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Solar Radiation Forecasting under Asymmetric Cost Functions

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***Abstract*—Grid operators are tasked to balance the electric grid such that generation equals load. In recent years renewable energy sources have become more popular since they are both clean and sustainable. Because of intermittency of renewable energy sources like wind and solar, the operators are required to predict renewable generation and allocate some operating reserves to mitigate errors. If they overestimate the renewable generation during scheduling, they do not have enough generation available during operation. So overestimation of resources create a more serious problem than underestimation. However, many researchers who study the solar radiation forecasting problem evaluate their methods using symmetric criteria like root mean square error (RMSE) or mean absolute error (MAE). In this paper, we investigate solar radiation forecasting under LinLin and LinEx which are asymmetric cost functions that are better fitted to the grid operator problem. We formulate the problem as an optimization problem and we use linear programming and steepest descent algorithm to find the solution. Simulation results show substantial cost saving using these methods.**

I. Introduction

Balance of load and generation is necessary for the electric grid. Independent system operators (ISO) at each hour estimate the loads and schedule for generation of conventional power plants. Integration of renewable generation to the grid have been increasing because renewable energy sources are both clean and sustainable. However, renewable generation like solar and wind are intermittent and change with time. For this reason grid operators need to predict the intermittent generation as well as load. To mitigate forecasting errors they allocate some operating reserves to ensure that during operation, generation always meets load.

Underestimation means that true renewable generation during operation time is more than what the ISO scheduled for it. So the scheduled generation is more than load. In this case during operation by automatic generation control (AGC) the desired generation of conventional power plants is decreased such that generation equals load [1].

On the other hand, overestimation means that true renewable generation during operation time is less than what the ISO scheduled for it. So the scheduled generation is not enough to meet load. Small overestimation errors could be compensated by using of operating reserves, but for larger errors the ISO is forced to decrease the load to keep the balance between load and generation. The act of disconnecting customers power to keep the stability of grid is called load shedding. Unscheduled load shedding is very undesirable for customers and it must be avoided as much as possible [2].

As a result, in case of overestimation of generation, the ISO may encounter shortages of generation and may be forced to do load shedding, however, in case of underestimation of generation, they can curtail the excess power. So the overestimation is more serious than underestimation. Therefore the solar and wind generation forecasting problem in the ISOs' view is not symmetric.

There has been a lot of research on solar radiation forecasting using different methods based on statistical time series methods like autoregressive (AR) [3] and auto-regressive with moving average (ARMA) [4] or artificial intelligence techniques such as neural networks [5],[6] and recurrent neural networks[7],[8] . However, they tried to minimize the root mean square error (RMSE) or relative root mean square error (rRMSE) in their methods [9] which is symmetric for both underestimation and overestimation.

Previous researchers studied the forecast value of solar radiation based on market price in California [10], similarly in [11] cost of wind generation prediction errors in electricity market was analyzed. Holttinen discussed handling of wind power forecast errors in the Nordic power market [12]. The value of wind forecasting was also studied in [13].

There are many factors in determining market price that help the system operates economically and increases the revenue of both producers and customers. However, market price is not always available in electric systems managed by utility monopoly. For example in Hawaii, in Oahu, Maui and the Big island only one company is responsible for electricity and all ancillary services. For this reason we use insight from economics and market intuition which leads us to an asymmetric cost function. Insufficient generation is very costly for the utility monopoly, similarly in a real market when available generation is scarce the price surges to a very high value. On the other hand, in case of abundant generation the price only decreases to marginal cost of more efficient generators for both market and the utility monopoly.

In 1969, Granger mentioned that in many practical problems in economics the cost function is asymmetric. He introduced LinLin function as an asymmetric linear function and suggest a useful although sub-optimal way for considering asymmetry by adding a constant bias value to the predictor [14]. The LinLin loss function is the simplest asymmetric cost function we can use to distinguish between overestimation and underestimation, but per unit cost does not depend on magnitude of error.

The second popular asymmetric loss function is LinEx

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which was originally introduced for real estate assessment [15] and comprehensively discussed by Zellner. Many other applications require the use of asymmetric cost function. For example in dam construction underestimation of peak water level is more serious than overestimation[16]. In estimation of average life of the components of a spaceship, overestimation is usually more serious than underestimation [17]. In this study we also have that overestimation of renewable generation is more serious than underestimation.

In this paper we consider both LinLin and LinEx cost functions as more suitable functions for the utility problem. Since solar radiation is a non-stationary process (i.e. its mean and variance change with time) we show that a biased forecasting method which consider loss function gives better result than biasing an unbiased forecast. However, we used both biased forecasting and biasing the unbiased forecast to emphasize the importance of using biased forecasting in this problem.

The rest of the paper is organized as follows. In section II, the forecasting problem is formulated based on both LinLin and LinEx cost functions. Section III discusses suboptimal solution by adding constant bias value to unbiased prediction and an optimal solution using direct optimization for both LinLin and LinEx cost functions. To find optimal solution, we used linear programming for LinLin and steepest descent algorithm for LinEx . Section IV is dedicated to the simulation results and discussion. Summary of results and conclusion is presented in section V.

II. Problem statement

Our objective is to minimize expected loss by adjusting forecasting hypothesis parameters. Let the actual solar radiation at time n be x_n and corresponding forecast be $\hat{x}_n$. We are interested in k step ahead forecasting using a window of past observations.

$$X_n = [x_n, x_{n-1}, ..., x_{n-m+1}]^T$$

where m is window size.

Let us assume that k step ahead forecast is a function of past observations

$$\hat{x}_{n+k} = h(X_n)$$

$$\min_h \sum_{i=1}^{M} Loss(h(X_i) - x_{i+k})$$

Where $Loss$ is loss function either LinLin or LinEx and M is total number of samples.

Here we use forecasting method using zenith angle which was introduced in [21]. So our hypothesis h is linear combination of past data converted to the time of prediction:

$$\hat{x}_{n+k} = (\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + ... + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) \quad (1)$$

where $\theta_z(n)$ is solar zenith angle at time n and $\alpha_0, \alpha_1, ..., \alpha_m$ are the adjusting parameters.

If the ISO ignores all intermittent generation and schedules for the grid, there are enough operating reserves at any time. However, those reserves cost about 20% of per unit price of energy (i.e. in Hawaii about \$0.06/kWh). So forecasting of intermittent generation is useful to avoid that cost. On the other hand if the generation is overestimated, they may encounter shortage of generation and are forced to do load shedding. The ISOs consider load shedding cost very expensive. The value of lost load (VOLL) due to load shedding is different for various cities and reported around \$8/kWh to \$24/kWh [18][19][20]. For this study we assume VOLL to be \$10/kWh.

Let ϵ be forecast error given by

$$\epsilon_{n+k} = \hat{x}_{n+k} - x_{n+k}$$

So overestimation which means the predicted value exceeds the actual value, corresponds to a positive error and underestimation which means the forecasted value is less than actual generation, corresponds to a negative error. So the asymmetric trade off between underestimation (\$0.06/kWh loss of revenue) and overestimation (\$10/kWh penalty fee) lead to a LinLin loss function.

$$LinLin(\epsilon) = \begin{cases} C_1\epsilon & \text{if } \epsilon > 0, \quad C_1 \approx 10\$/kWh \\ -C_2\epsilon & \text{if } \epsilon \le 0, \quad C_2 \approx 0.06\$/kWh \end{cases} \quad (2)$$

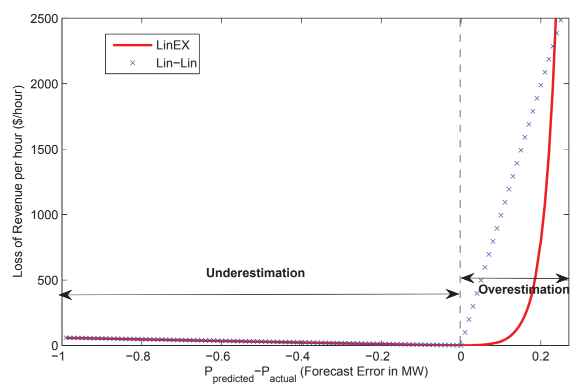

Fig. 1. Lin-Lin ($C_1 = 10\$/kWh, C_2 = 0.06\$/kWh$)and Linex ($b = 2, a = 0.03\$/kWh$) cost functions

The system usually is robust so that it can tolerate small errors, hence we assume the load shedding cost is exponentially distributed among errors in the way that small errors pay less penalty fee but larger errors pay a more expensive penalty fee. In this case we have the LinEx loss function given by

$$LinEx(\epsilon) = b(e^{a\epsilon} - a\epsilon - 1) \quad (3)$$

In Fig. 1 the two cost functions in equation (2) and (3) are shown.

III. Methods

In the previous section we formulated the problem as a minimization problem based on loss function. If the loss function is symmetric like squared error, there is no difference between negative and positive errors, so the predictor is unbiased and mean of errors equals zero. The least square predictor is a popular unbiased predictor which has analytical solution and is extensively used in many applications. The bias is a constant value added to unbiased predictor to compensate effect of asymmetry in loss function. The biased forecast means that we considered asymmetric cost function at the beginning and directly solved the optimization problem.

This section is divided into two subsections. Subsection A is devoted to solutions for LinLin cost functions. We find a suboptimal solution by selection of optimal bias value added to unbiased forecast to compensate effect of asymmetry in loss function. The optimal solution is also given by direct optimization using linear programming. Subsection B uses LinEx and both suboptimal and optimal solution are discussed.

A. Lin-Lin cost function

Adding bias to unbiased forecast: Let our unbiased forecast error be ϵ and cumulative distribution function (CDF) of error be F_ϵ and probability density function of errors be $f(\epsilon)$. If we add bias value β to the unbiased forecast, the error also add with the β so cumulative loss with LinLin cost function becomes:

$$Loss_{total} = \int_{-\infty}^{+\infty} LinLin(\epsilon+\beta) f(\epsilon) d\epsilon$$
$$= -C_2 \int_{-\infty}^{-\beta} (\epsilon+\beta) f(\epsilon) d\epsilon + C_1 \int_{-\beta}^{+\infty} (\epsilon+\beta) f(\epsilon) d\epsilon$$

For unbiased forecast mean of errors equal to zero so

$$\int_{-\infty}^{-\beta} \epsilon f(\epsilon) d\epsilon = -\int_{-\beta}^{+\infty} \epsilon f(\epsilon) d\epsilon$$

$$Loss_{total} = -(C_1+C_2)(\beta F_\epsilon(-\beta) + \int_{-\infty}^{-\beta} \epsilon f(\epsilon) d\epsilon) + C_1\beta$$

To find bias value β which minimizes cumulative loss, we have:

$$\frac{\partial Loss_{total}}{\partial \beta} = -(C_1+C_2) F_\epsilon(-\beta) + C_1$$
$$\Rightarrow \beta = -F_\epsilon^{-1}\left(\frac{C_1}{C_1+C_2}\right)$$

Direct biased forecasting: Our objective is

$$\min_{\alpha_0,\alpha_1,\ldots,\alpha_m} \sum_{n=1}^{M} LinLin(\hat{x}_{n+k} - x_{n+k})$$

where $\hat{x}_{n+k}$ computed using equation (1).

The LinLin loss function could be expressed by

$$LinLin(\epsilon) = \lambda_1 |\epsilon| + \lambda_2 \epsilon$$

So we have

$$\min_{\alpha_0,\alpha_1,,\ldots,\alpha_m} \sum_{n=1}^{M} \{\lambda_1 |(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k}| + \lambda_2 [(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k}]\}$$

In order to get rid of absolute value segment, let us introduce new decision variables such that

$$|(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k}| \le w_n$$

So we have

$$\min_{\substack{w_1, w_2, \ldots, w_M \\ \alpha_0, \alpha_1, \ldots, \alpha_m}} \sum_{n=1}^{M} \{\lambda_1 w_n + \lambda_2 [(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k}]\}$$

subject to $\forall n$

$$w_n \ge 0$$
$$(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k} \le w_n$$
$$(\alpha_0 + \frac{\alpha_1 x_n}{\cos\theta_z(n)} + \ldots + \frac{\alpha_m x_{n-m+1}}{\cos\theta_z(n-m+1)}) \cos\theta_z(n+k) - x_{n+k} \ge -w_n$$

which is a linear programming problem.

B. LinEx cost function

Adding bias to unbiased forecast: Again let our unbiased forecast error be ϵ and probability density function of errors be $f(\epsilon)$. If we add bias value β to the unbiased forecast, the error also add with the β so cumulative loss with LinEx cost function becomes:

$$Loss_{total} = \int_{-\infty}^{+\infty} LinEx(\epsilon+\beta) f(\epsilon) d\epsilon$$
$$= b \int_{-\infty}^{+\infty} (e^{a(\epsilon+\beta)} - a(\epsilon+\beta) - 1) f(\epsilon) d\epsilon$$

To find optimal bias value β which minimizes cumulative loss, we have:

$$\frac{\partial Loss_{total}}{\partial \beta} = abe^{a\beta} \int_{-\infty}^{+\infty} e^{a\epsilon} f(\epsilon) d\epsilon - ab$$
$$\Rightarrow \beta = -\frac{1}{a} log\left(\int_{-\infty}^{+\infty} e^{a\epsilon} f(\epsilon) d\epsilon\right)$$

similar to Zellner's suggestion [16] :

$$\beta = -\frac{1}{a} log(E_\epsilon e^{a\epsilon})$$

Direct biased forecasting: Here again we use forecasting method using zenith angle as in equation(1), however for selection of parameters we consider LinEx cost function. We want adjust $\alpha_0, \alpha_1, ..., \alpha_m$ such that the following objective function be minimized.

$$J = \sum_{n=1}^{M} LinEx(\hat{x}_{n+k} - x_{n+k})$$

where $\hat{x}_{n+k}$ computed using equation (1). Since this optimization does not have analytical answer we used steepest descent algorithm. Let $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, ...\alpha_m]^T$ then the $\boldsymbol{\alpha}$ iteratively updated by following equation.

$$\boldsymbol{\alpha}_{i+1} = \boldsymbol{\alpha}_i - \eta \nabla J$$

where η is step size and ∇J is gradient vector and is computed by following equations

$$\nabla J = [\frac{\partial J}{\partial \alpha_0}, \frac{\partial J}{\partial \alpha_1}, ..., \frac{\partial J}{\partial \alpha_m}]^T$$

$$\frac{\partial J}{\partial \alpha_0} = ab \sum_{n=1}^{M} [\cos \theta_z(n+k)(e^{a(\hat{x}_{n+k} - x_{n+k})} - 1)]$$

similarly for $j = 0, 1, 2, ..., m-1$

$$\frac{\partial J}{\partial \alpha_{j+1}} = ab \sum_{n=1}^{M} [\frac{x_{n-j} \cos \theta_z(n+k)}{\cos \theta_z(n-j)} (e^{a(\hat{x}_{n+k} - x_{n+k})} - 1)]$$

IV. Simulation Results

For simulation, we downloaded solar irradiation of several sites from http://www.nrel.gov/midc/. The name and specification of sites are shown in table 1. Resolution of the original data for LaOla and Los Angeles is one minute and for Elizabeth City is five minutes. The data removed night hours and low irradiation times in the morning and the evening . So only nine hours per day is considered.

Hawaii La Ola Lanai	Latitude:20.76685 N Longitude:156.92291 W Elevation: 381 meters AMSL	1/1/2010 to 12/31/2011
North Carolina Elizabeth City	Latitude:36.28 N Longitude:76.22 W Elevation: 26 meters AMSL	1/1/2005 to 12/31/2013
California Los Angeles	Latitude:33.966674 N Longitude:118.42282 W 27 meters AMSL	1/1/2011 to 12/31/2013

The revenue is the annual cost which is avoided by using forecasting method (i.e. annual cost without using forecast minus annual cost using the forecasting method). We use revenue of perfect forecast as a base line of per unit revenue. So the per unit revenue is the revenue of forecasting method divided by maximum possible revenue (perfect forecast).

To compare the benefit of forecasting using both biasing unbiased forecast and biased forecasting we used one year data for training to forecast an hour ahead and the next year for validation. The Average annual revenue for LinLin cost function is shown in Fig. 2 and for LinEx loss function is shown in Fig. 3. As it is clear from the figures in both cases revenue of biased forecasting is significantly more than revenue from adding bias to an unbiased prediction.

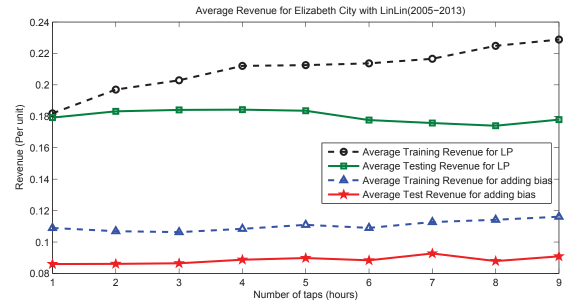

Fig. 2. The linear programming method have more advantage over adding bias to unbiased forecast.(LinLin loss function)

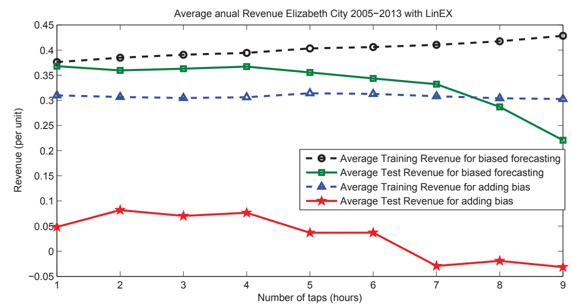

Fig. 3. Direct biased forecasting method have more benefit than adding bias to unbiased forecast.(LinEx loss function)

For simulation of LinLin loss function, one year of data is used for training and the next year for testing. The graphs in Fig. 2 are results of averaging per unit revenue of 2005-2013 of Elizabeth City data set. The black dashed line is used for training revenue for biased forecasting using linear programming and the blue dashed line is training revenue of adding bias to the unbiased forecast. The green and red solid lines are respectively used for validation of biased forecasting and adding bias to the unbiased forecast. Although increasing number of taps improves the training revenue for linear programming method, the test revenue decreases for more than four taps due to over fitting. So by using linear programming approach we can reach to about 18% of maximum possible benefit which is around twice as good as the 9% achieved from adding bias to unbiased forecast.

For simulation of LinEx loss function, again one year of data is used for training and the next year for testing. Fig. 3 shows averaging per unit revenue of 2005-2013 of Elizabeth City. While training revenue slightly improves by increasing number of taps, the validation revenue decreases for more than one tap.

Because of large difference between test and train revenue specially for adding bias to unbiased forecast, we find out that one year data is not sufficient for training. So by using cross validation technique, we used eight years for training and one other year for testing. The results of the cross validation are shown in Fig. 4. In this way there is fair agreement

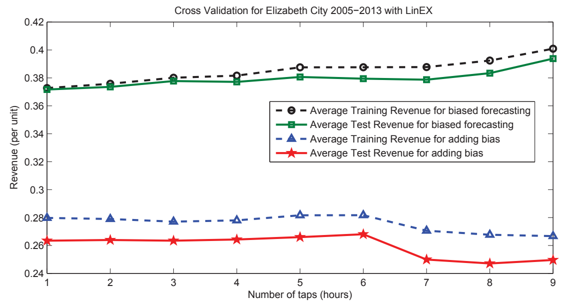

Fig. 4. More training data gives better agreement between training and testing results (nine fold cross validation)

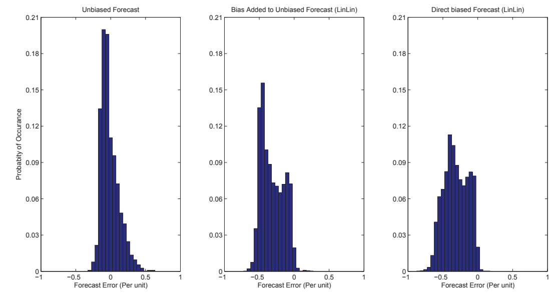

Fig. 5. Histograms of forecasting Errors for for three different scenarios for LinLin loss function (unbiased, adding bias to unbiased, and linear programming)

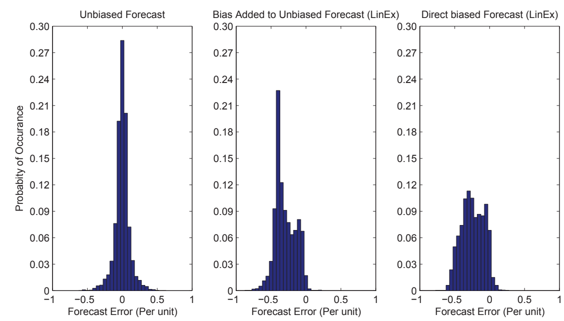

Fig. 6. Histograms of forecasting Errors for for three different scenarios for LinEx loss function (unbiased, adding bias to unbiased, and direct biased forecasting)

between training and test. We have achieved about 38% of maximum possible revenue using direct biased forecasting which is significantly more than 26% achievement of adding bias to unbiased forecast.

It worth noting that LinEx loss have less penalty for small errors which gives us opportunity to use intermittent generation more efficiently; on the other hand by having huge penalty for larger errors prevents from serious problems that may lead to load shedding hence we have more stable grid operation.

Distribution of errors for LinLin shown in Fig. 5. For unbiased forecast we have many positive errors as well as negative errors since there is no difference between positive and negative errors, also a large portion of errors are located around -0.12. By adding bias (negative number) to unbiased forecast total graph shifted to the left since underestimation have less cost than overestimation. So the large portion which was located around -0.12, is located around -0.45 now. On the other hand, linear programming method shifted errors to the left so that small portion of errors are positive; at the same time by effective use of input features errors deviated from zero less.

Again in Fig. 6 distribution of errors for unbiased forecasting are symmetric and large portion of errors are located around zero. By adding bias (negative number) to unbiased forecast total graph shifted to the left since underestimation have less cost than overestimation, however the large portion which was located around zero, is located around -0.4 now. On the other hand, direct biased forecasting shifted errors to the left so that small portion of errors are positive; at the same time by effective use of input features errors deviated from zero less.

V. CONCLUSION

While many researchers studied the problem of forecasting of solar radiation, they evaluated their methods using symmetric criteria like root mean square error(RMSE) or mean absolute error (MAE). However, grid operators have more concern about shortage of production rather than its abundance, i.e. overestimation of resources have more serious problem than underestimation. So in ISO's view the cost function is not symmetric. For this reason we discussed solar radiation forecasting under Lin-Lin and LinEx as asymmetric cost functions which are better fitted to the grid operator problem. For each of these loss functions we used two scenarios i.e. adding bias to unbiased forecast or formulating biased forecasting which consider the loss function at the beginning. Under LinLin loss the forecasting is formulated as linear programming and for LinEx loss we formulated the problem as convex optimization and solved by steepest descent algorithm. Our simulations showed that direct biased forecasting have significantly more advantage. The methods used are batch algorithms and on-line biased forecasting methods are also interesting and left for future research.

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