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A Computationally Efficient Complete Area Coverage Algorithm for Intelligent Mobile Robot Navigation

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***Abstract*— Complete area coverage navigation (CAC) requires a special type of robot path planning, where the robots should visit every point of the state workspace. CAC is an essential issue for cleaning robots and many other robotic applications. Real-time complete area coverage path planning is desirable for efficient performance in many applications. In this paper, a novel vertical cell-decomposition (VCD) with convex hull (VCD-CH) approach is proposed for real-time CAC navigation of autonomous mobile robots. In this model, a vertical cell-decomposition (VCD) methodology and a spanning-tree based approach with convex hull are effectively integrated to plan a complete area coverage motion for autonomous mobile robot navigation. The computational complexity of this method with minimum trajectory length planned by a cleaning robot in the complete area coverage navigation with rectangle obstacles in the Euclidean space is $O(n \log n)$. The performance analysis, computational validation and comparison studies demonstrate that the proposal model is computational efficient, complete and robust.**

I. INTRODUCTION

COMPLETE area coverage (CAC) navigation requires the robot path to pass through every region of the workspace. In addition to cleaning robots [1]-[3], many other robotic applications also require CAC, e.g., vacuum robots [4], painter robots, autonomous underwater covering vehicles, demining robots [6], mine detectors [6], lawn mowers [7], automated harvesters [8], crop harvesting equipment, and window cleaners [9].

CAC motion planning of mobile robots has been extensively studied such as template-based methodology [3-4], distance transformation approaches [3-4, 13], potential field methodology [11], approximate cellular decomposition [12-13, 18,25], spanning tree method [14-15], neural network models [2,16-17], graph-based approaches [18], Depth-First Search [19], spatial cell diffusion (SCD) [20], sensor-based approaches [21-22], sweep-line approach [23-24], etc.

Oh *et al.* [3] and Oh *et al.* [4] developed a triangular cell decomposition approach for unknown environments; where triangular cell map representation enables the cleaning robot navigate along shorter and more flexible paths. The cleaning robot might move to eight potential directions in the rectangular or square cell map model, while the triangular map representation increases the navigational directions to twelve. Seven templates are proposed for coverage path planning. This method combines triangular cell decomposition a template-based approach and a wall-following navigation algorithm for CAC [4]. Pirzadeh and Snyder [11] proposed an indirect control strategy to deal with the coverage and search problem, where the complete coverage is accomplished using potential field methodology. The underlying idea of the algorithm is to discretize the workspace and the robot motion. The robot movement is designated with four orthogonal directions, up, down, left, and right, without considering any diagonal neighbors. Moravec and Elfes [12] initially proposed an approximate cellular decomposition model, where the workspace is decomposed into cells with the same size and shape. Zelinsky [13] developed another approximate cellular decomposition approach using a grid based complete coverage model. A distance transform algorithm is used to assign a specific number to each grid element, which is a function of the distance to the goal. The complete coverage is then achieved by a gradient descent rule. The generated path may include some unexpected turns. Recently, Gabriely and Rimon [14] proposed a spanning tree covering approach by subdividing the workspace into discrete cells and following a spanning tree of a graph induced by the cells. The robot is able to cover every point precisely once, and travel an optimal path in a grid-like representation in the workspace. Agmon *et al.* [15] successfully extended spanning tree based complete coverage oath planning for multi-robot system.

Neural network method is an efficient approach for robotics navigation. Yang and Luo [16] proposed a biologically inspired neural network approach for real-time complete coverage path planning with obstacle avoidance of a mobile robot and. The planned robot motion in a static environment is globally optimal although there is no explicit optimization of any global cost functions. The optimality of the real-time robot trajectory is planned through the dynamic activity landscape of the neural network without any prior knowledge of the dynamic environment, without explicitly searching over the free workspace or the collision paths, and without any learning procedures. Therefore, it is computationally efficient. Luo *et al.* [17] extended the neural dynamics model to coverage-type motion planning of an autonomous mobile robot and this approach is applied to solve vicinity problems of obstacles in complete coverage navigation. However, the neural network models described previously are only suitable for navigation in non-stationary

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environments without map building. Luo and Yang [2] recently developed heuristic algorithms based on a biologically inspired neural network model, which perform concurrently complete coverage navigation and map building under unknown environments.

Some researchers integrated two or more methodologies to take advantage of properties of two approaches. For instance, Mannadiar and Rekleitis [18] combined Reeb graph based method and Boustrophedon cellular decomposition for complete coverage path planning on free space, in which, cells to be covered as edges of the Reeb graph are encoded. Additionally, Chinese Postman Problem (CPP) is utilized to compute an Euler tour with optimal solution that ensures complete coverage of the available free space while minimizing the trajectory of the robot. However, their model is not computational efficient. Zuo *et al.* [19] suggested a hybrid system for complete coverage motion planning of an agricultural robot by combining Depth-First Search (DFS) and sub-region methods. The configuration space is decomposed into several sub-regions thus the agricultural robot is capable of cover every area in the order of sub-regions by performing Depth-First Search (DFS). The entire workspace is covered by the robot by visiting every sub-region. However, there are overlapping areas in their model. Ryu *et al.* [20] combined modeling method and spatial cell diffusion (SCD) search algorithm for complete area coverage of a mobile robot. The path planned by their model has overlapping areas. The entire environment to be navigated is not fulfilled completely by their model thus their coverage algorithm is incomplete. Choset *et al.* [21] suggested a sensor-based CCN approach for demining robots by combining sensor information with an exact cell decomposition. The approaches enable robots to simultaneously cover a cell and search critical points of the entire unknown environment. Acar *et al.* [22] combined their previous sensor-based work with a generalized Voronoi diagram approach. This sensor- based coverage algorithm guides the robot to pass over all points in vast workspaces. Park and Lee [23] proposed a CCN algorithm in unknown environments that are composed of three components: a sweeping algorithm, a point-to-point moving algorithm, and a corner work algorithm. This model builds an information bitmap and applies these three algorithms to generate coverage paths. The cleaning robots may overlap some areas and miss some corner areas. Huang [24] proposed a planar line sweep algorithm for complete coverage motion planning. The configuration space is initially decomposed into sub-regions. Afterward, the robot covers the entire environment by visiting every sub-region in turn. In the model, a minimal sum of altitudes (MSA) decomposition methodology is developed, in which the algorithm assigns a different sweep direction to each sub- region. The MSA decomposition is performed by multiple line sweeps and dynamic programming.

In this paper, a novel vertical cell-decomposition (VCD) with convex hull (VCD-CH) approach is proposed for real-time CAC navigation of autonomous mobile robots. The underlying strategy of this algorithm is that a vertical cell-decomposition (VCD) methodology and a spanning-tree based approach with convex hull are effectively integrated to plan a complete area coverage motion for autonomous mobile robot navigation [26]. Outperformance extracted from both algorithms makes this hybrid model computational efficient, complete and robust. The computational complexity of this method with minimum trajectory length planned by a cleaning robot in the complete area coverage navigation with rectangle obstacles in the Euclidean space is $O(n \log n)$. To the best of our knowledge, this is the fastest algorithm with minimum trajectory length addressed in the complete area coverage problem with rectangle obstacles, in comparison with other models. The computation complexity of the proposed algorithm is proved in the paper indicate the proposed hybrid model is energy-time-efficient.

The rest of this paper is organized as follows: Section II introduces convex hull and sweep line for CAC navigation algorithm. Section III derives CAC navigation algorithm based on vertical cell decomposition, sweep-line and spanning tree methodologies, illustration of the CAC algorithm of cleaning mobile robot navigation is presented in Section IV, Section V concludes the paper.

II. Convex Hull and Sweep Line for CAC

In computational geometry, a sweep line (SL) algorithm is a sort of efficient algorithm that a vertical line is swept or moved across the plane, stopping at some points in a graph [23-24]. This SL algorithm solves various problems in Euclidean space by making use of a conceptual sweep line or sweep surface. A vertical line, termed, *sweep-line*, sweeps from left to right through a bounded workspace populated with polygonal obstacles illustrated in Figure 1 [24]. In case of the sweep-line that intersects a vertex of a polygon, cells are constructed through a sequence of actions that the sweep-line encounters a vertex and moves away from the vertex (see Figure 1) [21-22].

The vertical cell decomposition is a path planning technique that the free space in configuration workspace is decomposed into cells (grids) so that the set of cells becomes the original free space in Figure 1(A). An adjacency graph is defined by the resulting cell decomposition graph that each cell is represented as a node in this graph whereas an edge of adjacent cells connects their corresponding nodes. The boundary of cells is dependent on geometric critically. The motion planning of a cleaning robot in this workspace is complete as the constructed cells are either fully free or completely occupied shown in Figure 1(B). The advantage of this methodology is that the motion planning of a cleaning robot depends only on the capability of the robot that it transverses from a free cell to another no matter where the robot is in a cell [21-22].

The major disadvantage of this vertical cell decomposition of motion planning is that the overall computational efficiency is dependent on the density and complexity of obstacles in the workspace. In other words, the amount of cells impacts on the computational efficiency of the motion planning of a cleaning robot.

Polygons are one of the most all-encompassing shapes in geometry consisting of simple triangle, squares, rectangles, trapezoids, and dodecagons [25]. A planar polygon is convex if it contains all the line segments connecting any pair of its points, in which its interior is a convex set. Thus, a regular pentagon is convex illustrated in Figure 2.

Convex hull is an important issue in computational geometry that has extensive of applications [26]. The *convex hull* of a set of points $\boldsymbol{S}$ in n dimensions is the intersection of all convex sets containing $\boldsymbol{S}$. For $\boldsymbol{N}$ points $q_1, q_2, \ldots, q_N$, the convex hull $\boldsymbol{C}$ is then given by the expression (1)

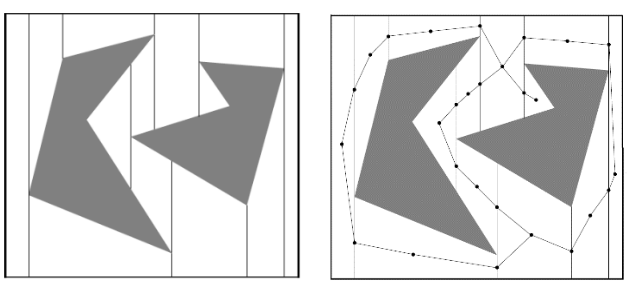
$$C = \sum_{i=1}^{N} \delta_i \, q_i : \delta_i \geq 0 \text{ for all } i \text{ and } \sum_{i=1}^{N} \delta_i = 1 \qquad (1)$$

The complete area coverage motion planning takes advantage of the convex polygon. In Figure 2, every internal angle in convex polygon is less than or equal to 180 degrees, whereas every line segment between two vertices remains inside or on the boundary of the polygon [25].

III. The Proposed CAC Algorithm

In this section, the hybrid CAC algorithm that integrates the vertical cell decomposition and convex hull methods is described and simulation study is shown. The configuration space is initially decomposed by vertical cell decomposition approach into sub-regions like cells. The adjacency graph is re-constructed by proposed new techniques. The CAC algorithm based on the vertical cell decomposition and convex hull approaches is depicted as follows.

Step 1:
Initialization of each critical point of obstacles in the configuration workspace;



(A) (B)

Fig. 1 The vertical cell decomposition. A: Sweep line for the cell decomposition; B: The possible planned robot trajectory in the roadmap [25].

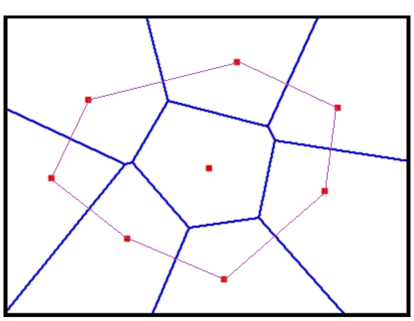


Fig. 2 Illustration of a convex hull and convex polygon

Step 2:
Via vertical cell decomposition method described previously, a road map of motion planning vertical cell decomposition is constructed;

Step 3:
The adjacency graph for the workspace is formed. Create the initial point of the mobile robot in the workspace of the cell decomposition;

Step 4:
Re-construct an adjacency graph $\boldsymbol{G}$ by deconstructing those vertices with degree >3. The set of vertices is defined as $\boldsymbol{V}$;

Step 5:
In the re-constructed adjacency graph $\boldsymbol{G}$, temporarily remove those vertices with degree = 1 from the set $\boldsymbol{V}$ to generate a new set $\boldsymbol{V}'$;

Step 6:
In the re-constructed adjacency graph $\boldsymbol{G}$, a maximal cycle graph $\boldsymbol{G}^C$ is constructed;

Step 6.1: A convex hull is created by set $\boldsymbol{V}'$;
Step 6.2: Obtain the cycle of graph $\boldsymbol{G}^C$ by the convex hull as large as possible.

Step 7:
Re-connect the vertex deleted in **Step 5** to the convex point in the convex hull to create a new $\boldsymbol{G}^T$ by spanning tree technique. A new graph $\boldsymbol{G}'$ is resulted by combining $\boldsymbol{G}^C$ and $\boldsymbol{G}^T$.

Step 8:
Complete area coverage (CAC) global motion planning: based on the graph $\boldsymbol{G}'$, a trajectory is generated by the clock-wise direction;

Step 9:
Complete area coverage (CAC) local (detailed) motion planning: in each cell, the CAC motion planning algorithm in Step 8 is applied to generate detailed complete coverage trajectory.

In this section, in Steps 4 and 6, with regard to the deconstruction algorithm on vertices, a new adjacency graph is constructed after the rectangles are divided into two portions bases on their focuses (degree/2) with the vertices with degree >3. According to various adjacency graphs, the CAC motion planning of a mobile robot is performed in different format accordingly.

IV. Illustration of the CAC Algorithm.

A detailed motion planning of a mobile robot in the adjacency graph $\boldsymbol{G}^C$ (degree = 2 and degree = 3) is illustrated in Figure 3 whereas a detailed motion planning of a mobile robot in the adjacency graph $\boldsymbol{G}^T$ (degree = 1, degree = 2 and degree = 3) is depicted in Figure 4. The mobile robot traverses along the zigzag patterned trajectory planned by the CAC algorithm in Figures 3 and 4. The adjacency graphs ***G'***, $\boldsymbol{G^C}$, and $\boldsymbol{G^T}$ are defined as follows.

$$G' = G_i^C \cup G_j^T \quad (2)$$

$$G^C = \{G_i^C | 0 \le i \le k_C,\ k_C \in N\} \quad (3)$$

$$G^T = \{G_j^T | 0 \le j \le k_T,\ k_T \in N\} \quad (4)$$

The CAC algorithm with several steps described above is depicted in Figure 5.

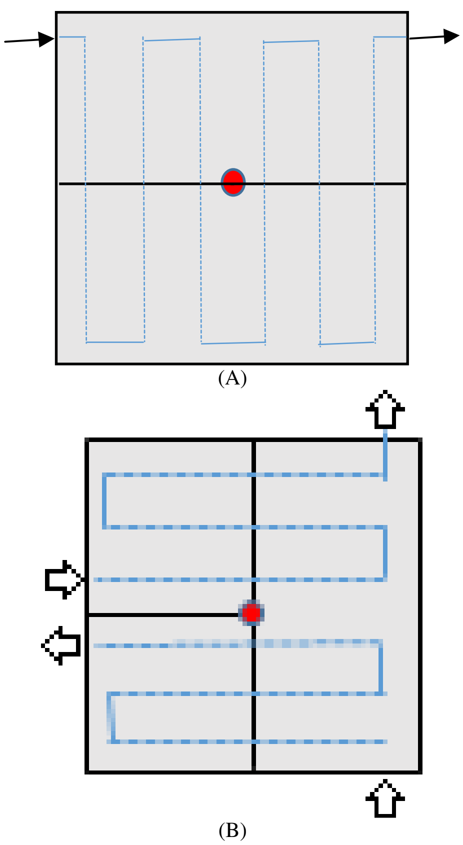


Fig. 3 Illustration of detailed motion planning of adjacency graph $\boldsymbol{G^C}$.
A: Degree = 2; B: Degree = 3.

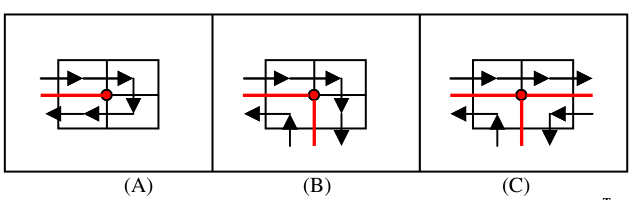


Fig. 4 Illustration of the detailed motion planning of adjacency graph $\boldsymbol{G^T}$.
A: Degree = 1; B: Degree = 2; C: Degree = 3.

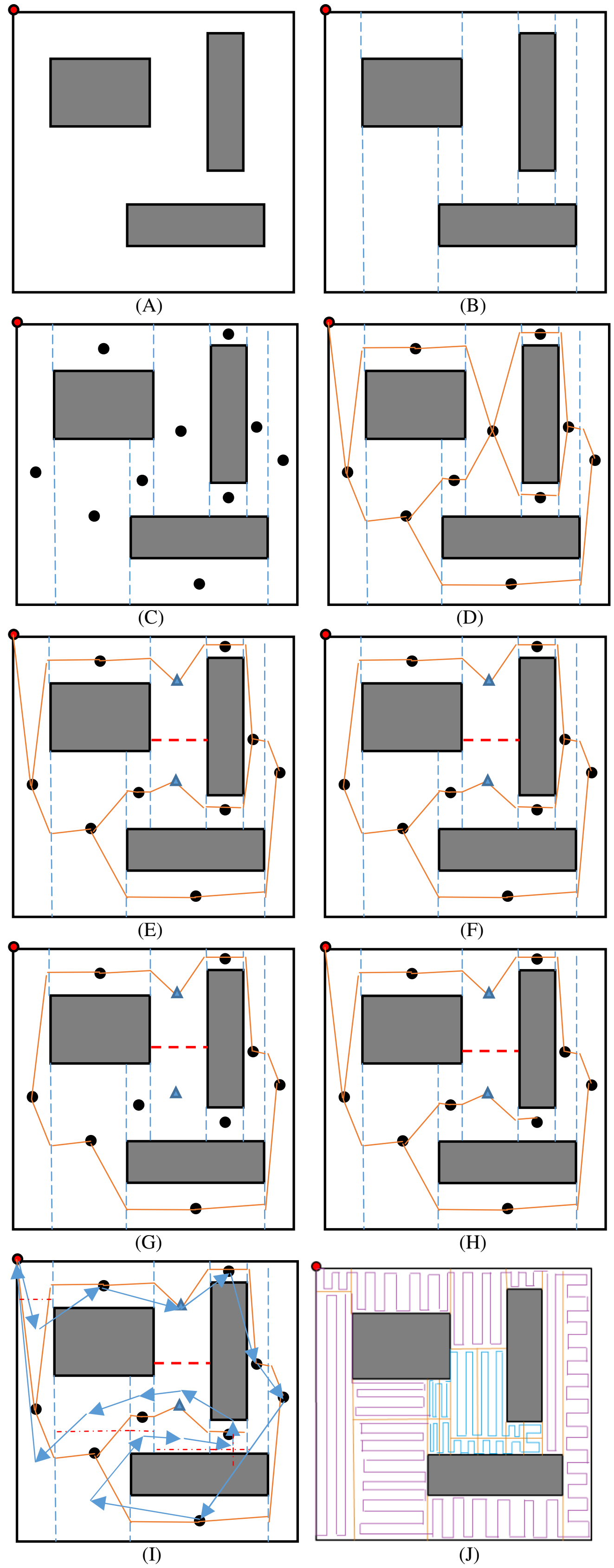


Fig. 5 Illustration of the CAC algorithm of cleaning mobile robot navigation. (A): Initialization; (B): Construction of the vertical cell decomposition; (C): Addition of the focus of cell; (D): Generation of the roadmap; (E): Deconstruction of vertices with degree >3; (F): Delete the vertices with degree =1; (G):Construction of the convex hull; (H): Connection of the rest

vertices to the nearest convex point; (I): Global motion planning by CAC; (J): Detailed motion planning by CAC

The CAC algorithm is then applied for a mobile robot simulation that is illustrated in Figure 5(J). In the configuration space of a mobile robot, there are three rectangular-shaped obstacles populated in the workspace. The robot starts at the red point to plan a zigzag back-and-forth trajectory to reach the low-right corner that is the target point. In the simulation result, the mobile robot is capable of planning a collision-free trajectory to cover every point.

TABLE I
COMPARISON OF VARIOUS ALGORITHMS IN COMPLETE COVERAGE NAVIGATION OF A MOBILE ROBOT

Algorithms	Gabriely [14]	Mannadiar [18]	Zuo [18]	Ryu [19]	Ours
Time complexity	*N*	Postman problem (NP-hard)	NA	*N*	*n* log *n*
W/O concave polygons	Without	With	NA	NA	Without
Complete/ Approximate	Approximate	Complete	Complete	Approximate	Complete
Overlap/ Non-overlap	Non-overlap	Non-overlap	Over-lap	Over-lap	Non-overlap
Return to the starting point	Yes	Yes	No	No	Yes
Shortest length	NA	Yes	No	No	Yes
Turn penalty	Under ave.	Ave.	Above ave.	Ave.	Ave.

V. CONCLUSION

In this paper, a novel vertical cell-decomposition (VCD) with convex hull (VCD-CH) approach is proposed for real-time CAC navigation of an autonomous mobile robot. A hybrid model is developed that integrates a vertical cell-decomposition (VCD) methodology and a spanning-tree based approach with convex hull. Two methodologies are effectively integrated to plan a complete area coverage motion for autonomous mobile robot navigation. The computational complexity of this method with minimum trajectory length planned by a cleaning robot in the complete area coverage navigation with rectangle obstacles in the Euclidean space is $O(n \log n)$. The performance analysis, computational validation and comparison studies demonstrate that the proposal model is computational efficient, complete and robust. Some properties are worth mentioning about the proposed CAC approach, which outperforms other models: (1) The entire configuration space is covered *completely*; (2) The uncovered areas are visited without any overlapping area. (3) The mobile robot driven by the proposed CAC algorithm moves back to the initial position with energy-time-efficient mode. (4) To the best of our knowledge, this is the fastest algorithm with minimum trajectory length in complete coverage path planning of a mobile robot with rectangle obstacles, in comparison with other models.

REFERENCES

[1] G. Lawitzky, "A navigation system for cleaning robots," *Autonomous Robots*, vol. 9, 2000, pp. 255–260.

[2] C. Luo and S. X. Yang, "A bioinspired neural network for real-time concurrent map building and complete coverage robot navigation in unknown environments," *IEEE Trans. on Neural Networks*, vol. 19, no. 7, 2008, pp. 1279–1298.

[3] J. S. Oh, J. B. Park, and Y. H. Choi, "Complete coverage navigation of clean robot based on triangular cell map," in *Proc .of IEEE Intl. Symp. on Industrial Electronics*, Pusan, Korea, 2001, pp. 2089–2093.

[4] J. S. Oh, Y. H. Choi, J. B. Park, and Y. F. Zheng, "Complete coverage navigation of cleaning robots using triangular-cell-based map," *IEEE Trans. on Ind. Elec.*, vol. 51, no. 3, 2004, pp.718–726.

[5] F. Yasutomi, D. Takaoka, M. Yamada, and K. Tsukamoto, "Cleaning robot control," in *Proc. of IEEE Intl. Conf. on Robotics and Automation*, Philadelphia, USA, 1988, pp. 1839–1841.

[6] D. W. Gage, "Randomized search strategies with imperfect sensors," in *Proc. of SPIE, Mobile Robots VIII - The Intl. Society for Optical Engineering*, Boston, USA, 1994, pp. 270–279.

[7] H. Najjaran and N. Kircanski, "Path planning for a terrain scanner robot," in *Proc. of the 31st Intl. Symp. on Robotics*, Montreal, Canada, 2000, pp. 132–137.

[8] M. Ollis and A. Stentz, "Vision-based perception for an automated harvester," in *Proc. of IEEE/RSJ Intl. Conf. on Intelligent Robot and Systems*, Grenoble, France, 1997, pp. 1838–1844.

[9] M. Ollis and A. Stentz, "First results in vision-based crop line tracking," in *Proc. of IEEE Intl. Conf. on Robotics and Automation*, Minneapolis, USA, 1996, pp. 951–956.

[10] M. Farsi, K. Ratcliff, P. J. Johnson, C. R. Allen, K. Z. Karam, and R. Pawson, "Robot control system for window cleaning," in *Proc. of 11th Intl. Symp. on Automation and Robotics in Construction*, Brighton, UK, 1994, pp. 617–623.

[11] A. Pirzadeh and W. Snyder, "A unified solution to coverage and search in explored and unexplored terrains using indirect control," in *Proc. IEEE Int. Conf. Robotics Automation*, Raleigh, NC, 1990, pp. 2113–2119.

[12] H. P. Moravec and A. Elfes, "High resolution maps from wide angle sonar," in *Proc. IEEE Int. Conf. Robotics Automation*, St. Louis, MO, 1985, pp. 116–121.

[13] A. Zelinsky, R. A. Jarvis, J. C. Byarne, and S. Yuta, "Planning paths of complete coverage of an unstructured environment by a mobile robot," in *Proc. IEEE Int. Conf. Robotics Automation*, Tokyo, Japan, 1993, pp. 533–538.

[14] Y. Gabriely and E. Rimon, "Spanning-tree based coverage of continuous areas by a mobile robot," *Ann. Math. Artificial Intelligence*, vol. 31, no. 1–4, pp. 77–98, 2001.

[15] N. Agmon, N. Hazon, and G. A. Kaminka, "Constructing spanning trees for efficient multi-robot coverage," in Proc. of IEEE Intl. Conf. on Robotics and Automation, Orlando, USA, 2006, pp. 1698–1703.

[16] S. X. Yang, and C. Luo "A neural network approach to complete coverage path planning", IEEE Transactions on Systems, Man, and Cybernetics, Part B, 34(1), pp. 718-725, 2004.

[17] C. Luo, S. X. Yang, D. A. Stacey, and J. C. Jofriet, "A solution to vicinity problem of obstacles in complete coverage path planning", Proc. of IEEE Intl. Conf. on Robotics and Automation (ICRA). Washington D.C., USA, May 11-15, pp. 612-617, 2002.

[18] R. Mannadiar and A. I. Rekleitis, "Optimal Coverage of a Known Arbitrary Environment," in Proc. of IEEE/RSJ Intl. Conf. on Intelligent Robots and Automation, 2010, pp. 5525–5530.

[19] G. Zuo, P. Zhang, and J. Qiao, "Path planning algorithm based on sub-region for agricultural robot," in *Proc. of the 2nd international Asia conf. on Informatics in control*, automation and robotics, vol. 2, 2010, pp. 197–200.

[20] S. W. Ryu, Y. H. Lee, T. Y. Kuc, S. H. Ji, and Y. S. Moon, "A Search and Coverage Algorithm for Mobile Robot," in *Proc. of the 2nd International Conf. on Ubiquitous Robots and Ambient Intelligence*, Incheon, Korea, Nov. 23–26, 2011.

[21] H. M. Choset, E. U. Acar, A. A. Rizzi, and J. E. Luntz, "Sensor-based planning: Exact cellular decompositions in terms of critical points," in *Proc. SPIE—Int. Soc. Opt. Eng.*, Boston, MA, 2001, pp. 204–215.

[22] E. U. Acar, H. Choset, and P. N. Atkar, "Complete sensor-based coverage with extended-range detectors: A hierarchical decomposition in terms of critical points and Voronoi diagrams," *in Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Maui, HI, 2001, pp. 1305–1311.

[23] J. Y. Park and K. D. Lee, "A study on the cleaning algorithm for autonomous mobile robot under the unknown environment," in *Proc.*

6^{th} *IEEE Int. Workshop Robot Human Commun*., Sendai, Japan, 1997, pp. 70-75.
[24] W. H. Huang, "Optimal line-sweep-based decompositions for coverage algorithms", in *Proc. IEEE International Conference on Robotics and Automation*, pp. 27 - 32 2001.
[25] H. Choset, "Coverage for robotics - a survey of recent results," *Annals of Mathematics and Artificial Intelligence*, vol. 31, 2001, pp. 113–126.
[26] B. Chazelle, "Approximation and decomposition of shapes," in J. T. Schwartz and C.-K. Yap, Editors, Advances in Robotics 1: Algorithmic and Geometric Aspects of Robotics, Lawrence Erlbaum Associates, Hillsdale, NJ, 1987, pp. 145-185.
[27] C. O'Dunlaing and C. K. Yap., "A retraction method for planning the motion of a disc," *Journal of Algorithms*, 6:104–111, 1982.