Neural Approach for Bearing Fault Classification in Induction Motors by Using Motor Current and Voltage

W. F. Godoy, I. N. da Silva, A. Goedtel, R. H. C. Palácios, W. S. Gongora

Abstract—The induction motor is considered one of the most important elements in manufacturing processes. The use of strategies based on intelligent systems capable to classify the presence or absence of failures and also to determine its origin for the diagnosis and faults prediction is widely investigated in three phase induction motors. Thus, the aim of this paper is to present a methodology of bearing failures classification based on artificial neural networks, by using voltage and electric currents values in the time domain. Experimental results collected at real industrial process are presented to validate this proposal.

I. INTRODUCTION

THE three-phase induction motor is frequently used in the most diverse industrial sectors. Brazilian industrial sector consumes 47% of the country's electricity, and 50% of this energy is used in rotating electrical machines [1], which highlights the importance of three-phase induction motor in several industrial applications.

In Brazil, during the past nine years, the application of resources with predictive maintenance has averaged 17.42%, as for the corrective maintenance it was reached 28.8%, and estimated total cost of maintenance due to the gross sales was 3.95% [2].

Aiming to maximize process efficiency and also productivity, electric motors, and other equipments at industrial premises, require maintenance plans aligned with predictive techniques of diagnosis and prevention of failures that can lead to unwanted process stops [3].

Thus, it can consider that the profitability of a process is related to the availability of its equipments, environmental preservation and also keeping people and processes integrity.

The continuous search for cost reduction requires the

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development of plans and strategies that can be used to predict and eliminate potential failures and unwanted machinery breaks, among others, reducing costs caused by unscheduled shutdowns.

Hence, it can be employed various types of maintenance, such as predictive maintenance. This technique allows evaluating the real operating conditions of machines through the analysis of the data collected directly from the machine, allowing to minimize the occurrence of unexpected failures [4].

According to [5], reports concerning induction motor reliability shows that bearing faults are the major cause of failure in asynchronous motors. In the event of failure, the induction motor shows signs of defects in various ways, allowing its proper identification still under evolution.

In this context, intelligent systems have been used for the identification and also resolution of several issues related to the control and actuation of electrical machines being able to address the failure source still in its early stage [6]-[7]-[8]-[9].

Intelligent systems applied to machines diagnosis are based on Artificial Neural Networks (ANN), Fuzzy Logic and also Hybrid Systems [10]-[11]-[12]-[13].

The works of [14]-[15], deals with the evaluation of two types of incipient faults in a single-phase squirrel-cage induction motor: stator winding fault and bearing wear under constant load torque conditions.

Thus, following the structure proposed by [14], the original input space was composed by effective values of current and speed $\{I, W\}$, and it was expanded to 5 dimensions $\{I, W, I^2, W^2, WxI\}$ to achieve better network accuracy and to reduce training time.

The objective of this paper is investigate a low cost alternative strategy to detect bearings faults in induction motors by using current (I) and voltage (V) signals in the time domain by combining the structures of the network inputs based on [14].

A comparative study between two neural structures is also presented in this paper. The first network consider as input only $\{I, V\}$ while the second network is composed by $\{I, V, I^2, V^2, IxV\}$.

This paper is organized as follows: Section 2 presents a description of the main faults in induction motors. In Section 3, ANN data applied in this work are described. In Section 4, methodology proposed to evaluate the performance of networks and also present the results obtained from experimental data. Finally, in Section 5, the final conclusions of this work are presented.

II. ASPECTS RELATED TO FAULTS IN INDUCTION MOTORS

The monitoring of the operating conditions of an induction motor enables fault diagnosis and also estimating its operation conditions attracting attention of many researchers in the recent years [16].

This is due to the considerable influence of induction motor on the continuity of many industrial processes.

The early detection and accurate diagnosis of incipient faults minimizes the occurrence of process damage, increases equipment availability and consequent maintenance of financial results.

Electric motors are subject to various types of failures, which can be divided into two distinct groups: i) electrical faults and ii) mechanical faults.

The main fault types in which electrical faults are highlighted problems relating to stator winding, rotor winding, which are present in some models of motors; broken bars in the rotor, broken rotor rings; connections among others.

Moreover, the mechanical failures may be derived from problems of bearings, eccentricity, wear coupling misalignment among others as reported by [16]-[17].

According to the failures described in the literature [16]-[17]-[18]-[19], it is estimated that the bearings are responsible for approximately 40% of the unwanted stops in the induction motors, as can be followed in Fig. 1.

Thus, this paper addresses investigation related with bearings faults, since this refers to the type incipient failure more common in the induction motors.

Bearings are subject to deterioration caused by inadequate or contaminated lubrication, misalignment and mainly to incorrect assemblies.

According to [20], the deterioration of bearings can also occur due to common mode currents that move through the same due to electrostatic charge induced on the motor shaft.

Other factor to be considered is related with the torque pulsations caused by the existence of low-order harmonics in power or for possible broken bars.

Traditional methods consider the monitoring of temperature and bearings vibration to estimate its operating conditions. The cost of sensors for monitoring vibration devices associated with signal processing also restricts its large use at industrial scale.

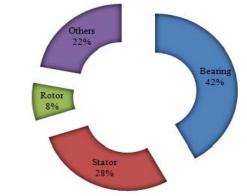


Fig. 1. Distribution of induction motor faults

In this work, voltages and currents were measured in three induction motors of 1, 7.5 and 12.5 HP respectively. The 1 HP motor is used in the laboratory, in perfect operating condition, and the other ones used in the heating and milling sugar cane processes, both with bearing faults.

Initially, data was collected at motors on failure conditions in normal process operation. After corrective maintainability, new measures of both mechanical vibration and current were taken and no vibrations were noticed.

As an example, Figure 2 (a) shows the normalized currents measured in a motor applied to the heating process operating with bearing fault. This problem has been detected by the conventional method of analysis of mechanical vibration.

Even the machine showing such disturbances, still in operation, there was collections of data and can be observed in detail that distortion exists between the motor currents.

Thus, as showed in Fig. 2 (b), it can be inferred the proper operation of the machine by restoring the standard current signal, as it was not observed distortions. Figure 2 (c) presents the motor under analysis in the industrial process.

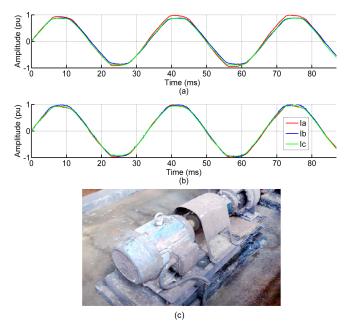


Fig. 2. (a) Current before maintenance (b) Current after maintenance (c) Motor under analysis.

III. ARTIFICIAL NEURAL NETWORKS

Identification using artificial neural network has shown promise for the solution of a series of problems involving power systems [21].

More specifically, the use of ANN has provided alternative schemes to handling problems related to electrical machines [12]-[18]. In this study, ANNs were applied to bearing fault identification in TIM.

For such purpose, a multilayer perceptron network was used, which was trained with a backpropagation algorithm. This training algorithm has two basic steps: the first one, called propagation, applies values to the ANN inputs and verifies the response signal in its output layer [21]-[22]. This value is then compared with the desired signal for that output. The second step occurs in the reverse way, i.e., from the output to the input layer. The error produced by the network is used in the adjustment process of its internal parameters (weights and bias) [21].

The basic element of a neural network is the artificial neuron (Fig. 3), which is also known as the node or processing element.

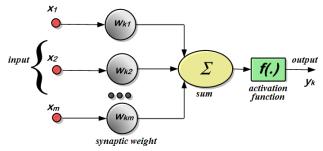


Fig. 3. Representation of the artificial neuron.

The artificial neuron illustrated in Fig. 3 can be modeled mathematically as follows:

$$v_j(k) = \sum_{i=1}^n X_i . w_i + b$$
 (1)

$$y_j(k) = \varphi_j(v_j(k)) \tag{2}$$

where:

n is the number of input signals of the neuron;

 X_i is the *i*-th input signal of the neuron;

w_i is the weight associated with the *i*-th input signal;

b is the threshold associated with the neuron;

 $v_j(k)$ is the weighted response (summing junction) of the *j*-th neuron with respect to the instant *k*;

 $\varphi(.)$ is the activation function of the *j*-th neuron;

 $y_j(k)$ is the output signal of the *j*-th neuron with respect to the instant *k*.

The adjust of the network weights (w_j) , associated with the *j*-th output neuron, is done by computing of the error signal linked to the *k*-th iteration or *k*-th input vector (training example). This error signal is provided by:

$$e_{i}(k) = d_{i}(k) - y_{i}(k)$$
 (3)

where $d_j(k)$ is the desired response to the *j*-th output neuron. Adding all squared errors produced by the output neurons of the network with respect to *k*-th iteration, we have:

$$E(k) = \frac{1}{2} \sum_{j=1}^{p} e_j^2(k)$$
(4)

where *p* is the number of output neurons.

For an optimum weight configuration, E(k) is minimized with respect to the synaptic weight w_{ji} . Therefore, the weights associated with the output layer of the network are updated using the following relationship:

$$w_{ji}(k+1) \leftarrow w_{ji}(k) - \eta \frac{\partial E(k)}{\partial w_{ji}(k)}$$
 (5)

where w_{ji} is the weight connecting the *j*-th neuron of the output layer to *i*-th neuron of the previous layer, and η is a constant that determines the learning rate of the backpropagation algorithm.

The adjustment of weights belonging to the hidden layers of the network is carried out in an analogous way. The necessary steps for adjusting the weights associated with the hidden neurons can be found in [21].

The objective of the backpropagation algorithm is minimizing E(k) and E_m through the adjustment of w_i and b.

To achieve the objective of reducing the error, the training algorithm presents the input data set to the neural network and the output is then computed as described in (1) and (2).

The error calculated in each iteration is used as a parameter to the weight adjustment.

After presentation of all training data set to the network, the mean error can be calculated using (5).

The parameter E_m estimates the convergence of the algorithm and determines if the algorithm should stop when it reaches the mean desired error.

The backpropagation algorithm is based on the Least Mean Square (LMS) method and it applies a correction in the synaptic weights, called $\Delta w_{ji}(k)$, to the synaptic weight $w_{ji}(k)$.

This correction is proportional to the partial derivative $\partial E(k)/\partial w_{ji}(k)$ as described in [21]-[22]. Using the chain rule, it is possible to express this gradient in the following form:

$$\frac{\partial E(k)}{\partial w_{ij}(k)} = \frac{\partial E(k)}{\partial e_j(k)} \cdot \frac{\partial e_j(k)}{\partial y_i(k)} \cdot \frac{\partial y_j(k)}{\partial v_i(k)} \cdot \frac{\partial v_j(k)}{\partial w_{ij}(k)} \cdot \frac{\partial v_j(k)}{\partial w_{ij}(k)}$$
(6)

This partial derivative is called "sensitivity factor" and indicates the search direction with respect to weight $w_{ji}(k)$ [21]. The terms in equation (6) are given by:

$$\frac{\partial E(k)}{\partial e_i(k)} = e_j(k) \tag{7}$$

$$\frac{\partial e_j(k)}{\partial y_j(k)} = -1 \tag{8}$$

$$\frac{\partial y_j(k)}{\partial v_i(k)} = \dot{\varphi_j}(v_j(k)) \tag{9}$$

$$\frac{\partial v_j(k)}{\partial w_{ii}(k)} = y_i(k) \tag{10}$$

Equation (8) is calculated through the derivative of (4) with respect to $e_j(k)$. The derivative of the error function in (3) with respect to the *j*-th output, i.e. $y_i(k)$, results in (9).

The derivative of (2) with respect to $v_j(k)$ results in (9). Equation (10) is the result of the derivative of (1) considering w_{ji} the weight connecting the *j*-th neuron of the output layer to *i*-th neuron of the previous layer.

Equation (11) is the result of the grouping of (7)-(10) and it

is described as follows:

$$\frac{\partial E(k)}{\partial w_{ii}(k)} = -e_j(k)\dot{\varphi_j}(v_j(n))y_i(n) \tag{11}$$

The synaptic correction $\Delta w_{ji}(k)$ with respect to the weight $w_{ji}(k)$ is described through the delta rule as described in (6), i.e.:

$$\Delta w_{ji}(k) = -\eta \frac{\partial E(k)}{\partial w_{ii}(k)} \tag{12}$$

The use of the negative signal in (12) indicates the descendent gradient in relation to the search of synaptic weights to reduce E(k). The substitution of (11) in (12) results in the following equation:

$$\Delta w_{ji}(k) = -\eta \delta_j(k) y_i(k) \tag{13}$$

where $\delta_i(k)$ is the local gradient defined by:

$$\delta_j(k) = -\frac{\partial E(k)}{\partial v_j(k)} = -\frac{\partial E(k)}{\partial e_j(k)} \frac{\partial e_j(k)}{\partial y_j(k)} \frac{\partial y_j(k)}{\partial v_j(k)} = e_j(k) \dot{\varphi_j}(v_j(k)) \quad (14)$$

The local gradient shows the direction of the synaptic weights in order to reduce E(k).

IV. THEORETICAL APPROACH AND RESULTS

The purpose of this work is the usage of the induction motor currents and voltage signals in the time domain presented to an ANN capable to classify the existence or absence of bearing failure. Figure 4 illustrates acquisition and processing data routine.

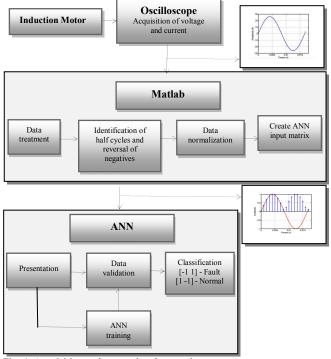


Fig. 4. Acquisition and processing data routine

Differently from traditional methods of mechanical

vibration analysis, which requires specific sensors for data acquisition, and even the method proposed by [14] which uses the effective values of current in each sample and speed, this proposal is based on data collected by a digital oscilloscope of four isolated channels Model TPS 2014 Tecktronix® with current ferrules A622 100 Amp AC/DC.

This unit has a memory card to storage the signals viewed on screen, where samples are stored as a datasheet of 2500 points.

The sampling rate is variable according to the selector sec/div which is adjusted as a function of the signal displayed on the screen.

The signals are separated by a half cycle and normalized by its peak value to hold then in the time domain disregarding machine scale.

Based on the collected data and with a proper import routine, information is evaluated and manipulated in the MATLAB software.

A. Input Treatment Data

In order to proper classify the bearing functioning, voltage and currents signals were sampled in the three-phase for analysis. The signals were divided into 50 points per semi-cycle, resulting in a periodic sampling rate, and the amplitude value of each point presented as the ANN input, as per [23].

This method considers the input signal of a sinusoidal waveform in continuous time. In this way, each semi-cycle is divided into a number of samples required for the composition of input vectors which will be presented to the ANN thus making linear signal discretization, as per Fig. 5.

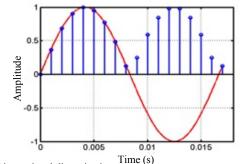


Fig. 5. Linear signal discretization

Using a sampling of 10 ms/div on the oscilloscope, it is possible to obtain a total of 12 semi-cycles of sampled waveform each half-cycle of the signal. Thus a subsampling of 50 points is capable of translating the necessary information without spoiling the waveform.

Another processing aspect is the fact that the signals are the currents (I_a, I_b, I_c) and voltages (V_a, V_b, V_c) of a three-phase induction motor. Hence, it is just necessary to create a column vector with points of each phase of the collected system, subsequent from each other.

Since the purpose of this work is still the combination of five input structures vectors of voltage and current replicated for each condition $\{I, V, I^2, V^2, IxV\}$ assembled from the previous conditions, thus creating an array of 750 entry points by the amount samples of each phase.

In terms of signals usage, collected from real industrial processes, currents and voltages of motors from different power and different functional states, it is necessary to perform the normalization of these data. Then, it was recognized the maximum voltage and current value of each sample for this purpose.

The analyzes are performed for each wave semi-cycle, then it is considered the absolute values of the sampling signals having only the positive semi-cycle signals, as showed in Fig. 6, which presents the experimental curves normalized for the unit amplitude.

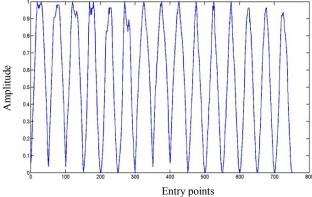


Fig. 6. Positive signals semi-cycles

B. Neural Structure

ANN has been efficiently used to solve problems of engineering. In this case specifically, ANN were used to identify the existence or not of bearing faults in induction motors.

To work with the proposed networks of machines only within formation in real applications without simulation results, divide up the vectors of samples randomly collected from the engines into two classes, one for training and one for validation, and were divided according to Table I.

TABLE I Division of collected data				
Classes	(%)	Samples		
Training samples	68	17		
Validation samples	32	8		

Thus, the data used in network training are not presented for validation, allowing the evaluation of their ability to generalize and actual response of the system.

In this work, it was considered a Multilayer Perceptron network with supervised training composed by two neurons in both hidden and output layers. For activation of the hidden layer, it was used the hyperbolic tangent function with a range varying from -1 to 1.

Evaluation were focused on the incipient bearing fault under conditions of constant load torque. Entries for Network 1 were defined as the stator current (I) and voltage (V).

For the second network, the voltage and current signals were combined into five different structures and it were presented to the network, the electric current (I), motor supply voltage (V), the square of the current (I^2), the square of the

voltage (V^2) and the product of these two quantities (IxV). The desired outputs are the qualitative conditions (with and without fail), which were classified by the ANN as per Fig. 4.

The training algorithm applies the gradient based on the method of least square for nonlinear models which, when incorporated into the training process, enhancing the efficiency of the synaptic weights adjustment. The characteristics and topologies of both networks are described in Table II.

TABLE II
NETWORK PARAMETERS

Туре	Network 1	Network 2
Architecture	MLP	MLP
Training	S	S
Number of layers	2	2
Neurons in the 1° layer	2	2
Neurons in the 1° layer	2	2
Training Algorithm	LM	LM
Network function - 1° layer	HT	HT
Network function - 2° layer	HT	HT
Output network function	Linear	Linear

(S) Supervised; (LM) Levenberg-Maquardt; (HT) Hyperbolic Tangent

C. Classification Results

The proposed networks were subjected to training within input signals as described in the Input Treatment Data section, with a learning rate η =0.01. As stopping criterion was established the mean squared error (MSE) of 5.10⁻⁵.

Network 1 reached the stopping criterion with 10 epochs, achieving 75% of accuracy. Network 2 has the same neural structure as the first network, however; it reached the stopping criterion with 8 epochs.

In validation, it was obtained 100% of accuracy, confirming the generalization capability of the network, as showed in Table III.

TABLE III Network results

THET WORK RESOLUTS				
Туре	Network 1	Network 2		
Training samples	17	17		
Validation samples	8	8		
AQE	5.10-5	5.10-5		
Learning coefficient	0,01	0,01		
Epochs	10	8		
Positive false	0	0		
Negative false	2	0		
Classification error	2/8	0/8		
Accuracy percentage	75	100		

V. CONCLUSION

This work presents a comparative study between two neural structures used to classify, from the current and voltage signals in the time domain, the occurrence of incipient bearings faults.

This proposal provides an indication of faults allowing proper decisions in real time without the need to resort to conventional methods of analysis.

Combination of the input data allows to increase the computational efficiency, enhancing detectability as the

results obtained by using this artifice increased reliability and accuracy in 100% of cases, which can be considered satisfactory in real applications.

As for a possible hardware implementation, proposed structure can be implemented in low-cost processors, and mainly, it can be used in induction motors of varied potencies and in different regimes of operation.

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