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Compressive Direction-of-Arrival Estimation via Regularized Multiple Measurement FOCUSS Algorithm

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***Abstract*—The recently developed Compressed Sensing (CS) theory has made the super-resolution of spectrum estimation possible. In this paper, we exploit the joint sparsity of received signals to develop a new Compressive Direction-of-Arrival Estimation approach via a new Regularized Multiple Measurment FOCal Underdetermined System Solver (RMM-FOCUSS) Algorithm. It can overcome the resolution limitation of traditional spatial energy spectrum estimation algorithm, such as MUSIC algorithm, and present more accurate estimation of direction of multiple sources when there are a few numbers of antenna units. Some experiments are taken to validate the performance of our proposed method.**

Keywords—compressed sensing; DOA estimation; array signal processing

I. Introduction

Direction-of-arrival (DOA) estimation is an important issue in the field of array signal processing [1]. Several high resolution technologies have been advanced for more accurate DOA estimation, including MUSIC [2] and MVDR [3] algorithm. MUSIC algorithm is characteristic of accurate estimation when there are a large number of samples, however, it will suffer from high computational complexity because it needs to calculate the eigen-decomposition of a large matrix. MVDR algorithm assumes that the gain in the steering direction is unity, and minimize the output power to estimate the directions of sources. However, it is very sensitive to the array noises and the uncertainty in the look direction. Some variants of these algorithms have been proposed to improve these classical algorithms, however, they are all limited to the Nyquist's rate, that is, the spatial resolution is decided by the number of antenna units.

The information level of radio signals is often far lower than the actual bandwidth. So Compressive Sensing (CS) can be used for lower rate sampling of signals [4], which states that a compressible signal that has a sparse representation in some dictionary can be recovered from a small number of linear measurements. Compressed sensing is a new emerging theoretical framework for signal acquiration and processing. The past decade has witnessed prosperity in it. Two conditions should be satisfied before performing a compressive sensing of the signal. The first is the signal is compressible, that is, the signal can be sparsely represented under some dictionary. The second condition is the compressive measurement matrix should be incoherent with the dictionary. As soon as the two conditions are satisfied, and there are sufficient number of compressive measurements, one can recover signals from a few compressive measurements of signals.

With the rapid development of CS, it has successfully used in array signal processing, including the direction-of-arrival estimation, beamforming and so on. The sensed signals in the array has sparsity, so can be used for compressive and super-resolution DOA estimation when there are a limited number of antenna units. Many new methods based on CS have been proposed and show better performance than traditional ones. In [5], a focal underdetermined system solver (FOCUSS) was proposed for obtaining signal measurement sparse solutions so that we could have access to multiple measurement vectors with sparsity. In [6], CS algorithms including CS-BF, CS-MUSIC and CS-RMUSIC are presented. [7] proposed three novel DOA models including covariance matrix CS, interpolated array CS and beam space CS. In this paper, we exploit the joint sparsity of received signals to develop a new Compressive Direction-of-Arrival Estimation approach via a new Regularized Multiple Measurement FOCUSS (RMM-FOCUSS) Algorithm. It can overcome the resolution limitation of traditional spatial energy spectrum estimation algorithm, such as MUSIC algorithm, and present more accurate estimation of direction of multiple sources when there are a few numbers of antenna units. Some experiments are taken to validate the performance of our proposed method.

II. Signal Model and Our Method

In this section we consider a field consisting of a linear array of M sensors and p sources. Each sensor receives a superposition of the time-domain source signals,

$$\begin{cases} s_j(t) = u_j(t)\, e^{j(\omega_0 t + \varphi_j(t))} \\ s(t-\tau) = u_j(t-\tau)\, e^{j(\omega_0(t-\tau) + \varphi_j(t-\tau))} \end{cases} \tag{1}$$

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where $u_j(t)$ is the amplitude of the j-th received signal, and φ_j is the phase of the j-th received signal, ω_0 is the frequency of the j-th received signal; $\omega_0 = 2\pi f_0 = 2\pi c/\lambda$; f_0 is the central frequency of the received signal. Under the assumption of narrow-band signals, we have,

$$\begin{cases} u_j(t-\tau) = u_j(t) \\ \varphi_j(t-\tau) = \varphi_j(t) \end{cases} \tag{2}$$

So we can derive

$$s_j(t-\tau) \approx s_j(t)e^{-j\omega_0\tau} \qquad j = 1,2,...,p \tag{3}$$

And the receipt signal of the i-th unit is,

$$x_i(t) = \sum_{j=1}^{p} g_{ij}s_j(t-\tau_{ij}) + n_i(t) \qquad i = 1,2,...,M \tag{4}$$

where g_{ij} is the gain of the i-th unit to the j-th source, $n_i(t)$ is the noise and τ_{ij} is the time delay of the j-th source arriving at the i-th unit. The received signal of the M units can be written as,

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} g_{11}e^{-j\omega_0\tau_{11}} & g_{12}e^{-j\omega_0\tau_{12}} & \cdots & g_{1p}e^{-j\omega_0\tau_{1p}} \\ g_{21}e^{-j\omega_0\tau_{21}} & g_{22}e^{-j\omega_0\tau_{22}} & \cdots & g_{2p}e^{-j\omega_0\tau_{2p}} \\ \cdots & \cdots & \cdots & \cdots \\ g_{M1}e^{-j\omega_0\tau_{M1}} & g_{M2}e^{-j\omega_0\tau_{M2}} & \cdots & g_{Mp}e^{-j\omega_0\tau_{Mp}} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \cdots \\ s_M(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \cdots \\ n_M(t) \end{bmatrix} \tag{5}$$

Assuming the gains are the same, and we can get a simplified version of

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} e^{-j\omega_0\tau_{11}} & e^{-j\omega_0\tau_{12}} & \cdots & e^{-j\omega_0\tau_{1p}} \\ e^{-j\omega_0\tau_{21}} & e^{-j\omega_0\tau_{22}} & \cdots & e^{-j\omega_0\tau_{2p}} \\ \cdots & \cdots & \cdots & \cdots \\ e^{-j\omega_0\tau_{M1}} & e^{-j\omega_0\tau_{M2}} & \cdots & e^{-j\omega_0\tau_{Mp}} \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \cdots \\ s_M(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \cdots \\ n_M(t) \end{bmatrix} \tag{6}$$

A matrix of (6) can be written as,

$$\mathbf{X}(t) = \mathbf{AS}(t) + \mathbf{N}(t) \tag{7}$$

where $\mathbf{A} = [a_1(\omega_0), a_2(\omega_0), ..., a_p(\omega_0)]$, and

$$a_j(\omega_0) = \begin{bmatrix} \exp(-j\omega_0\tau_{1j}) \\ \exp(-j\omega_0\tau_{2j}) \\ \cdots \\ \exp(-j\omega_0\tau_{Mj}) \end{bmatrix} \qquad j = 1,2,...,p \tag{8}$$

In the compressive DOA estimation approach, we denote $\mathbf{X} = [x(1), x(2), \cdots, x(T)] \in R^{N\times T}$ and $\mathbf{A} \in R^{N\times K}$, $\mathbf{S} = [s(1), s(2), \cdots, s(T)] \in R^{K\times T}$; $\mathbf{W} = [w(1), w(2), \cdots, w(T)] \in R^{N\times T}$.

Assuming there are N_s sources with their DOAs being $\theta_1, \theta_2, ..., \theta_{N_s}$. Define the angle related matrix $\mathbf{\Psi} = [a(\theta_1), a(\theta_2), \cdots, a(\theta_{N_s})]$ and a sparse vector $z(t) = [z_{\theta_1}(t), z_{\theta_2}(t), \cdots, z_{\theta_{N_s}}(t)]^T \in R^{N_s\times 1}$, where there are K nonzero coefficients in the locations of $z_\theta(t) = s_k(t)$ and the left $N_s - K$ coefficients are zeros. So the formula can be rewritten as,

$$\mathbf{X} = \mathbf{\Psi Z} + \mathbf{W} \tag{9}$$

with $\mathbf{Z} = [z(1), z(2), \cdots, z(T)] \in R^{N_s\times T}$. So the DOA estimation can be reduced to the estimation of matrix $\mathbf{Z}$. In the Focuss algorithm, we can solve $\mathbf{Z}$ by solving a minimization of l_0-norm of solution. That is,

$$\begin{cases} \min_{\mathbf{Z}} \|\mathbf{Z}\|_{row,0} \\ s.t \|\mathbf{X} - \mathbf{\Psi Z}\|_2^2 < \varepsilon \end{cases} \tag{10}$$

where $\|\mathbf{Z}\|_{row,0}$ indicates the number of non-zero rows of the matrix and ε is the error tolerance. It is a non-convex optimization problem, so we use a measure $J^{(p,q)}(\mathbf{Z})$ to evaluate the difference [5-6],

$$J^{(p,q)}(\mathbf{Z}) = \sum_{i=1}^{n} \left(\|\mathbf{Z}^i\|_q\right)^p \tag{11}$$

We let $q = 2$ to obtain

$$J^{(p,2)}(X) = \sum_{i=1}^{n} \left(\|X^i\|_2\right)^p = \sum_{i=1}^{n} \left(\sum_{i=1}^{L} |X^i(l)|^2\right)^{p/2} \tag{12}$$

When $p \to 0$, the above function approaches an approximation of l_0-norm . On the other hand, it can reduce the computation complexity in minimizing the l_0-norm .

In order to minimize (12) ,we let

$$W^{(k+1)} = diag\left(\left(c_i^{(k)}\right)^{1-p/2}\right), p \in [0,1] \tag{13}$$

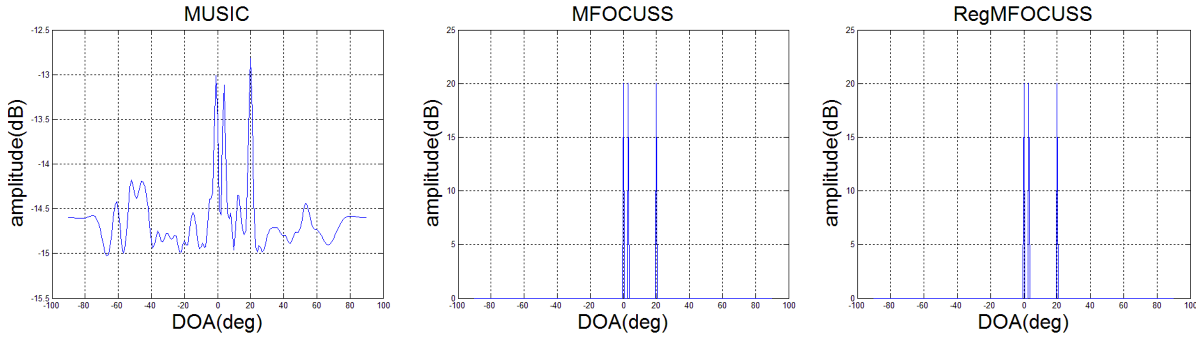
where $c_i^{(k)} = \|X_i^{(k)}\| = \left(\sum_{i=1}^{L} |X_i^{(k)}(l)|^2\right)^{1/2}$. So we can obtain

$$Q^{(k+1)} = \left(A^{(k+1)}\right)^+ \left(A^{(k+1)}\left(A^{(k+1)}\right)^+ + \lambda I\right)^{-1} Y, \lambda \ge 0 \tag{14}$$

where $A^{(k+1)} = AW^{(k+1)}$ and $X^{(k+1)} = W^{(k+1)}Q^{(k+1)}$. The parameter makes a balance between the sparsity and estimation error. In our method we let $\lambda = \sigma^2$, where σ^2 is the variance of noise.

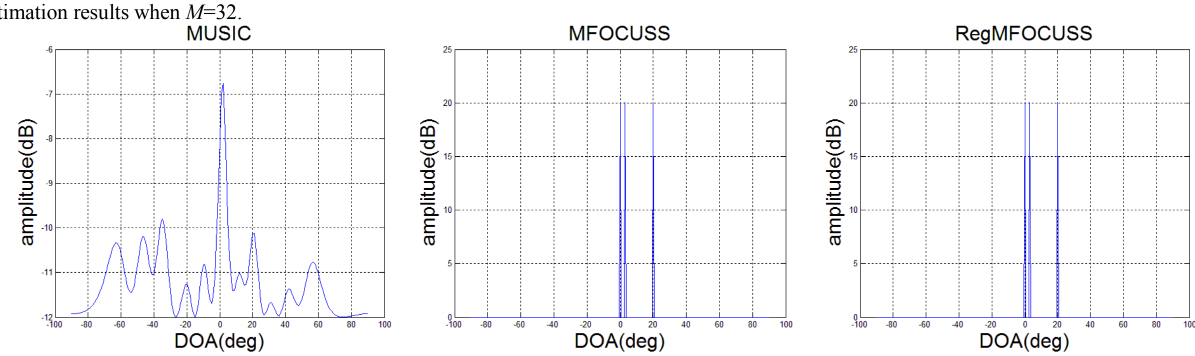
III. EXPERIMENTAL RESULTS

In this section, some experiments are taken to validate the performance of our proposed method. The operating environment for all experiments is MATLAB7.0, and computer is configured to Intel Core 2/2.13G/2G.



(a) (b) (c)

Fig.2. Estimation results when *M*=32.



(a) (b) (c)

Fig.3. Estimation results when *M*=16.

A. Experiment 1: comparison results with different M's

In this test we compare our proposed RMM-FOCUSS method with the traditional MUSIC algorithm, the Multiple FOCal Underdetermined System Solver (MFOCUSS) algorithm, when there are 32, 16 equally spaced antenna units and three 420MHz far field narrow band sources. Two sources are coherent and the directions are randomly selected as 0^o、3^o and 20^o while SNR=80dB. The number of samples is $T=100$. Fig. 1 plots the energy spectrum.

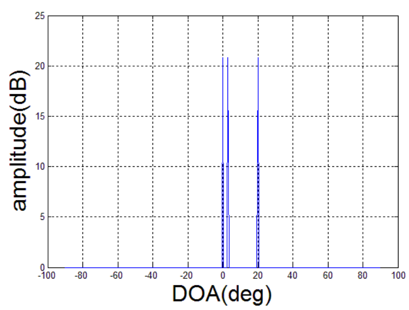


Fig.1. The energy spectrum of three sources.

The estimation results of MUSIC, MFOCUSS and our method are shown in Fig. 2(a)-(c) respectively. From the result we can see that an accurate estimation can be obtained for these three algorithms when they are lots of antenna units.

Then we reduce the number of antenna units and let *M*=16. The estimation results of MUSIC, MFOCUSS and our method when $M=16$ are shown in Fig. 3(a)-(c) respectively. From the result we can see that with the decrease of the number of antenna units, the estimation results of MUSIC degrades remarkably. However, both MFOCUSS and RMM-FOCUSS outperform MUSIC algorithm.

B. Experiment 2: comparison results with low SNR

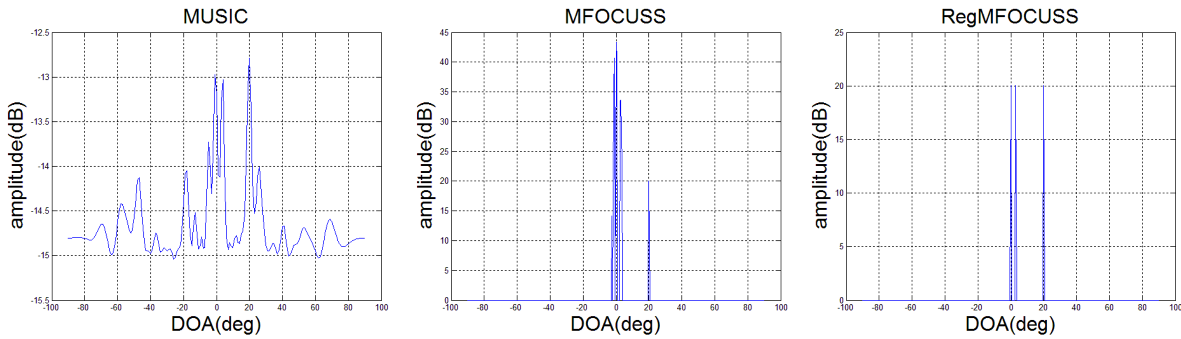
In this test we compare our proposed RMM-FOCUSS method with the traditional MUSIC algorithm, the Multiple FOCal Underdetermined System Solver (MFOCUSS) algorithm, when there are 32 or 16 antenna units, 3 sources the same as experiment 1and SNR=30dB.

The estimation results of MUSIC, MFOCUSS and our method when $M=32$ are shown in Fig. 4(a)-(c) respectively. From the result we can see that our proposed method can achieve more accurate estimation than MUSIC and MFOCUSS. Then we reduce the number of antenna units and let *M*=16. The estimation results of MUSIC, MFOCUSS and our method when $M=16$ are shown in Fig. 5(a)-(c) respectively. From the result we can see that our proposed method outperforms MUSIC and MFOCUSS remarkably.

C. Experiment 3: comparison results with different SNRs

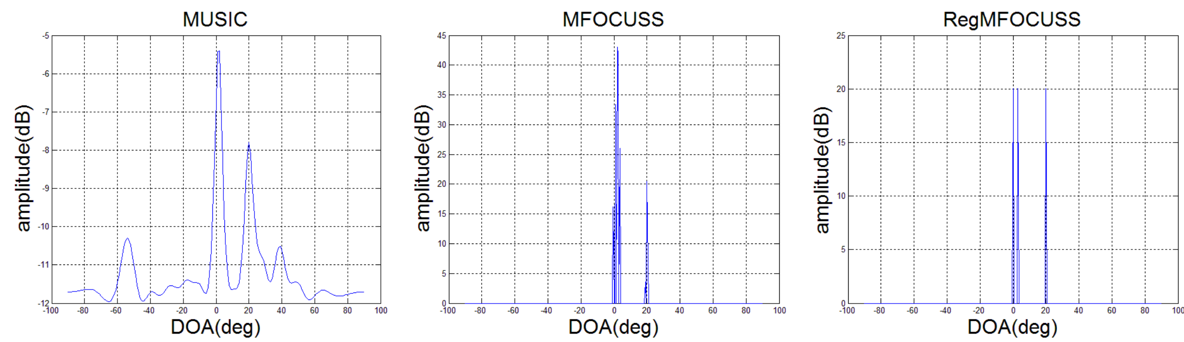
In this test, we investigate the performance of our proposed method when SNR=5, 10, 20, 30, 40, 50, 60, 70 and 80dB. Fig. 6(a) shows the MSE of the estimation result of three methods when *M*=16 and Fig. 6(b) shows the result of three methods when *M*=8. The MSE is defined as,

$$MSE = E\left(\frac{\left\|\mathbf{Z}-\mathbf{Z}_0\right\|_2^2}{\left\|\mathbf{Z}_0\right\|_2^2}\right) \tag{15}$$



(a) (b) (c)

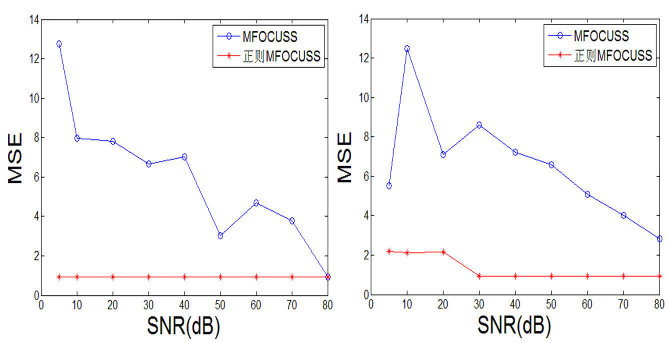
Fig.4. Estimation results when when M=32 and SNR=30dB.



(a) (b) (c)

Fig.5. Estimation results when when M=16 and SNR=30dB.

where $\mathbf{Z}$ and $\mathbf{Z}_0$ are the real and estimated coefficients respectively. From the result we can see that our proposed method outperforms MFOCUSS remarkably at different SNRs.



(a) (b)

Fig.6. (a) MSE of the Estimation results when M=16, (b) MSE of the Estimation results when M=8.

IV. CONCLUSION

In this paper, we have proposed a Regularized Multiple Measurement FOCal Underdetermined System Solver algorithm. Results show that it can obtain higher angle resolution than traditional methods when there are few antenna units. When the SNR is low, our proposed method is more robust and accurate. Meanwhile, more research will be devoted to building space-time dictionary to achieve high resolution spectrum estimation with less antenna units and snapshots.

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