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Distributed control for second-order leader-following multi-agent systems with heterogeneous leader

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***Abstract*—In this paper, distributed control for second-order leader-following multi-agent systems has been solved. All the outputs of the agents reach a common trajectory. The dynamic of the leader agent is different with the follower agents. If the digraph contains a spanning tree, then the problem is solved using an appropriate control law. A compensator is designed to making the closed-loop system matrix stable. Simulation results are further presented to show the effectiveness and performance of our work.**

I. Introduction

Alot of interests in consensus control for multi-agent systems have been taken by the researchers in the recent years[1]-[9]. This is due to the fact that the multi-agent systems have broad applications in many areas, such as formation control, flocking, attitude alignment, etc. The consensus of multi-agent systems means that all the agents' states reach a common trajectory after the appropriate consensus protocol based on the information of itself and its neighbors. Some results of consensus algorithms can be applied in many areas including cooperative control of mobile robots, unmanned air vehicles, autonomous underwater vehicles, automated highway systems, and so on.

Usually, the multi-agent systems can be classified into leaderless and leader-follower systems. First-order consensus problems for networked dynamic systems have been extensively studied by many researchers. Ref [10] studied the leader-following consensus problem of multi-agent systems, where the leader is a particular agent, whose motion is independent of the other agents and followed by the other ones. The state of the leader is unmeasurable for all the agents. Thus a dynamic neighbor-based system is given to estimate it. Further extensions of this work were presented in [11]-[15] with second-order dynamics. Consensus algorithms for double-integrator dynamics were proposed and analyzed in [11] with a bounded control input and without relative velocity measurements. The consensus of second-order multi-agent dynamical systems with exogenous disturbances is studied in [12] via a pinning control strategy which can bring all the agents' states to reaching the expected consensus track under the influence of the disturbances. Ref [13] extended the results on the unknown nonlinearities in which the agents can have different dynamics and the treatment is for directed graphs with fixed communication topologies, and the leader is nonlinear and unknown. Some necessary and sufficient conditions were given in [15] for the consensus of second-order multi-agent dynamical systems.

An important topic in the study of the consensus problem is output regulation problem. Output regulation theory is to control one or more than one plants so that their output tracks a reference signal(and/or rejects a disturbance) produced by an exsystem. The conventional output regulation for both linear and nonlinear systems has been studied for many decades [16],[17]. Output regulation for multi-agent systems has been studied by a lot of researchers [18]-[24]. Output regulation problem of state-coupled linear certain and uncertain multi-agent systems with globally reachable topologies was investigated by [18], in which exosystem is same for all the nodes but only partial nodes have the state information channel with it. Ref [19] solved the regulation problem in the case that only partial agents could be measured by itself and its neighbors, and a full order Luenberger observer is given for the unmeasurable part. Heterogeneous agents were considered by [23], [24], and the compensator method was used in [23]. Small gain method was first used in solving output regulation problem in [24].

In our paper, output regulation for second-order multi-agent systems has been solved. All the outputs reach a common trajectory with the reference signal. The network topology structure is presented by a digraph which contains a spanning tree and the exosystem is as its root node. We give the assumption that the system matrix B is full row rank instead of transmission zeros condition. A compensator is designed to make the closed-loop system matrix stable. Then a distributed dynamic state feedback control law is established to solve the regulator problem.

The rest of this paper is organized as follows: Section II introduces some framework and formulate the base problem. Section III gives the main results about output regulation of second-order multi-agent systems. Some simulation results are presented in Section IV to show its performance. Finally, some concludes about this paper is given in Section V.

II. Framework and Problem Formulation

We consider a platoon of n identical agents modeled by the following second-order dynamics

$$\begin{aligned} \dot{x}_{1i}(t) &= x_{2i}(t) \\ \dot{x}_{2i}(t) &= Ax_{2i}(t) + Bu_i(t) + \delta_i(t) \\ y_i(t) &= Cx_{1i}(t), i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_{1i}(t) \in R^n, x_{2i}(t) \in R^n, i = 1, 2, \ldots, N$ are the states of the agents, and $y_i(t) \in R^p$ is the output of agent i. Assume that B and C are full row rank in our paper. $\delta_i(t) = E_i\omega(t)$ is the disturbance which is generated by the following exosystem

$$\begin{aligned} \dot{\omega}(t) &= S\omega(t), \\ y_0(t) &= Q\omega(t), \end{aligned} \tag{2}$$

where $\omega(t) \in R^q$ is the state of the exosystem, and $y_0(t) \in R^p$ is the tracking signal.

Assumption 1: All the eigenvalues of Γ do not have negative real parts.

The error between the output of the $i-$th agent with the tracking signal is given as

$$e_i(t) = y_i(t) - y_0(t) = Cx_{1i}(t) - Q\omega(t), i = 1, \ldots, N. \tag{3}$$

The communication topology between agents and the exosystem is represented by a weighted graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{v_0, v_1, v_2, \ldots, v_N\}$ is the set of nodes. v_0 represents the dynamic of the exosystem. $\mathcal{A} = [a_{ij}]$ is a weighted adjacency matrix, where $a_{ii} = 0$ and $a_{ij} \geq 0$ for all $i \neq j$. In our paper, $a_{ij} > 0$ if and only if there is an edge from vertex j to vertex i. The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. $\mathcal{N}_s = \{v_1, v_2, \ldots, v_N\}$ represents the node set of the subgraph $\mathcal{G}_s = (\mathcal{N}_s, \mathcal{E}_s, \mathcal{A}_s)$. A diagonal matrix $\mathcal{D}_s = \text{diag}\{d_1, d_2, \ldots, d_N\}$, where $d_i = \Sigma_{j=1}^n a_{ij}, i = 1, 2, \ldots, N$ is called a degree matrix of $\mathcal{G}_s$. The Laplacian with the directed graph $\mathcal{G}_s$ is defined as $\mathcal{L}_s = \mathcal{D}_s - \mathcal{A}_s$. Obviously, all the row sums of $\mathcal{L}$ are zero. An edge of $\mathcal{G}$ denoted by $e_{ij} = (v_i, v_j) \in \mathcal{E}$ means that node v_i receives information from node v_j. There is a sequence of edges with the form $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_j}, v_j) \in \mathcal{E}$ composing a direct path beginning with v_i ending with v_j, then node v_j is reachable from node v_i. A node is reachable from all the other nodes of graph, the node is called globally reachable.

We assume that not all the agent could access the information from the exosystem. But the aim is to regulate all the agents' output tracking to the reference output signal. Thus, the external state measurements relative to its neighbors and the output-coupling variable relationship between agent i and $j \in \mathcal{N}_i$ is defined as

$$\phi_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) - y_j(t)) + a_{i0}(y_i(t) - y_0(t)). \tag{4}$$

Then a dynamic state feedback control law is introduced based on the output-coupling variable relationship is given as

$$\begin{aligned} u_i(t) &= K_1(\sum_{j \in \mathcal{N}_i} a_{ij}(x_{1i}(t) - x_{1j}(t)) + a_{i0}x_{1i}(t)) \\ &+K_2(\sum_{j \in \mathcal{N}_i} a_{ij}(x_{2i}(t) - x_{2j}(t)) + a_{i0}x_{2i}(t)) + K_3\zeta_i(t) \\ \dot{\zeta}_i(t) &= G_1\zeta_i(t) + G_2\phi_i(t), \end{aligned} \tag{5}$$

in which $K_j, j = 1, 2, 3$ are the gain matrices. $\zeta_i \in R^s$ is the compensator to be designed later.

Let $x_k = (x_{k1}(t)^T, x_{k2}(t)^T, \ldots, x_{kN}(t)^T)^T, k = 1, 2$, and $\zeta = (\zeta_1(t)^T, \zeta_2(t)^T, \ldots, \zeta_N(t)^T)^T$. By control law (5), the system dynamics (1) have the following closed-loop form

$$\begin{aligned} \dot{x}_1 &= I_{nN}x_2, \\ \dot{x}_2 &= (\mathcal{H}_s \otimes BK_1)x_1 + (I_N \otimes A + \mathcal{H}_s \otimes BK_2)x_2 \\ &\quad +(I_N \otimes BK_3)\zeta + E\omega, \\ \dot{\zeta} &= (\mathcal{H}_s \otimes G_2C)x_1 + (I_N \otimes G_1)\zeta \\ &\quad -(\mathcal{H}_s \otimes G_2Q)(1_N \otimes I_q)\omega, \end{aligned} \tag{6}$$

where $\mathcal{H}_s = \mathcal{L}_s + \mathcal{A}_0$, and $\mathcal{A}_0 = \text{diag}\{a_{01}, a_{02}, \ldots, a_{0N}\}$. $E = (E_1^T, E_2^T, \ldots, E_N^T)^T$. Thus (6) can be rewritten as

$$\dot{\xi} = A_c\xi + B_c\omega, \tag{7}$$

where $\xi = (x_1^T, x_2^T, \zeta^T)^T$, and

$$A_c = \begin{pmatrix} 0 & I_{nN} & 0 \\ \mathcal{H}_s \otimes BK_1 & I_N \otimes A + \mathcal{H}_s \otimes BK_2 & I_N \otimes BK_3 \\ \mathcal{H}_s \otimes G_2C & 0 & I_N \otimes G_1 \end{pmatrix}$$

$$B_c = \begin{pmatrix} 0 \\ E \\ -(\mathcal{H}_s \otimes G_2Q)(1_N \otimes I_q) \end{pmatrix}.$$

Definition 1: Cooperative output regulation of multi-agent systems can be solved if the following two conditions satisfied:

a) The closed-loop system $\dot{\xi} = A_c\xi$ is exponentially stable, i.e., all the eigenvalues of matrix A_c are assigned in the open left half plane.

b) For all initial condition $x_{1i}(0), x_{2i}(0)$ and $\omega(0)$, all the tracking errors satisfy: $\lim_{t\to\infty} e_i(t) = 0, i = 1, \ldots, N$, i.e., $\lim_{t\to\infty} e(t) = C_c\zeta + Q_c\omega = 0, i = 1, \ldots, N$, in which $e(t) = (e_1(t)^T, e_2(t)^T, \ldots, e_N(t)^T)^T$ and $C_c = (I_N \otimes C, \mathbf{0}, \mathbf{0}), Q_c = (1_N \otimes Q)$.

III. Main Result

In this section, a main result will be given based on some useful lemmas.

Lemma 1: [25] All the eigenvalues of $\mathcal{H}_s = \mathcal{L}_s + \mathcal{A}_0$ defined in (7) have positive real parts if and only if the digraph $\mathcal{G}$ contains a spanning tree and node v_0 as its root.

Lemma 2: [26] Under the Assumption 1, if the pair (Φ_a, Φ_b) and (G_1, G_2) are stable, and for all $\lambda \in \sigma(G_1)$, matrix

$$\begin{pmatrix} \Phi_a - \lambda I & \Phi_b \\ \Phi_c & 0 \end{pmatrix}$$

is full row rank, then the pair

$$\left(\begin{pmatrix} \Phi_a & 0 \\ G_2\Phi_c & G_1 \end{pmatrix} \begin{pmatrix} \Phi_b \\ 0 \end{pmatrix} \right)$$

is stabilizable.

Lemma 3: [27] Given the stabilizable pair $(\mathcal{A}, \mathcal{B})$, the following algebraic Riccati equation

$$\mathcal{A}^{\mathcal{T}}\mathcal{P} + \mathcal{P}\mathcal{A} + I_n - \mathcal{P}\mathcal{B}\mathcal{B}^{\mathcal{T}}\mathcal{P} = 0 \tag{8}$$

has a unique solution $\mathcal{P} = \mathcal{P}^T > 0$, and for all $a \geq 1$ and $b \in R$, matrix $\mathcal{A} - (a + jb)\mathcal{B}\mathcal{B}^{\mathcal{T}}\mathcal{P}$ is Hurwitz.

Theorem 1: If the digraph $\mathcal{G}$ contains a spanning tree, and the following regulator equation

$$\begin{aligned} XS &= A_cX + B_c \\ \mathbf{0} &= C_cX + Q_c \end{aligned} \tag{9}$$

exists a unique matrix solution X, then the controller (5) solves the output regulation problem of second-order multi-agent systems.

Proof: According to the Definition 1, we first proof A_c is stable. A transformation is used as

$$\begin{aligned} \hat{A}_c &= T^{-1}A_cT \\ &= \begin{pmatrix} 0 & I_{nN} & 0 \\ \mathcal{H}_s \otimes BK_1 & \Upsilon & \mathcal{H}_s \otimes BK_3 \\ I_N \otimes G_2C & 0 & I_N \otimes G_1 \end{pmatrix}, \end{aligned} \tag{10}$$

where $T = \text{block diag}\{I_{nN}, I_{nN}, \mathcal{H}_s \otimes I_s\}$, and $\Upsilon = I_N \otimes A + \mathcal{H}_s \otimes BK_2$. $\hat{A}_c$ can be rewritten as

$$\hat{A}_c = I_N \otimes \hat{A}_{ca} + \mathcal{H}_s \otimes \hat{A}_{cb}K, \tag{11}$$

in which

$$\hat{A}_{ca} = \begin{pmatrix} 0 & I_n & 0 \\ 0 & A & 0 \\ G_2C & 0 & G_1 \end{pmatrix}, \hat{A}_{cb} = \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix},$$
$$K = \begin{pmatrix} K_1 & K_2 & K_3 \end{pmatrix}.$$

Since B has full row rank, it is easy to see that

$$\left(\begin{pmatrix} 0 & I_n \\ 0 & A \end{pmatrix} \begin{pmatrix} 0 \\ B \end{pmatrix} \right)$$

is stabilizable..

For all $\lambda \in \mathbb{C}$, the matrix pair

$$\begin{pmatrix} -\lambda I_n & I_n & 0 & 0 \\ 0 & A - \lambda I_n & 0 & B \\ C & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

has full row rank. Matrix pair (G_1, G_2) in (5) is chosen as a stable pair. Then according to Lemma 2, $(\hat{A}_{ca}, \hat{A}_{cb})$ is stable, i.e., there exists appropriate K such that $\hat{A}_{ca} + \hat{A}_{cb}K$ is Hurwitz.

Equation (11) is Hurwitz if and only if

$$\hat{A}_{ca} + \lambda_i(\mathcal{H}_s)\hat{A}_{cb}K \tag{13}$$

is Hurwitz, in which $\lambda_i(\mathcal{H}_s), i = 1, 2, \ldots, N$ are the eigenvalues of $\mathcal{H}_s$. Therefore, according to Lemma 3, choosing $K = -(min\ Re(\lambda_i(\mathcal{H}_s)))^{-1}\hat{A}_{cb}^T P$, in which P is the solution of the following equation

$$\hat{A}_{ca}^T P + P\hat{A}_{ca} + I - P\hat{A}_{cb}\hat{A}_{cb}^T P = 0. \tag{14}$$

Then A_c is stable.

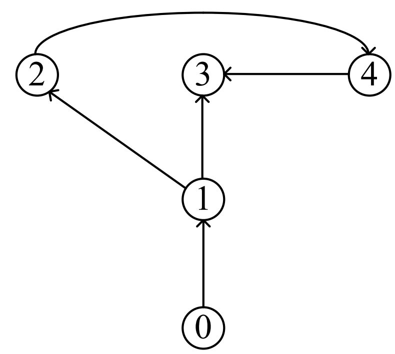

Fig. 1. The network of the four agents and exosystem

Since the regulator equation (9) has solution X, let $\bar{\xi} = \xi - X\omega$. Then

$$\begin{aligned} \dot{\bar{\xi}} &= A_c\bar{\xi} \\ e(t) &= C_c\bar{\xi} + (C_cX + Q_c)\omega. \end{aligned} \tag{15}$$

Thus, $\lim_{t\to\infty} e(t) = C_c\xi + Q_c\omega = 0$.

Output synchronization of second-order multi-agent systems could also be solved by the output regulation method. The definition about synchronization is given as follows

Definition 2: The system (1) with $N-$agents is said to be synchronized if the following requirements are satisfied:

Asymptotic synchronization: For specific initial states $x_{1i}(0), x_{2i}(0)$ and $\omega(0)$, all outputs $y_i(t)$ reach a common trajectory, i.e.,

$$\lim_{t\to\infty}(y_i(t) - y_j(t)) = 0. \tag{16}$$

According to the Theorem 1,

$$\begin{aligned} & \lim_{t\to\infty}(y_i(t) - y_j(t)) \\ = & \lim_{t\to\infty}(y_i(t) - Q\omega(t) - (y_j(t) - Q\omega(t))) \\ = & \lim_{t\to\infty}(y_i(t) - Q\omega(t)) - \lim_{t\to\infty}(y_j(t) - Q\omega(t)) \\ = & \ 0. \end{aligned} \tag{17}$$

Then, the following result is given as

Corollary 1: Under the Assumption 1, output synchronization of second-order multi-agent systems is solved if the regulator equation designed in (9) has a unique solution and the digraph contains a spanning tree.

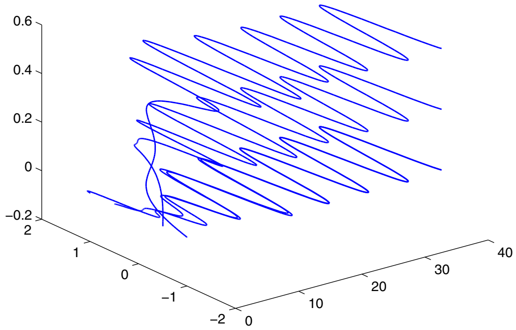

Fig. 2. Transient behavior of $x1$.

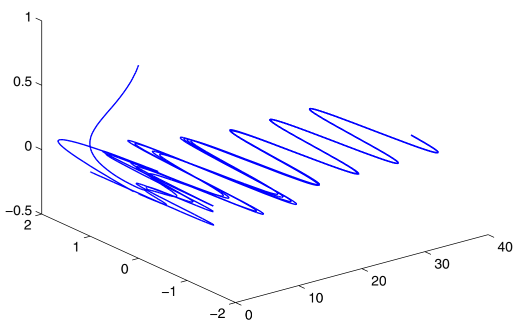

Fig. 3. Transient behavior of $x2$.

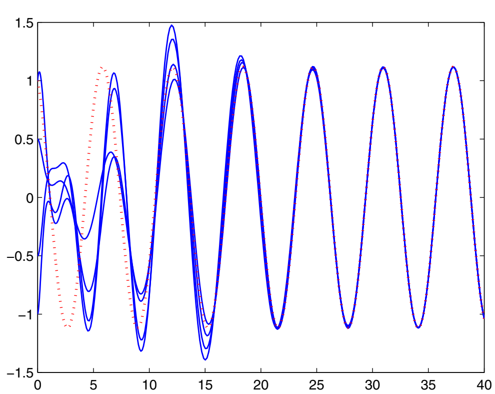

Fig. 4. The output of four agents and the reference signal in red color.

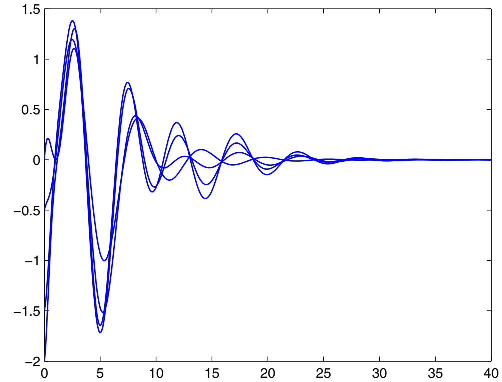

Fig. 5. The errors of four agents and the reference signal.

IV. Simulation Result

Consider the following multi-agent systems with 4 nodes

$$\begin{aligned} \dot{x}_{1i}(t) &= x_{2i}(t) \\ \dot{x}_{2i}(t) &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} x_{2i}(t) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} u_i(t) + E_i \omega(t) \\ y_i(t) &= \begin{pmatrix} 1 & 0 \end{pmatrix} x_{1i}(t), i = 1, 2, 3, 4, \end{aligned}$$

where

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$E_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, E_4 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

and the exosystem is given as

$$\dot{\omega}(t) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \omega(t).$$

The exosystem could be seen as the leader and contains the reference output to be tracking.

Since the eigenvalues of S is $\lambda_1 = i, \lambda_2 = -i, \lambda_3 = 0$, then the Assumption 1 is satisfied. The reference signal is given as

$$y_0(t) = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \omega(t).$$

The topology structure of four agents and exosystem is given in Fig. 1. Then we have

$$\mathcal{A}_s = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \mathcal{H}_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

The eigenvalues of $\mathcal{H}_s$ are calculated as $\lambda_1 = 2, \lambda_2 = \lambda_3 = \lambda_4 = 1$. The matrices G_1, G_2 in (5) can be chosen as

$$G_1 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, G_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Obviously, (G_1, G_2) is stabile. Thus, the gain matrices K_1, K_2, K_3 are obtained by solving the algebraic Riccati equation (14):

$$\begin{aligned} K_1 &= \begin{pmatrix} -4.4883 & 0 \\ 0 & -1 \end{pmatrix} \\ K_2 &= \begin{pmatrix} -4.3131 & 0 \\ 0 & -1.7321 \end{pmatrix} \\ K_3 &= \begin{pmatrix} 1.3742 & 0.3340 & -2.7884 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned}$$

Therefore, the simulation result about the state x_1 and x_2 are shown in Fig. 2 and Fig. 3. The output of the four agents and the reference signal are shown in Fig. 4.

V. Conclusions

Distributed control for second-order leader-following multi-agent systems had been solved. All the outputs of the agents reach a common trajectory. The dynamic of the leader agent is different with the follower agents. If the digraph contains a spanning tree. A compensator is designed to making the closed-loop system matrix stable. Then the problem was solved using an appropriate control law in Theorem 1, and a corollary was given to show that out result also contains the output synchronization problem. At last, an example was given to show the effectiveness of our results.

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References

[1] A. Jadbabaie, J. Lin, A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control.* vol. 48, pp. 988-1001, 2003.

[2] Y. Hong, L. Gao, D. Cheng, J. Hu, "Lyapunov-Based Approach to Multiagent Systems With Switching Jointly Connected Interconnection," *IEEE Trans. Autom. Control.* vol. 52, pp. 943-948, 2007.

[3] J. A. Fax, R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control.* vol. 49, pp. 1465-1476, 2004.

[4] W. Ren and R.W. Beard, "Decentralized scheme for spacecraft formation flying via the virtual structure approach," *AIAA J. Guidance, Control, Dynamics* vol. 27, pp. 73-82, 2004.

[5] H. Zhang, L. Cai, Z. Bien, "A fuzzy basis function vector-based multivariable adaptive controller for nonlinear systems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.* vol. 30, pp. 210-217, 2000.

[6] H. Zhang, D. Liu, Y. Luo, D. Wang, "Adaptive Dynamic Programming for Control-Algorithms and Stability." London: Springer-Verlag, 2013.

[7] Y. Wang, H. Zhang, X. Wang, D. Yang, Networked Synchronization Control of Coupled Dynamic Networks With Time-Varying Delay,*IEEE Trans. Syst., Man, Cybern. B, Cybern.* vol. 40, pp. 1468-1479, 2010.

[8] R. Olfati-Saber, R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control* vol. 49, pp. 1520-1533, 2004.

[9] R. Olfati-Saber, J. A. Fax, R.M.Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE* vol. 95, pp. 215-223, 2007.

[10] Y. Hong, J. Hu, L. Gao. "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica* vol. 42, pp. 1177-1182, 2006.

[11] W. Ren, On consensus algorithm for double-intergrator dynamics, *IEEE Trans. Automatic Contr.,* vol. 53 pp. 1503-1509, 2008.

[12] H. Yang, Z. Zhang, and S. Zhang, Consensus of second-order multiagent systems with exogenous disturbances, *Int. J. Robust Nonlinear Contr.,* vol. 21, pp. 945-956, 2011.

[13] A. Das and F. Lewis, Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities, *Int. J. Robust Nonlinear Contr.,* vol. 21, pp. 1509-1524, 2011.

[14] F. Sun, Z. Guan, "Finite-time consensus for leader-following second-order multiagent system," *International Journal of Systems Science,* vol. 44, pp. 727-738, 2013.

[15] W. Yu, G. Ghen, and M. Cao, Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems, *Automatica,* vol. 46, pp. 1089-1095, 2010.

[16] B. A. Francis, "The linear multivariable regulator problem," *SIAM J. Control Optimiz.*, vol. 15, pp. 486-505, 1977.

[17] A. Isidori, CI. Byrnes, "Output regulation of nonlinear systems,' *IEEE Trans. Autom. Control.* vol. 35 pp. 131-140, 1990.

[18] H. Liang, H. Zhang, Z. Wang and J. Wang, "Output regulation of state-coupled linear multi-agent systems with globally reachable topologies," *Neurocomputing*, vol. 123, pp. 337–343, 2014.

[19] H. Liang, H. Zhang, Z. Wang and J. Wang, "Output regulation for linear multi-agent systems with unmeasurable nodes," *Chin. Phys. B*, vol. 23, pp. 018902, 2014.

[20] H. Liang, H. Zhang, Z. Wang and J. Wang, " Consensus Robust Output Regulation of Discrete-time Linear Multi-Agent Systems," *Acta Automatica Sinica.* Accepted.

[21] X. Wang, Y. Hong, J. Huang, and Z. Jiang, "A distributed control approach to a robust output regulation problem for multi-agent linear systems," *IEEE Trans. on Automatic Control*, vol. 55, pp. 2891-2895, 2010.

[22] X. Wang, "Distributed formation output regulation of switching heterogeneous multi-agent systems," *International Journal of Systems Science,* DOI:10.1080/00207721.2012.683831

[23] P. Wieland, R. Sepulchre, F. Allgwer, "An internal model principle is necessary and suffcient for linear output synchronization," *Automatica* vol. 47, 1068-1074, 2011.

[24] L. Yu J. Wang, "Distributed output regulation for multi-agent systems with norm-bounded uncertainties," *International Journal of Systems Science,* DOI:10.1080/00207721.2013.769647

[25] Z. Li, Z. Duan, G. Chen, L. Huang, "Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint," *IEEE Transactions on Circuits and Systems I* , vol. 57, 213-224, 2012.

[26] J. Huang, *Nonlinear Output Regulation: Theory and Applications.* Phildelphia, PA: SIAM, 2004.

[27] Tuna SE. "LQR-based coupling gain for synchronization of linear systems," Available from: http://arxiv.org/abs /0801.3390.