# Sideslip angle soft-sensor based on neural network left inversion for multi-wheel independently driven electric vehicles

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Abstract-Effective estimation of vehicle states such as the yaw rate and the sideslip angle is important for vehicle stability control. Unfortunately the devices are very expensive to measure the sideslip angle directly and are not suitable for ordinary vehicle. Therefore, it must be estimated. A novel sideslip angle soft-sensor using neural network left inversion (NNLI) is presented for the in-wheel motor driven electric vehicle (EV). The innovation of the presented algorithm is not only little concerned with reference model parameters identification, but also uses the characteristic of the in-wheel motor driven EV. Longitudinal acceleration, lateral acceleration, yaw rate, longitudinal velocity, steering angle, the torque of in-wheel motor which can be acquired by ordinary sensors are used as inputs. Co-simulations are carried out to demonstrate the effectiveness of the proposed soft-sensor with Simulink and CarSim.

#### I. INTRODUCTION

DUE to energy conservation and environmental protection, electric vehicles (EVs) have been popular in both academic and industry. Compared with the conventional vehicles, EVs with in-wheel motors have several advantages in motion control [1]-[3]. The torque generated by in-wheel motor is accurate and fast. The driving torque of each wheel can be easily measured from the motor current and can be controlled independently.

Nowadays vehicle stability control systems have been introduced to handle the problem that it's difficult to drive a vehicle at low adhesion well. Such as the direct yaw-moment control (DYC) can be easier to realize because of the unique power trains of EVs, epically for multi-wheel independently driven EVs. In DYC system, it is required to accurately measure the yaw rate and the sideslip angle. Unfortunately, the direct measurement of the sideslip angle is only provided by special devices such as optical sensors or global positioning system (GPS) inertial sensors, which are very expensive and however unsuitable for the ordinary vehicles. Thus the sideslip angle must be estimated in real-time.

Many kinds of estimation methods have been used to estimate the sideslip angle. Based on the linear vehicle model a state observer with kalman filter is designed to estimate the sideslip angle [4], [5]. But these methods are not robust against the parameters changes in tire-road friction and driving conditions. To deal with the above problem, a fuzzy observer was proposed with non-linear vehicle model in [6]. In [7], extended and unscented kalman filters were presented to deal with its non-linearity. Based on the tire-road friction adaptation, an adaptive sideslip angle observer was presented in [8-9]. A novel method which combined the vehicle-model-based method and the kinematics-based method was proposed in [10]. In recent years, several researchers have presented the estimation using GPS without knowing the vehicle model [11].

To estimate the sideslip angle, some attempts of applying the neural network (NN) technique can be found in [12-14]. But the complex layout of the estimator generated a high computational effort. In [13], the function of the yaw rate and the lateral acceleration at instants was used to estimate the sideslip angle at the instant, but it's unsuitable when changes happened in vehicle speed and tire-road friction coefficient. In [14] the NN-based observer with the non-linear vehicle model was obtained, but it was not ideal when the vehicle speed was changed. Furthermore, these NN-based methods were seldom theoretically proved for the parameters selection during the training.

Given the above considerations, this paper will theoretically validate the input parameters selection of NN left inversion (NNLI). In the meantime, the presented soft-sensor considers the characteristics of EVs and imports the torque of motor as the input of the NNLI. This paper is organized as follows. Section II introduces the non-linear model of the multi-wheel independently driven EV. Sections III describes briefly the theory of NNLI and design a sideslip angle soft-sensor. In Section IV, the performance of the proposed soft-sensor is compared with CarSim. Finally, some conclusions are presented in Section V. The symbols are list in the appendix.

# II. VEHICLE MODEL

## A. Lateral Dynamics Model

The non-linear two-track yaw plane vehicle is shown in Fig. 1.

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Fig. 1. Non-linear two-track yaw plane vehicle model. The vehicle speed obtains from  $v = (v_x^2 + v_y^2)^{1/2}$  with the body sideslip angle  $\beta = \arctan(v_x / v_y)$ . appears per page. The vehicle is assumed to have a low vehicle centre of gravity (CG) height with stiff suspension, and also the suspension dynamics and roll dynamics have no affect on the vehicle plane dynamics.

The equations for the vehicle speed, the sideslip angle and the yaw rate dynamics are displayed in the follows:

$$\dot{v} = \frac{1}{m} ((F_{x1} + F_{x2}) \cos(\beta - \delta) + (F_{x3} + F_{x4}) \cos \beta + (F_{y1} + F_{y2}) \sin(\beta - \delta) + (F_{y3} + F_{y4}) \sin \beta)$$
(1)  
$$\dot{\beta} = \frac{1}{m} (-(F_{x1} + F_{x2}) \sin(\beta - \delta) - (F_{x3} + F_{x4}) \sin \beta + (F_{y1} + F_{y2}) \cos(\delta - \beta) + (F_{y3} + F_{y4}) \cos \beta) - \gamma$$
  
$$\dot{\gamma} = \frac{1}{J_z} (F_{x1} (I_f \sin \delta - \frac{d}{2} \cos \delta) + F_{x2} (I_f \sin \delta + \frac{d}{2} \cos \delta) + \frac{d}{2} (F_{x4} - F_{x3}) + F_{y1} (\frac{d}{2} \sin \delta + I_f \cos \delta) + F_{y2} (-\frac{d}{2} \sin \delta + I_f \cos \delta) - I_r (F_{y3} + F_{y4}))$$

The coordinate frame of the equations is based on the CG as its origin fixed and with the center line of vehicle as the *x*-axis.

## B. Tire Force and Tire Slip Angle

To better understand the motion of vehicles, we must study the tire force. The longitudinal force of the vehicle is generated by the in-wheel motor and the wheel rotational motion is shown in Fig. 2.





The longitudinal force is shown in the following equation:

$$F_{xi} = \frac{1}{R_i} (T_i - J_i \cdot \dot{\omega}_i) \tag{2}$$

With the assumption that the tire as a rigid body. So the rolling radius and rotational inertia are considered to be

constant.

As this study is mainly focused on the estimation of the sideslip angle in non-linear region, the Dugoff's model is used as the tire model. The simplified non-linear lateral tire forces are shown in the follow:

$$F_{vi} = -C_i \tan \alpha_i f(\lambda) \tag{3}$$

Where  $C_i$  is the lateral stiffness,  $\alpha_i$  is the slip angle, and  $f(\lambda)$  is shown as:

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda & if \quad \lambda < 1\\ 1 & if \quad \lambda \ge 1 \end{cases}$$
(4)

$$\lambda = \frac{\mu F_{zi}}{2C_i |\tan \alpha_i|} \tag{5}$$

 $\mu$  is the road coefficient and  $F_{zi}$  is the normal load on the tires. In order to simplify the model we assume the conditions is pure slip.

The normal load of the tire has three parts due to the effect of the lateral acceleration, the longitudinal acceleration and the roll of the vehicle. Without considering the roll of the vehicle, so the bank of angle is defined as  $\Phi = 0$ . The normal load of each tire can be described as:

$$\begin{cases} F_{z1} = \frac{mgl_r}{2(l_f + l_r)} - \frac{ma_xh}{2(l_f + l_r)} - \frac{K_{f\Phi}}{K_{f\Phi} + K_{r\Phi}} \cdot \frac{ma_yh}{d} \\ F_{z2} = \frac{mgl_r}{2(l_f + l_r)} - \frac{ma_xh}{2(l_f + l_r)} + \frac{K_{f\Phi}}{K_{f\Phi} + K_{r\Phi}} \cdot \frac{ma_yh}{d} \\ F_{z3} = \frac{mgl_f}{2(l_f + l_r)} + \frac{ma_xh}{2(l_f + l_r)} - \frac{K_{r\Phi}}{K_{f\Phi} + K_{r\Phi}} \cdot \frac{ma_yh}{d} \\ F_{z4} = \frac{mgl_f}{2(l_f + l_r)} + \frac{ma_xh}{2(l_f + l_r)} + \frac{K_{r\Phi}}{K_{f\Phi} + K_{r\Phi}} \cdot \frac{ma_yh}{d} \end{cases}$$

The tire slip angle  $\alpha_i$  is shown as follow:

$$\left\{ \begin{aligned} \alpha_{1} &= \delta - \arctan\left(\frac{\nu\beta + l_{f}\gamma}{\nu - \gamma d/2}\right) \\ \alpha_{2} &= \delta - \arctan\left(\frac{\nu\beta + l_{f}\gamma}{\nu + \gamma d/2}\right) \\ \alpha_{3} &= -\arctan\left(\frac{\nu\beta - l_{r}\gamma}{\nu - \gamma d/2}\right) \\ \alpha_{4} &= -\arctan\left(\frac{\nu\beta - l_{r}\gamma}{\nu + \gamma d/2}\right) \end{aligned} \right.$$
(7)

## III. NNLI SIDESLIP ANGLE SOFT-SENSOR

This section will illustrate the assumed inherent sensor and the NNLI sideslip angle soft-sensor [18-20].

# A. Left Inverse Soft-Sensor Theory

A linear or non-linear system  $\Sigma$  with *q*-dimensional input  $\mathbf{u}(t) = (u_1, u_2, \dots, u_q)^T$  and *d*-dimensional output  $y(t) = (y_1, y_2, \dots, y_d)^T$  can be represented by the state function:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}, \qquad \qquad x(t_0) = x_0 \tag{8}$$

The system statement is  $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$  and given a certain initial state  $x(t_0) = x_0$ .

For a general point of view, the mathematical model of this system is equivalent to a mapping from the input to the output. It can be described as the following:

$$y(\cdot) = \theta(x_0, u(\cdot)) = \theta u \tag{9}$$

Generally speaking, for a non-linear system, in its interior, it can be assumed that there exists an assumed inherent sensor subsystem as shown in Fig. 3. The variables to be estimated are the inputs of the subsystem while the variables which can be measured directly are the outputs. The variables are the input variables of the controlled system. If a left inversion of the assumed inherent sensor can be constructed and cascaded behind the assumed inherent sensor, then a so-called composite identity system whose outputs would be the identity mapping of its inputs is obtained. It means that the outputs of the assumed inherent sensor inversion will reproduce completely the inputs of the "assumed inherent sensor". Therefore, the non-directly measured variables of the controlled system can be estimated from the variables which can be measured directly [17], [20].



Fig. 3. The measuring principle based on left inversion.

#### B. NNLI Sideslip Angle Soft-Sensor

The static NN has the ability to approach the non-linear system. A NNLI sideslip angle soft-sensor based on the ability can be achieved. The basic idea of the NNLI sideslip angle estimator is as follows:

1) Construct an assumed inherent sensor.

2) Get the inversion of the assumed inherent sensor.

3) Using a static NN to approximate this inversion system.

4) Put the inversion system behind the assumed inherent sensor in series to produce a unified system, therefore, it is named as the NNLI.

5) Through this NNLI, we can realize the estimation of the non-directly measured sideslip angle.

For the system in (1), its state variable is

$$\boldsymbol{x} = (x_1, x_2, x_3, x_4)^{\prime} = (\gamma, \beta, v_x, a_x)^{\prime}$$
(10)

The input of the system is

$$u = [u_1, u_2, u_3]^T = [\delta, T_{d3}, T_{d4}]^T$$
(11)

The directly measured variable is

$$Z = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}]^T$$
  
=  $[\nu, \gamma, F_{x1}, F_{x2}, F_{x3}, F_{x4}, F_{y1}, F_{y2}, F_{y3}, F_{y4}]^T$  (12)

The non-directly measured variable is

$$x = [x_1]^T = [\boldsymbol{\beta}]^T \tag{13}$$

According to the modeling algorithm, the Jacobian matrix rank is obtained.

$$rank\left(\frac{\partial Z^{T}}{\partial x}\right) = rank\left(\frac{\partial Z^{T}}{\partial \beta}\right) = rank\left(\begin{array}{c}0\\0\end{array}\right) = 0 \quad (14)$$

Since the rank is not equal to the number of the non-directly measured variables, it can be further derivate.

$$rank\left(\frac{\partial(z^{T},\dot{z}^{T})}{\partial x}\right) = rank\left(\frac{\partial}{\partial\beta} \cdot \frac{\partial}{\partial\beta} \cdot \frac{\partial}{\partial\beta} \cdot 0 \cdot 0 \cdot 0 \cdot 0\right) \quad (15)$$

As is shown in the following equation referred to (1):

$$\frac{\partial \dot{v}}{\partial \beta} = -\frac{1}{m} ((F_{x3} + F_{x4}) \sin \beta + (F_{y1} + F_{y2}) \cos(\beta - \delta) + (F_{y3} + F_{y4}) \cos \beta) \neq 0$$

$$\frac{\partial \dot{\gamma}}{\partial \beta} = \frac{1}{J_z} F_{y1} (\frac{d}{2} \cos \delta - l_f \sin \delta) \neq 0$$
(16)

The rank is as following:

$$rank\left(\frac{\partial \left(z^{T}, \dot{z}^{T}\right)}{\partial x}\right) = 1$$
(17)

Therefore, the model of the assumed inherent sensor is left invertible. The left inversion model of the sideslip angle according to these functions can be achieved. And the inversion of the assumed inherent sensor for the sideslip angle can be written as:

$$\beta = \phi(v, a_x, a_v, T_{d1}, T_{d2}, T_{d3}, T_{d4}, \delta)$$
(18)

Since the vehicle is driven by two-rear-wheel, therefore

$$T_{d1} = T_{d2} = 0 \tag{19}$$

And the inversion of the assumed inherent sensor can be written as:

$$\boldsymbol{\beta} = \boldsymbol{\phi}(\boldsymbol{\nu}, \boldsymbol{a}_x, \boldsymbol{a}_y, T_{d3}, T_{d4}, \boldsymbol{\delta}) \tag{20}$$

As the system is complex in mathematical model, it is difficult to construct the inversion of the assumed inherent sensor by analytic means. Hence, a static NN is used to approximate the above nonlinear function in (20). Then, the NNLI dynamic soft-sensing model is finally completed, which is composed of a static NN and a series of differentiators. This simplifies the construction of the proposed NN-based soft-sensor in practical use, while it is strict enough in theory. The structure of the NNLI sideslip angle soft-sensor is shown in Fig. 4.



#### IV. SIMULATION RESULT

The performance of the proposed soft-sensor is evaluated by Matlab/Simulink-CarSim co-simulation. A vehicle with two in-motors at the rear wheels is developed using CarSim. The NNLI soft-sensor is constructed by using Matlab/Simulink.

# A. NNLI Train

A NN should be trained and tested by means of numerical data in the first step. In order to highlight the performance of the NNLI sideslip angle soft-sensor in non-linear condition, the range of vehicle speed is chosen comparatively high one.

At the same time, the variable steering wheel angle is necessary for inspiring the NN sufficiently. The set of maneuvers for NN training is shown in Figs. 5 and 6.



Meanwhile, a static back propagation NN with the structure of 11-18-1 with "tan sigmoid" transfer function on the nodes of one hidden layer, and "linear" transfer function on the node of output layer is adopted. The training mean squared error is 9.999e-4 after 515 times of training by using

Levenberg-Marquardt algorithm. In addition, to improve the training performance and enhance the soft-sensing performance, all the data are normalized within [-1, 1].

## B. NNLI Sideslip Angle Result

Two Simulations are carried out under a single lane change maneuver and a step maneuver. In two maneuver simulations, the vehicle travels on a wet road ( $\mu$ =0.4) at a constant speed of 90 km/h. The estimated sideslip angle is compared with the output of CarSim in order to reflect the superiority of NNLI soft-sensor.

As shown in Figs. 7 and 8, simulation results verify that the actual (denoted as CarSim) and estimated values (denoted as NNLI) of the sideslip angle agree well. Therefore, the NNLI soft-sensor can accurately predict the sideslip angle. The presented algorithm is proved perfect in theory verification and the simulation.



(a) Front steering angle.



(b) Comparison of actual and estimated values of sideslip angle. Fig. 8. Simulation under step maneuver

### V. CONCLUSION

The system of the vehicle is non-linear and the common observe methods can hardly track the state of the vehicle. But the soft-sensor based on NNLI is perfect in estimating the non-linear state of the vehicle. In this paper, to estimate the sideslip angle, a new method for sideslip angle estimation has been presented. The NNLI has been treated as the dynamic soft-sensor, which consists of a NN and a series of differentiators. And its performance has been investigated through CarSim-Matlab/Simulink co-simulation. The vehicle with the proposed NNLI soft-sensor can successfully follow the sideslip angle trajectory.

# APPENDIX

	I ABLE I
	NOMENCLATURE
Symbol	MEANING
$a_x$	Longitudinal acceleration at CG (center of gravity)
$a_y$	Lateral acceleration at CG
Vx	Longitudinal speed at CG
$V_y$	Lateral speed at CG
d	Track width
g	Acceleration due to gravity
$l_f$	Distance from CG to front axle
$l_r$	Distance from CG to rear axle
i	1,2,3 and 4 corresponding to front left, front right,
	Rear left and rear right
т	Total mass of vehicle
$C_i$	Tire cornering stiffness at the ith tire
$R_i$	Rolling radius at the ith tire
$F_{xi}$	Longitudinal tire force at the ith tire
$F_{xi}$	Lateral tire force at the ith tire
V	Vehicle speed at CG
$J_z$	Vehicle inertia around the z-axis
$J_i$	Wheel angular moment of inertia at the ith tire
$T_i$	In-wheel motor torque applied to the ith tire
Vi	Wheel speed at the ith tire
$\omega_i$	Wheel angular velocity at the ith tire.
$\beta$	Vehicle sideslip angle
δ	Front steering angle
Y	Yaw rate
K <sub>f</sub> ø	Roll stiffness coefficient of front axle
K <sub>r</sub> ø	Roll stiffness coefficient of rear axle

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