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Statistical approach for reconstruction of dynamic brain dipoles based on EEG data

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Abstract—**In this paper, we propose a statistical approach to reconstruct the brain neuronal activity based only on recorded EEG data. The brain zones with the strongest activity are expressed at a macro level by a few number of active brain dipoles. Normally, for solving the EEG inverse problem, fixed dipole locations are assumed, independently of the different stimuli that excite the brain. The proposed particle filter (PF) framework presents a shift in the current paradigm by estimating dynamic brain dipoles, which may vary from one location to another in the brain depending on internal/external stimuli that may affect the brain. Also, in contrast to previous solutions, the proposed PF algorithm estimates simultaneously, the number of the active dipoles, their moving locations and their respective oscillations in the three dimensional head geometry.**

I. INTRODUCTION

Electroencephalography (EEG) is a widely used technology for brain study because it is non-invasive, relatively cheap, portable and has an excellent temporal resolution [1]. However, the full potentiality of EEG has not yet been exploited, mainly owing to the complexity of EEG data analysis. The main issue in EEG investigation is the solution of the inverse problem of determining the spatio-temporal evolution of the neural currents from the dynamic EEG measurements at different locations outside the skull. This problem is difficult for many reasons. First, it is ill-posed and numerically unstable problem. Second, the problem has to be solved at many different time points and this makes computational effectiveness a crucial issue in the data analysis process. Third the EEG data is characterized by a very low signal to noise ratio (SNR), the row signal is contaminated by both neural background and sensor's noise [2].

Most brain source reconstruction techniques belong to one of the two approaches, [3] : i) *imaging models* (distributed source) which explain the data with thousands of equivalent current dipoles and ii) *dipole models* (point source models) which use a small number of dipoles.

The imaging techniques are by far more researched, they provide a detailed map of the brain neuronal activity. However, these are computationally heavy procedures, it is difficult to explain the data by the obtained complex models and to make inter-subject statistics. Moreover there are other imaging technologies like Magneto-encephalography (MEG) Magneto Resonance Images (MRI), functional MRI that provide the same brain models.

The few-dipole models obtained less attention. Their main advantage is that they represent a direct mapping from scalp topology to a small number of parameters. Dipole solutions provide simple interpretations that explain the data. Furthermore it is easy to report statistics of dipole parameters over different subjects. Summarizing distributed brain activity with a small number of active dipoles simplifies analysis of connectivity among those sources. It is only recently, and due to the increase in the available computational power, that statistical methods, such as the Kalman filter and the Particle Filters, seem feasible as brain source localization tools, [4], [5]. However, these techniques are still at a very initial explorative stage and further investigations are required.

Statistical approach for estimation of a few-active-dipole model that summarizes the distributed brain activity based only on EEG measurements is the focus of this paper and recent work by the same authors [6], [7]. We propose a particle filter – based (PF) algorithm to reconstruct the brain neuronal activity. The brain zones with the strongest activity are expressed at a macro level by a few number of active brain dipoles.

Normally, for solving the EEG inverse problem, fixed dipole locations are assumed, independently of the different stimuli that excite the brain. Recently, Miao *et.al.*, [8] have proposed a Bayesian approach to track dynamic dipoles in real time and Independent Component Analysis (ICA)

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technique to define the number of the independent dipoles. However, in order to reduce the computational intensity, each dipole is tracked by a separate particle filter. In [7], we proposed a more general PF framework to track all dynamic dipoles assuming their number is known a priori. The present work is an extension of [7] where the PF algorithm estimates simultaneously, the number of the active dipoles, their moving locations and their respective amplitudes in the three dimensional head geometry.

The paper is organized as follows: In section 2 the PF framework is outlined. In section 3, based on physiological specifications, the EEG state-space model is defined. A novel approach for estimation of the number of active brain dipoles is introduced in section 4 based on information theoretic criteria. The major contribution of this work, namely the PF algorithm for dynamic EEG source estimation is detailed in section 5. In sections 6 and 7 the feasibility of the proposed PF method is demonstrated on generated and real EEG data. Section 8 concludes the paper.

II. Particle Filter

Many problems in statistical signal processing can be stated in a state space form. A system transition function describes the prior distribution of a hidden Markov process according to the model:

$$\boldsymbol{x}_{k+1} = f_k\left(\boldsymbol{x}_k, \boldsymbol{w}_k\right). \tag{1}$$

Here, f_k is the system transition function and $\boldsymbol{w}_k$ is a zero-mean, white noise sequence of known pdf, independent of past and current states. Measurements z_k are available at discrete times *k*, relating to the state vector $\boldsymbol{x}_k$ via the observation equation:

$$z_k = h_k\left(\boldsymbol{x}_k, \boldsymbol{v}_k\right), \tag{2}$$

where h_k is the measurement function and $\boldsymbol{v}_k$ is another zero-mean, white noise sequence of known pdf, independent of past and present states and the system noise.

Within a Bayesian framework, all relevant information about the state vector, given observations up to time *k*, can be obtained from the posterior distribution $p\left(\boldsymbol{x}_k \middle| z_{1:k}\right)$, where $z_{1:k} = \left\{z_1, \cdots, z_k\right\}$. This distribution may be obtained recursively in two steps: prediction and update. Suppose that the posterior distribution at the previous time index $k-1$, $p\left(\boldsymbol{x}_{k-1} \middle| z_{1:k-1}\right)$, is available. Then, using the system transition model, we can obtain the prior pdf of the state at time *k* as follows:

$$p\left(\boldsymbol{x}_k \middle| z_{1:k-1}\right) = \int p\left(\boldsymbol{x}_k \middle| \boldsymbol{x}_{k-1}\right) p\left(\boldsymbol{x}_{k-1} \middle| z_{1:k-1}\right) d\boldsymbol{x}_{k-1}. \tag{3}$$

When a measurement z_k, at time step *k*, is available, the prior is updated via Bayes rule:

$$p\left(\boldsymbol{x}_k \middle| z_{1:k}\right) = \frac{p\left(z_k \middle| \boldsymbol{x}_k\right) p\left(\boldsymbol{x}_k \middle| z_{1:k-1}\right)}{p\left(z_k \middle| z_{1:k-1}\right)}, \tag{4}$$

where the denominator is a normalizing factor and the conditional pdf of z_k given $\boldsymbol{x}_k$ is defined by the measurement model in (2).

The recurrence equations in (3) and (4) constitute the solution to the Bayesian recursive estimation problem. If the functions f_k and h_k are linear and the noises $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are Gaussian with known variances, then an analytic solution to the Bayesian recursive estimation problem is given by the well-known Kalman filter. In the EEG inverse problem, however, the measurement function h_k is non-linear, because the EEG measurements z_k are non-linear functions of the source locations.

In order to deal with the non-linear and/or non-Gaussian realities, two main approaches have been adopted: parametric and non-parametric. The parametric techniques are based on extensions of the Kalman filter by linearizing non-linear functions around the predicted values. The non-parametric techniques are based on sequential Monte Carlo methods and particularly the particle filter (PF). Unlike the Kalman filter, which propagates the mean and covariance of the Gaussian posterior density, the PF uses a set of random samples, called particles, to estimate the posterior distribution of the state. Specifically, the posterior is approximated by a set of weighted particles (hence the name particle filter) as:

$$p\left(\boldsymbol{x}_k \middle| z_{1:k}\right) \approx \sum_{i=1}^{N} \pi_k^{(l)} \delta\left(\boldsymbol{x}_k - \boldsymbol{x}_k^{(l)}\right), \tag{5}$$

Here, N is the total number of particles, $\pi_k^{(l)} = w_k^{(l)} / \sum_{l=1}^{N} w_k^{(l)}$ is the normalized weight for particle *l* at time *k*. Ideally, the particles are required to be sampled from the true distribution $p\left(\boldsymbol{x}_k \middle| z_{1:k}\right)$, which is not available. Therefore, another distribution, referred to as the importance distribution, or the proposal distribution $q(\mathrm{x}_n \mid \mathrm{x}_{n-1}, \mathrm{y}_n)$, is used. Theoretically, the only condition on the importance distribution is that its support includes the support of the posterior distribution. In practice, the number of particles is finite and the importance distribution should be chosen to approximate the posterior distribution. The importance weights are given by:

$$w_k^{(l)} = w_{k-1}^{(l)} \frac{p\left(z_k \middle| \boldsymbol{x}_k^{(l)}\right) p\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}\right)}{q\left(\boldsymbol{x}_k^{(l)} \middle| \boldsymbol{x}_{k-1}^{(l)}, \boldsymbol{z}_{1:k}\right)}, \tag{6}$$

For instance, if the importance distribution is given by the prior density,

$$q\left(\boldsymbol{x}_k^{(l)}\left|\boldsymbol{x}_{k-1}^{(l)}, z_{1:k}\right.\right) = p\left(\boldsymbol{x}_k^{(l)}\left|\boldsymbol{x}_{k-1}^{(l)}\right.\right), \tag{7}$$

then Eq. (6) reduces to:

$$w_k^{(l)} = w_{k-1}^{(l)} p\left(\boldsymbol{z}_k\left|\boldsymbol{x}_k^{(l)}\right.\right). \tag{8}$$

Given a discrete approximation to the posterior distribution, one can then proceed to a filtered point estimate such as the mean of the state at time *k*:

$$\hat{\boldsymbol{x}}_k = \sum_{l=1}^{N} \pi_k^{(l)} \boldsymbol{x}_k^{(l)}. \tag{9}$$

The main advantage of the particle filter is that no restrictions are placed on the functions f_k and h_k, or on the distribution of the system and measurement noise.

III. EEG State-Space Model

In order to apply the particle filtering framework, described in Section 2, we need to define the state-space model of the EEG source localization problem based on physiological constraints.

A. EEG measurement model

The main source of EEG potentials, measured at the scalp, derive from simultaneous postsynaptic current flows (i.e., graded synaptic activity) of many neighboring neurons with similar orientations. In particular, these clusters of similar oriented neurons are mainly found in the cortical areas of the brain associated with the pyramidal cells. The total electric current in an activated region is often modeled by a mathematical current dipole with an adequate dipole moment. Furthermore, many of those current dipoles representing microscopic current flows with the same orientation are replaced by an equivalent current dipole [1]. In this paper, it is assumed that the electric activity of the brain at any given time can be modelled by only a small number of dipoles having arbitrary location and orientation within the source volume. The focal current models seem particularly suitable in BCIs where strong assumptions about the areas involved in a specific mental task are available (e.g., is well-known that the components of the EEG used for discriminating imaginary movements originate in the motor cortex).Assuming that the brain electrical activity has been originated by *M* active dipoles, the observation vector z_k that represents the EEG measurements collected from all sensors at time *k* (the forward EEG model) [1] can be expressed by

$$z_k = L(d_k)s_k + v_k \tag{10}$$

Where for each dipole $d_k = [d_{xk}, d_{yk}, d_{zk}]^t$ is a three dimensional coordinate vector at time *k*, $s_k = [s_{xk}, s_{yk}, s_{zk}]^t$ is the source oscillation amplitude in the space at the same time and *t* is the transpose operator. *L* is the lead field dipole matrix at time *k*, and v_k is a white Gaussian noise with variance σ_v^2. From Eq. (10), the likelihood of each measurement can be obtained:

$$\mathrm{L}\left(\boldsymbol{z}_k \left|(\boldsymbol{x}_k, \boldsymbol{s}_k)\right.\right) \propto \exp\left[-\frac{\left(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k\right)^t \boldsymbol{R}_{z_k}^{-1}\left(\boldsymbol{z}_k - \boldsymbol{L}(\boldsymbol{x}_k)\boldsymbol{s}_k\right)}{2}\right], \tag{11}$$

where $\boldsymbol{R}_{z_k}$ is the covariance matrix of the measurement vector $\boldsymbol{z}_k$, $\propto$ denotes "proportional to". The goal is to estimate d_k and s_k and given $\boldsymbol{z}_k$ (the multichannel EEG signal).

B. EEG state transition model

In the PF framework of the dipole localization problem, the states x_k that have to be estimated are the geometrical positions and neural activity signals of *M* dipoles. That is, for one dipole, we have six state variables: $x_k = [d_{xk}, d_{yk}, d_{zk}, s_{xk}, s_{yk}, s_{zk}]^t$. In the adopted scenario, we have no a priori knowledge of the dipole locations or signals. Such a model is valid in practice, especially for characterizing brain sources of atypical brain activity like seizure in epilepsy. We therefore assume the state transition to be a random walk (first-order Markov chain) in the source localization space [9],

$$x_k = x_k + w_k \tag{12}$$

where w_k is a zero-mean, Gaussian white noise with covariance $\sigma_w^2 I$, where I denotes the identity matrix. The process w_k is assumed to be independent of past and current states. Thus, the complete state-space model of the dipole reconstruction problem is the following:

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{w}_k & \text{state transition model} \\ \boldsymbol{z}_k = \boldsymbol{L}\left(\boldsymbol{x}_k\right)\boldsymbol{s}_k + \boldsymbol{v}_k & \text{observation model.} \end{cases} \tag{13}$$

C. State-space dynamics

In the above section, the number of active brain dipoles *M* is assumed to be known a priori. In this section, we include the estimation of the number of active dipoles within the PF framework. We therefore consider the vector $y_k = \{x_k, m_k\}$, where m_k is the number of active dipoles at time *k*. m_k is modeled by the following birth-death process [10],

$$m_k = m_{k-1} + \varepsilon_{mk} \tag{14}$$

where ε_{mk} is a discrete process having the following probability

$$\begin{aligned}
&\Pr\left(\varepsilon_{m_k} = 1\right) = p_b \\
&\Pr\left(\varepsilon_{m_k} = 0\right) = 1 - p_b - p_d \\
&\Pr\left(\varepsilon_{m_k} = -1\right) = p_d
\end{aligned} \tag{15}$$

where $p_b, p_d \in [0,1]$ are, respectively, the probability of birth and death of a source. Observe that the model in Eqs. (14) and (15) implies that the number of sources can change by no more than one at a given time. Assuming that the dipoles move independently, we have the following joint dynamics for the dipoles' location, amplitudes and number

$$\begin{aligned}
p\left(y_k \middle| y_{k-1}\right) &= p\left(x_k, m_k \middle| x_{k-1}, m_{k-1}\right) = \\
&= p\left(x_k \middle| x_{k-1}, m_k, m_{k-1}\right) p\left(m_k \middle| m_{k-1}\right)
\end{aligned} \tag{16}$$

The prior dynamics for x_k is therefore given by

$$p\left(x_k \middle| x_{k-1}, m_k, m_{k-1}\right) =$$
$$= \begin{cases} p_0\left(x_k(m_k)\right) \prod_{m=1}^{m_{k-1}} p\left(x_k(m) \middle| x_{k-1}(m)\right), & \text{if } m_k = m_{k-1} + 1; \\ \prod_{m=1}^{m_k} p\left(x_k(m) x_{k-1}(m)\right) & \text{if } m_k = m_{k-1}; \\ \prod_{m=1, m=m^*}^{m_{k-1}} p\left(x_k(m) \middle| x_{k-1}(m)\right) & \text{if } m_k = m_{k-1} - 1. \end{cases} \tag{17}$$

where $p_0\left(x_k(m_k)\right)$ is the distribution of the new dipole and m^* is the dipole to be removed.

IV. Source Number Estimation

We propose to estimate the number of uncorrelated brain dipoles by analyzing the structure of the covariance matrix of the observations. We assume an array of n_z electrodes sensing signals from m dipoles. The EEG recordings span T time points to form a spatio-temporal data vector $Z = L(X)S + V = \left[(z_1)^t, \ldots.(z_T)^t\right]$. The covariance matrix of the observations can be decomposed as

$$R = R_s + R_n = R_s + \sigma_v^2 I \tag{18}$$

where R_s and $R_n = \sigma_v^2 I$ are the covariance of the source signal $L(X)S$ and the noise V. We assume that the observations are generated from m uncorrelated dipoles; thus the rank of R_s is m. Since R_s is also positive semi-definite, R_s has m non-zero positive eigenvalues $\lambda_s \geq 0$. The eigenvalues of R can be written as $\lambda = \lambda_s + \sigma_v^2$ and can be ordered as $\lambda_1 \geq \ldots. \geq \lambda_m \geq \lambda_{m+1} \geq \ldots. \geq \lambda_{n_z}$. Hence, the problem of estimating the number of independent dipoles reduces to the problem of finding the smallest eigenvalues of the observation covariance matrix. In practice, however, the covariance matrix R is unknown. Instead, an estimate of the covariance matrix is available from a finite sample set. In this case, determining the smallest eigenvalue may not be straightforward. Information theoretic criteria (IC) methods [11] offer a better alternative to determining the smallest eigenvalues by using a penalized likelihood function of the data. The IC criteria for determining the number of signals can generally be expressed as

$$IC = -\log f\left(Z \middle| \Theta(m)\right) + C_f(T) P_f \tag{19}$$

where f is a family of conditional probability density functions, m is the assumed number of signals, $\Theta(m)$ is a system function of m parameters, P_f is the number of free parameters in Θ, and $C_f(T)$ is the penalty function, which depends on the number of sample points T.
The likelihood function in (19) is based on the minimum descriptive length (MDL) criterion proposed by Wax and Kailath [12]. The IC value derived by Wax and Kailath can be obtained from the eigenvalues of the observation covariance matrix R as follows [11], [12]

$$\begin{aligned}
IC(m) = T(n_z - m) \log\left[\frac{1}{n_z - m} \sum_{i=m+1}^{n_z} \lambda_i\right] - \\
T \sum_{i=m+1}^{n_z} \log(\lambda_i) + 2m(2n_z - m + 1) \log(T)
\end{aligned} \tag{20}$$

where m is the number of assumed independent active dipoles and n_z is the number of the EEG channels. The number of dipoles with minimum IC is selected as the estimated number of active brain dipoles.

IC-based estimation algorithm of independent active dipoles

The IC algorithm to estimate the number of independent brain dipoles with strong neural activity can be summarized in the following steps:

1. Calculate the covariance matrix R from the measurement data matrix Z.
2. Use SVD to compute the eigenvalues of the covariance matrix R, $\lambda_1 \geq \ldots. \geq \lambda_m \geq \lambda_{m+1} \geq \ldots. \geq \lambda_{n_z}$.
3. Use the eigenvalues of R to compute the information criterion $IC(m)$ in Eq. (20) for different values of m.
4. The number of dipoles with minimum IC is selected as the estimated number of active dipoles.

V. PARTICLE FILTER ALGORITHM

The PF algorithm for estimating the number of active dynamic (moving) dipoles and their locations is described below, where N denotes the total number of particles.

PF algorithm for reconstruction of dynamic brain dipoles

I. Initialization

1. Estimate the initial number of active dipoles using the *IC* algorithm described in Section 4 for $m = 1,...M_{\max}$, where $M_{\max}$ is the expected maximum number of active brain dipoles. The initial number (m_0) corresponds to the minimum of IC.

2. $k = 0$,

generate $l = 1,...N$ samples $\boldsymbol{x}_0^{(l)} \sim p(\boldsymbol{x}_0)$ and set initial weights $\pi_0^{(l)} = 1/N$.

II. Estimation for k = 1, 2, . . . ,

1. Prediction Step

Generate $m_k^{(l)}$ samples according to the prior in Eq. (17) the to track $x_k^{(l)} = p\left(\tilde{x}_k \middle| \tilde{x}_{k-1}^{(l)}, m_k^{(l)}\right)$.

2. Measurement Update

- Compute the lead field matrix $L(x_k)$ by solving the Maxwell equations in [1].
- Evaluate the particle weights:
  (a) for $l = 1,...N$, on the receipt of a new measurement, compute the weights

$$w_k^{(l)} = w_{k-1}^{(l)} \mathrm{L}\left(\boldsymbol{z}_k \middle| \left(\boldsymbol{x}_k^{(l)}, \boldsymbol{L}\left(\boldsymbol{x}_k^{(l)}\right), s_k^{(l)} \right) \right). \tag{21}$$

  The likelihood $\mathrm{L}\left(\boldsymbol{z}_k \middle| \left(\boldsymbol{x}_k^{(l)}, \boldsymbol{L}\left(\boldsymbol{x}_k^{(l)}\right), s_k^{(l)} \right) \right)$ is calculated using Eq. (10).

  (b) for $l = 1,...N$, normalise the weights,

  $\pi_k^{(l)} = w_k^{(l)} / \sum_{l=1}^{N} w_k^{(l)}$.

3. *Evaluate the posterior mean* as the estimate of the state at time k:

$$\tilde{y}_k = E[y_k \mid z_{1:k}] = E\left\{ \left[x_k^t, m_k \right]^t \middle| z_{1:k} \right\} = \sum_{l=1}^{N} \pi_k^{(l)} y_k^{(l)} \tag{22}$$

4. *Compute the effective sample size* $N_{eff} = 1/\sum_{l=1}^{N} \left(\pi_k^{(l)}\right)^2$.

5. *Selection step (resampling)* if $N_{eff} < N_{tresh}$: Multiply/ suppress samples $\left\{y_k^{(l)}\right\}$ with high/low weights $\pi_k^{(l)}$, in order to obtain N new random samples approximately distributed according to the posterior state distribution.

III. Estimate the # of active dipoles (for k = k + epoch)

Estimate periodically (once per *epoch*) the number of active dipoles as

$$\hat{m}_k = \arg\min\{IC(m_{k-1} - 1), IC(m_{k-1}), IC(m_{k-1} + 1)\}, \tag{23}$$

where *epoch* is the number of time samples over which it is assumed the number of active dipoles does not change, e.g., during a focused brain activity, as described in Section 6.

Birth (of a new active dipole):

- Increment the number of dipoles to $m_k = m_{k-1} + 1$
- Initialize the particles for the new dipole, $\tilde{x}_0^{(l)} \approx p(\tilde{x}_0)$, $l = 1,...N$.
- Insert the new particles to form an augmented state vector $\tilde{x}_{k-1}^{(l)} = \left[\left(x_{k-1}^{(i)}\right)', \left(\tilde{x}_0^{(l)}\right)' \right]$

Death (of an active dipole):

- Decrement the number of dipoles to $m_k = m_{k-1} - 1$.
- Remove the dipole corresponding to the smallest average weight, m^*, and all its particles, i.e.,
- $\tilde{x}_{k-1}^{(l)} = \left[\left(x_{1,k-1}^{(i)}\right)', ..., \left(x_{m^*-1,k-1}^{(i)}\right)', \left(x_{m^*+1,k-1}^{(i)}\right)', ..., \left(x_{m_{k-1}^{(l)},k-1}^{(i)}\right)' \right]$

Maintenance (keep the same number of active dipoles): Keep $m_k = m_{k-1}$ and generate samples according to the prior in Eq. (17).

The PF resampling step is necessary in order to avoid the degeneracy of the algorithm, where the variance of the particles' weights can only increase over time, [13]. In practice, after a certain number of iterations, all but one of the normalized weights are very close to zero. The purpose of resampling is to eliminate estimates which have small normalized weights and to concentrate on estimates with large weights. A suitable measure of degeneracy of the algorithm is the effective sample size $N_{eff} = 1/\sum_{l=1}^{N} \left(\pi_k^{(l)}\right)^2$, [13]. When N_{eff} is below a fixed threshold N_{tresh}, the resampling procedure is used.

Remark: The IC criterion (20) is used as an initial guess for the number of active independent dipoles. In the course of the PF algorithm, each epoch, this number can increase or decrease smoothly only with one dipole.

VI. RESULTS ON SIMULATED EEG DATA

The simulated EEG data is based on the following assumptions:

• The scalp electrodes record the superposition of both brain sources and non-brain sources related to, for example, movements of muscles.
• The reference has a zero potential.
• The distribution of the electrodes on the scalp follows the standard 10/20 International system. An array of 30-electrodes is assumed (Fp1, AF3, F7, F3, FC1, FC5, C3, CP1, CP5, P7, P3, Pz, PO3, O1, Oz, O2, PO4, P4, P8, CP6, CP2, C4, FC6, FC2, F4, F8, AF4, Fp2, Fz, Cz).

The coordinates of a grid of 21012 dipoles uniformly distributed inside a spherical head model, with radius R = 10 cm, represent the discrete dipole state-space. A small number of moving dipoles (from 2 to 20) is randomly chosen from this grid. The neural activity in the dipoles is simulated by a sine wave with constant amplitude 0.1 and varying frequencies [10, 15, 20, 25]Hz. For the moving dipoles, the waves propagate along the moving directions of each dipole. EEG data, as a result of the neural activity in the chosen active dipoles and artificially added noise with varying SNR, is generated using the forward model (Eq. 8). The number and location of the active dipoles change in a piecewise mode, from one epoch to another. Over one epoch (which corresponds to 40 msec/20 samples in our simulations) they are kept constant. One epoch corresponds to a static (focused) period in the neural activity.

A. *Estimation of the number of active dipoles*

We first assess the performance of the information theoretic criterion (Eq. 20) in estimating the number of active dipoles. Fig. 1 shows the IC values computed for EEG generated by 4, 12 and 20 dipoles, in the presence of temporal correlation between some of the dipoles.

The temporal correlation $M_{i,j}(f)$ of a pair of (i,j) dipoles is quantified by the magnitude-squared cross spectrum $S_{i,j}(f)$ divided by the power spectra of the dipole waveforms $S_{i,i}(f)$ and $S_{j,j}(f)$:

$$M_{i,j}(f) = \frac{\left|S_{i,j}(f)\right|^2}{S_{i,i}(f)S_{j,j}(f)}. \qquad (24)$$

The correlation is bounded between 0 and 1, where $M_{i,j}(f)=1$ indicates a perfect linear relation between dipoles d_i and d_j at frequency *f*. The minimum of the four IC curves depicted on Fig.1 correspond to the number of uncorrelated dipoles ($M_{i,j}(f)=0$). Therefore we empirically confirm the theoretical proofs in [12] that IC can reliably infer only uncorrelated dipoles.

B. *Particle filter (PF) results*

For the moving dipoles, the initial and the final locations are selected. We simulated four sequential location stages (epochs) across 80 samples (20 samples per epoch). The dipoles move with constant speed between the initial and the final locations. PF reconstruct the dipole coordinates within the head geometry as described in Section 5. We consider the estimation of moving: (i) two uncorrelated dipoles, (ii) three uncorrelated dipoles, (iii) four uncorrelated dipoles and (iv) four (two by two) correlated dipoles. Very noisy EEG data was generated (SNR= 1 dB). Fig. 2 shows the Mean Square Distance Error (MSDE) over 100 Monte Carlo runs for the four cases, where

$$\text{MSDE}=\left(\sqrt{(\hat{x}-x)^2+(\hat{y}-y)^2+(\hat{z}-z)^2}\right) \qquad (25)$$

For uncorrelated dipoles over each subsequent moving stage (20 samples), the MSDE decreases. This is expected because the posterior at the previous stage is transferred as a prior for the next location. Only at the beginning of the first stage, an uninformed (flat) prior is assumed. The case of two by two correlated dipoles is more challenging and longer time is required to correctly localize the dipoles.

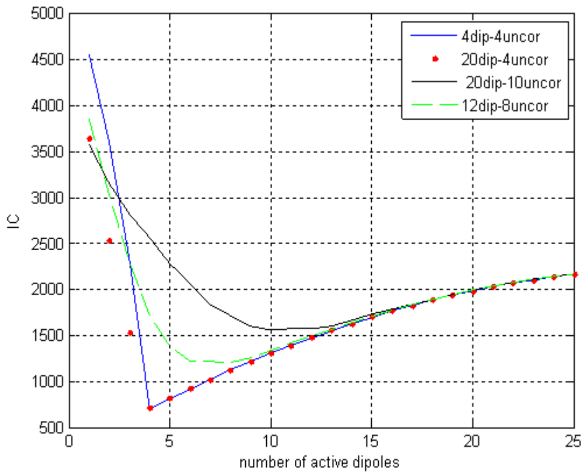

Fig. 1. Estimation of the # of active dipoles using the IC in Eq. (20)

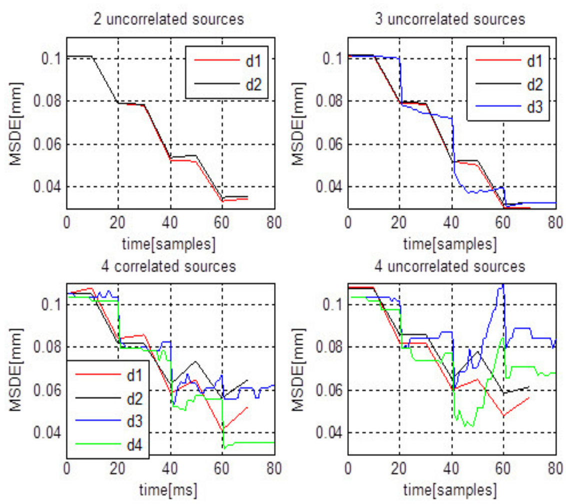

Fig. 2. MSDE of moving dipole localization, SNR=1dB (100 MC runs)

VII. RESULTS ON REAL EEG DATA

In this section, we validate the estimation accuracy of the proposed algorithm with real EEG data. The data corresponds to EEG Visually Evoked Potentials (VEP) extracted from thirteen female subjects (20- 28 years old). All participants had normal or corrected to normal vision and no history of neurological or psychiatric illness. Different facial expressions (neutral, fearful and disgusted) of 16 individuals (8 males and 8 females) were selected, giving a total of 48 different facial stimuli. Images of 16 different house fronts were superimposed on each of the faces. This resulted in a total of 384 grayscale composite images (9.5 cm wide by 14 cm high) of transparently superimposed face and house. Participants were seated in a dimly lit room, where a computer screen was placed at a viewing distance of approximately 80 cm coupled to a PC equipped with software for the EEG recording. The images were divided into two experimental blocks. In the first, the participants were required to attend to the houses (ignoring the faces) and in the other they were required to attend to the faces (ignoring the houses). The participant's task was to determine, on each trial, if the current house or face (depending on the experimental block) is the same as the one presented on the previous trial. Stimuli were presented in a sequence of 300 ms each and were preceded by a fixation cross displayed for 500 ms. The inter-trial interval was 2000 ms.EEG signals were recorded from 20 electrodes (Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2; F7, F8, T3, T6; P7, P8, Fz, Cz, Pz, Oz) according to the 10/20 International system. Electrooculogram (EOG) signals were also recorded from electrodes placed just above the left supra orbital ridge (vertical EOG) and on the left outer canthus (horizontal EOG). VEP were calculated off-line averaging segments of 400 points of digitized EEG (12 bit A/D converter, sampling rate 250 Hz). These segments covered 1600 ms comprising a pre-stimulus interval of 148 ms (37 samples) and post-stimulus onset interval of 1452 ms. The EEG signals were visually inspected, prior to processing, and those segments with excessive EOG artifacts were manually eliminated. Only trials with correct responses were included in the data set. The experimental setup was designed by Santos et al. [14] for their study on subject attention and perception using VEP signals.

Our goal is to reconstruct the principal brain zones (expressed by the location of the most active dipoles) that have originated the positive EEG peaks in the range of 80-120 milliseconds, known as P100. P100 corresponds to the perception of the sensory stimulus, a brain activity that is known to happen in the primary visual cortex. Therefore, we expect to localize the most active dipoles in the visual cortex and also that the occipital channels (O1, 02, Oz) and the parietal channels (Pz, P3, P4), located around the visual cortex, will respond with stronger VEPs.

A. *Estimation of the number of active dipoles*

First the number of dipoles with strongest neural activity that may have produced the recorded EEG data is estimated following the IC in Eq. (20). Fig. 3 depicts the overlapped IC curves computed from the EEG data for all 148 trials. Each trial corresponds to the EEG data recorded while one subject was repeating the same experiment described above 148 times. It is intriguing to observe that the minimums of the IC are obtained for a range of 2 to 9 (uncorrelated) dipoles .

This variation in the number of the active dipoles, when apparently the subject was doing the same task, can be explained by the fact that the patient may not have been completely focused on the repetitive task at hand (the patient may get bored and think of other issues while performing the same task), or that the presented images bring up some personnel memories, experiences or feelings to the patient (e.g., if a face looks familiar or the patient likes a particular house, etc). The human brain, far from being stiff and rigid, is flexible and adaptable; and thus, even a focused brain activity may stimulate different neuronal sources each time it is performed. For 18 of the trials (the bottom plot of Fig.3) the IC minimum is obtained for 2 dipoles that we consider as the dipoles (d_1 and d_2) with the strongest neuronal activity.

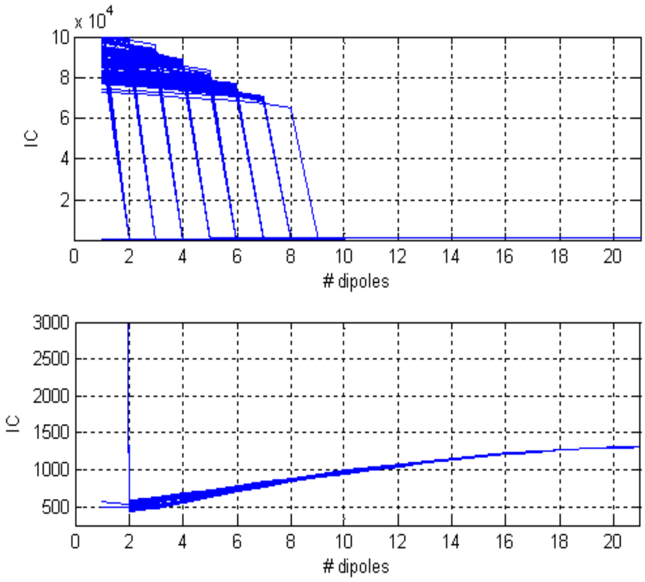

Fig. 3. Estimation of the # of active dipoles by IC criterion (Eq. 20) for real EEG data: 148 trials, min IC for 2 to 9 dipoles (top); 18 trials, min IC for 2 dipoles (bottom).

B. *Particle filter (PF) results*

We apply the proposed PF framework to estimate the two strongest dipoles that may have produced the P100 peak. The filtering is repeated 100 runs starting from randomly selected particles. The results of the averaged estimation are summarized in Fig. 4. It is very promising to see that the estimated coordinates d_1 (3.6 mm,-5.5 mm, -1.03 mm) and d_2 (-3.3 mm, -2.2 mm, -1.02 mm) correspond to the zone of the primary visual cortex.

These biologically plausible results for real EEG data encourage us that the proposed statistical framework for reconstruction of dynamic brain dipoles is feasible and can be extended for more complex brain processes.

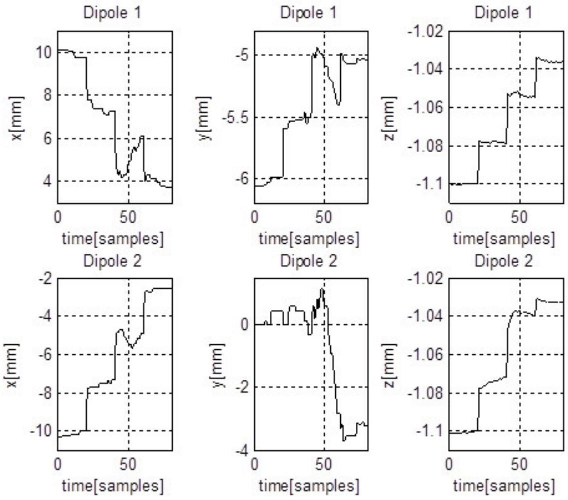

Fig. 4. Estimation of the coordinates of the strongest dipoles d_1 and d_2 that originated P100 wave in the occipital channels. (100 MC runs)

VIII. CONCLUSION

In this paper, we propose a particle filter (PF)-based algorithm to simultaneously estimate the number of the most active dipoles that originated recorded EEG data, their locations and corresponding oscillations in the 3D head geometry. Our main contribution is that the PF algorithm explicitly considers dynamic (moving) dipoles. The dipole localization problem is formulated as the estimation of the probability density function of the state vector (moving over time-space coordinates and the respective amplitudes of the dipole signals) based on all available observations (EEG measurements). This framework takes into account the non-linear relationship between the state vector and the observations as well as the noisy nature of the signals. Computer simulations, based on generated and real EEG data, show that the proposed PF approach is feasible to estimate the most active uncorrelated brain dipoles which move over time to new space locations in the brain.

Current research is focused on implementing the proposed technique to more complex applications, i.e. EEG data with more affected brain zones and respectively higher number of active dipoles (more than two). Moreover, we are aware that the assumption of uncorrelated brain dipoles is too strong, therefore the algorithm is under modifications to explicitly consider possible correlations between the dipoles.

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