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Neural-Network-Based Optimal Control for a Class of Complex-Valued Nonlinear Systems with Input Saturation

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***Abstract*— This paper proposes an optimal control scheme based on adaptive dynamic programming (ADP) algorithm for complex-valued systems with input saturation. The equivalence transformation is used to obtain the real dynamic system. Then the performance index function is defined. Based on the transformed system, an ADP optimal control method is established. The update methods for critic network neural network and action network are given. It is proved that the closed-loop system is uniformly ultimately bounded based on Lyapunov approach. Finally, the simulation study was given to show the effectiveness of the proposed optimal control scheme.**

I. Introduction

In recent years, more and more researchers paid their attention to complex-valued systems and neural networks [2], [3]. Especially, [9] and [10] studied the complex-valued filter problems for complex signals and systems. Using observational input/output data, [11] proposed complex-valued B-spline neural network to model the complex-valued Wiener system. On the other side, as the nonanalytic nature of the actuator nonlinear dynamics, the optimal control problem of complex-valued system with input saturation is a challenge for control engineers.

It is well known that, dynamic programming is a very useful tool in solving optimization and optimal control problems by employing the principle of optimality. However, solving the associated Hamilton-Jacobi-Bellman (HJB) equation demands a large (rather infeasible) number of computations and storage space. This prevented the implementation of dynamic programming. An innovative idea was proposed in [12] to get around this numerical complexity by using an adaptive/approximate dynamic programming (ADP) formulation. During last decade, ADP has played an important role in the area of optimal control [13], [14], [15], [16], [17], [18]. By now, ADP has successfully solved the nonlinear zero-sum differential games problem [20], on-line optimization problem [22], optimal tracking control problem [21], multi-objective optimal control problem [19] etc. Especially, in [5], the near-optimal control problem for a class of nonlinear discrete-time systems with control constraints was solved by iterative adaptive dynamic programming algorithm. Based on previous research, we will discuss the optimal control problem for complex-valued nonlinear systems with input saturation.

This paper will study the optimal control problem based on ADP algorithm. First, the complex-valued system is transformed into the real system. Then the performance index function is presented. Based on the transformed system, an optimal control is established by ADP algorithm. The update methods for critic network neural (NN) and action NN are given. And then, it is proven that the closed-loop system is uniformly ultimately bounded (UUB) based on Lyapunov approach. Finally, the simulation study is given to show the effectiveness of the proposed control scheme.

The rest of this paper is organized as follows: Section II derives motivations and preliminaries. Section III introduces the neural-network-based optimal control method. The stability analysis is presented in Section IV. Section V presents the simulation results. In Section VI, the conclusion is drawn.

II. Motivations and Preliminaries

Consider a continuous-time complex-valued nonlinear system

$$\dot{\eta}(t) = f(\eta(t)) + g(\eta(t))u, \tag{1}$$

where $\eta \in \mathcal{C}^n$. $f(\eta(t)) \in \mathcal{C}^n$ and $f(0) = 0$. $f(\eta(t)) = (f_1(\eta(t)), f_2(\eta(t)), \cdots, f_n(\eta(t)))^T$. $g(\eta(t)) \in \mathcal{C}^{n\times n}$ is the input gain and $g(\eta(t))$ is bounded. u is the input vector and $u \in \mathcal{C}^n$. And the input $u = [u_1, u_2, \cdots, u_n]^T$, where $\alpha_i \leq ||u_i|| \leq \beta_i$, α_i and β_i are constants.

This paper is desired to find u, which minimizes a generalized nonquadratic functional

$$\Lambda(\eta(t)) = \int_t^{\infty} U\left(\eta(\tau), u(\eta(\tau))\right)d\tau, \tag{2}$$

where the utility function U is positive definite. It can be expanded as follows:

$$\begin{aligned}\Lambda(\eta(t)) &= \int_t^{T} U\left(\eta(\tau), u(\eta(\tau))\right)d\tau + \int_T^{\infty} U\left(\eta(\tau), u(\eta(\tau))\right)d\tau \\ &= \int_t^{T} U\left(\eta(\tau), u(\eta(\tau))\right)d\tau + \Lambda(\eta(T)). \end{aligned}\tag{3}$$

Define $\eta = \eta^R + i\eta^I$, $f = f^R + if^I$, $g = g^R + ig^I$ and $u = u^R + iu^I$, then we have

$$\dot{x} = F(x) + G(x)v, \tag{4}$$

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where $x = [\eta^R; \eta^I]$, $F = [f^R; f^I]$, $G = [g^R, -g^I; g^I, g^R]$ and $v = [u^R; u^I]$.

Then we can define

$$\Lambda(x(t)) = \int_t^T U(x(\tau), v(\tau))d\tau + \Lambda(x(T)), \tag{5}$$

where $U(x, v) = x^T Qx + 2\int_0^v \varphi^{-1}(\bar{v})Rd\bar{v}$, Q and R are positive definite matrices, φ is the bounded and monotone odd function, i.e., $||\varphi|| < \varphi_M$.

The infinitesimal version of (3) is

$$0 = \Lambda_x^T(F(x) + G(x)v) + U(x, v). \tag{6}$$

Defining the Hamiltonian function as follows

$$H(x, v, \Lambda_x) = \Lambda_x^T(F(x) + G(x)v) + U(x, v). \tag{7}$$

So we can define the optimal performance index function satisfying

$$0 = \min_v H(x, v, \Lambda_x^*), \tag{8}$$

where

$$\Lambda^*(x) = \min_v \int_t^\infty U\left(x(\tau), v(x(\tau))\right)d\tau, \tag{9}$$

and the optimal control is

$$v^* = -\varphi\left(\frac{1}{2}R^{-1}G^T\Lambda_x^*\right). \tag{10}$$

III. NEURAL-NETWORK-BASED OPTIMAL CONTROL DESIGN METHOD

In the following part, the neural-network-based optimal control design method using the ADP will be established. As we all know the critic NN and the action NN, are two vital modules of ADP method. The NNs are used to be as the critic NN and action NN. In the NN, the number of hidden layer neurons is L, the weight matrix between the input layer and hidden layer is Y, the weight matrix between the hidden layer and output layer is W, the input vector is X. Then the output is represented as $F_N(X, Y, W) = W^T\sigma(YX)$, where $\sigma(YX)$ is the activation function. For convenience of analysis, only the output weight W is updating during the training, while the hidden weight is kept fixed. Hence, in the following part, the NN function can be simplified by the expression $F_N(X, W) = W^T\bar{\sigma}(X)$, where $\bar{\sigma}(X) = \sigma(YX)$.

In the following subsections, the detailed design methods for the critic NN and the action NN will be given.

A. Critic NN design method

In this paper, the performance index function $\Lambda(x)$ is obtained by the critic NN. The ideal critic NN is expressed as

$$\Lambda(x) = M_c^T\phi_c(x) + \epsilon_c, \tag{11}$$

where $\phi_c(x)$ is the activation function, M_c is the ideal critic NN weight matrix and ϵ_c is the critic NN approximation error.

So we have

$$\Lambda_x = \nabla\phi_c^T(x)M_c + \nabla\epsilon_c, \tag{12}$$

and if we define $\epsilon_H = -\nabla\epsilon_c^T(F + Gv)$, then we can get

$$H(x, v, M_c) = M_c^T\nabla\phi_c(F + Gv) + U(x, v) - \epsilon_H. \tag{13}$$

For the actual NNs, let the estimate of M_c be $\hat{M}_c$, then the actual output of the critic NN is

$$\hat{\Lambda}(x) = \hat{M}_c^T\phi_c(x). \tag{14}$$

So we can get

$$H(x, v, \hat{M}_c) = \hat{M}_c^T\nabla\phi_c(F + Gv) + U(x, v). \tag{15}$$

Define the weight estimation error of the critic NN as follows

$$\tilde{M}_c = M_c - \hat{M}_c. \tag{16}$$

Let $e_c = H(x, v, \hat{M}_c) - H(x, v, M_c)$, then we have

$$e_c = -\tilde{M}_c^T\nabla\phi_c(x)(F(x) + G(x)v) + \epsilon_H. \tag{17}$$

Define the weight update law $\dot{\hat{M}}_c$ is given as

$$\dot{\hat{M}}_c = -\alpha_c\frac{\xi_1(\xi_1^T\hat{M}_c + U(x, v))}{(\xi_1^T\xi_1 + 1)^2}, \tag{18}$$

where α_c is the adaptive gain of the critic network and is positive, $\xi_1 = \nabla\phi_c(F + Gv)$.

Define $\xi_2 = \dfrac{\xi_1}{\xi_3}$ and $\xi_3 = \xi_1^T\xi_1 + 1$, we have

$$\dot{\hat{M}}_c = -\alpha_c\xi_2\xi_2^T\tilde{M}_c + \alpha_c\xi_2\frac{\epsilon_H}{\xi_3}. \tag{19}$$

B. Action NN design method

The action NN is used to obtain the control error policy v. Define the action NN as follows

$$v = M_a^T\phi_a(x) + \epsilon_a, \tag{20}$$

where M_a is the ideal weight matrix of the action network, $\phi_a(x)$ is the activation function and ϵ_a is the action network approximation error.

Define the actual output of action network as

$$\hat{v}(x) = \hat{M}_a^T\phi_a(x), \tag{21}$$

where $\hat{M}_a$ is the actual weight of the action network. Define

$$e_a = \hat{M}_a^T\phi_a + \varphi\left(\frac{1}{2}R^{-1}G^T\hat{\Lambda}_x\right). \tag{22}$$

The update method of $\hat{M}_a$ is given as follows

$$\dot{\hat{M}}_a = -\alpha_a\phi_a\left(\hat{M}_a^T\phi_a + \varphi\left(\frac{1}{2}R^{-1}G^T\hat{\Lambda}_x\right)\right)^T, \tag{23}$$

where α_a is the adaptive gain.

Define the weight estimation error of the action network as

$$\tilde{M}_a = M_a - \hat{M}_a. \tag{24}$$

Then the update law of $\tilde{M}_a$ is

$$\dot{\tilde{M}}_a = \alpha_a \phi_a (-\tilde{M}_a^T \phi_a + \varphi(\frac{1}{2} R^{-1} G^T \nabla \phi_c^T (M_c - \tilde{M}_c)) + M_a^T \phi_a)^T. \quad (25)$$

IV. Stability Analysis

For the proposed method, we will give the detail stability analysis.

Theorem 1: Let the weight updating laws of the critic network and the action network be given as in (18) and (23), respectively. Then the closed-loop error system (4), the weight estimation errors $\tilde{W}_c$ and $\tilde{W}_a$ are UUB.

Proof: Define Lyapunov function candidate as follows:

$$\Gamma(t) = \Gamma_1(t) + \Gamma_2(t) + \Gamma_3(t), \quad (26)$$

where $\Gamma_1(t) = \frac{1}{2\alpha_c} \tilde{W}_c^T \tilde{W}_c$, $\Gamma_2(t) = \frac{l_2}{2\alpha_a} tr\{\tilde{W}_a^T \tilde{W}_a\}$, $\Gamma_3(t) = x^T x + l_3 J(x)$, $l_2 > 0, l_3 > 0$.

So the time derivative of the Lyapunov function candidate (26) along the trajectories of the closed-loop systems (6) is computed as $\dot{\Gamma}(t) = \dot{\Gamma}_1(t) + \dot{\Gamma}_2(t) + \dot{\Gamma}_3(t)$. According to (18), we have

$$\dot{\Gamma}_1(t) = -(\tilde{W}_c^T \xi_2)^T \tilde{W}_c^T \xi_2 + (\tilde{W}_c^T \xi_2)^T \frac{\epsilon_H}{\xi_3}. \quad (27)$$

Based on (23), we obtain

$$\begin{aligned} \dot{\Gamma}_2(t) = & - l_2(\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a - l_2(\tilde{W}_a^T \phi_a)^T (v - \epsilon_a) \\ & + l_2(\tilde{W}_a^T \phi_a)^T \varphi(\frac{1}{2} R^{-1} G^T \nabla \phi_c^T (M_c - \tilde{M}_c)) \\ \le & - l_2(\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a + \frac{l_2}{4} (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a + l_2 \varphi_M^2 \\ & + \frac{l_2}{4} (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a + l_2 v^T v \\ & + l_2(\tilde{W}_a^T \phi_a)^T \epsilon_a. \end{aligned} \quad (28)$$

As

$$\begin{aligned} \dot{x} &= F + G\hat{W}_a^T \phi_a \\ &= F + G(W_a - \tilde{W}_a)^T \phi_a \\ &= F + Gv - G\tilde{W}_a^T \phi_a - G\epsilon_a. \end{aligned} \quad (29)$$

Then the time derivative of Γ_3 is calculated as follows

$$\dot{\Gamma}_3 = 2x^T (F + Gv - G\tilde{W}_a^T \phi_a - G\epsilon_a) - l_3 U(x, v). \quad (30)$$

As $2x^T F \le 2k \cdot ||x||^2$, and

$$-2x^T G^T \tilde{W}_a^T \phi_a \le ||x||^2 + ||G^T||^2 (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a, \quad (31)$$

$$2x^T G^T v \le ||x||^2 + ||G^T||^2 ||v||^2, \quad (32)$$

$$-2x^T G^T \epsilon_a \le ||x||^2 + ||G^T||^2 \epsilon_{aM}^2. \quad (33)$$

So (30) can be rewritten as

$$\begin{aligned} \dot{\Gamma}_3 \le & (2k + 3 - l_3 \lambda_{\min}(Q))||x||^2 + (||G^T||^2 - l_3 \lambda_{\min}(R))||v||^2 \\ & + ||G^T||^2 (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a + ||G^T||^2 \epsilon_{aM}^2. \end{aligned} \quad (34)$$

Therefore, we have

$$\begin{aligned} \dot{\Gamma}(t) \le & - (\tilde{W}_c^T \xi_2)^T \tilde{W}_c^T \xi_2 - \frac{1}{2} l_2 (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a \\ & + ||G^T||^2 (\tilde{W}_a^T \phi_a)^T \tilde{W}_a^T \phi_a \\ & + l_2 (\tilde{W}_a^T \phi_a)^T \epsilon_a + l_2 \varphi_M^2 \\ & + (\tilde{W}_c^T \xi_2)^T \frac{\epsilon_H}{\xi_3} + (2k + 3 - l_3 \lambda_{\min}(Q))||x||^2 \\ & + (||G^T||^2 - l_3 \lambda_{\min}(R) + l_2)||v||^2 \\ & + ||G^T||^2 \epsilon_{aM}^2. \end{aligned} \quad (35)$$

Let $Y = \begin{bmatrix} e \\ v \\ \tilde{W}_c^T \xi_2 \\ \tilde{W}_a^T \phi_a \end{bmatrix}$, $N_\Gamma = \begin{bmatrix} 0 \\ 0 \\ \frac{\epsilon_H}{\xi_3} \\ l_2 \epsilon_a \end{bmatrix}$ and $\epsilon_\Gamma = ||G^T||^2 \epsilon_{aM}^2 + l_2 \varphi_M^2$, where $M_\Gamma = diag$ $(l_3 \lambda_{\min}(Q) - 2k - 3,\ l_3 \lambda_{\min}(R) - ||G^T||^2 - l_2,\ 1,\ \frac{1}{2} l_2 - ||G^T||^2)$, then we can get

$$\begin{aligned} \dot{\Gamma} \le & - Y^T M_\Gamma Y + Y^T N_\Gamma \\ \le & - ||Y||^2 \lambda_{min}(M_\Gamma) + ||Y|| ||N_\Gamma|| + \epsilon_\Gamma. \end{aligned} \quad (36)$$

So if the parameters l_2 and l_3 satisfying

$$l_2 > 2||G||, \quad (37)$$

and

$$l_3 > \max \left\{ \frac{||G||^2 + l_2}{\lambda_{\min}(R)}, \frac{2k + 3}{\lambda_{\min}(Q)} \right\}. \quad (38)$$

Then the Lyapunov derivative is negative if $||Y|| > \frac{||N_\Gamma||}{2\lambda_{\min}(M_\Gamma)} + \sqrt{\frac{N_\Gamma^2}{4\lambda_{\min}^2(M_\Gamma)} + \frac{\epsilon_\Gamma}{\lambda_{\min}(M_\Gamma)}} \equiv Y_B$. It is now straightforward to demonstrate that if Γ exceeds a certain bound, then, $\dot{\Gamma}$ is negative. Therefore, according to the standard Lyapunov extension theorem the analysis above demonstrates the state and the weight errors are UUB [1].

V. Simulation Study

We consider the following nonlinear oscillator [4]:

$$\begin{aligned} \dot{\eta}_1 &= \eta_1 + \eta_2 - \eta_1(\eta_1^2 + \eta_2^2) \\ \dot{\eta}_2 &= - \eta_1 + \eta_2 - \eta_2(\eta_1^2 + \eta_2^2) + u \end{aligned} \quad (39)$$

where $\eta = [\eta_1, \eta_2]^T \in \mathcal{C}^2$, $\eta_j = x_j^R + iy_j^I$ and $u = u^R + iu^I$, $||u|| \le 1.5$.

For the infinite-horizon optimal control problem, in the utility function $\varphi = tanh$, $Q_1 = I_4$, $R_1 = I_2$. The critic network and action network are $\hat{\Lambda}(x) = \hat{W}_c^T \phi_c(Y_c x)$ and $\hat{v}(x) = \hat{W}_a^T \phi_a(Y_a x)$, where Y_c and Y_a are constant matrices. The activation functions in the critic network and the action network are hyperbolic tangent functions. The structures of critic network and action network are 4-4-1 and 4-8-2, respectively. The initial weights W_c and W_a are selected arbitrarily from $(-1, 1)$, respectively. The adaptive gains for the critic network and the action network are selected as $\alpha_c = \alpha_a = 0.1$. The initial state of system (39) is $[-1 + i, 1 - i]^T$. After 100 time steps, the simulation results are obtained in Figs. 1-2. The simulation results reveal that the proposed optimal controller can be applied to complex-valued nonlinear systems and obtain satisfying control performance.

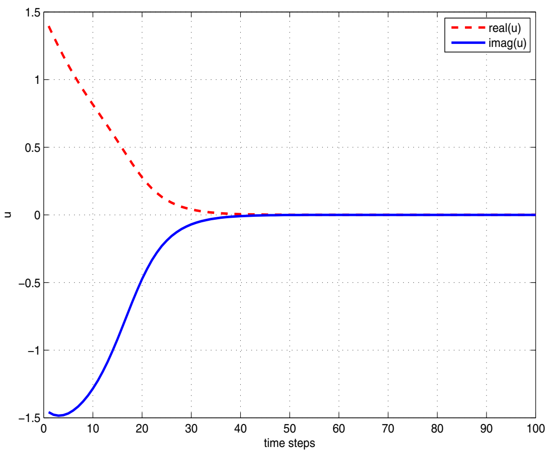

Fig. 1. The control trajectories of system (39).

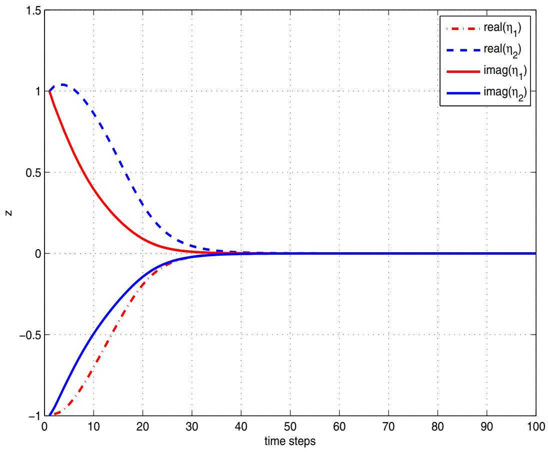

Fig. 2. The state trajectories of system (39).

VI. Conclusion

In this paper, the optimal control scheme based on ADP algorithm for complex-valued systems with input saturation has been established for the first time. First, the performance index function is defined. Then the equivalence transformations are used to obtain the corresponding real dynamic system and performance index function. Based on the transformed system, an ADP optimal control method is established. Finally, the simulation study is given to show the effectiveness of the proposed optimal control scheme.

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