components) is proposed based on the decomposition model which regards the time series as the weighted combination of trend, seasonality and other components (e.g., level component). Specifically, the Holt's two-parameter exponential smoothing (HTPES) method [6][10] is improved (i.e., IHTPES method) for evaluating the trend with linearly declining increments, and a method of multiple sine functions decomposition (MSFD) is proposed and developed to evaluate the seasonality. Then the global time series is estimated by the weighted combination of the evaluations, which means that the proposed WCC method has the superiority of estimating the trend and the seasonality of the global time series in a manner of simultaneously processing the original time series. By exploiting the WCC method on the NN3 forecasting competition dataset, the numerical experiment results verify the effectiveness and superiority of the proposed WCC method in terms of modeling and forecasting time series.

II. Methodology Description

In this paper, the decomposition model is used to describe the time series Y_t with trend T_t, seasonality S_t and the remainder of other components denoted by R_t, i.e.,

$$Y_t = wT_t + (1-w)S_t + R_t, \tag{1}$$

where $w \in [0,1]$ is used to adjust the weights for the two main components of the time series. It is worth pointing out that, for Y_t, the proposed WCC method is with $w \in (0,1)$, the IHTPES method is with $w = 1$, and the MSFD method is with $w = 0$.

Note that we choose the level of the time series L_t (i.e., the average value of the time series) as R_t throughout the paper; and that the WASD (weights and structure determination) neuronet [11] will be exploited to estimate R_t in the our further work for higher accuracy. In the rest of this section, the IHTPES, MSFD, and WCC methods are investigated for trend estimation, seasonality estimation, and global time series estimation, respectively.

A. Trend Estimation

In this subsection, the improved Holt's two-parameter exponential smoothing (i.e., IHTPES) method is used to evaluate the trend of the time series. Firstly, the original Holt's two-parameter exponential smoothing method [6][10] is described as follows:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + T_{t-1}), \tag{2}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}, \tag{3}$$

$$\hat{Y}_{t+k} = L_t + kT_t, \tag{4}$$

where $\alpha \in (0,1)$ denotes the smoothing parameter for the level L_t of the time series data, $\beta \in (0,1)$ denotes the smoothing

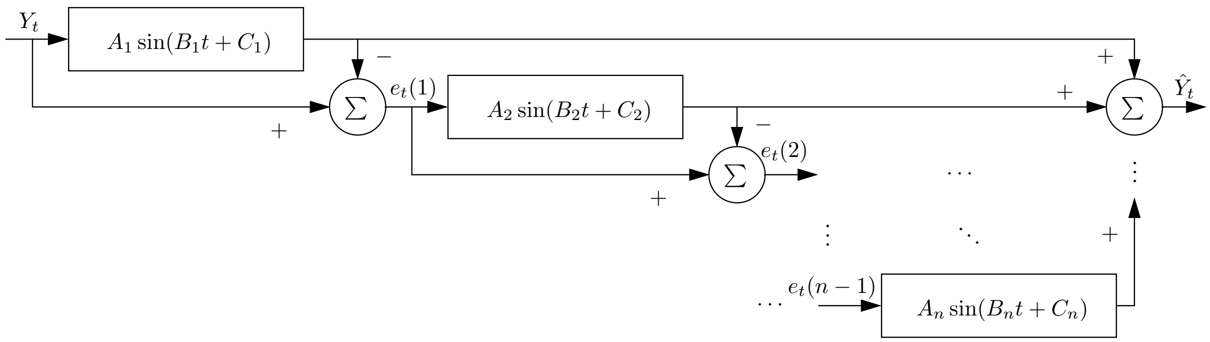


Fig. 1. The principle diagram of the MSFD (multiple sine functions decomposition) method for seasonality estimation

parameter for the trend T_t, and $\hat{Y}_{t+k}$ denotes the evaluation of the kth-step-ahead value with trend.

As seen from (2) and (3), the level L_t is a smoothed estimation of the value of the time series data at the end of each period (i.e., observation instant), and the trend T_t is a smoothed estimation of average growth at the end of each period. Then the $\hat{Y}_{t+k}$ is the evaluation of the kth-step-ahead value which assumes that the increments of trend are constant values as time goes on, which is not practical enough. In most practices [12], the increments of the trend are linearly-declining. Motivated by this, the increment parameter γ is used to replace k while estimating the future trend of the time series, and thus the specific formula of γ to use is proposed as

$$\begin{aligned}\gamma &= 1 + (1-r) + (1-2r) + \cdots + (1-(k-1)r) \\ &= \frac{(2-(k-1)r)k}{2},\end{aligned}$$

where $r \in (0,1)$ denotes the ratio of the linearly-declining increments of the trend. Besides, the averages of the last m estimations of level and trend [i.e., $L_t(m)$ and $T_t(m)$] are used to replace L_t and T_t of (4) respectively in the evaluation process of the future value, for avoiding the noise influence of the time series data. So, the trend of the kth-step-ahead value is estimated as

$$T_{t+k} = \gamma T_t(m) = \frac{(2-(k-1)r)k}{2} T_t(m),$$

where $T_t(m)$ denotes the average of the last m estimated trend values (i.e., T_t to T_{t-m+1}). Thus, (4) is improved as

$$\hat{Y}_{t+k} = L_t(m) + \gamma T_t(m), \tag{5}$$

where $L_t(m)$ denotes the average of the last m estimated level values (i.e., L_t to L_{t-m+1}).

As compared with Holt's damped-trend exponential smoothing method [9][10], the IHTPES method assumes that the increments of the trend are linearly-declining rather than exponentially-declining. Besides, the formulas of the smoothing estimations of level and trend [i.e., (2) and (3)] are exploited in the IHTPES method directly.

B. Seasonality Estimation

In this subsection, the multiple sine functions decomposition (i.e., MSFD) method is developed for estimation of the seasonality of the time series. Seasonality is a periodical and recurrent component caused by factors such as weather, holidays, and repeating promotions [7]. Conventional seasonal adjustment methods generally assume that the length of the seasonality is a constant value (e.g. 12 for monthly time series and 4 for quarterly time series). Therefore, it is unsuitable for some special cases, for example, the length of seasonality (or termed, quasi-period) is inconstant. Motivated by the reason, the MSFD method is proposed, investigated and developed for estimating the seasonality.

It is worth pointing out that the MSFD method is carried out by determining the parameters of multiple sine functions and updating the residual data for decomposition. More specifically, we firstly use sine function $A_1 \sin(B_1 t + C_1)$ to approximate the time series Y_t (i.e., the original data for the first-time decomposition), and the parameters (i.e., A_1, B_1 and C_1) are determined based on the least-squares method. Then we have

$$e_t(1) = Y_t - A_1 \sin(B_1 t + C_1),$$

where $e_t(1)$ denotes the residual error time series after the first-time decomposition using a sine function. Note that $e_t(1)$ is used as the "original" data of the second-time decomposition. After n times of similar decompositions, the estimation of seasonality is obtained as well as the estimation of the level of the time series. The estimation expression can be shown as follows:

$$\hat{Y}_t = \sum_{i=1}^{n} A_i \sin(B_i t + C_i) = S_t + L_t,$$

where S_t denotes the estimation of seasonality at observation instant t, L_t denotes the estimation of level, and n denotes the number of times of decompositions using sine functions. It is worth noting that all parameters $\{B_i\}$ (with $i = 1, 2, \cdots, n$) are 0 when the time series does not contain seasonality (i.e., $S_t = 0$). For better understanding, the principle diagram of MSFD method is shown as Fig. 1. Thus, the evaluation of the kth-step-ahead value $\hat{Y}_{t+k}$ with seasonality and level can be obtained as

$$\hat{Y}_{t+k} = S_{t+k} + L_{t+k} = \sum_{i=1}^{n} A_i \sin(B_i(t+k) + C_i). \tag{6}$$

C. Global Time Series Estimation

In order to model and forecast trend and seasonal time series, inspired by the research [13], the weighted-combination-of-components (i.e., WCC) method is proposed and investigated in this subsection. More specifically, based on the

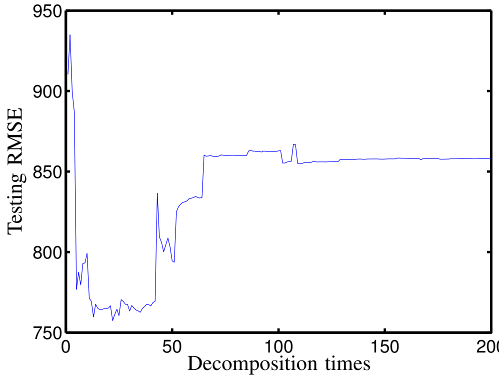


Fig. 2. The relationship between the testing RMSE and the decomposition times of multiple sine functions

decomposition model (1) with $w \in (0, 1)$, the IHTPES and MSFD methods are exploited to evaluate the trend, seasonality, and level of the time series simultaneously; then the weighted combination of the evaluations is obtained to model the global time series. Furthermore, according to (1), (5) and (6), the estimation via the proposed WCC method for the kth-step-ahead future value is achieved by

$$\begin{aligned}\hat{Y}_{t+k} &= wT_{t+k} + (1-w)S_{t+k} + R_{t+k} \\ &= w(L_t(m) + \gamma T_t(m)) + \\ &(1-w)(\sum_{i=1}^{n} A_i \sin(B_i(t+k) + C_i)),\end{aligned} \tag{7}$$

where R_{t+k} denotes the level of the kth-step-ahead value. Note that the level of the time series is considered in both IHTPES and MSFD methods. Thus, the estimation of R_{t+k} can be effectively obtained generally via the WCC method.

III. Numerical Experiments

In order to assess the modeling and the forecasting performances of the proposed WCC method, we conduct a large number of numerical experiments. Before the presentations of the results, data for experiments are described briefly. Besides, two performance measurements are provided for evaluating the overall performance. Furthermore, the procedures of determination for the parameters of the WCC method are described in detail. Finally, the experimental results are given and analyzed to verify the modeling and forecasting performances of the proposed WCC method.

A. Data Description

In this paper, the dataset of 111 time series (i.e., the complete dataset) from the NN3 competition [9][14] is used for validating the effectiveness of the proposed WCC method. Data of NN3 competition can be obtained from http://www.neural-forecasting-competition.com/NN3/datasets.htm. With the historical observations of each time series $Y_1, Y_2, \cdots, Y_t$ [14], the objective of the competition is to forecast the future 18 values (i.e., to forecast Y_{t+k}, where $k = 1, 2, \cdots, 18$). Due to space limitation and results similarity, numerical results are shown only for 6 time series, namely, NN3_023 and NN3_109 (dominated by trend), NN3_068 and NN3_104 (dominated by seasonality), NN3_011 (dominated by both trend and seasonality), and NN3_106 (with a high level of noise).

B. Performance Measures

In order to evaluate the overall performance of the proposed WCC method, we employ the symmetric mean absolute percentage error (SMAPE) and the root mean squared error (RMSE) as performance measures in this paper.

The formula of SMAPE is defined as below [11][15]:

$$E_{\text{SMAPE}} = \frac{1}{K} \sum_{j=1}^{K} \frac{|Y_j - \hat{Y}_j|}{(|Y_j| + |\hat{Y}_j|)/2} \times 100\%, \tag{8}$$

where Y_j and $\hat{Y}_j$ are the real and the forecasted value at jth instant, respectively, and K is the number of forecasted values. As seen from (8), the SMAPE is a relative-error measure, and we can use it to compare errors computed for different time series by different methods. In this paper, 18 future values (i.e., $K = 18$) are forecasted using SMAPE for evaluating the performance of the proposed new method.

Besides, the expression of RMSE [16][17] is shown as

$$E_{\text{RMSE}} = \sqrt{\frac{\sum_{j=1}^{K}(Y_j - \hat{Y}_j)^2}{K}}. \tag{9}$$

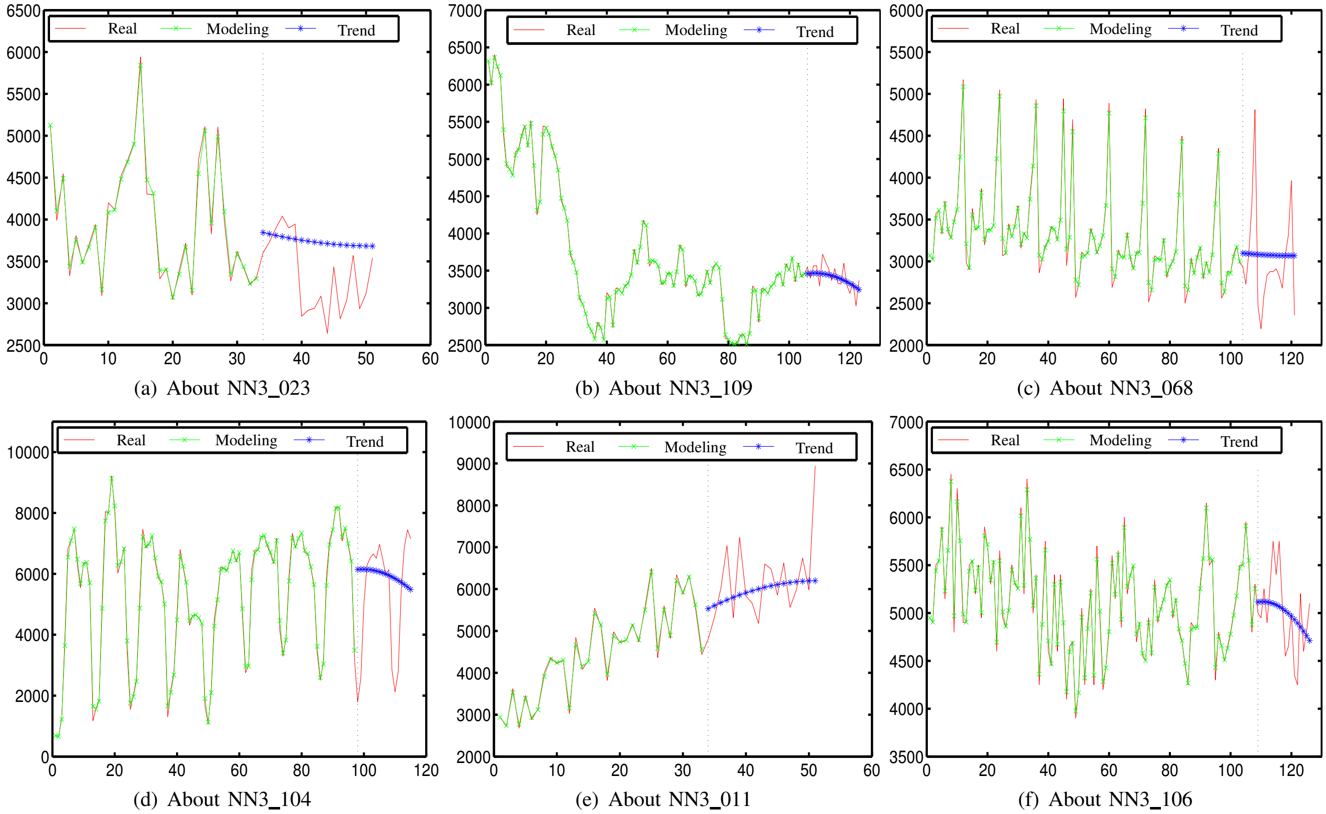
As seen from (9), RMSE can be used to measure the differences between the real value and the forecasted value. By exploiting RMSE as an additional exact tool, we can compare the modeling and forecasting performances of models with different parameters for a particular time series.

C. Parameters Determination

By exploiting the proposed WCC method (7) for multi-step-ahead forecasting of the NN3 competition, the future 18 values can be forecasted via (7) with specific parameters. Besides, to determine parameters of the WCC method, for each of the 111 time series, the last 18 historical observations are reserved as forecasting-type testing samples and the remaining historical observations are used for modeling the time series. In this subsection, the procedures of parameters determination of the WCC method are described in detail.

Through analysis and comparison of the modeling (i.e., smoothing) performances measured via (9), the smoothing parameters α and β are selected for the smoothing estimations of trend and level of the IHTPES method. Specifically, the optimal smoothing parameters α and β are determined within $(0, 1)^2$ via search gap 0.001 when achieving the optimal smoothing performance (i.e., the minimal modeling RMSE). Besides, r (i.e., the ratio of the linearly-declining increments of the trend) and m (i.e., the number of averages of level and trend) are determined based on the RMSE of testing samples and within $(0, 1) \times [1, 24]$ via search gaps 0.001 and 1 respectively.

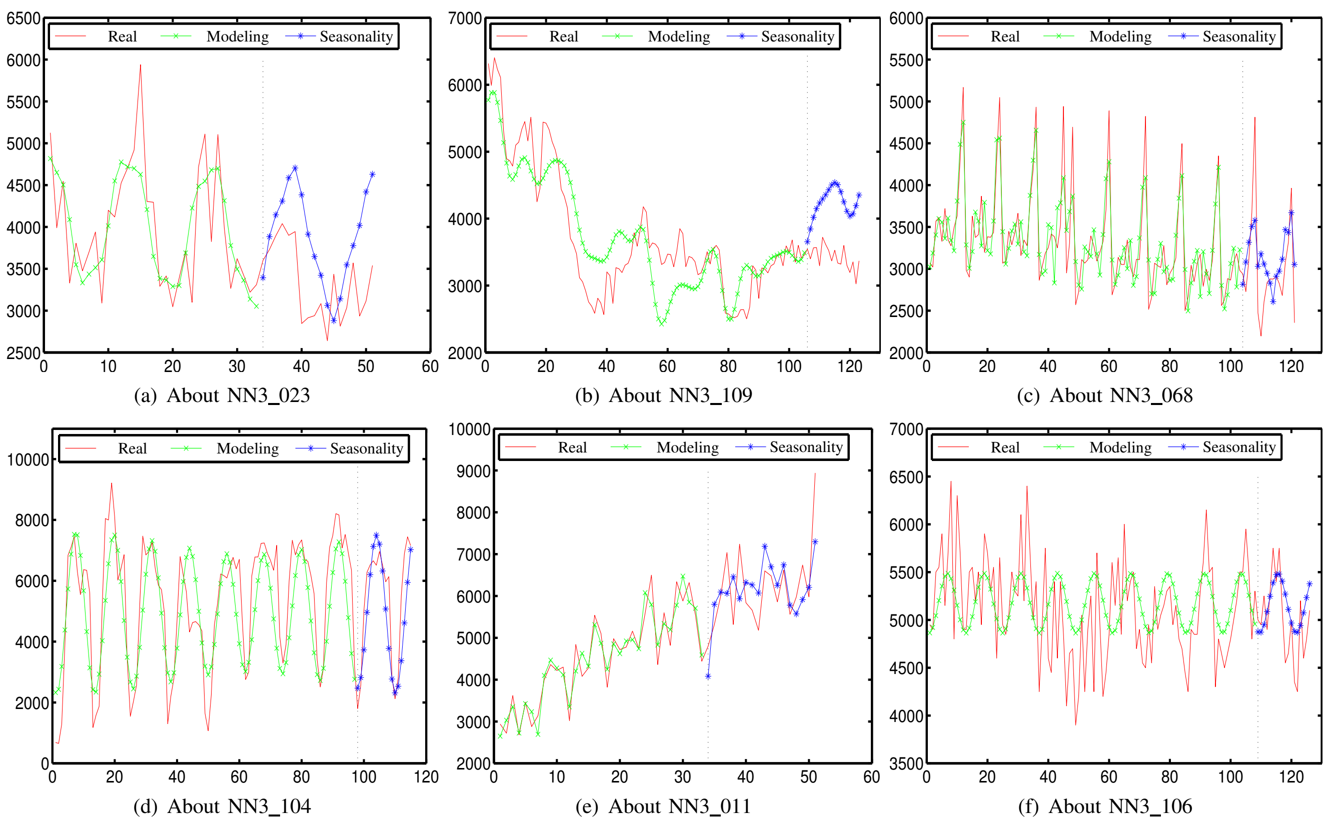
Additionally, the relationship between the estimation performance (i.e., testing RMSE) and the decomposition times of multiple sine functions is investigated. As seen from the results of the experiments shown in Fig. 2 (with just the result of the NN3_011 shown for space limitation and results similarity), too few times of decomposition may lead to failure on achieving the seasonality with desired accuracy, while too many times of decomposition may result in over-fitting



(a) About NN3_023 (b) About NN3_109 (c) About NN3_068

(d) About NN3_104 (e) About NN3_011 (f) About NN3_106

Fig. 3. Performances of modeling and trend estimation via the IHTPES method



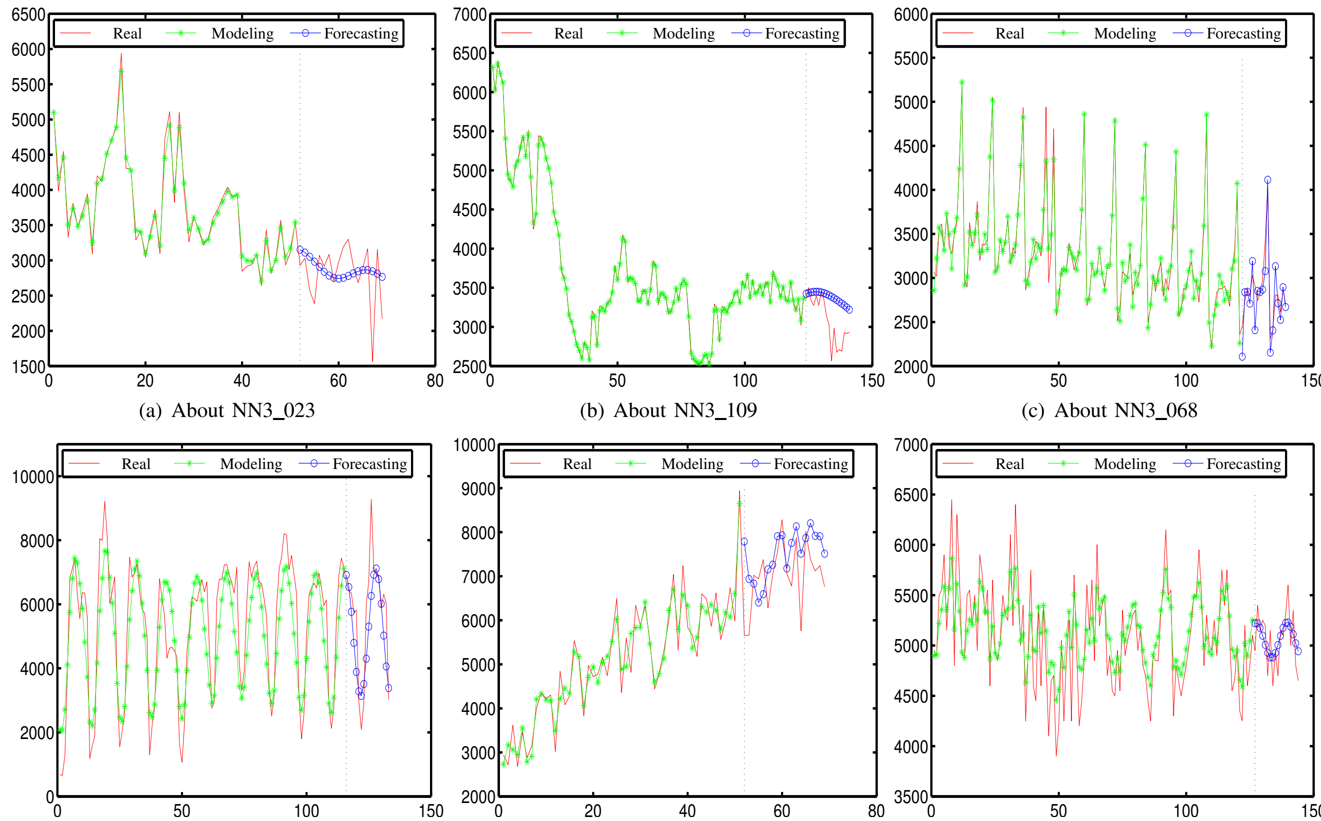
(a) About NN3_023 (b) About NN3_109 (c) About NN3_068

(d) About NN3_104 (e) About NN3_011 (f) About NN3_106

Fig. 4. Performances of modeling and seasonality estimation via the MSFD method

TABLE I. MODELING AND FORECASTING RESULTS OF THE PROPOSED WCC METHOD (7)

Time series	Modeling SMAPE	Forecasting SMAPE	Modeling RMSE	Forecasting RMSE	Weight w for (7)
NN3_023	2.2914%	12.4729%	110.8280	443.5291	0.865
NN3_109	0.6569%	9.7376%	32.4787	382.1495	0.973
NN3_068	2.6774%	5.2811%	120.7233	199.3757	0.001
NN3_104	19.7738%	17.8799%	1053.6161	1211.7580	0.196
NN3_011	6.2717%	10.3545%	370.7007	902.1311	0.213
NN3_106	4.2694%	3.8002%	271.7639	216.3399	0.451



(a) About NN3_023 (b) About NN3_109 (c) About NN3_068

(d) About NN3_104 (e) About NN3_011 (f) About NN3_106

Fig. 5. Performances of modeling and forecasting via the proposed WCC method (7)

phenomenon, which may introduce the noise at the same time. Therefore, the decomposition times of multiple sine functions is set via searching for the optimal testing RMSE. Specifically, we search for the optimal decomposition times of the MSFD method via the following simple process. 1) Record the optimal decomposition times with the minimal testing error during the process of decomposition. 2) Terminate the procedure when achieving the limitation of the decomposition times. It is worth mentioning that the numbers of the data of each time series from the NN3 competition are less than 150. Besides, most experiments show that the testing error does not decrease with the number of the decomposition times increasing after 100 times decomposition. Thus, the limitation of the decomposition times is set to 200 in the experiments throughout the paper.

Before the weighted combination of trend, seasonality and other components, the weight should be determined. In this paper, the weight (i.e, w) is obtained based on the testing errors (i.e., testing RMSE) of weighted combination of evaluations via the IHTPES method and the MSFD method. More specifically, firstly, each of the time series is modeled by the IHTPES method and the MSFD method simultaneously. Secondly, the value of w is selected from 0.001 to 0.999 with gap 0.001 to search for the optimal combination of estimations resulted; i.e., select the optimal weight w which achieves the minimal testing RMSE.

D. Experiment Results

For the trend estimation, to verify the effectiveness of the IHTPES method for evaluating the trend of the time series, numerical results of using (5) to model (or to say, smooth) the aforementioned 6 time series are presented in Fig. 3. As seen from the figure, the 6 time series are modeled and the trend can be estimated well by the IHTPES method, especially in the case that the time series contain the trend (e.g., NN3_023, NN3_109, NN3_011 and NN3_106).

For the seasonality estimation, to test the effectiveness of the MSFD method, experiment results are shown in Fig. 4. From the figure, we can find that the MSFD method can model time series well (except for the NN3_023 and NN3_109 which are mainly dominated by trend). Besides, the seasonality can be effectively estimated by (6), especially in the case that the time series contain the seasonality (e.g., NN3_068, NN3_104, NN3_011 and NN3_106).

For the global time series estimation, the optimal weight determined via the above-presented procedure is exploited for the proposed WCC method (7) to model and forecast, and the numerical results are presented in Table I and Fig. 5. Specifically, as shown in the table, the modeling and forecasting SMAPEs of the 6 presented time series [calculated by (8)] are less than 20%. Besides, the modeling and forecasting RMSEs [calculated by (9)] are much smaller in comparison with the values of the original time series. Additionally, Fig. 5 shows illustratively that the shape of the curve of modeling values estimated by the WCC method (7) closely resembles the curve of real values. Thus, we can conclude from Table I and Fig. 5 that the WCC method possesses excellent modeling ability even for some time series with a high level of noise (e.g., NN3_106). It is worth noting that such good characteristics can be found in the numerical experiment results of many other time series from NN3 competition.

Following the above numerical experiments on the modeling ability, we further investigate the forecasting ability of the WCC method. The corresponding numerical results of 6 time series are also shown in Table I and Fig. 5. As seen from the table, all of forecasting SMAPEs are less than 20% and most of forecasting RMSEs are much smaller, which substantiates the efficacy of the proposed WCC method on forecasting. Moreover, Figs. 5(c), 5(e) and 5(f) show that the WCC method can accurately forecast the future data of the time series with trend and seasonality. Furthermore, the variation tendency of future values can be forecasted even when the time series has a high level of noise.

In summary, the proposed WCC method (7) can achieve good performances in terms of modeling and forecasting trend and seasonal time series.

IV. Conclusions

In this paper, we have proposed a novel method, namely the weighted-combination-of-components (WCC) method, which is based on the decomposition model. Such a model regards the time series as the weighted combination of trend, seasonality and other components (e.g., level component). Specifically, the Holt's two-parameter exponential smoothing (HTPES) method has been improved (i.e., IHTPES method) for evaluating the trend of the time series; and a method of multiple sine functions decomposition (MSFD) has been proposed and developed to evaluate the seasonality. Then the global time series has been estimated by weighted combination of the evaluations. Numerical experiment results have further demonstrated the effectiveness and superiority of the proposed WCC method in terms of modeling and forecasting time series by applying the method to data from the NN3 forecasting competition. As a future research direction, the proposed WCC method may be applied to weather forecasting (e.g., monthly average temperature of a city); and the systematical research results are expected to be summarized and presented then.

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